The effect of perturbations on the magnetic field of the W VII-Stellarator

S. Rehker, H. Wobig

IPP 2/218 Januar 1974

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Abstract

Due to imperfections of the coil system different perturbation fields occur, which modify the unperturbed magnetic field of the W VII-Stellarator appreciably. The structure of islands, caused by these perturbations was investigated numerically. These islands exist on resonant surfaces with $\iota = \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \dots$. The width of the islands scales with the square root of the perturbation field, typical values are between 3 and 10 cm. If the coil system can be built with an accuracy of 0.1 cm, the width of the islands can be reduced to a tolerable level, but the islands cannot be avoided completely.

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Introduction

W VII is an l = 2-stellarator, which is under construction in Garching. As it has been shown in different papers, the structure of magnetic surfaces depends critically on the presence of unwanted perturbation fields [1], [2].
These perturbation fields can arise from all kinds of constructional errors. The simplest effect of perturbation fields is the occurrence of magnetic islands, which are localized to a particular resonant magnetic surface. More dangerous is an overlapping of the magnetic islands followed by a complete destruction of the magnetic surfaces [3], [4].

C. Gourdon et al. [5] have investigated the structure of magnetic surfaces under the effect of perturbation fields. The numerical calculations were done for a Torsatron and an l = 3-stellarator. According to their results and theoretical predictions the width of the islands scales with $\sqrt{\delta B / t'}$
(δB is the effective perturbation field and t' the shear). Therefore we expect a large effect in the l = 2-stellarator.

The major radius of the W VII-stellarator is 200 cm, the mean radius of the helix is 45.5 cm. In the numerical calculations we have represented the helix by 5 single wires with constant pitch ($\frac{d\Theta}{d\varphi}$ = const) Θ = poloidal angle, φ = azimuthal angle).
For comparison we have considered a case with variable pitch according to the formula:

$$\frac{d\Theta}{d\varphi} = 2.5(1 - \beta \cos \Theta)$$

In case of finite ß the shear of the magnetic surfaces is reduced. The main toroidal field is generated by 40 circular coils. The ohmic heating transformer and vertical field coils consist of circular toroidal coils. The whole set up can be seen in fig.1. Many deviations from the ideal case are possible. We have mainly concentrated our calculations on the m = 1-type perturbations. This means that a Fourier-expansion of the perturbation field δB in the azimuthal variable φ mainly contains an m = 1 component.

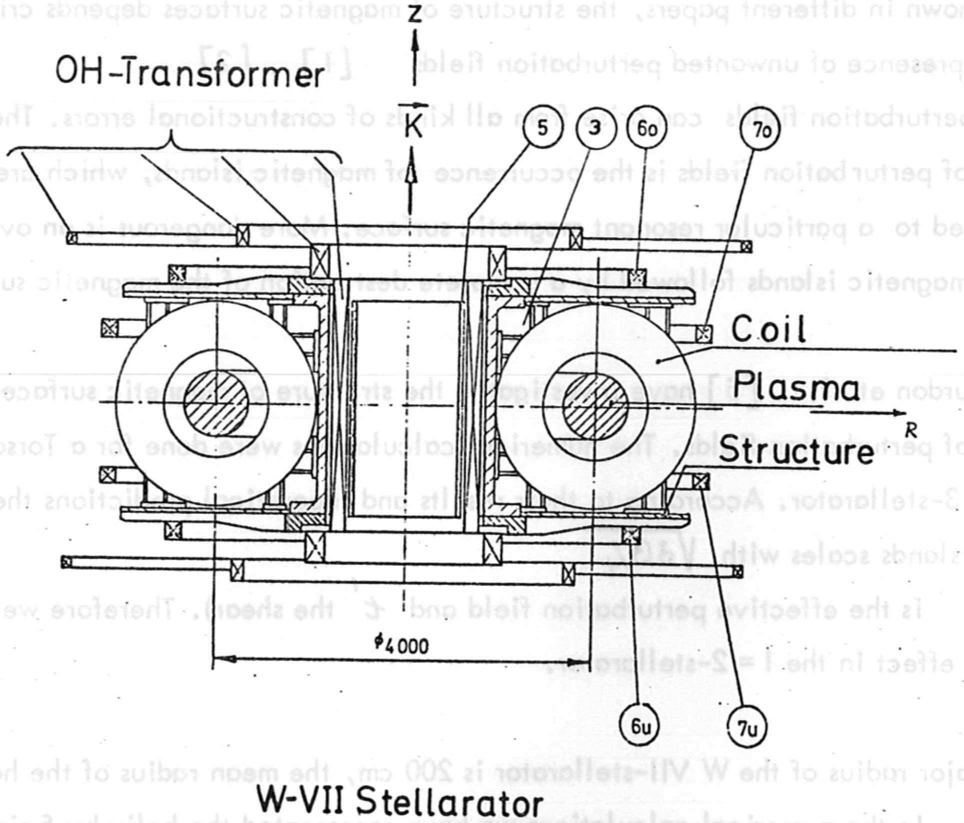

Fig. 1: Scheme of the W VII-Stellarator
Helical windings are not shown in this picture

- 3 -

The list of perturbations investigated by us is

1) Perturbation of the main field coil system
2) Perturbations of the helical windings
3) Perturbations due to current leads
4) Perturbations due to inaccurate positioning of the torus
5) Effect of superimposed horizontal field
6) Perturbations of the Ohmic heating transformer and the vertical field coils
7) Perturbations due to extra coils for neutral injection

The resonant surfaces investigated by us are ι = 1/2, 1/3, 1/4. Other surfaces (ι = 1/10) were also checked for comparison. According to the theory the m = 1-type perturbation is expected to cause $\frac{1}{\iota}$ = 2,3,4 islands on the resonant surfaces.

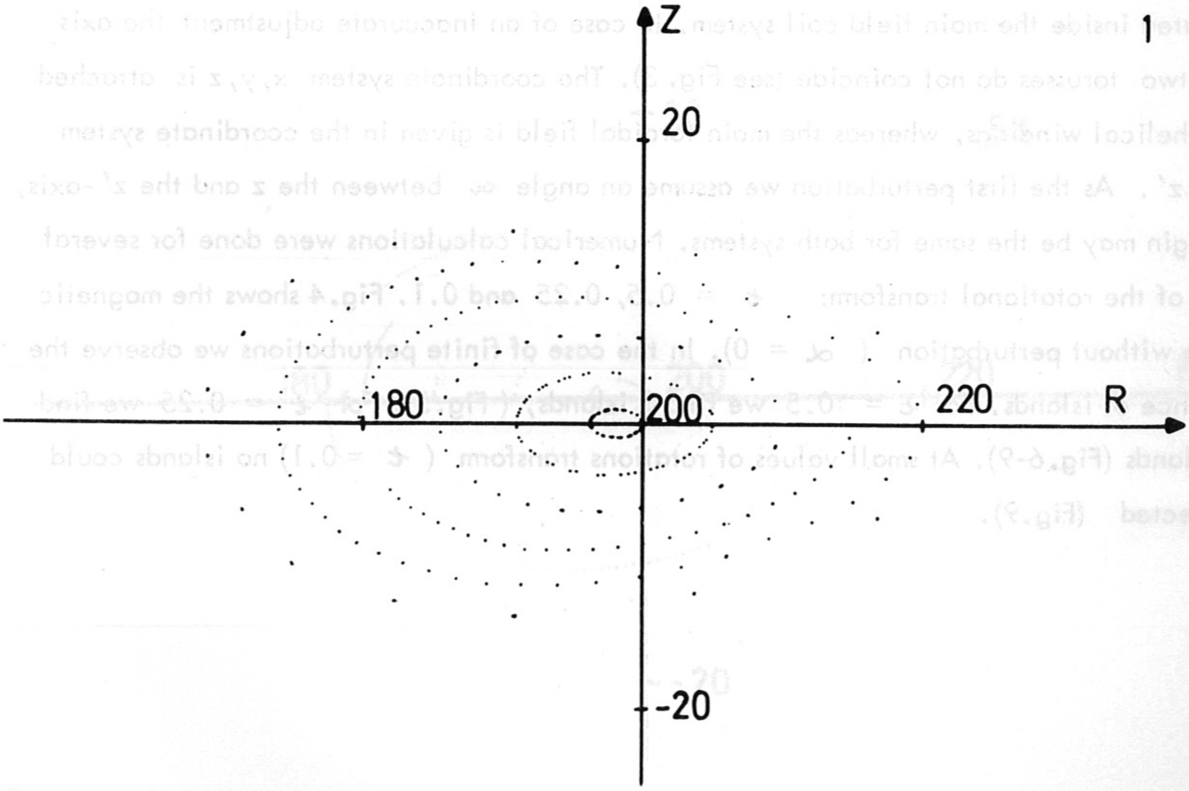

Fig.2: Unperturbed surface at ι = 0.5. The length scale is given in cm.

All our calculations confirm this prediction. Due to the toroidal curvature natural islands should occur on the resonant surfaces but this effect is too small to be detected in W VII. A check without perturbation fields did not show any islands at ι = 1/2 (fig.2). By changing the current in the helical windings the position of the resonant surface can be modified. Since the width of the magnetic islands critically depends on the position of the resonant surface, it is difficult to define this width at all. We have chosen the current so that the mean radius of the resonant surface is roughly 20 cm. This is an arbitrary choice, but it permits us to compare the effect of different perturbations.

1) Nonlocal Perturbations

1.1. Tilting of the torus

The helical windings of the stellarator are wound around the toroidal vacuum tube. This torus is situated inside the main field coil system. In case of an inaccurate adjustment the axis of the two torusses do not coincide (see Fig. 3). The coordinate system x,y,z is attached to the helical windings, whereas the main toroidal field is given in the coordinate system x',y',z'. As the first perturbation we assume an angle α between the z and the z'-axis, the origin may be the same for both systems. Numerical calculations were done for several values of the rotational transform: ι = 0.5, 0.25 and 0.1. Fig.4 shows the magnetic surface without perturbation (α = 0). In the case of finite perturbations we observe the occurence of islands. For ι = 0.5 we find 2 islands, (Fig.5). For ι = 0.25 we find four islands (Fig.6-9). At small values of rotations transform (ι = 0.1) no islands could be detected (Fig.9).

