RATE COEFFICIENTS FOR INTERACTION OF MONOENERGETIC PARTICLE BEAMS WITH MAXWELLIAN PLASMAS

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Abstract

Rate coefficients and mean free paths for the interaction of a monoenergetic hydrogen atom beam with a Maxwellian hydrogen plasma have been calculated taking into account charge exchange and impact ionisation by electrons and ions in the following temperature and energy range:

$$T = 1 \text{ eV} - 100 \text{ keV (nonrelativistic)}$$
$$E = 1 \text{ eV} - 100 \text{ keV}$$

1) Introduction

The interaction of monoenergetic beams of particles penetrating into high-temperature plasmas has become a field of increasing interest in the last few years. On the one hand there is the promising idea of heating plasmas by injection of powerful neutral beams (1,2); on the other hand there is the possibility of using particle beams as a diagnostic tool for plasma density, electron temperature (3,4,5,6), space charge, density, and current distribution (7). Furthermore, there are recent investigations into the problems related to recycling of neutral gas from the walls and its influence on density and temperature profiles as well as its significance for a stationary state plasma (8,9). In all of these cases it is necessary to know the mean free path of the penetrating particles with respect to the different types of interaction that may occur, like charge exchange, ionisation by electron and ion impact. Since this implies the knowledge of the related rate coefficients, in the following paper we describe a program for the required computations, which in contrast to former calculations (10) does not neglect the beam velocity. Similar work has been done by Riviere (11).

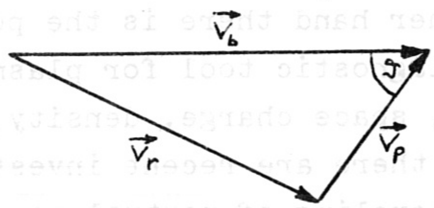
We first summarize the necessary formulas and inherent assumptions, then describe the code and finally consider the case of H^o-atoms undergoing charge exchange, ion and electron ionisation in a Maxwellian hydrogen plasma. Measured cross sections from literature are listed, rate coefficients and mean free paths are calculated.

2) Rate Coefficient <σv>

We consider a beam particle (mass m_b, velocity $\vec{v}_b$) interacting with an ensemble of plasma particles (mass m_p, individual velocities $\vec{v}_p$) with Maxwellian temperature distribution (temperature kT_p). The probability of a certain type of interaction is characterized by its cross-section $\sigma(v_r)$ which depends on the relative velocity v_r of beam and plasma particle:

$$v_r = |\vec{v}_b - \vec{v}_p| = \{v_b^2 + v_p^2 - 2v_b v_p \cos\vartheta\}^{1/2}$$

where ϑ is the angle between $\vec{v}_p$ and $\vec{v}_b$



If we consider the beam to be monoenergetic, the rate coefficient is defined by (12)

$$(1) \quad \langle\sigma v\rangle = \int_0^\infty \sigma(v_r)\, v_r\, f(v_p)\, d^3v_p$$

where $f(v_p)\, d^3v_p$ is the plasma distribution function. We assume $f(v_p)\, d^3v_p$ to be a Maxwellian:

$$(2) \quad f(v_p)\, d^3v_p = \int_{-1}^{+1} 2\pi \left(\frac{m_p}{2\pi k T_p}\right)^{3/2} e^{-\frac{m_p v_p^2}{2kT_p}}\, v_p^2\, dv_p\, d(\cos\vartheta)$$

For the purpose of computation it is convenient to perform the following transformation:

$$(3) \quad \langle\sigma v\rangle\, (E_b, T_p) = \frac{2}{\sqrt{\pi}} \int_{-1}^{+1} \int_0^\infty \sigma(E_b, u)\, \{A + Bu^2 + Cu\cos\vartheta\}^{1/2}\, u^2\, e^{-u^2}\, du\, d(\cos\vartheta)$$

where

$$E_b = \text{beam energy} = 1/2\, m_b v_b^2$$

$$u = \sqrt{\frac{m_p}{2kT_p}}\, v_p$$

$$A = \frac{2E_b}{m_b}$$

$$B = \frac{2kT_p}{m_p}$$

$$C = -4\sqrt{\frac{E_b\, kT_p}{m_p m_b}}$$

The cross-section σ is the total or integrated cross-section $\sigma \equiv \int_0^{4\pi} \frac{d\sigma}{d\Omega} d\Omega$. For processes, where this is divergent like in elastic scattering, our treatment has to be modified.
σ has been measured in many cases. However, taking these measured values, one has to draw attention to the following points:

Obviously, since σ depends on the relative velocity v_r, it depends on both the beam energy E_b and the thermal velocity v_p of the plasma particles. Cross-sections from the literature are generally measured with the target particle at rest, and in these measurements projectile and target particle may be reversed with respect to our application, (e.g. electron ionisation). Therefore, carrying out (for a given beam energy E_b) the integration over the normalized velocity u, one has to calculate the relative velocity

$$v_{rel} = \{A + B u^2 + C u \cos\vartheta\}^{1/2}$$

and then has to convert this into the energy scale E_{lab} which has been used in the measurement of the cross-section:

$$E_{lab} = m_b^* \cdot v_{rel}^2$$

where m_b^* = mass of the projectile in the cross-section measurement. This is the energy where the cross-section σ has to be taken in the integration.