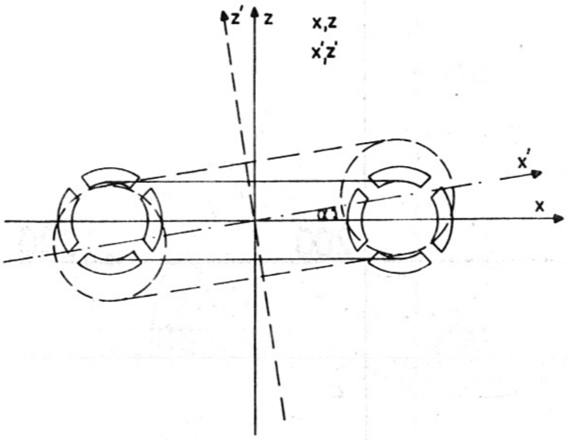

Fig.3: Tilt of the main field torus against torus with helix

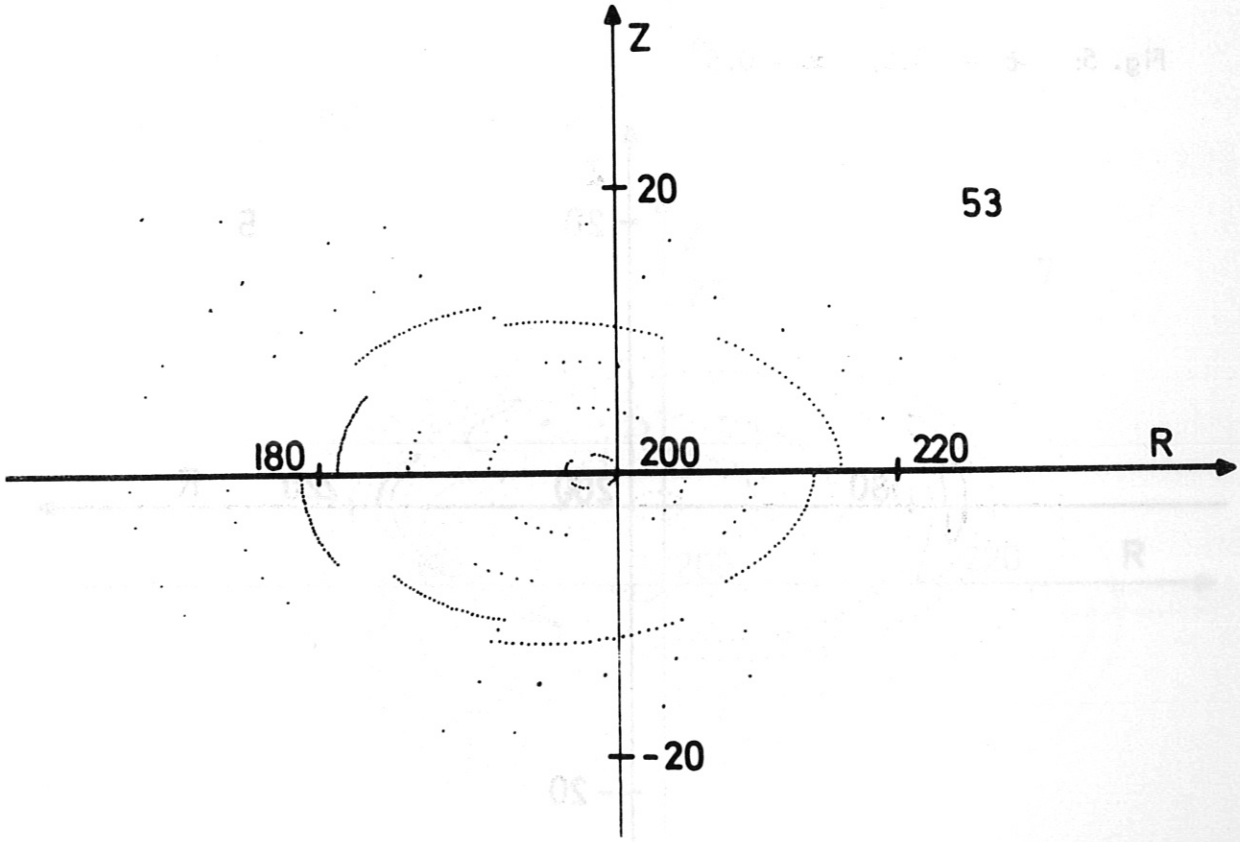

Fig. 4: Unperturbed magnetic surface. $\bar{\iota} = 0.25$, $\alpha = 0$

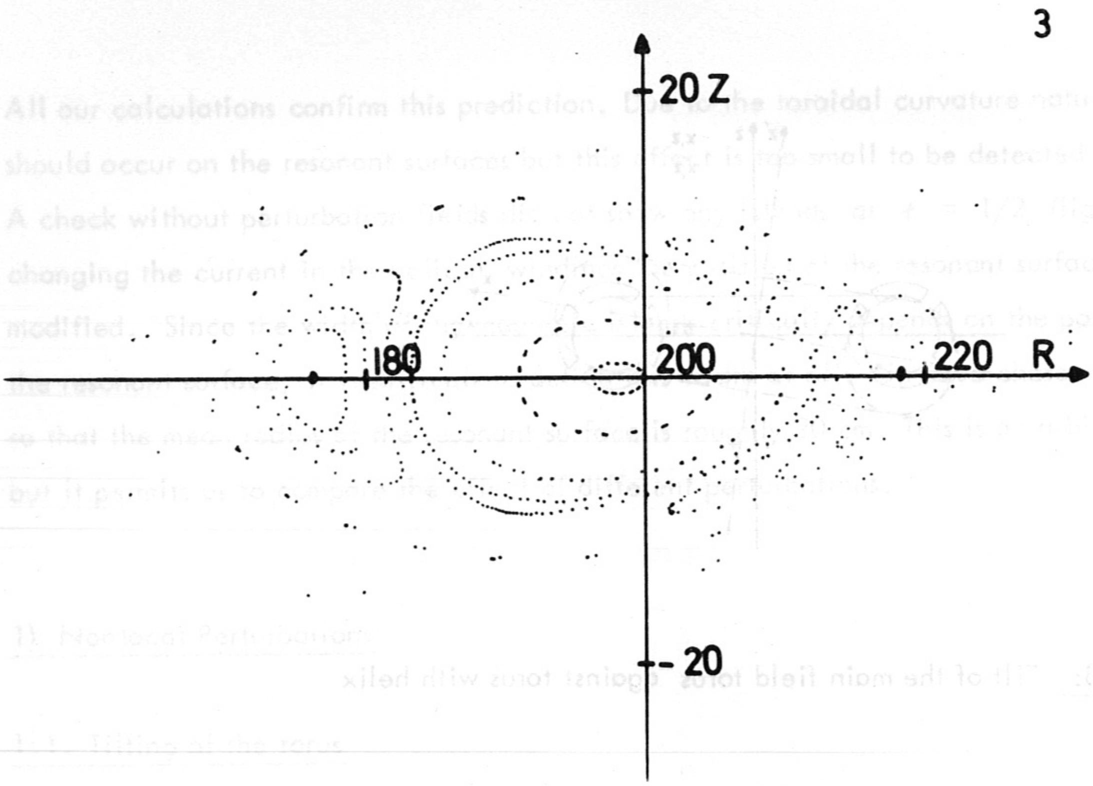

Fig. 5: ι = 0.5, α = 0.5°

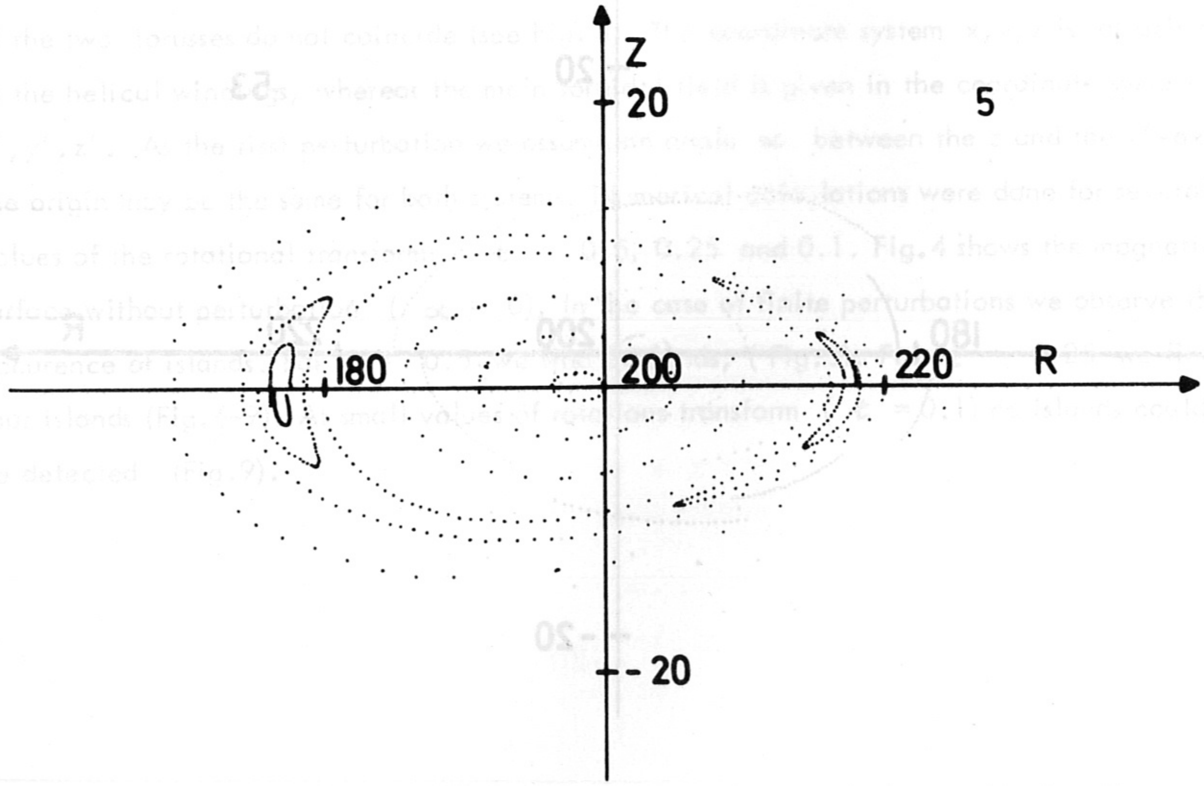

Fig. 6: ι = 0.5, α = 0.1°

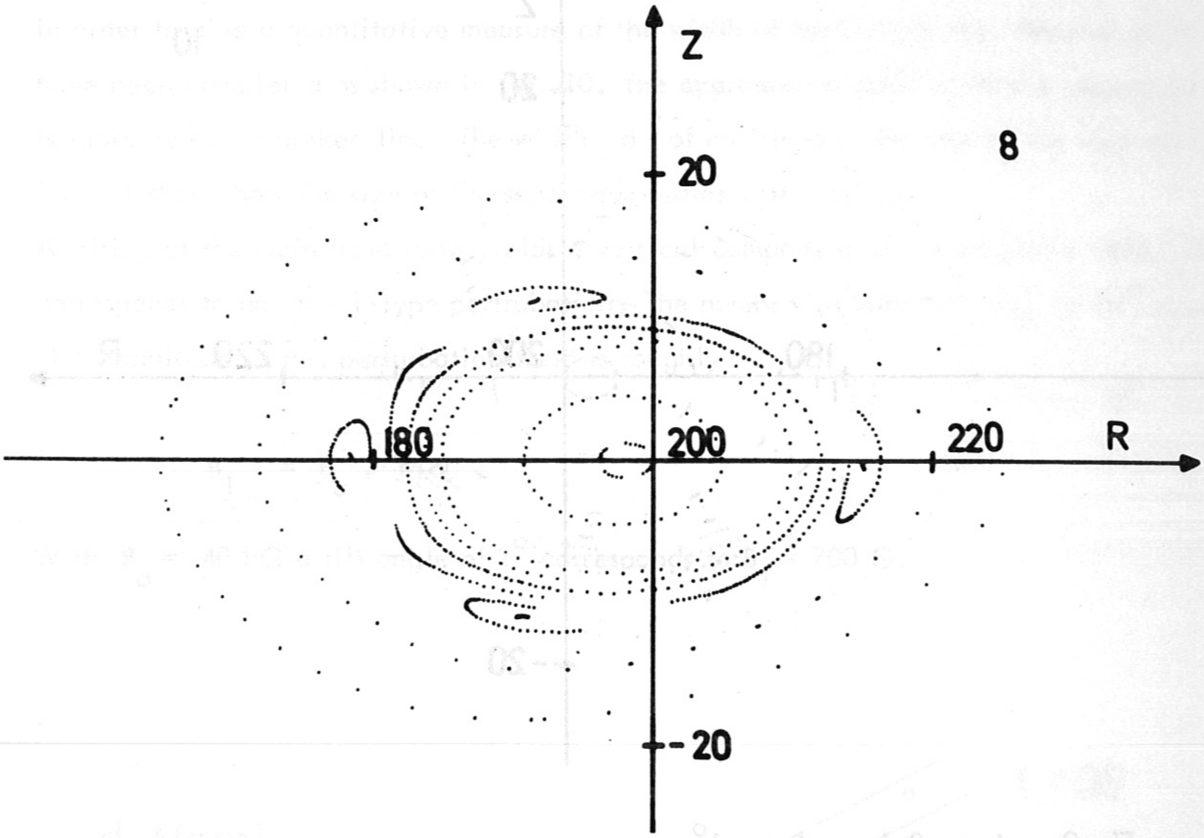

Fig.7: $t = 0.25$, $\alpha = 0.5^{\circ}$

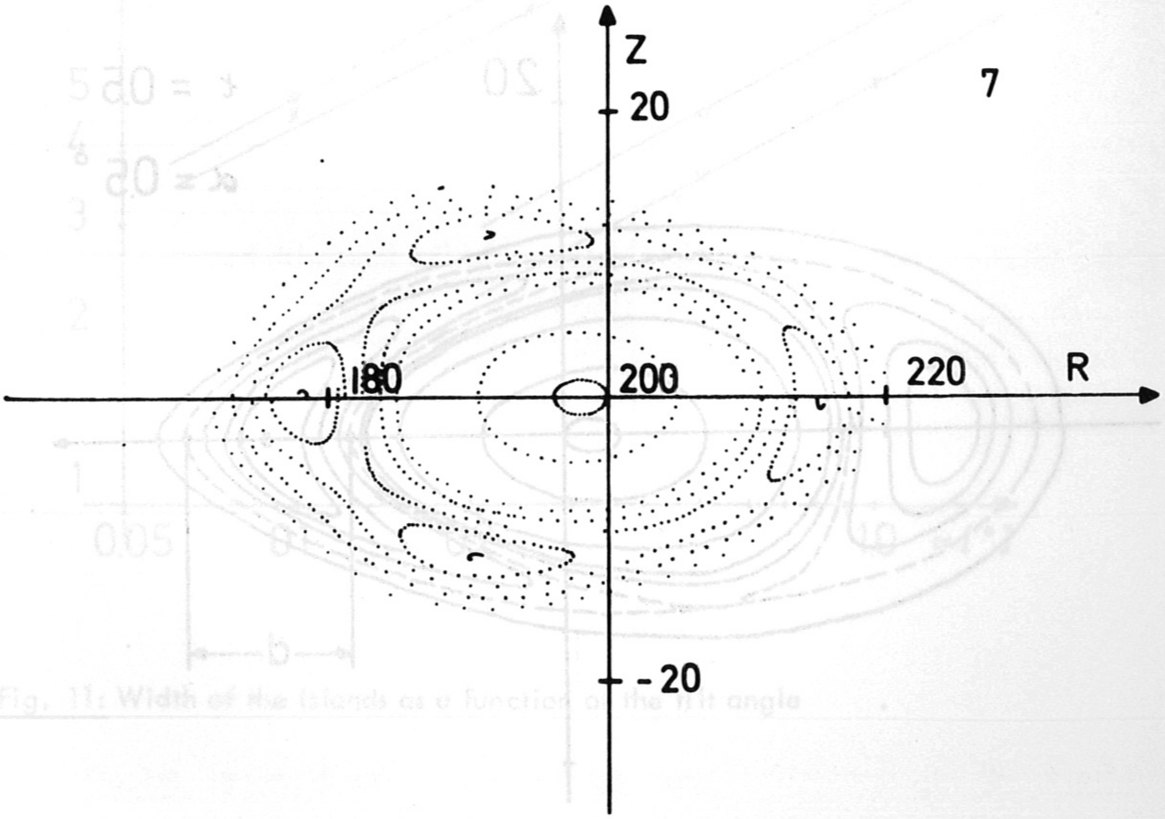

Fig.8: $t = 0.25$, $\alpha = 1^{\circ}$

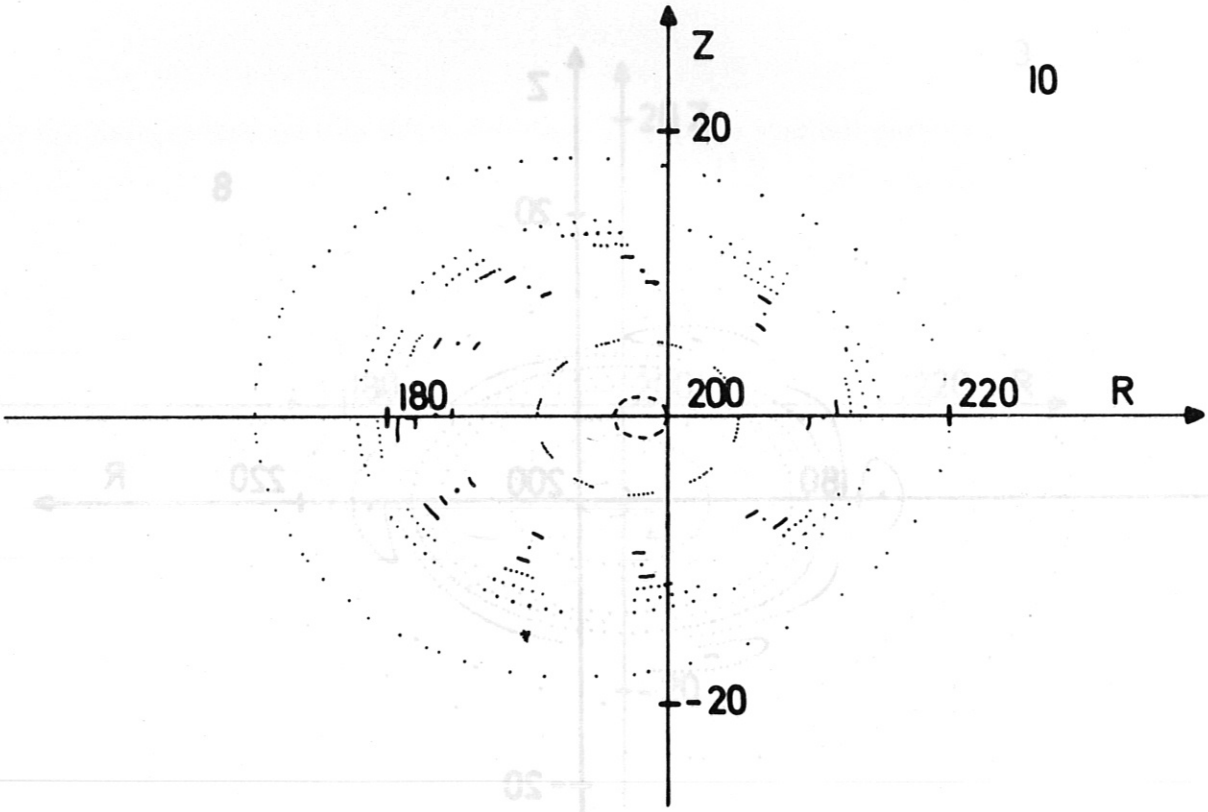

Fig.9: $\iota = 0.1 \quad \alpha = 1^\circ$

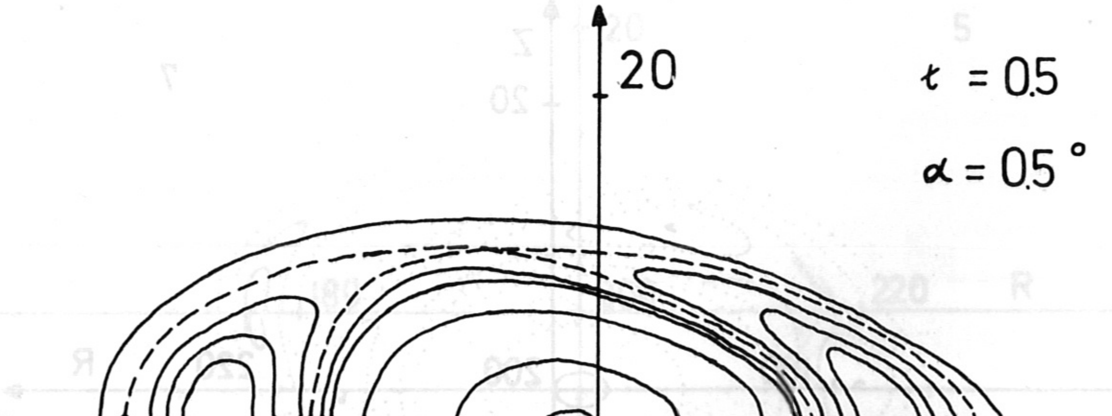

Fig.10: $\iota = 0.5$, $\alpha = 0.5^\circ$. Definition of the width of the islands

In order to give a quantitative measure of the width of the islands the computer plots have been completed as shown in fig. 10. The approximate position of the separatrix is indicated by a broken line. The width d of an island is defined by the separatrix. Fig. 11 shows how the size of the separatrix scales with angle α .
A tilting of the main field torus yields a vertical component of the magnetic field, which corresponds to an m = 1-type pertrubation. The number of islands is m/ι in this case. The magnitude of this perturbation field is roughly

$$B_1 = B_o \cdot \mathrm{tg}\alpha$$

With B_o = 40 kG a tilt angle of 1° corresponds to B_1 = 700 G.