3) Description of the code

We write equ. (3) in the form:

$$\langle\sigma v\rangle = \int_{-1}^{+1} d\alpha \int_0^{\infty} du\, e^{-u^2} f(u,\alpha)$$

where $f(u,\alpha)$ is the integrand which contains the cross-section and the relative velocity. For the integration over the normalized velocity u we use the Gauß-Hermite integration formula of N^H_{th} degree:

$$④ \quad \langle \sigma v \rangle \approx \int_{-1}^{+1} d\alpha \left\{ \sum_{j=1}^{N_H} W_j^H \ f(u_j^H, \alpha) \right\}$$

where W_j^H and u_j^H are the appropriate weights and abscissas, tabulated in (13). Similarly for the integration over dα we use the Gauß-Legendre integration formula:

$$⑤ \quad \langle \sigma v \rangle \approx \sum_{k=1}^{N_L} \left\{ W_k^L \sum_{j=1}^{N_H} W_j^H \ f(u_j^H, \alpha_k^L) \right\}$$

The computer code consists of a main program which performs the double sum. The main program calls a subroutine which calculates the integrand $f(u_j^H, \alpha_k^L)$. This subroutine in turn calls a subroutine "SIGMA", which provides the cross-section for the called routine. Interpolation out of tabulated values or computation out of semi-empirical formulas is possible. Although we take a rather high order for the integration (N_L = 25; N_H = 10), there are no problems in calculating eq. ⑤ on the computer.

4) Cross-sections, Rate Coefficients and Mean Free Paths for H^O into Hydrogen Plasma

We consider the following three processes:

(a) charge exchange $\underline{H}^O + H^+ \rightarrow \underline{H}^+ + H^O$

The cross-section has been measured in different energy ranges by various authors:

3.0 eV - 100 eV	by Belyaev et al (14)
200 eV - 40 keV	by Fite et al. (15,16)
2 keV - 117 keV	by McClure (17)

(b) proton ionisation $\underline{H}^o + H^+ \rightarrow \underline{H}^+ + H^+ + e^-$

7 keV - 40 keV Fite et al. (16)
60 keV - 400 keV Gilbody et al. (18)

(c) electron ionisation $\underline{H}^o + e^- \rightarrow \underline{H}^+ + 2\ e^-$

E_{Lab} < 750 eV summarized by Kieffer et al. (19)
E_{Lab} > 750 eV semiempirical formula by Lotz (10)

The measurements overlap in some energy intervals and the σ-values do not always coincide in magnitude. In Table 1 a "smoothed" cross-section is listed, the error of which may be as high as ± 15 % in some cases.

<u>Table 1</u>: Cross-sections for H^o into H^+, e^--plasma

charge exchange		proton ionisation		electron ionisation	
E_{Lab} [eV]	σ [cm^2]	E_{Lab} [eV]	σ [cm^2]	E_{Lab} [eV]	σ [cm^2]
3	$5.6\cdot10^{-15}$	(10^3	$4.3\cdot10^{-18}$)	13.6	0
5	$5.3\cdot10^{-15}$	($2\cdot10^3$	$1.1\cdot10^{-17}$)	20	$3\cdot10^{-17}$
10	$4.7\cdot10^{-15}$	($3\cdot10^3$	$1.8\cdot10^{-17}$)	30	$5.5\cdot10^{-17}$
20	$4.4\cdot10^{-15}$	($5\cdot10^3$	$3.3\cdot10^{-17}$)	40	$6.3\cdot10^{-17}$
30	$3.8\cdot10^{-15}$	10^4	$7.5\cdot10^{-17}$	50	$6.7\cdot10^{-17}$
50	$3.5\cdot10^{-15}$	$2\cdot10^4$	$1.4\cdot10^{-16}$	60	$6.75\cdot10^{-17}$
100	$3.1\cdot10^{-15}$	$3\cdot10^4$	$1.7\cdot10^{-16}$	70	$6.65\cdot10^{-17}$
200	$2.6\cdot10^{-15}$	$5\cdot10^4$	$1.8\cdot10^{-16}$	80	$6.5\cdot10^{-17}$
300	$2.5\cdot10^{-15}$	$6\cdot10^4$	$1.75\cdot10^{-16}$	100	$6.2\cdot10^{-17}$
500	$2.2\cdot10^{-15}$	$7\cdot10^4$	$1.7\cdot10^{-16}$	200	$4.4\cdot10^{-17}$
10^3	$2.0\cdot10^{-15}$	$8\cdot10^4$	$1.65\cdot10^{-16}$	400	$2.6\cdot10^{-17}$
2.10^3	$1.6\cdot10^{-15}$	$9\cdot10^4$	$1.55\cdot10^{-16}$	600	$1.75\cdot10^{-17}$
5.10^3	$1.2\cdot10^{-15}$	10^5	$1.4\cdot10^{-16}$	800	$1.35\cdot10^{-17}$
10^4	$8.6\cdot10^{-16}$	$2\cdot10^5$	$5\cdot10^{-17}$	10^3	$1.2\cdot10^{-17}$
2.10^4	$5\cdot10^{-16}$	$3\cdot10^5$	$6\cdot10^{-17}$	$2\cdot10^3$	$7.5\cdot10^{-18}$
3.10^4	$3\cdot10^{-16}$	($5\cdot10^5$	$4\cdot10^{-17}$)	$6\cdot10^3$	3.10^{-18}
5.10^4	$1.1\cdot10^{-16}$	(10^6	$2.3\cdot10^{-17}$)	10^4	$2\cdot10^{-18}$
10^5	$1\cdot10^{-17}$				

The values in brackets are extrapolations by the computer program.