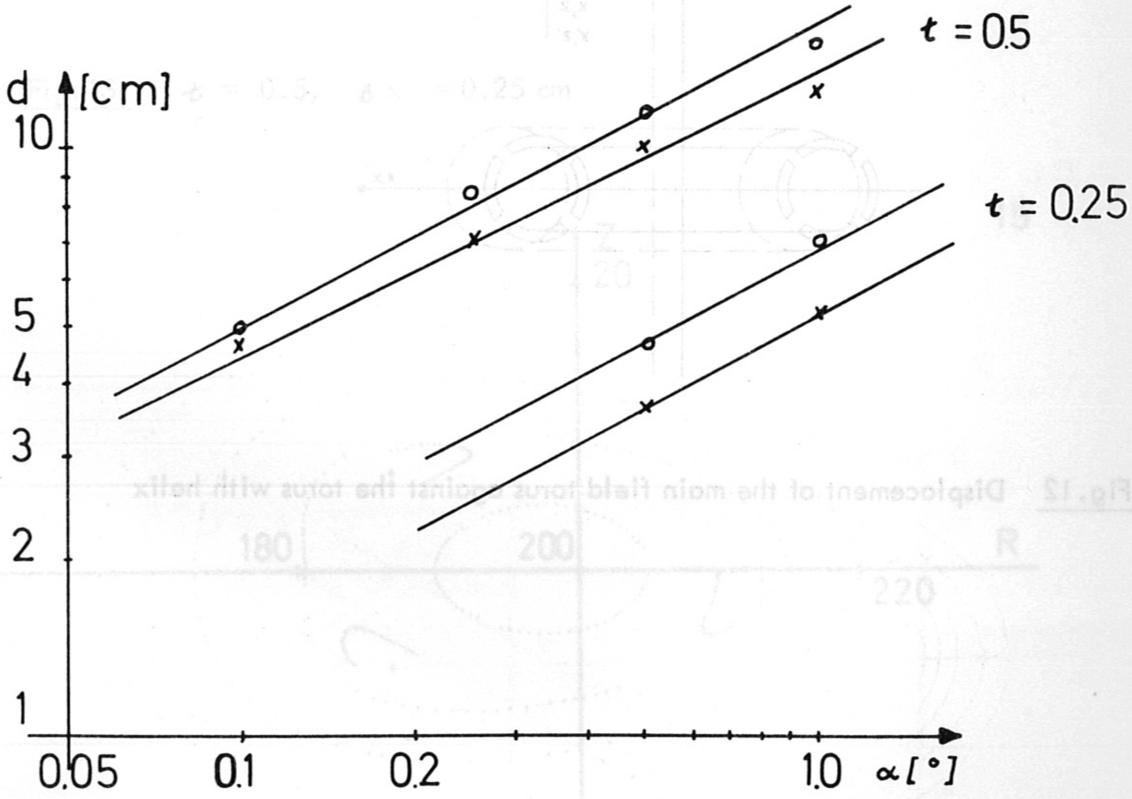

Fig. 11: Width of the islands as a function of the tilt angle .

1.2 Horizontal displacement

A horizontal displacement of the main field torus (fig. 12) is equivalent to a superimposed horizontal field. The z-axis of the coordinate systems remains parallel during the displacement. Fig. 13, 14, 15 show the effect of this perturbation on the magnetic surfaces at $\iota = 0.5$, 0.333, and 0.25. In fig. 16 the dependence of d on the shift is plotted. It can be seen that the size of the islands scales with the square root of the perturbation. According to the formula $B_h = \frac{\Delta x}{R} \cdot B_o$ a displacement of $\Delta x = 1$ cm corresponds to horizontal field of $B_h = 200$ G°. For $\iota = 0.1$ the effect of this perturbation is negligible, islands could not be found.

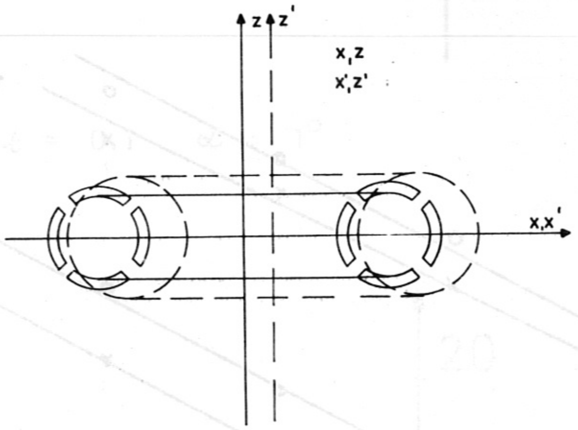

Fig. 12 Displacement of the main field torus against the torus with helix

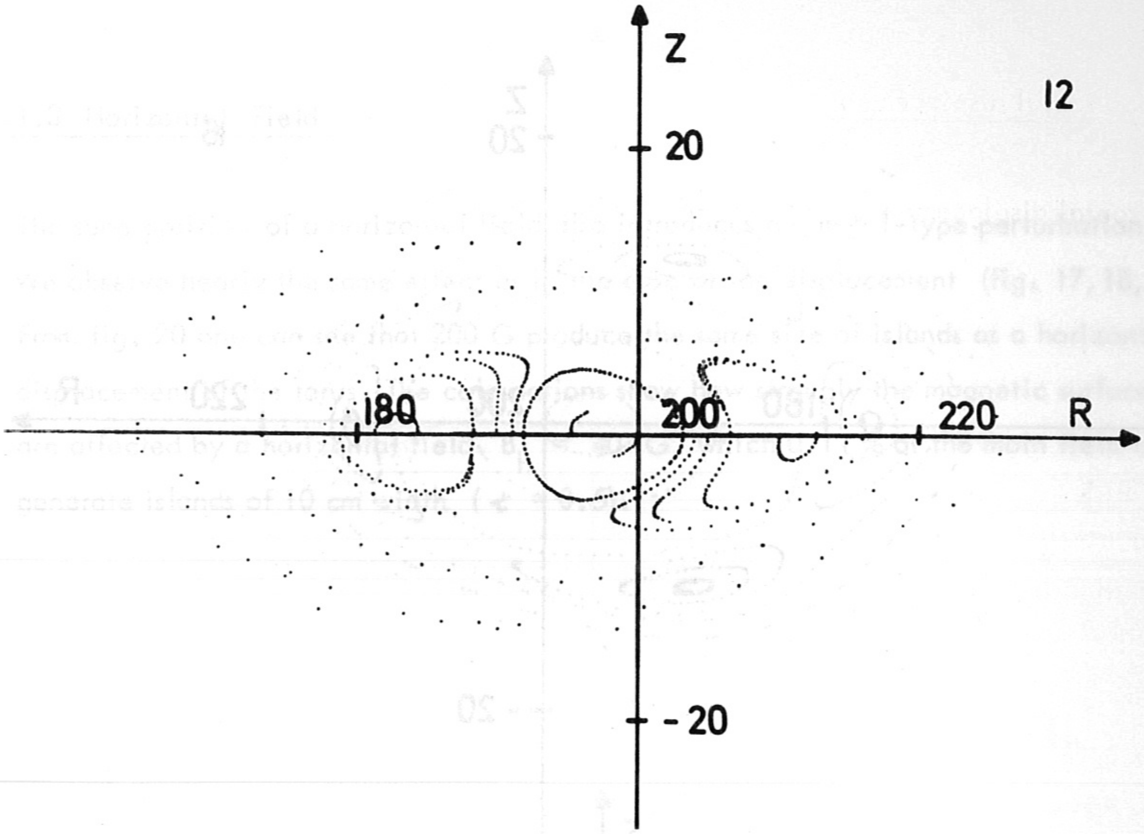

Fig.13: ι = 0.5, Δx = 0.25 cm

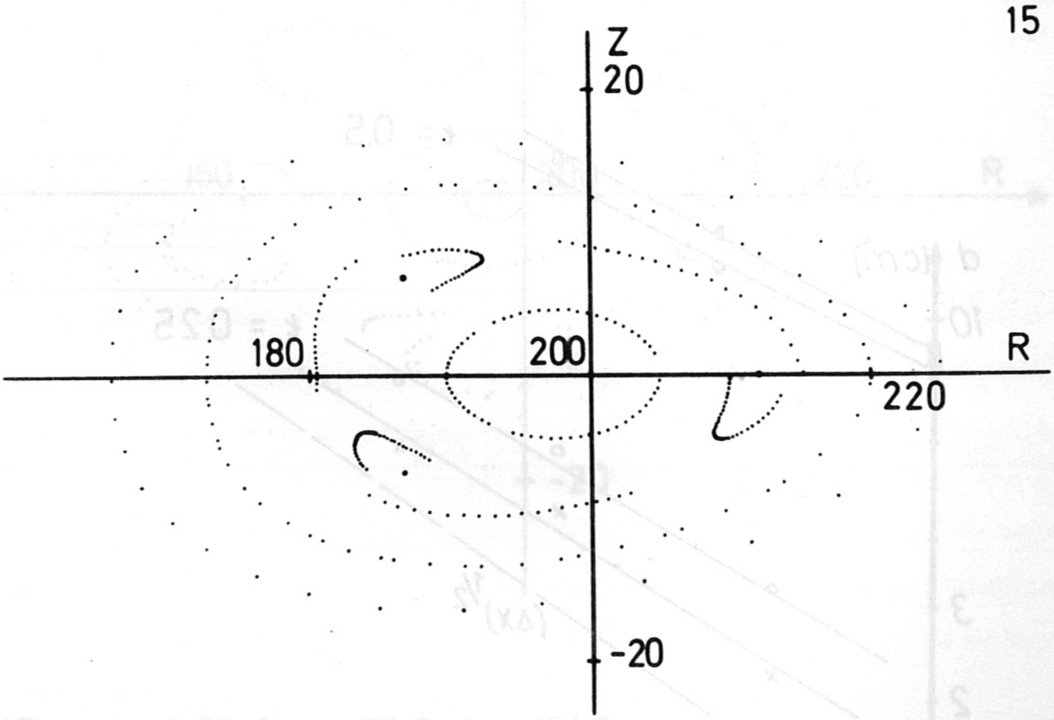

Fig.14: ι = 0.333, Δx = 0.25 cm

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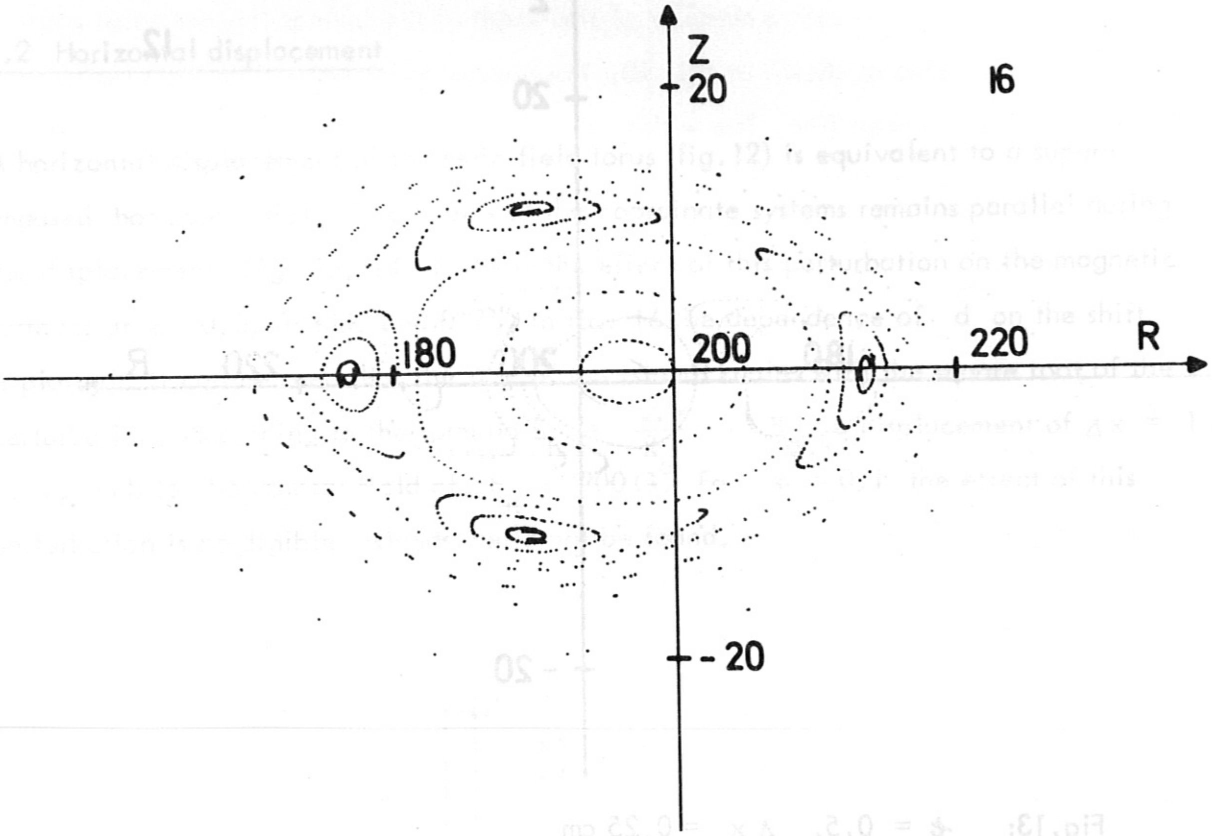

Fig. 15: ι = 0.25, Δx = 1 cm

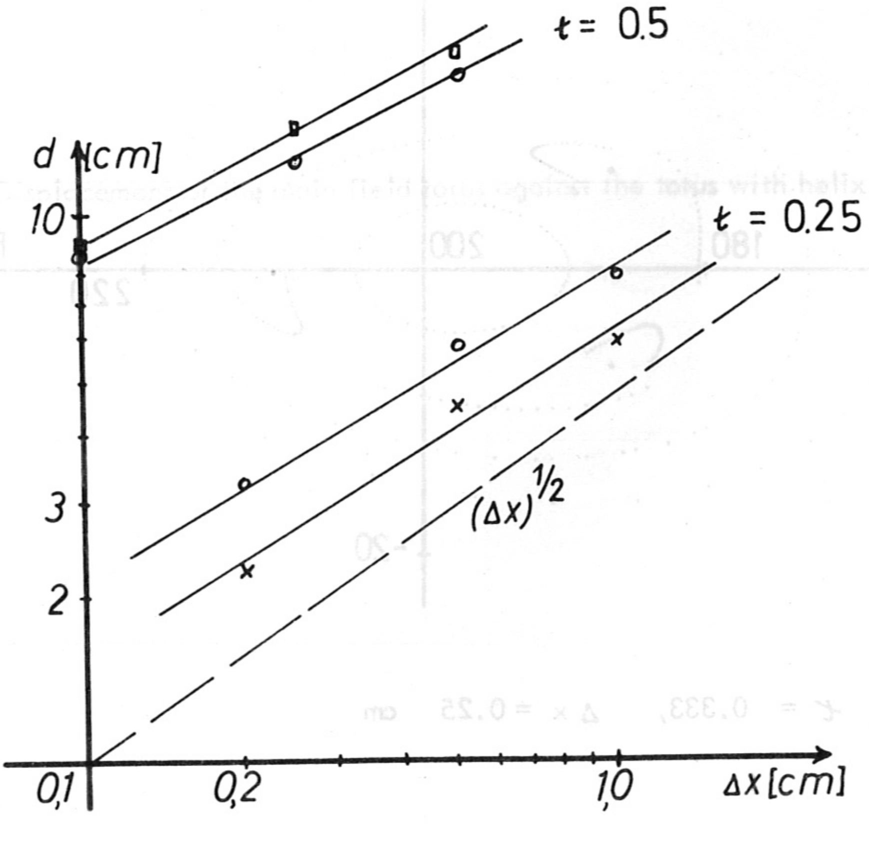

Fig. 16: Width of the islands as a function of Δx

1.3 Horizontal Field

The superposition of a horizontal field also introduces an m = 1-type perturbation. We observe nearly the same effect as in the case of the displacement (fig. 17, 18, 19). From fig. 20 one can see that 200 G produce the same size of islands as a horizontal displacement of the torus. The calculations show how strongly the magnetic surfaces are affected by a horizontal field. B_h = 40 G, which 0.1 % of the main field can generate islands of 10 cm width (ι = 0.5).

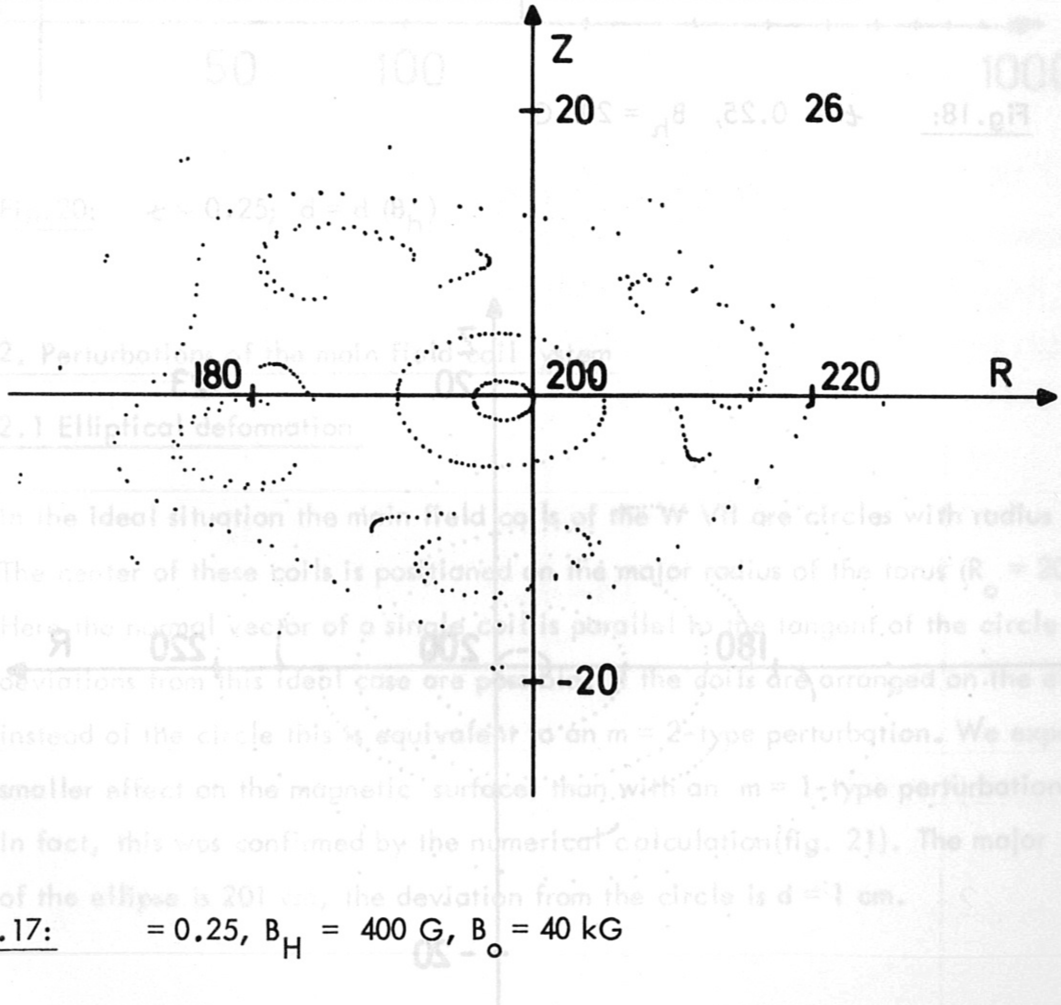

Fig. 17: = 0.25, B_H = 400 G, B_o = 40 kG

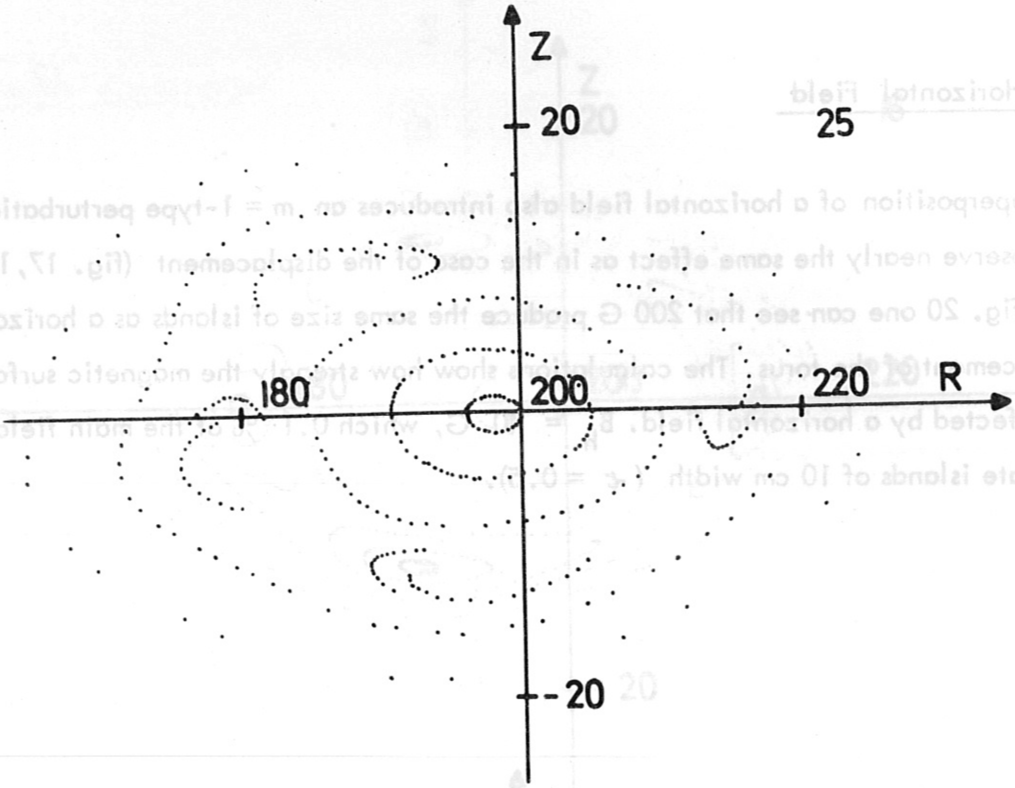

Fig.18: $\iota = 0.25$, $B_h = 200$ G

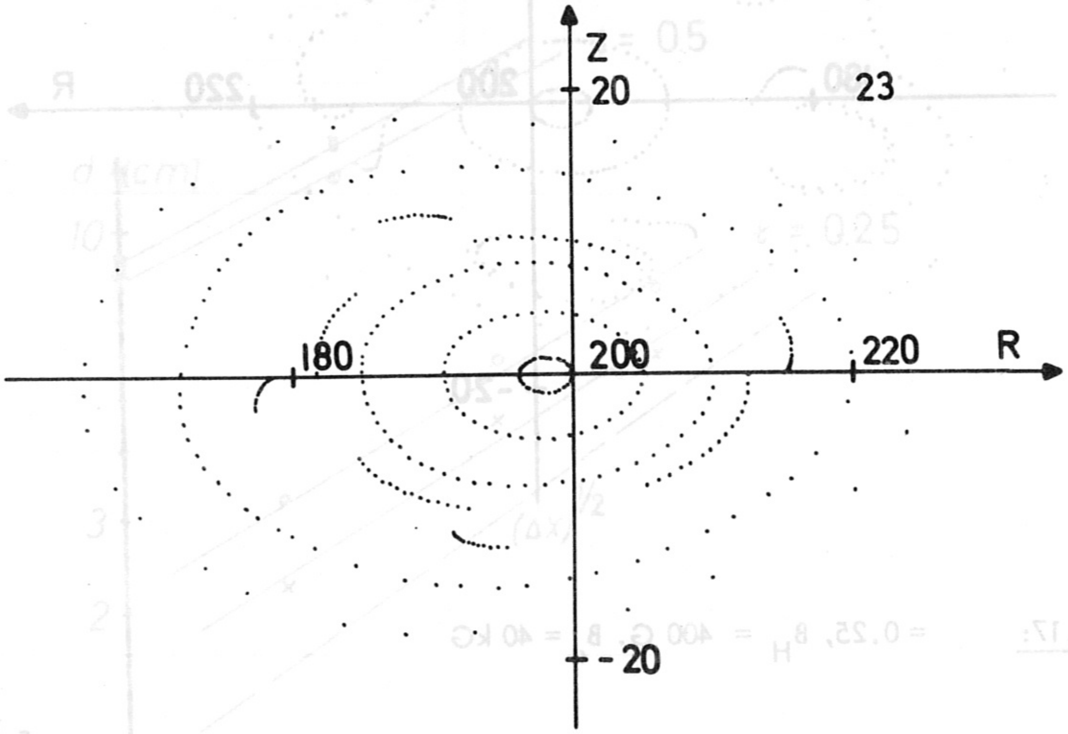

Fig.19: $\iota = 0.25$, $B_h = 50$ G

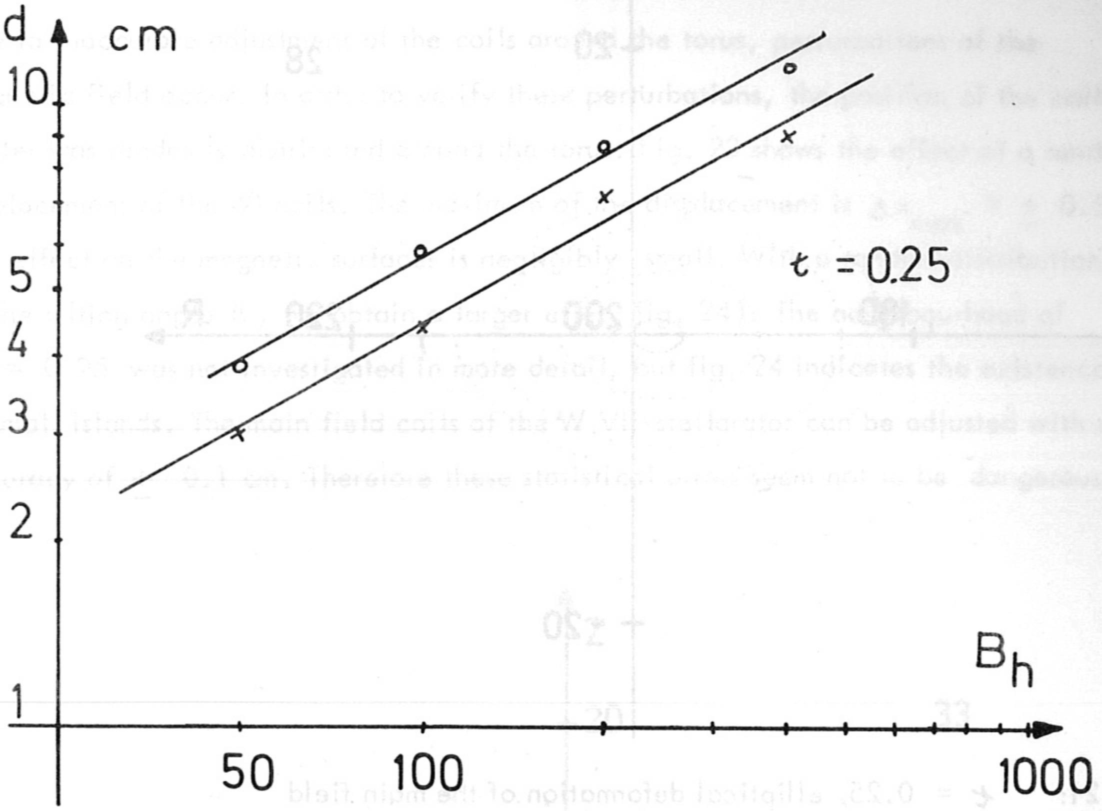

Fig.20: $\iota = 0.25$; $d = d\ (B_h)$

2. Perturbations of the main field coil system

2.1 Elliptical deformation

In the ideal situation the main field coils of the W VII are circles with radius r_o. The center of these coils is positioned on the major radius of the torus ($R_o = 200$ cm). Here the normal vector of a single coil is parallel to the tangent of the circle. Several deviations from this ideal case are possible. If the coils are arranged on the ellipse instead of the circle this is equivalent to an m = 2-type perturbation. We expect a smaller effect on the magnetic surfaces than with an m = 1-type perturbation. In fact, this was confirmed by the numerical calculation (fig. 21). The major axis of the ellipse is 201 cm, the deviation from the circle is d = 1 cm.

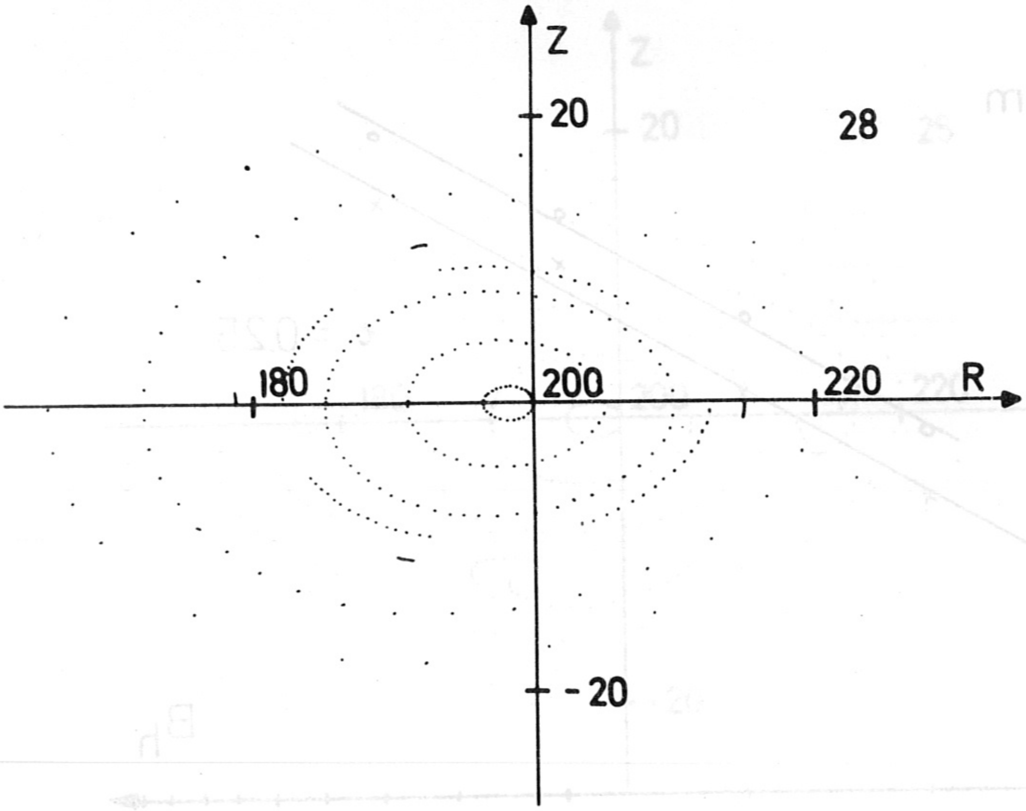

Fig.21: $\iota\!\!\!-$ = 0.25, elliptical deformation of the main field

2.2 Tilting of the coils

A systematic tilting of the coils with respect to the aquatorial plane introduces a vertical field. This only shifts the magnetic axis but does not cause any islands (fig.22). ß is the angle between the normal vector of the coils and the horizontal plane.

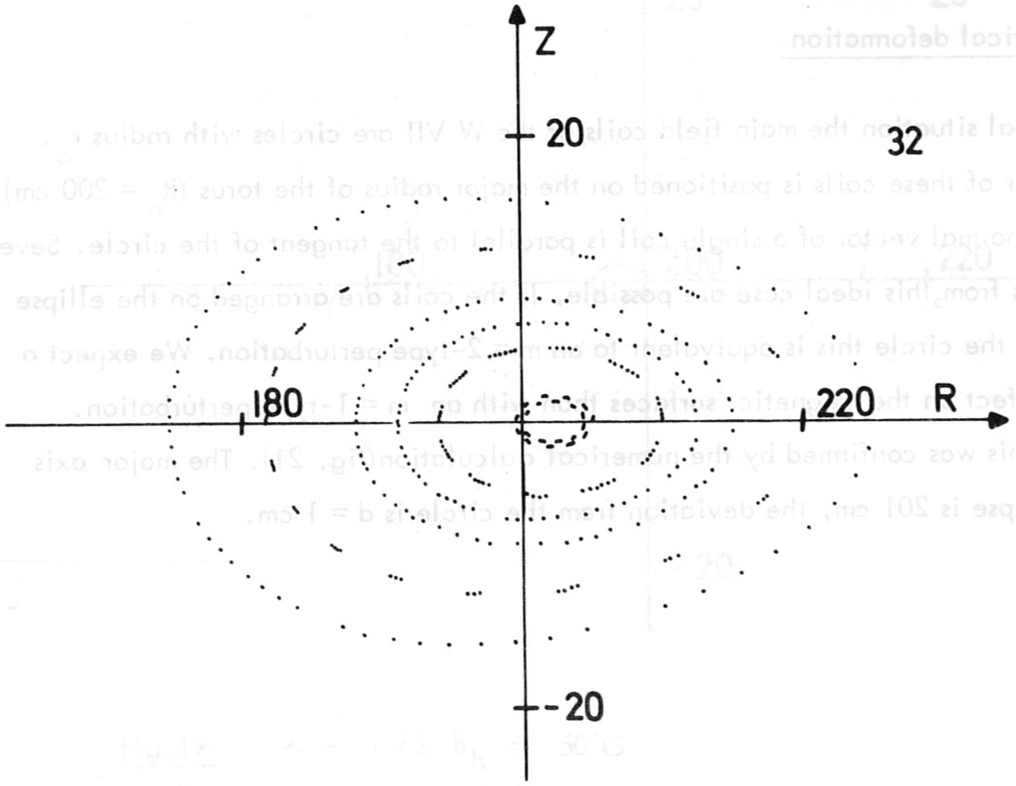

Fig.22:
$\iota\!\!\!-$ = 0.25
ß = 0.5°

2.3 Statistical errors of the coils

Due to inaccurate adjustment of the coils around the torus, perturbations of the magnetic field occur. In order to verify these perturbations, the position of the coil center was randomly distributed around the torus. Fig. 23 shows the effect of a random displacement of the 40 coils. The maximum of the displacement is $\Delta x_{max} = \pm 0.5$ cm. The effect on the magnetic surfaces is negligibly small. With a random distribution of the tilting angle β, we obtain a larger effect (fig. 24). The neighbourhood of $\iota = 0.25$ was not investigated in more detail, but fig. 24 indicates the existence of small islands. The main field coils of the W VII-stellarator can be adjusted with an accuracy of $\pm$ 0.1 cm. Therefore these statistical errors seem not to be dangerous.