The cross-sections of Table 1 are plotted vs. E_{Lab} in Fig. 1. Figs. 2,3,4 show the rate coefficients $<\sigma v>$ vs. ion or electron temperature, the parameter being the beam energy. The three sets of curves all exhibit the following common features:

1) As long as the beam velocity is large compared to the thermal velocity of the plasma particles, the rate coefficient is independent of plasma temperature, but depends on beam energy, because the relative velocity is mainly determined by the beam. The rate coefficient can be approximated by $\sigma \cdot v_b$.

2) When the beam velocity is small compared to the thermal velocities, the rate coefficient depends on plasma temperature but not on beam energy, since the relative velocity is now mainly determined by the thermal velocity of the plasma constituents. In this case $\langle \sigma v \rangle$ is approximately given by $\langle \sigma v \rangle$ for $v_b = 0$ (see Lotz (10)).

3) In the intermediate range where the velocities of beam and plasma particles are comparable, no approximations are valid.

In Figs. 5,6,7 finally the mean free path λ is plotted vs. $T_{e,i}$, (E_b again being the parameter), for a plasma density of $n_p = 10^{13}\ cm^{-3}$.

$$\lambda = \frac{v_{beam}}{n_{plasma} \langle \sigma v \rangle}$$

The mean free paths for different plasma densities can be scaled $\sim \frac{1}{n_p}$.

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Figure Captions

Fig. 1 Cross-sections for charge exchange, electron ionisation and proton ionisation of neutral hydrogen atoms in a hydrogen plasma. Indicated points are measured values, the continuous lines represent the smoothed values of Table 1. Broken lines are extrapolations.

Fig. 2 Rate coefficients for charge exchange vs. plasma temperature; parameter : neutral atom energy E_{HO}.

Fig. 3 Rate coefficients for electron ionisation vs. plasma temperature; parameter : neutral atom energy E_{HO}.

Fig. 4 Rate coefficients for proton ionisation vs. plasma temperature; parameter : neutral atom energy E_{HO}.

Fig. 5 Mean free paths for charge exchange vs. plasma temperature; parameter : neutral atom energy E_{HO}.

Fig. 6 Mean free paths for electron ionisation vs. plasma temperature; parameter : neutral atom energy E_{HO}.

Fig. 7 Mean free paths for proton ionisation vs. plasma temperature; parameter : neutral atom energy E_{HO}.