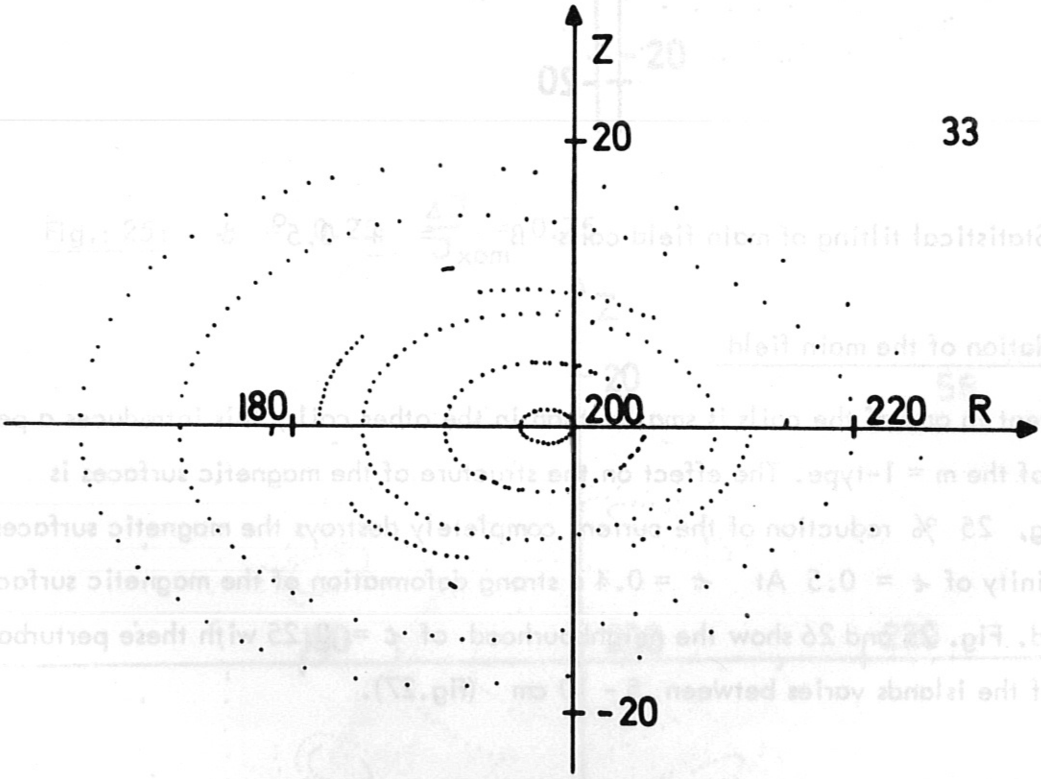

Fig. 23: Statistical displacement of coils $\Delta x_{max} = \pm 0.5$ cm, $\iota = 0.25$

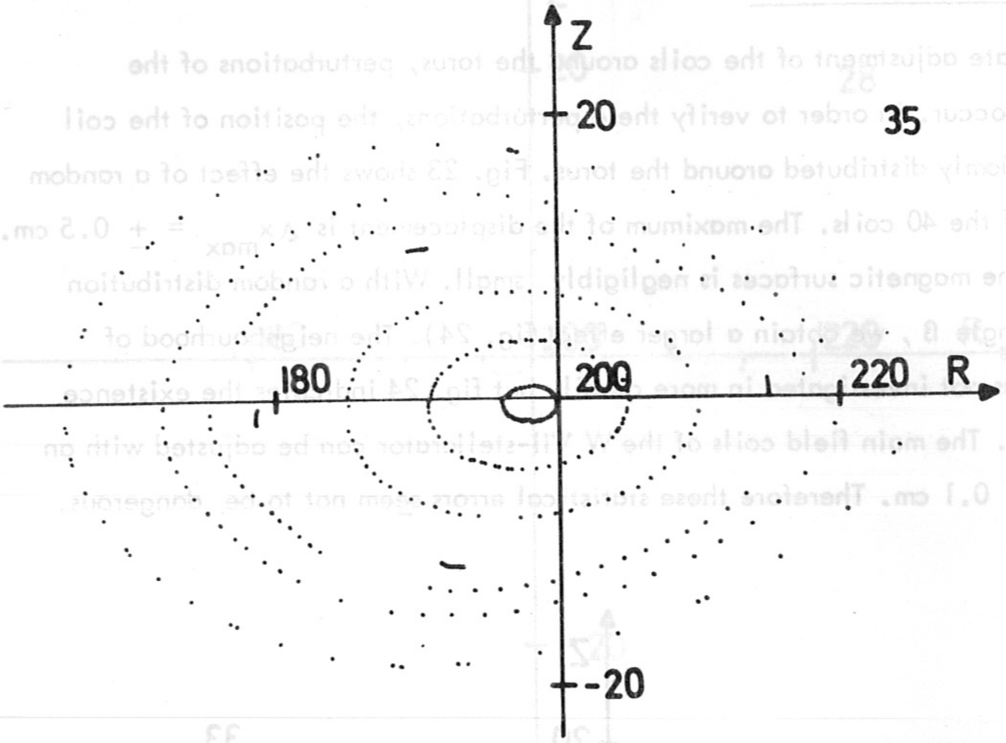

Fig.24: Statistical tilting of main field coils $\beta_{max} = \pm\ 0.5^{o}$

2.4 Modulation of the main field

If the current in one of the coils is smaller than in the other coils, this introduces a perturbation of the m = 1-type. The effect on the structure of the magnetic surfaces is very strong, 25 % reduction of the current completely destroys the magnetic surfaces in the vicinity of $\iota = 0.5$ At $\iota = 0.4$ a strong deformation of the magnetic surface is observed. Fig. 25 and 26 show the neighbourhood of $\iota = 0.25$ with these perturbations. The size of the islands varies between 5 - 10 cm (fig.27).

2.5 Modified coils for neutral injection

In order to give access to the neutral beam four modified coils are foreseen in the main coil system, two of them are shown in fig. 28. The current does not differ from the current in the other coils. Fig. 28 also shows the effect of this perturbation on the magnetic isobars B = const. in the equatorial plane. Due to these modified coils magnetic islands occur at $\iota = 0.5$ (fig. 29), at $\iota = 0.4$ no effect could be seen. Small islands ($d \leq 1$-2 cm) seem to exist at $\iota = 0.25$ (fig. 30). In conclusion the existence of these extra coils does not cause strong perturbations of the magnetic surfaces, except in the regime $\iota = 0.5$.

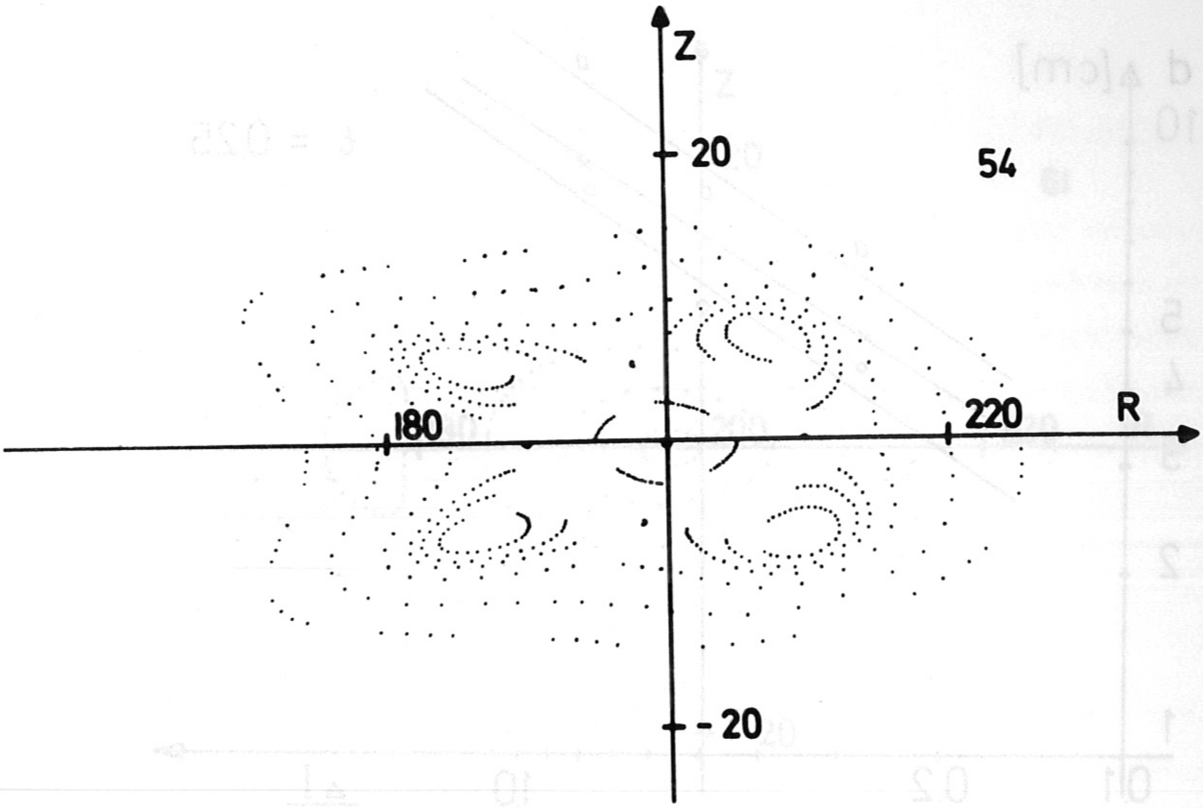

Fig.: 25: ι = 0.25; $\frac{\Delta J}{J}$ = 0.75

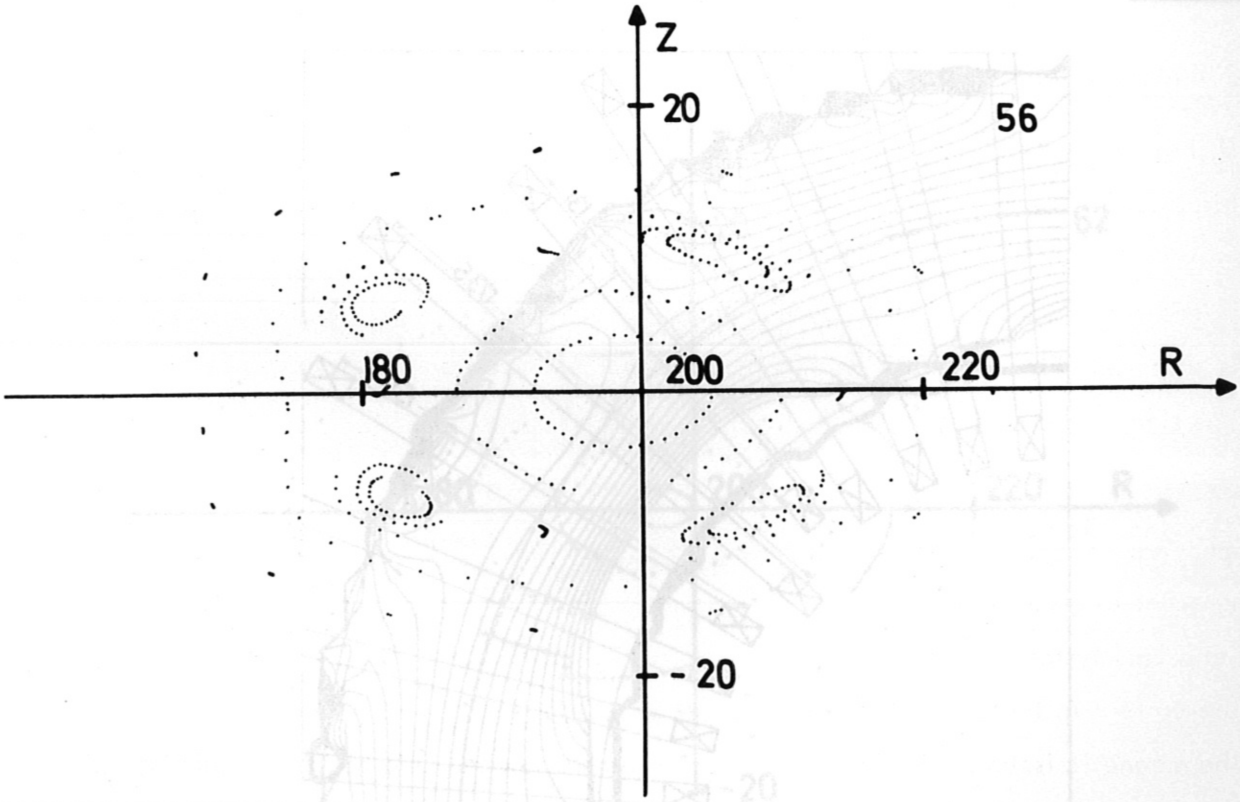

Fig.26: ι = 0.25 ; $\frac{\Delta J}{J}$ = 0.25

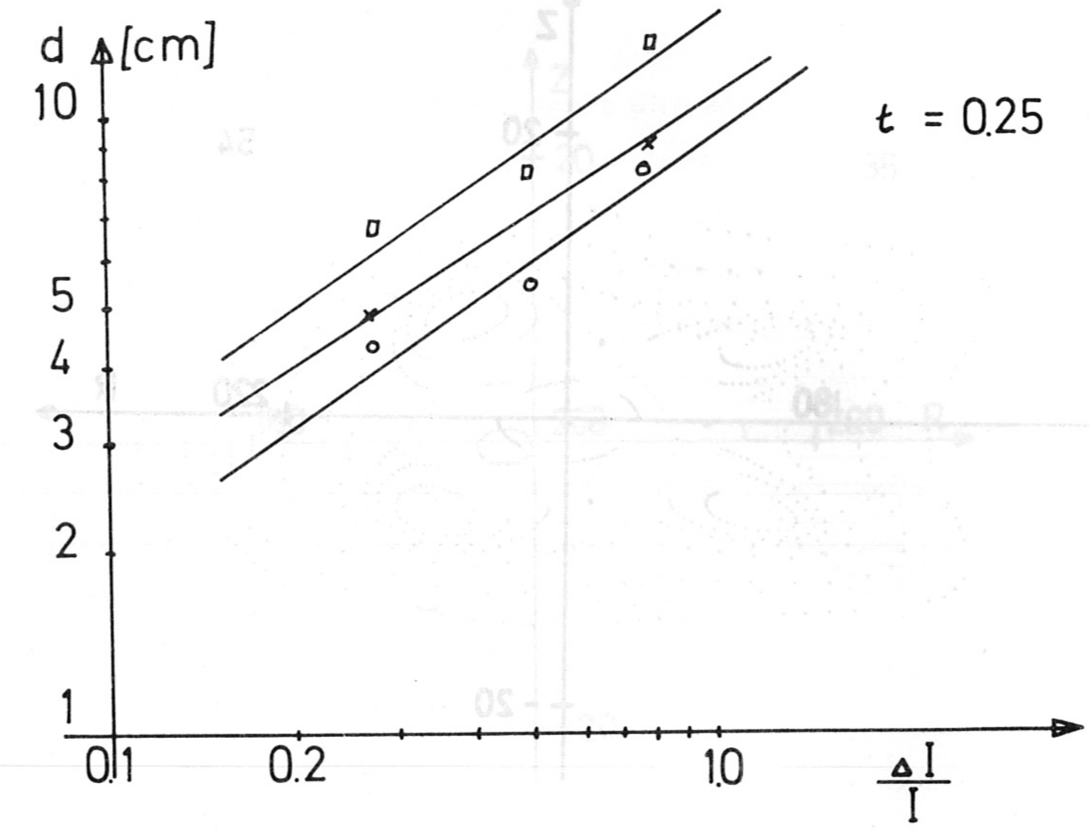

Fig. 27: = 0.25, modulation of the main field

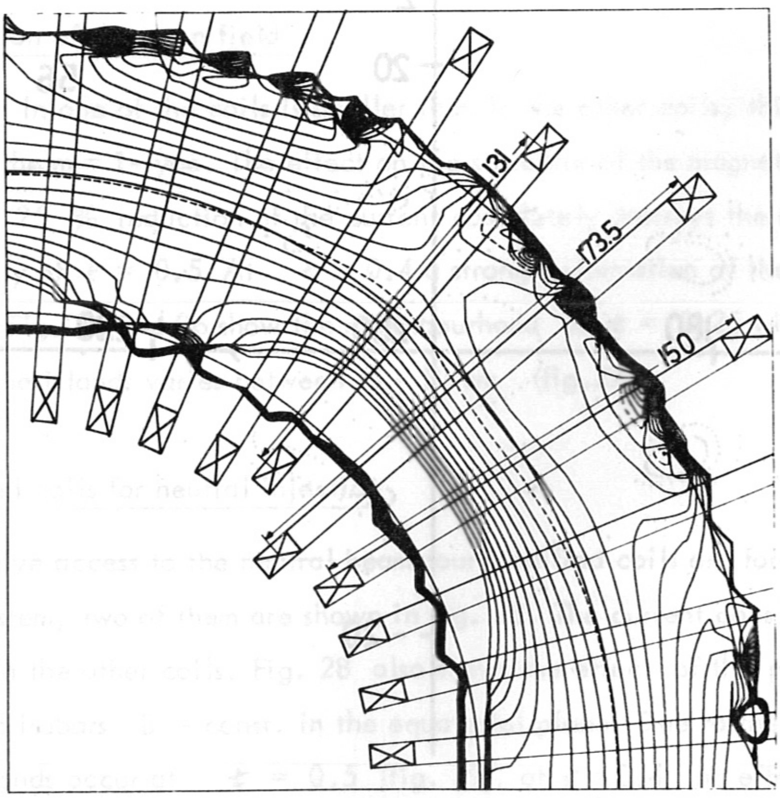

Fig. 28: Arrangement of special coils for neutral injection. The lines are B = const.-lines in the equatorial plane.

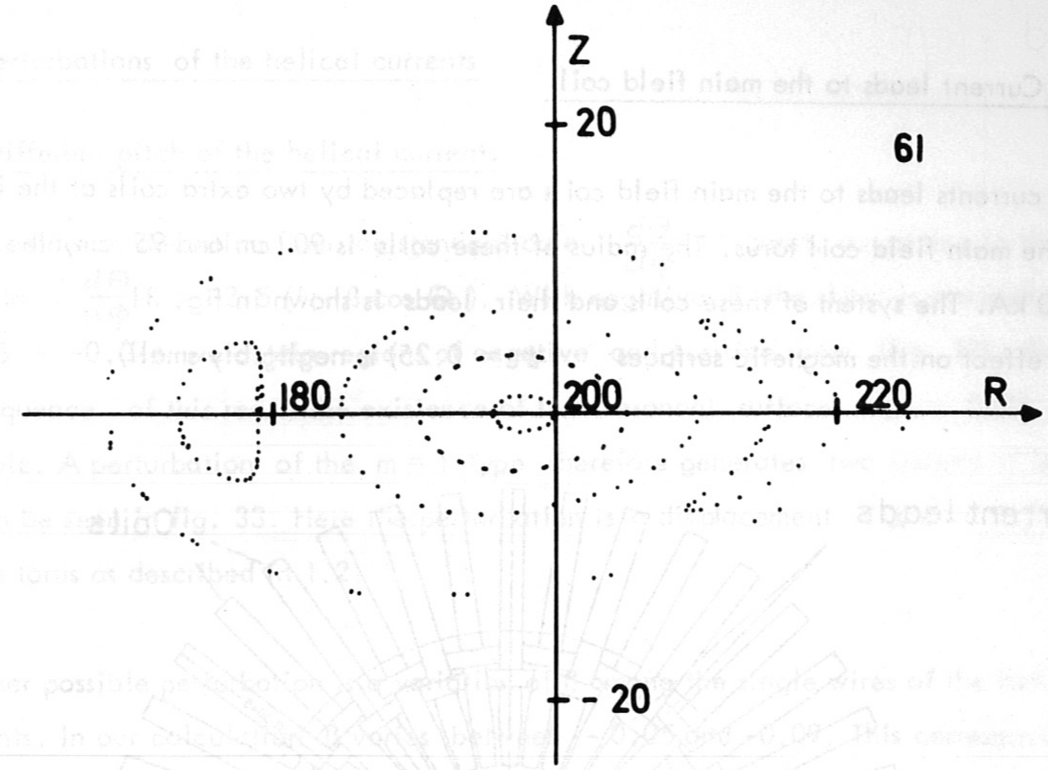

Fig. 29: ι = 0.5. Effect of neutral injection coils

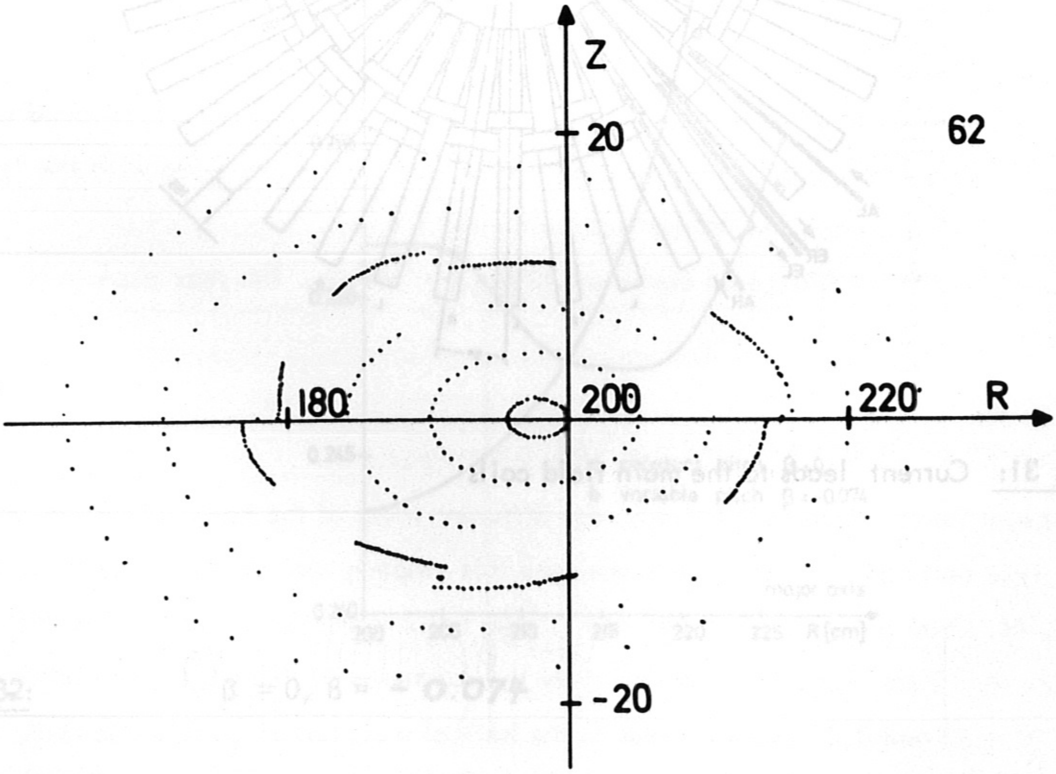

Fig. 30: ι = 0.25. Effect of neutral injection coils

2.6 Current leads to the main field coils

The currents leads to the main field coils are replaced by two extra coils at the inside of the main field coil torus. The radius of these coils is 90 cm and 95 cm, the current is 40 kA. The system of these coils and their leads is shown in fig. 31.
The effect on the magnetic surfaces (ι = 0.25) is negligibly small).

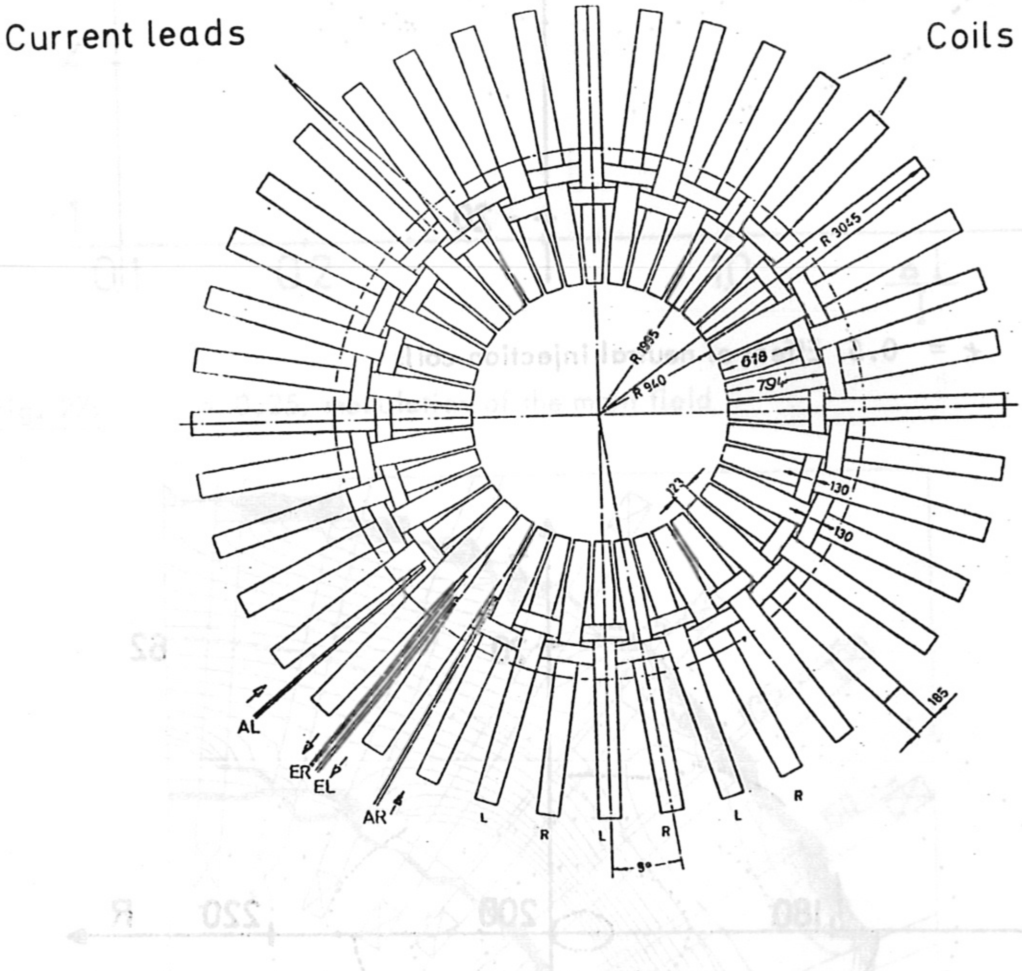

Fig. 31: Current leads to the main field coils

3. Perturbations of the helical currents

3.1 Different pitch of the helical currents

We consider a deviation from the standard case $\frac{d\Theta}{d\varphi}$ = const. according to the formula $\frac{d\Theta}{d\varphi} = 2.5\,(1 - \beta \cos\Theta)$. With negative β the shear is reduced, in the case $\beta = -0.074$ we obtain regions of negative and positive shear (fig. 32). As a consequence of this result the existence of two resonant surfaces $\iota = 0.25$ is possible. A perturbation of the m = 1-type therefore generates two systems of islands, as can be seen in fig. 33. Here the perturbation is a displacement ($\Delta x = 0.25$ cm) of the torus as described in 1.2.

Another possible perturbation is a variation of β among the single wires of the helical currents. In our calculation β varies between -0.05 and -0.09. This corresponds to a 1 - 2 cm displacement of the helical wires. This perturbation does not introduce any serious effect on the magnetic surfaces.

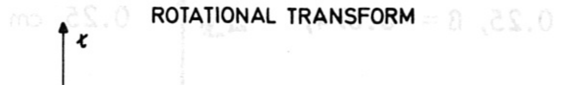

Fig.32: $\beta = 0$, $\beta = -0.074$

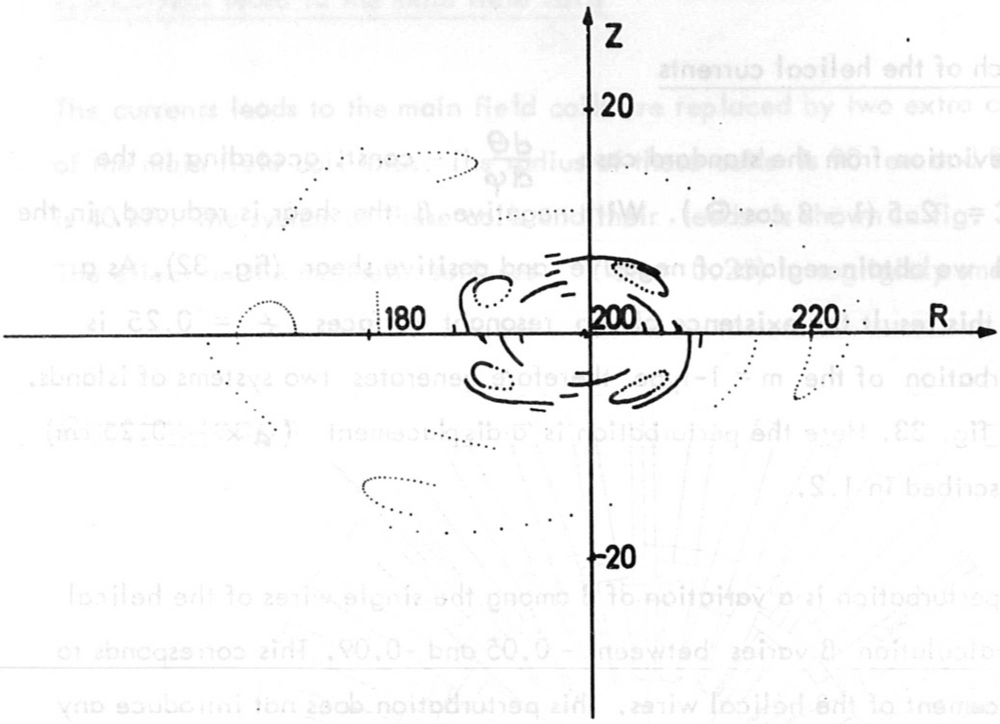

Fig. 33: ɫ = 0.25, ß = -0.074, Δx = 0.25 cm

A short circuit between the helical wires can result in different currents in these wires. Calculations were made with a 10 % increase of one of these currents. In the neighbourhood of ɫ = 0.25 no islands appear. The same result is found with the helical currents sitting at different radii. The calculation was made with the "plus"-currents at 45.6 cm and the "minus"-currents at 45.4 cm. All these perturbation discussed above are of the m = 5-type. This may explain the relatively small effect.

3.2 Current joints of the helical windings

The current joints ("bridges") between the different wires of the helix introduce an m = 1-type perturbation. In order to simulate this property we modify the helix as shown in fig. 34. Over an angle of $\Delta\varphi = 9^{o}$ the helix is shifted outward. This shift is ΔS = 2 cm. Fig. 35 and 36 show the existence of islands in the vicinity of ɫ = 0.25 and 0.5. Current leads to the helical windings do not generate islands, the perturbation is too small.