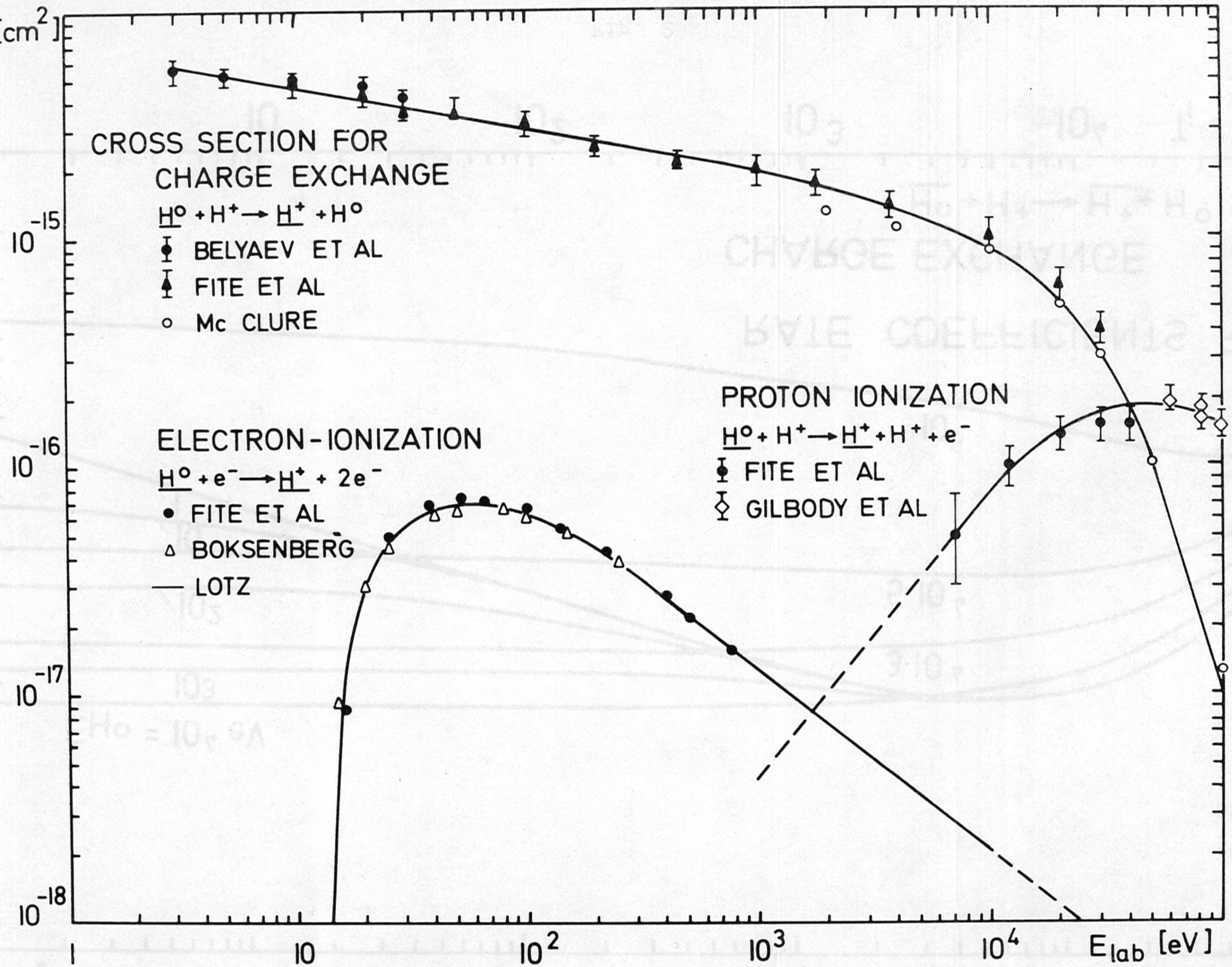
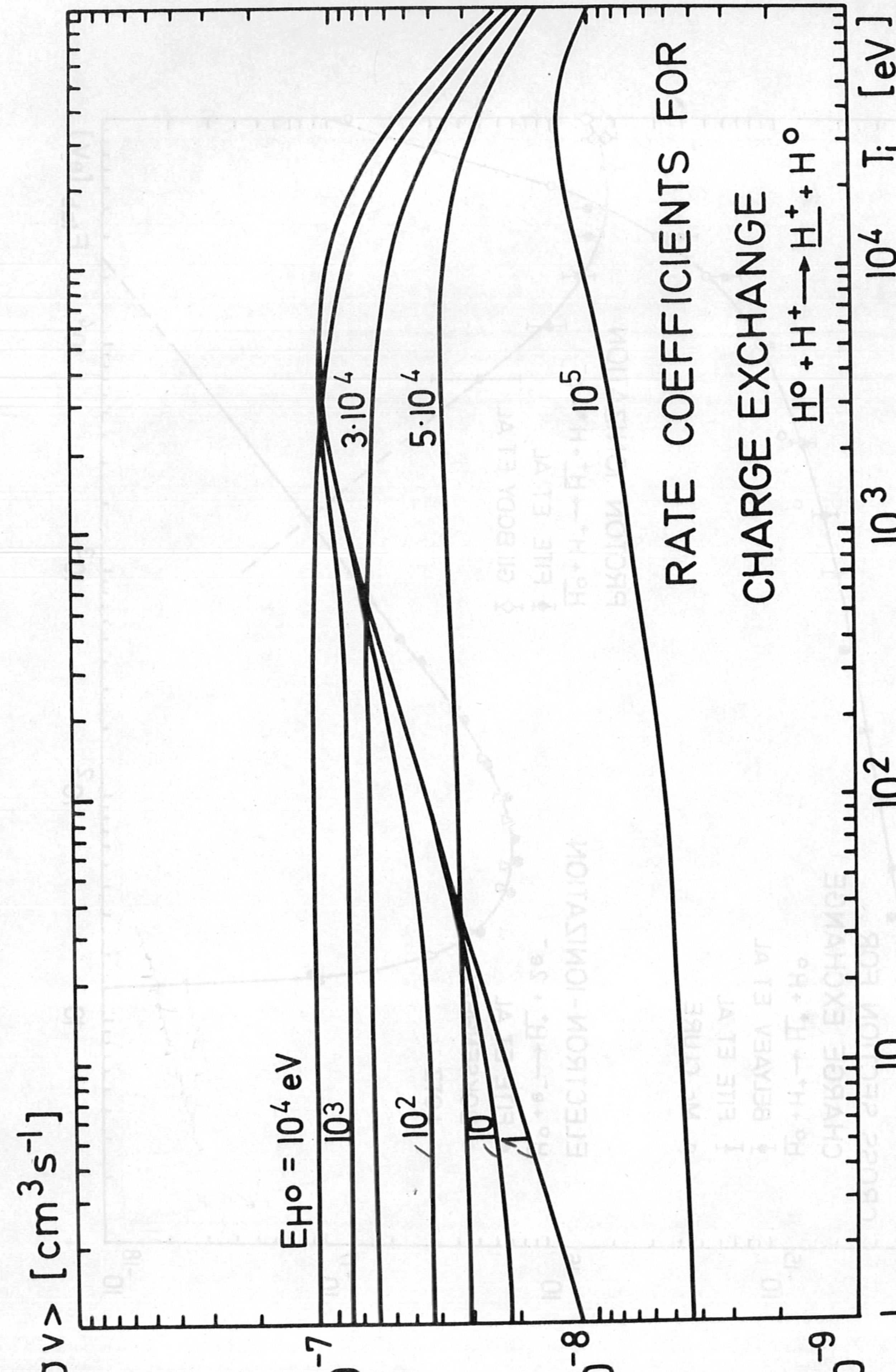
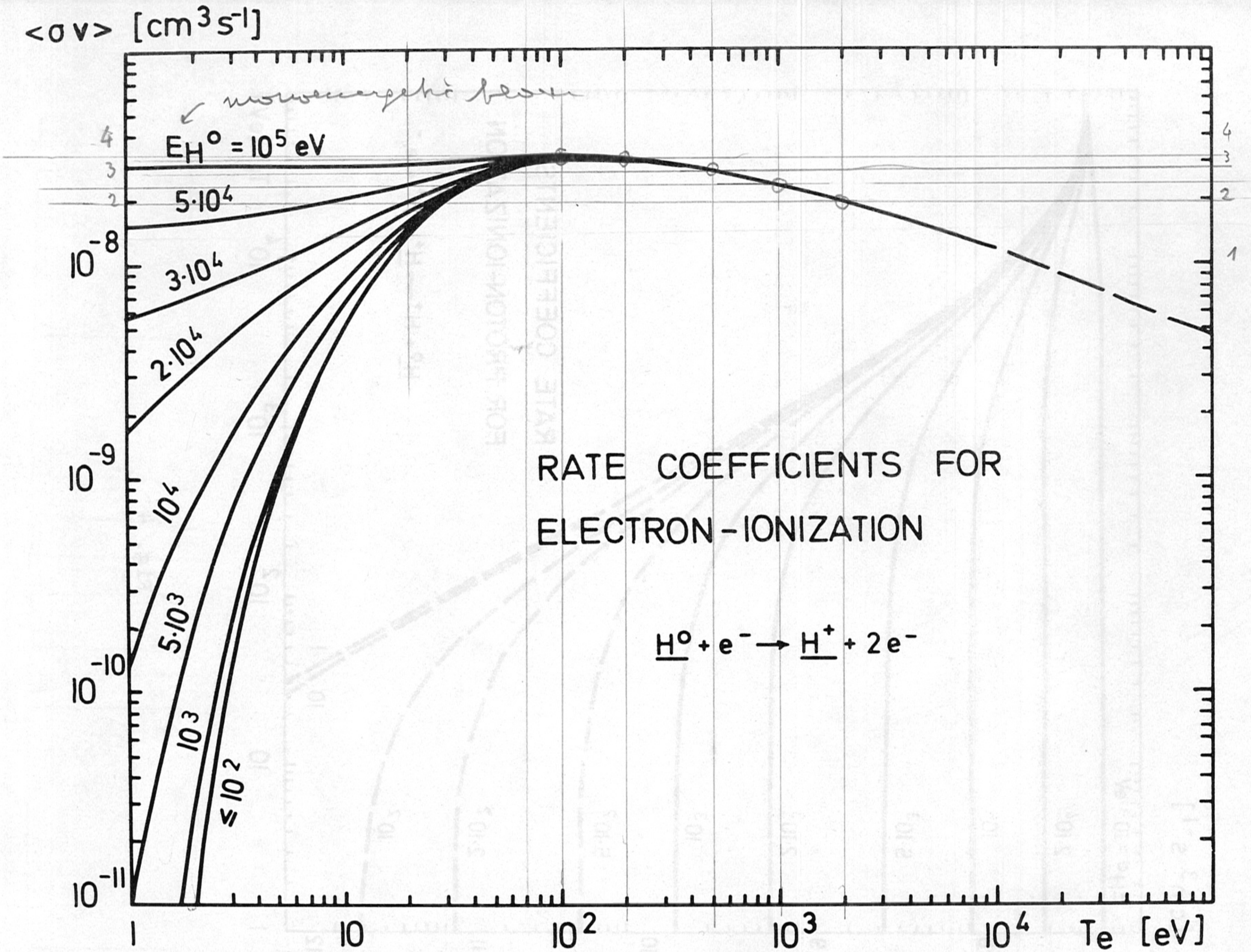