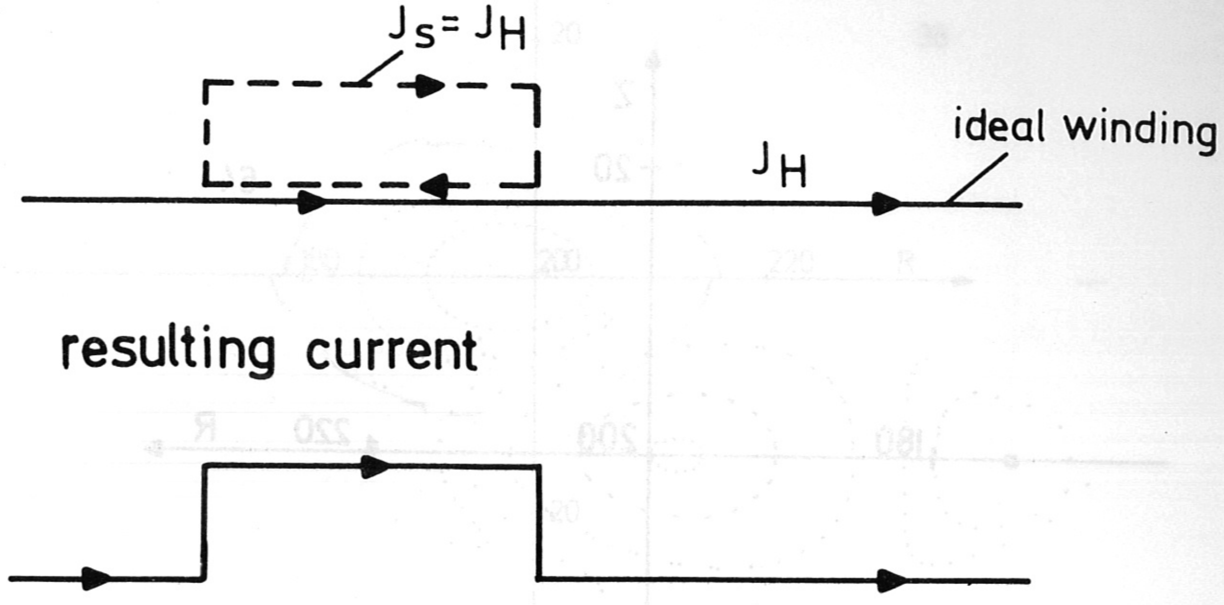

Fig.34: Scheme of the helix bridges

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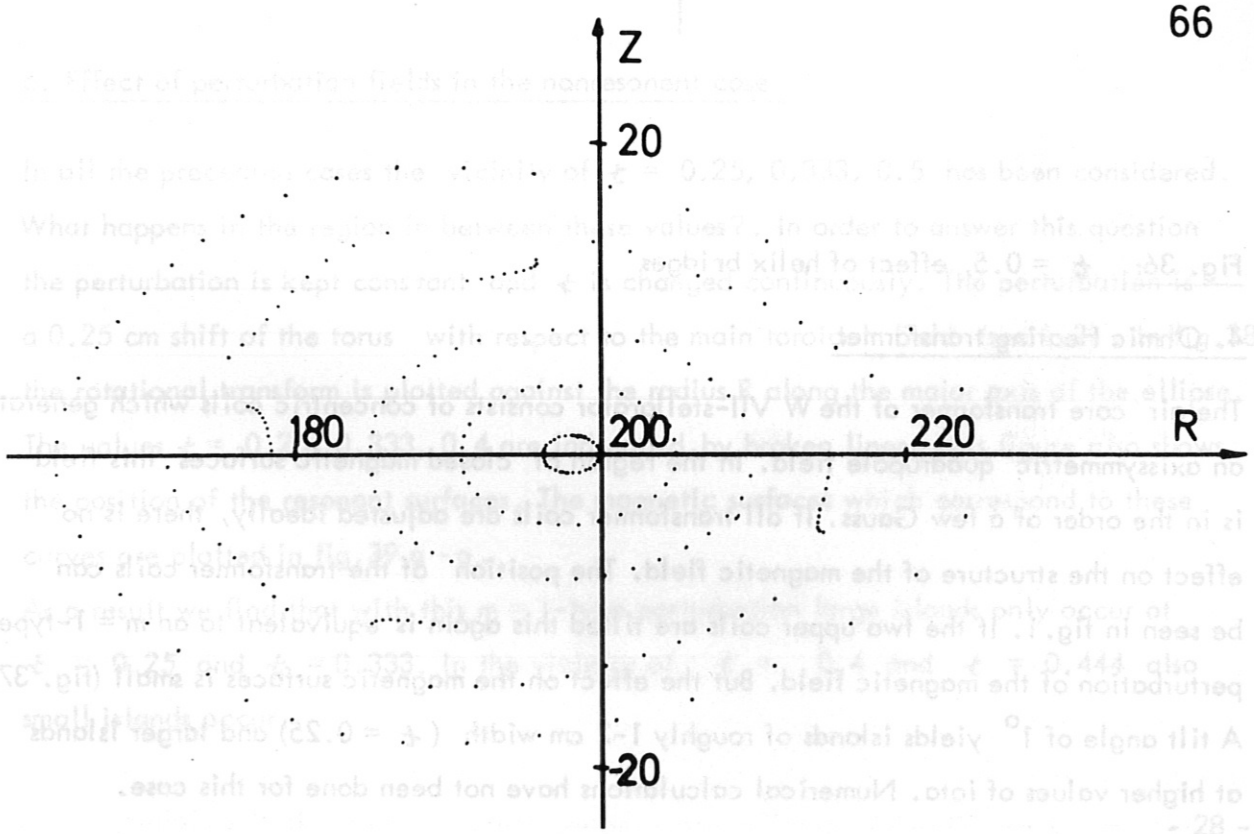


Fig. 35: ι = 0.25, effect of helix bridges

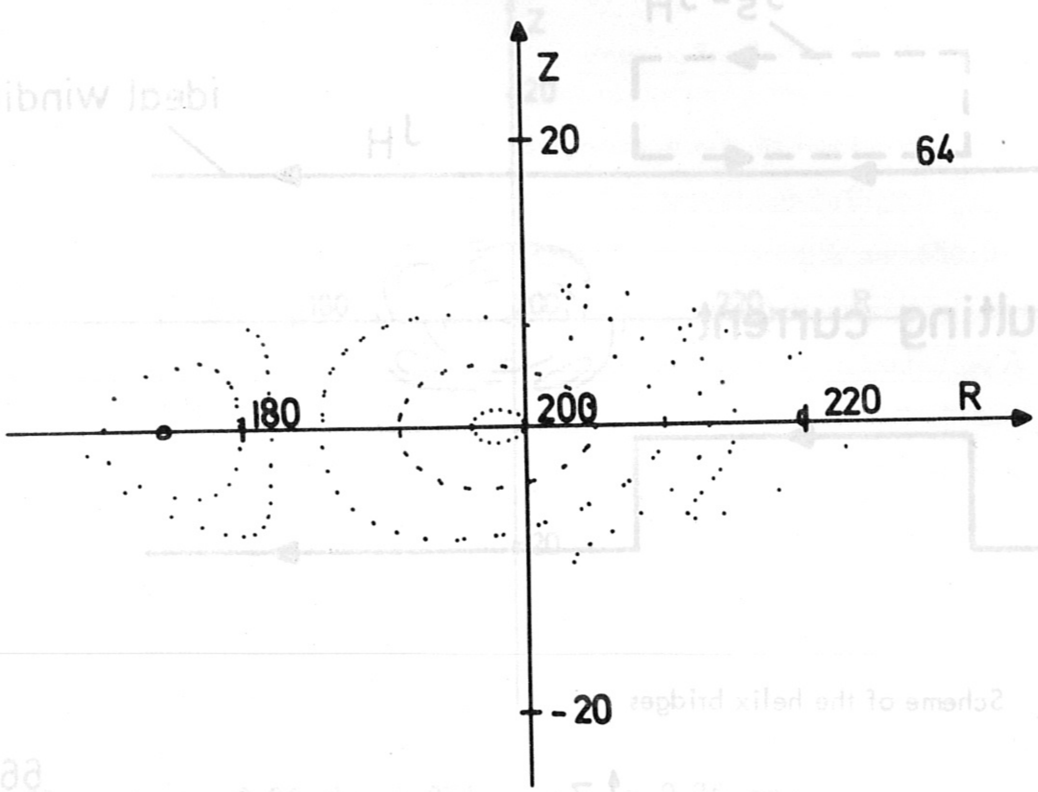


Fig. 36: ι = 0.5, effect of helix bridges

4. Ohmic Heating transformer

The air core transformer of the W VII-stellarator consists of concentric coils which generate an axissymmetric quadrupole field. In the region of closed magnetic surfaces this field is in the order of a few Gauss. If all transformer coils are adjusted ideally, there is no effect on the structure of the magnetic field. The position of the transformer coils can be seen in fig. 1. If the two upper coils are tilted this again is equivalent to an m = 1-type perturbation of the magnetic field. But the effect on the magnetic surfaces is small (fig. 37). A tilt angle of 1° yields islands of roughly 1-2 cm width (ι = 0.25) and larger islands at higher values of iota. Numerical calculations have not been done for this case.

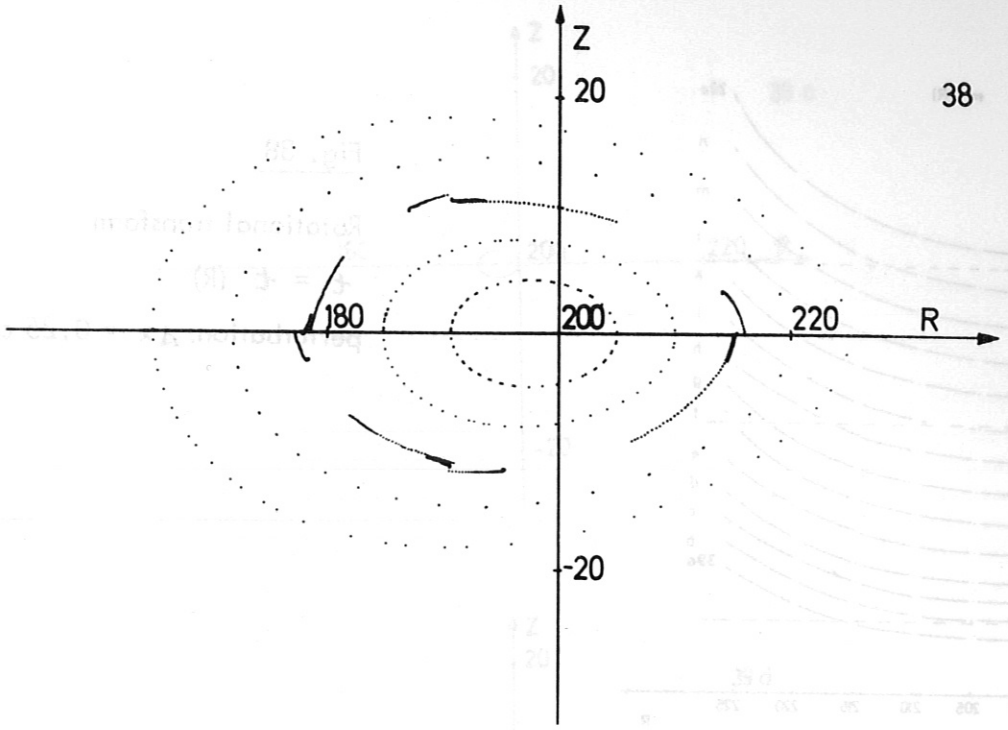


Fig. 37: Tilting of the transformer coils

6. Effect of perturbation fields in the nonresonent case

In all the preceding cases the vicinity of $\iota\!\!\!-$ = 0.25, 0.333, 0.5 has been considered. What happens in the region in between these values?. In order to answer this question the perturbation is kept constant and $\iota\!\!\!-$ is changed continuously. The perturbation is a 0.25 cm shift of the torus with respect to the main toroidal field (see 1.2) . In fig.38 the rotational transform is plotted against the radius R along the major axis of the ellipse. The values $\iota\!\!\!-$ = 0.25, 0.333, 0.4 are indicated by broken lines. This figure also shows the position of the resonant surfaces. The magnetic surfaces which correspond to these curves are plotted in fig.39 a - o .

As a result we find that with this m = 1-type perturbation large islands only occur at $\iota\!\!\!-$ = 0.25 and $\iota\!\!\!-$ = 0.333. In the vicinity of $\iota\!\!\!-$ = 0.4 and $\iota\!\!\!-$ = 0.444 also small islands occur.

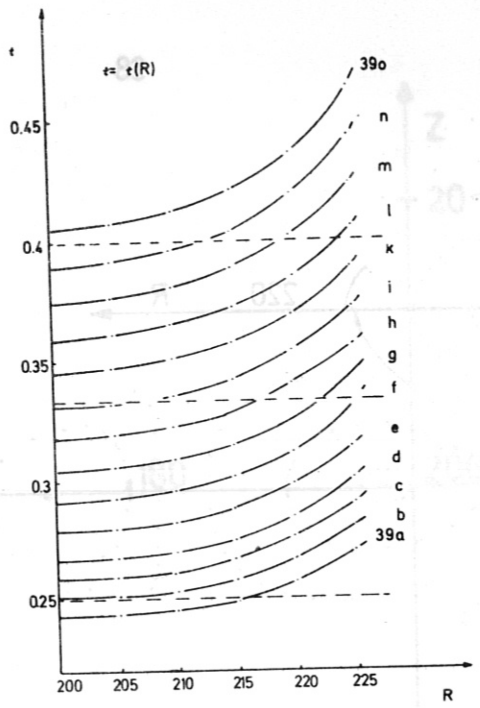


Fig. 38

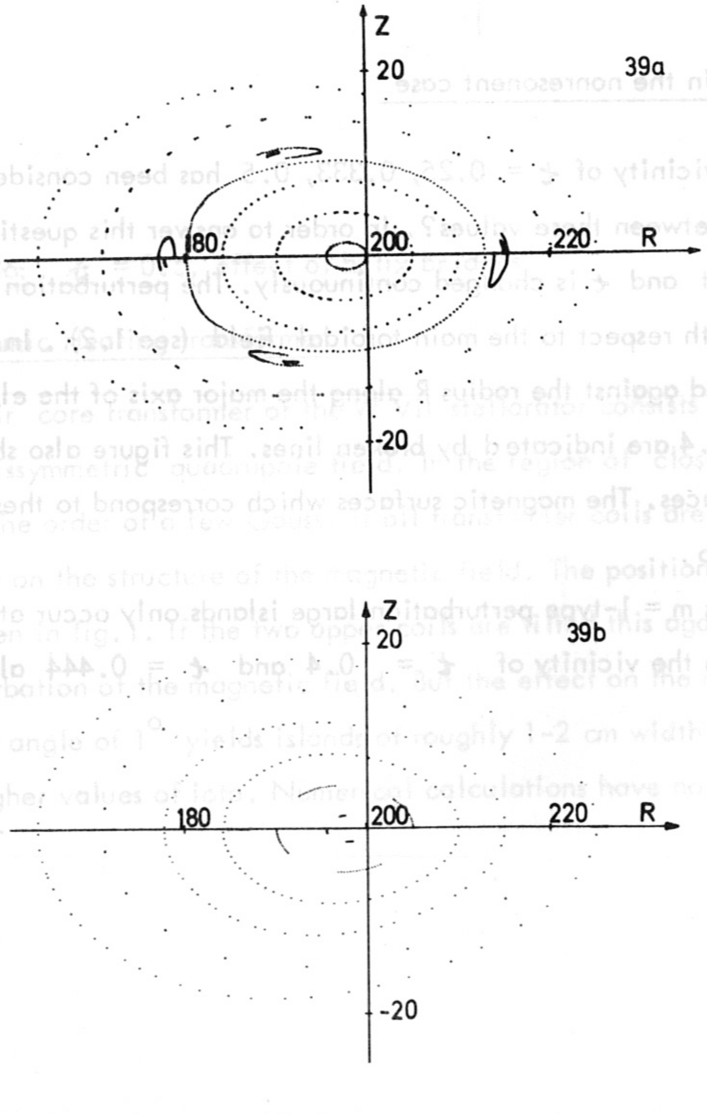
Rotational transform

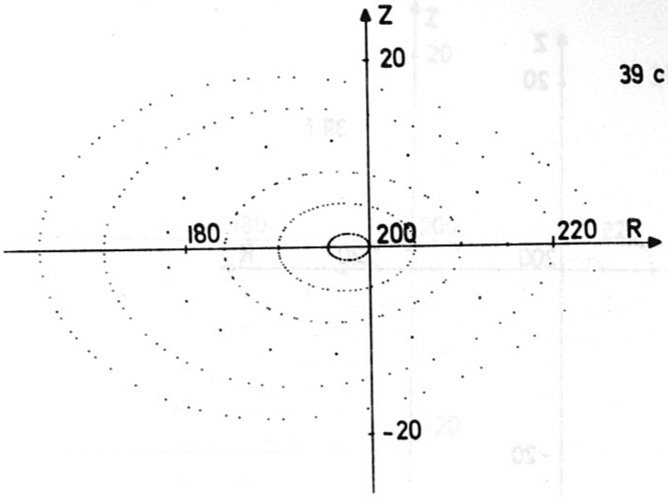
ꞇ = ꞇ (R)

perturbation: $\Delta x = 0.25$ cm

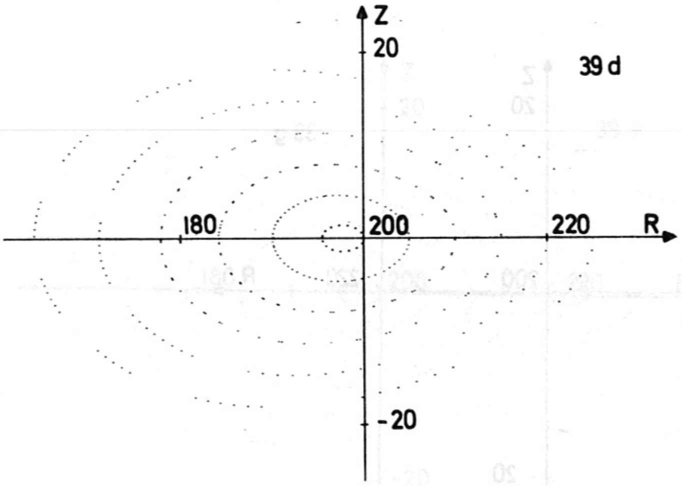
Fig. 39 a - o

Magnetic surfaces

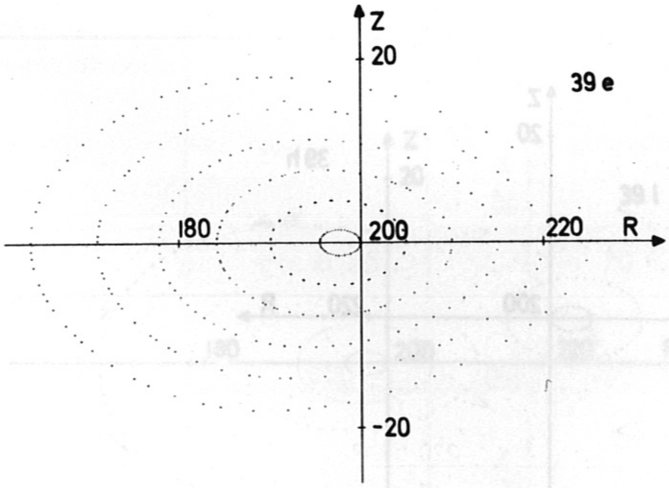




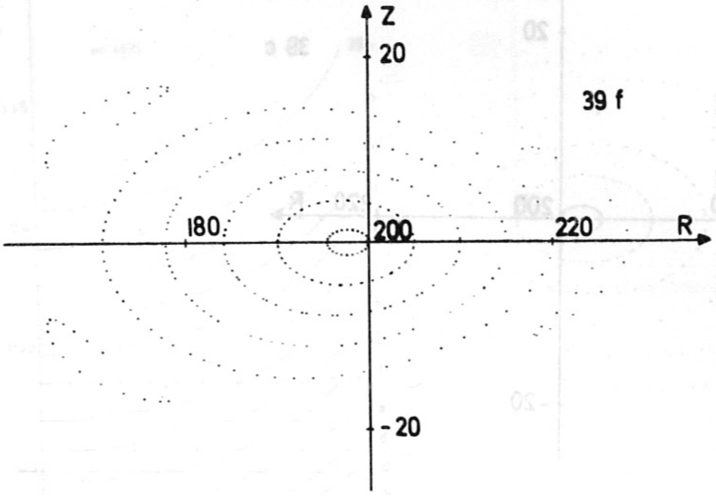
39 c



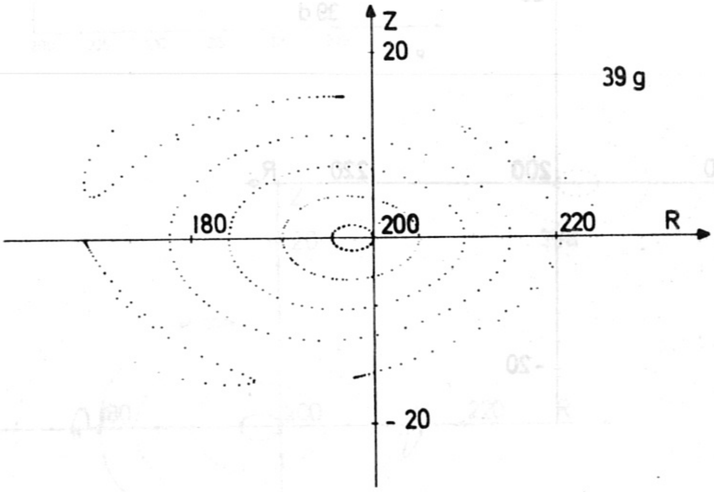
39 d



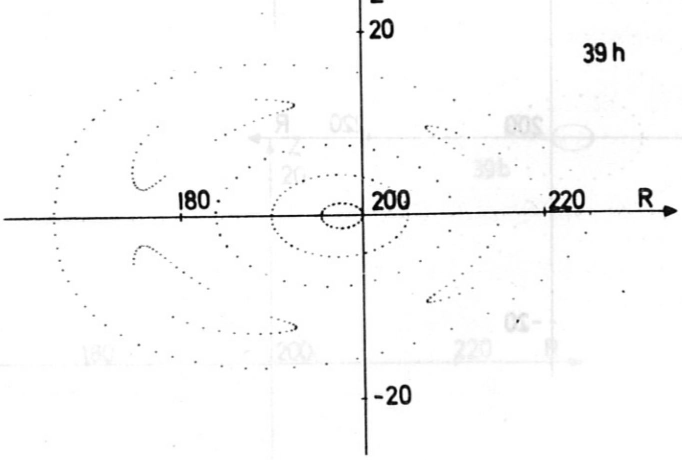
39 e



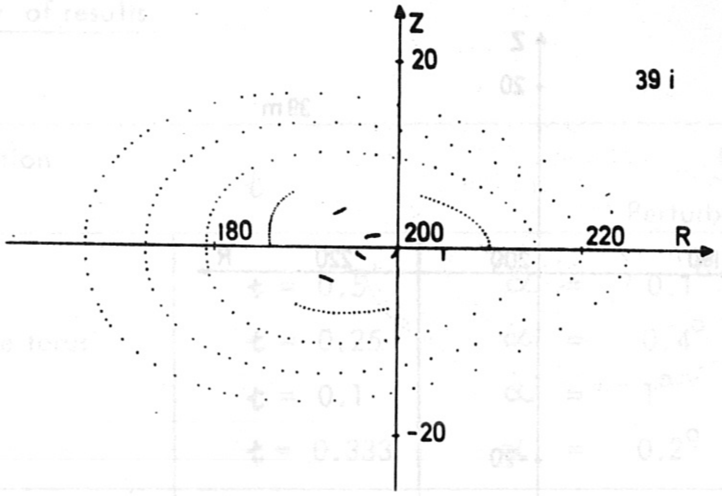
39 f



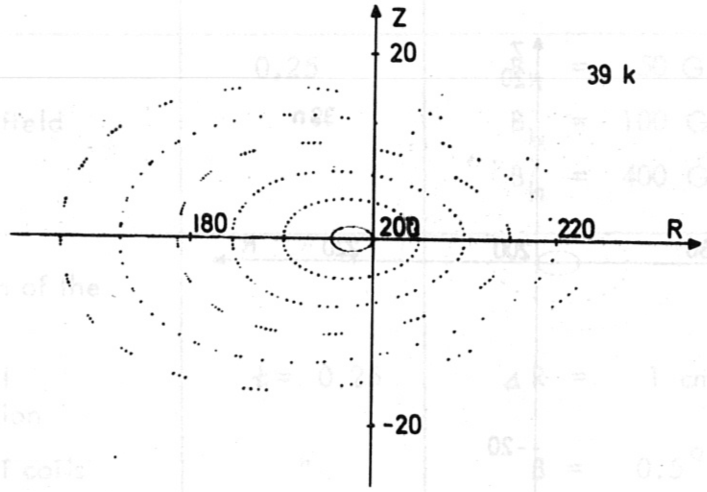
39 g



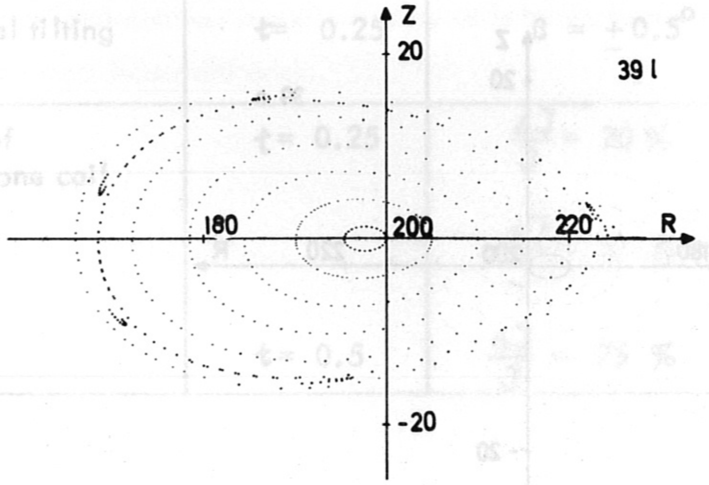
39 h



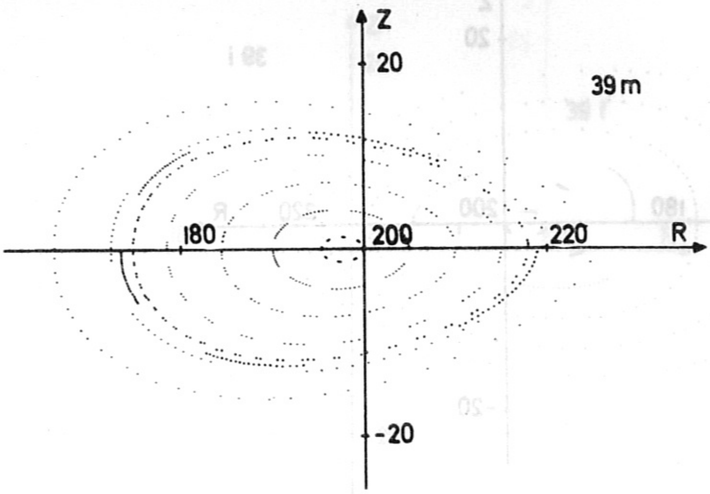
39 i



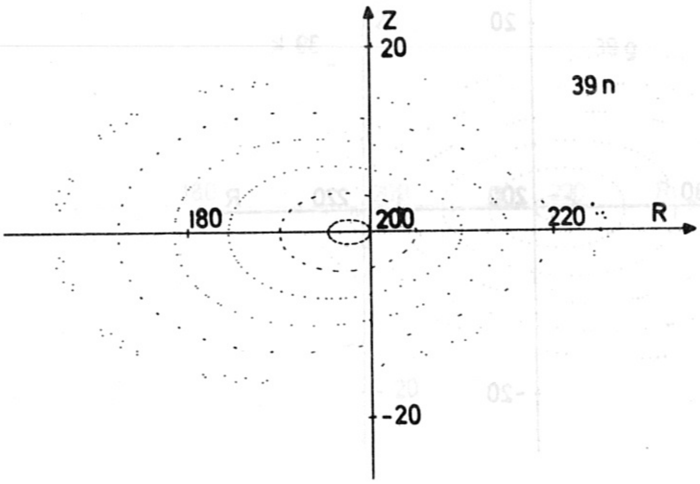
39 k



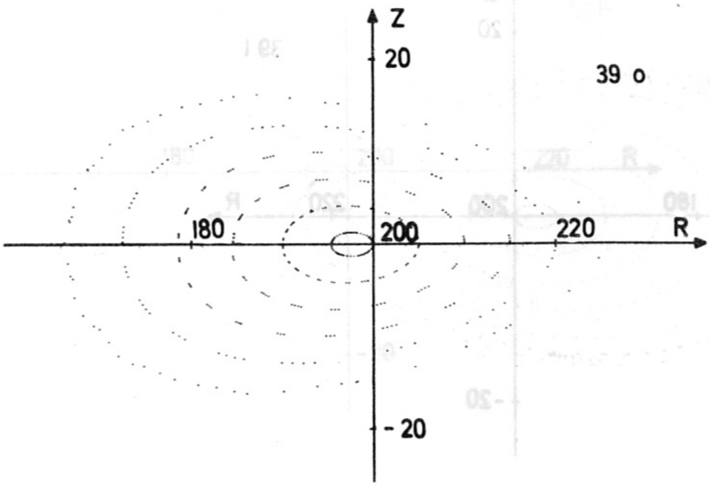
39 l



39m



39n



39 o

Summary of results

Perturbation	ι	Parameter of Perturbation	Effect
1) Tilting of the torus	$\iota = 0.5$	$\alpha = 0.1^\circ$	Islands (d = 4 - 5 cm)
	$\iota = 0.25$	$\alpha = 0.4^\circ$	d = 3-4 cm
	$\iota = 0.1$	$\alpha = 1^\circ$	no effect
	$\iota = 0.333$	$\alpha = 0.2^\circ$	d = 3-4 cm
2) Displacement of the torus	0.5	$\Delta x = 0.1$ cm	d = 8 - 9 cm
	0.25	$\Delta x = 0.2$ cm	d = 2 - 3 cm
	0.333	$\Delta x = 0$	d = 5 cm
3) Horizontal field	0.25	$B_h = 50$ G	d = 3 - 4 cm
		$B_h = 100$ G	d = 4 - 5 cm
		$B_h = 400$ G	d = 5 - 10 cm
4) Perturbation of the main field			
a) Elliptical deformation	$\iota = 0.25$	$\Delta R = 1$ cm	no effect
b) Tilting of coils	"	$\beta = 0.5^\circ$	no islands shift of magnetic surfaces
c) Statistical displacement of coils	"	$\Delta x_{max} = \pm 0.5$ cm	no effect
d) Statistical tilting of coils	$\iota = 0.25$	$\beta = \pm 0.5^\circ$	small effect d < 1 - 2 cm
5) Variation of current in one coil	$\iota = 0.25$	$\frac{\Delta J}{J} = 20$ %	d = 3 - 4 cm
		$\frac{\Delta J}{J} = 50$ %	d = 5 - 8 cm
	$\iota = 0.5$	$\frac{\Delta J}{J} = 75$ %	destruction of surfaces

6) Special coils for neutral injection	t = 0.5 t = 0.25 t = 0.4		d ≈ 10 cm d $\lesssim$ 2 cm no effect (d < 1 cm)
7) Current leads to coils			
a) Main field coils	t = 0.25		no effect
b) To helical windings	"		no effect
8) "Bridges"	t = 0.5 t = 0.25	Δs = 2cm	d = 13 - 16 cm
9) OH-Transformer	t = 0.25		no effect
	t = 0.5		small islands (d < 1 cm)
Tilting of the coils	t = 0.25	ß = 1°	small islands (d < 1 cm)
10) Perturbation the helix	t = 0.25		
a) Variation of pitch	"	ß = 0.05 - 0.09	no effect
b) Different currents in helical wires	"	$\frac{\Delta J}{J}$ = 10 %	no effect
c) "+" Currents at r = 45.6 cm "-" Currents at r = 45.4 cm	"		no effect

Conclusions

The numerical calculations show that due to perturbation fields large islands occur in the magnetic field of the W VII-stellarator. A comparison with the unperturbed case shows that natural islands, which are caused by the toroidal curvature, do not occur. As already noticed by Gourdon et al. [5] the displacement and tilting of the torus are one of the most dangeous perturbations. In the W VII-Stellarator the adjustment of the torus can be done with an accuracy of 0.1 cm. According to fig. 16 this is sufficient to generate islands with $d \approx 8 - 9$ cm (at ŧ = 0.5). For $t = 0.333$ we estimate $d \approx 4 - 5$ cm.

Since the width of the islands scales with the square root of the perturbation a further increase of the accuracy would result only in a small reduction of the islands.

The main field coils can also be adjusted with an accuracy of 0.1 cm. According to the calculations statistical errors of ± 0.5 cm do not introduce any serious effect. If one of the field coils is tilted by 0.1° this corresponds to a perturbation field of $B_h \approx 14$ G (see fig. 20). Perturbations of the helix seem not to be dangerous as far as the 5-fold symmetry is conserved. Care has to be taken of the current joints of the helical windings, these introduce perturbation of the $m = 1$-type.

The field of the ohmic heating transformer only has a small effect on the magnetic surfaces. The coils for neutral injection also introduce perturbation of the $m = 1$-type, but at $t = 0.25$ the effect seems not to be serious.

This allows us to extrapolate the calculation to small values of the perturbation. The choice of perturbation investigated by us is by no means complete, but it seems to cover the most important ones.

Acknowledgements:

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