σ [cm²]
CROSS SECTION FOR CHARGE EXCHANGE
H° + H⁺ → H⁺ + H°
BELYAEV ET AL
FITE ET AL
Mc CLURE
ELECTRON-IONIZATION
H° + e⁻ → H⁺ + 2e⁻
FITE ET AL
BOKSENBERG
LOTZ
PROTON IONIZATION
H° + H⁺ → H⁺ + H⁺ + e⁻
FITE ET AL
GILBODY ET AL
10^-15
10^-16
10^-17
10^-18
1
10
10^2
10^3
10^4
E_lab [eV]

Fig. 1



<σ v> [cm³s⁻¹]
10⁻⁷
10⁻⁸
10⁻⁹
E_H° = 10⁴ eV
10³
10²
10
1
3·10⁴
5·10⁴
10⁵
RATE COEFFICIENTS FOR
CHARGE EXCHANGE
H° + H⁺ ⟶ H⁺ + H°
1
10
10²
10³
10⁴
Tᵢ [eV]

Fig. 2



<σv> [cm³s⁻¹]
RATE COEFFICIENTS FOR
ELECTRON-IONIZATION
H° + e⁻ → H⁺ + 2e⁻
E_H° = 10⁵ eV
5·10⁴
3·10⁴
2·10⁴
10⁴
5·10³
10³
≤ 10²
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
1
10
10²
10³
10⁴
Te [eV]

Fig. 3

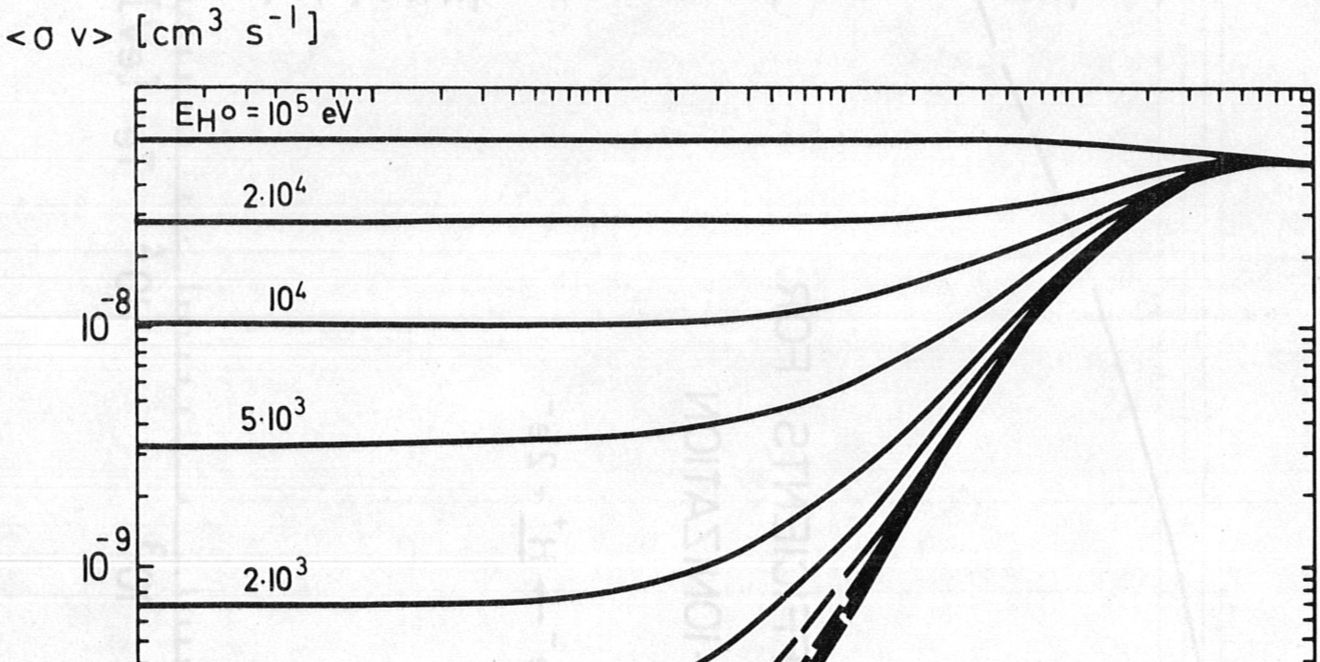

<σ v> [cm3 s-1]
EH° = 10⁵ eV
2·10⁴
10⁴
5·10³
2·10³
10³
5·10²
2·10²
10²
10
1
10-8
10-9
10-10
10-11
10-12
1
10
10²
10³
10⁴
Ti [eV]
RATE COEFFICIENTS
FOR PROTON-IONIZATION
H° + H⁺ → H⁺ + H⁺ + e⁻

Fig. 4

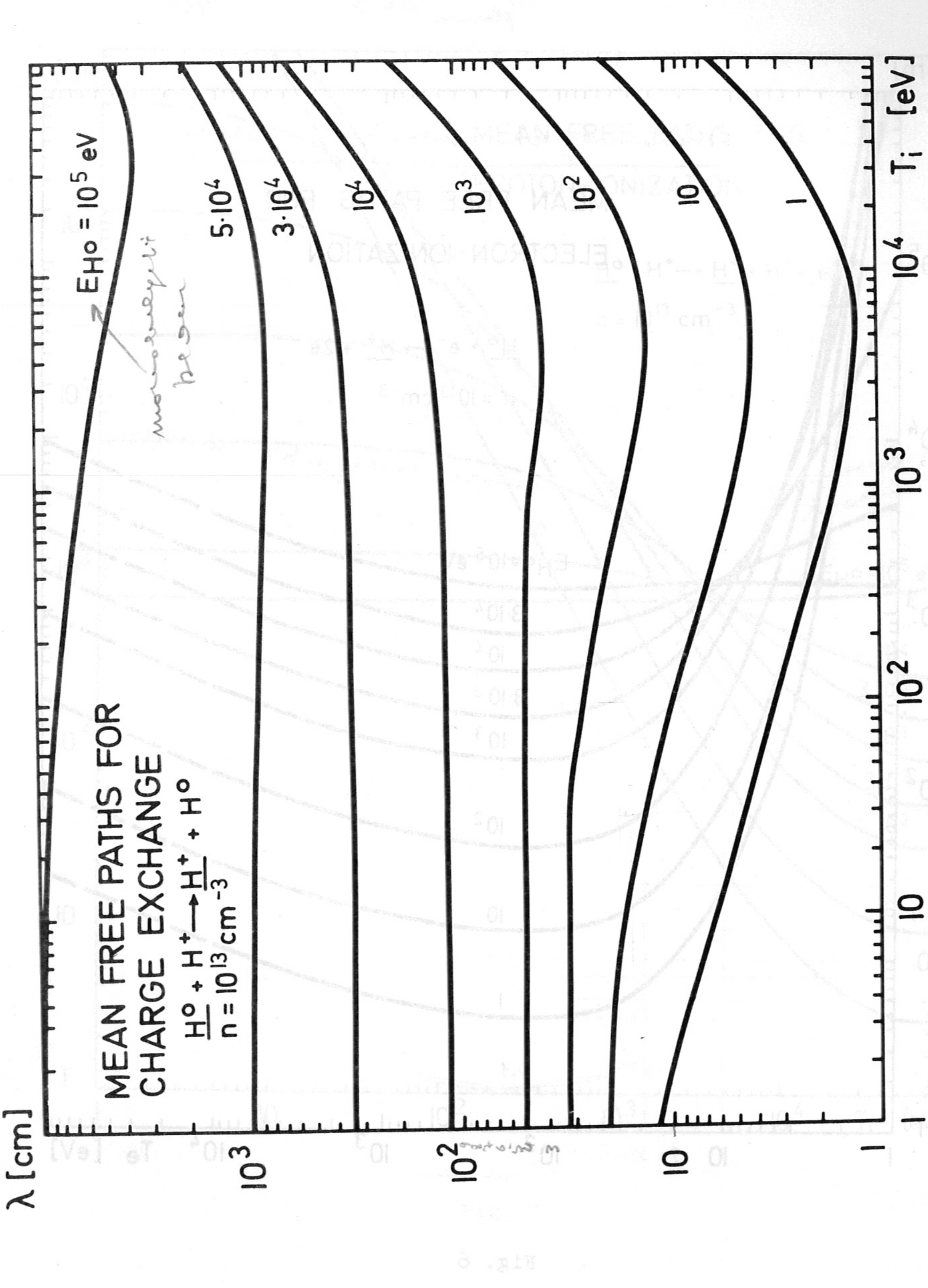

MEAN FREE PATHS FOR
CHARGE EXCHANGE
H° + H⁺ → H⁺ + H°
n = 10^13 cm^-3
λ [cm]
T_i [eV]
E_H° = 10^5 eV
5·10^4
3·10^4
10^4
10^3
10^2
10
1

Fig. 5

$$\lambda = \frac{v_{beam}}{n_{plasma} \langle \sigma v \rangle} \propto \frac{1}{n_{plasma}} \quad \text{(chap. 6)}$$

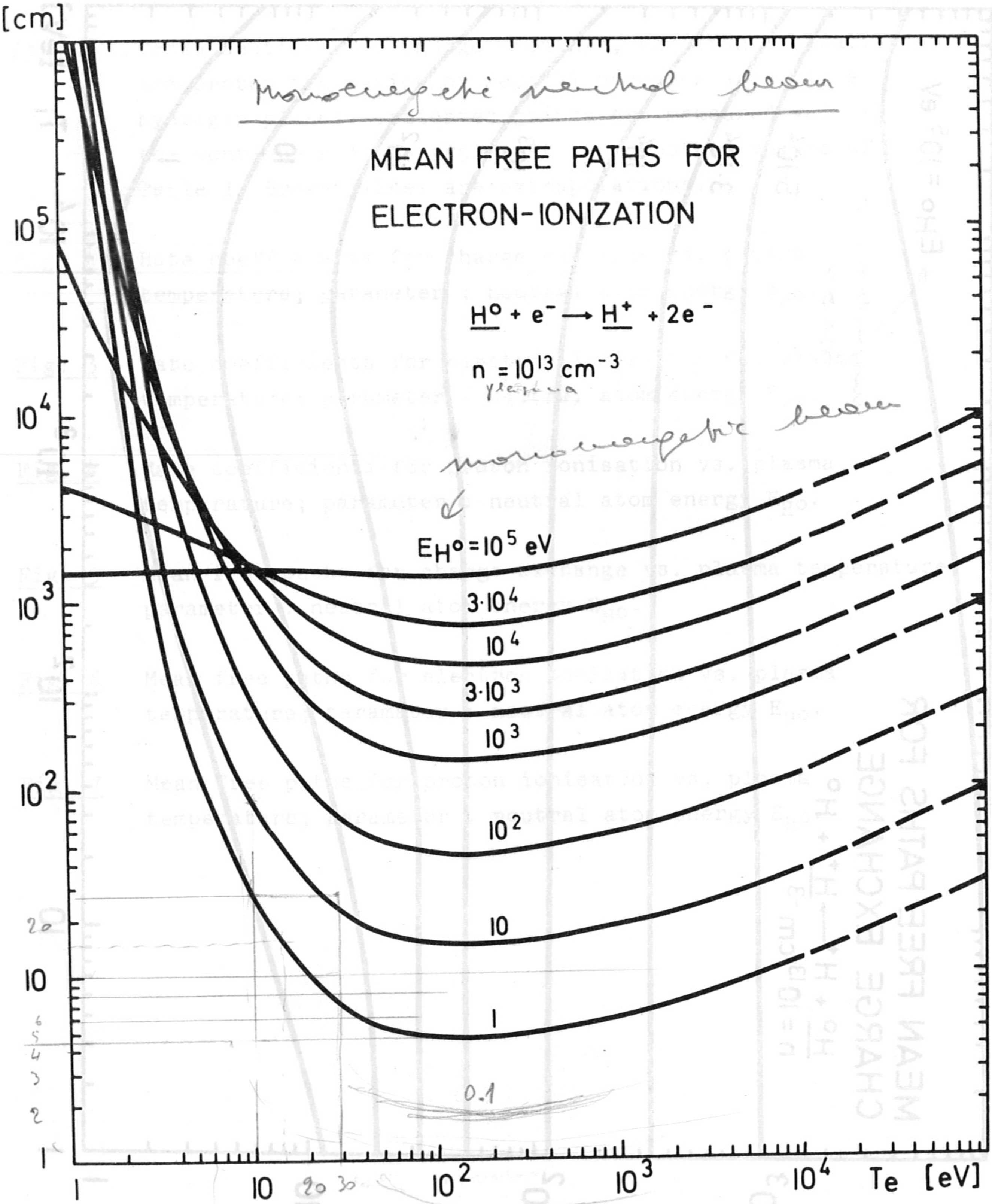


Fig. 6

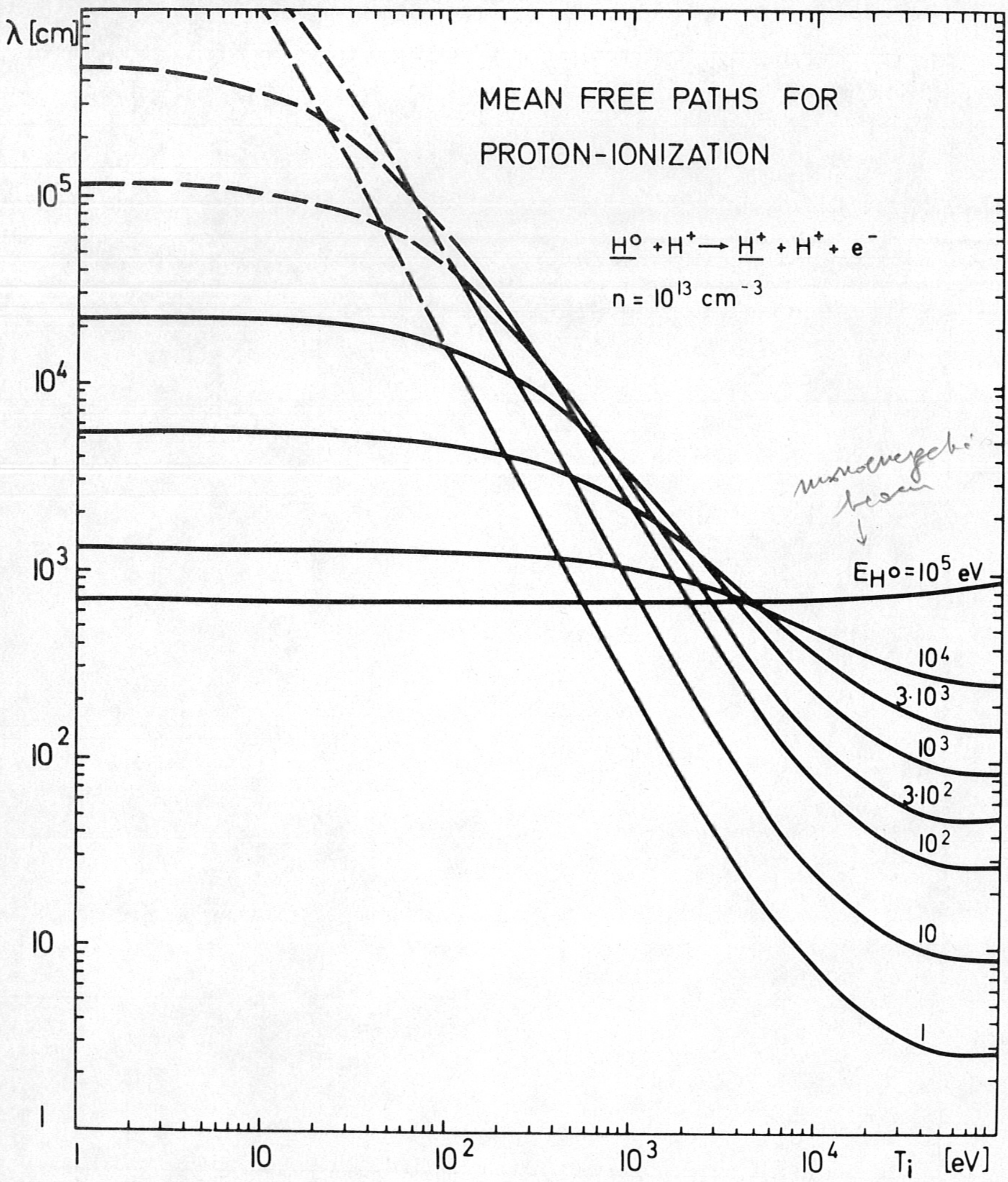

MEAN FREE PATHS FOR
PROTON-IONIZATION
$\underline{H^o} + H^+ \rightarrow \underline{H^+} + H^+ + e^-$
$n = 10^{13}\ cm^{-3}$
λ [cm]
10^5
10^4
10^3
10^2
10
1
1
10
10^2
10^3
10^4
T_i [eV]
$E_{H^o} = 10^5$ eV
10^4
$3 \cdot 10^3$
10^3
$3 \cdot 10^2$
10^2
10
1

Fig. 7