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Some Aspects of Plasma Diagnostics
by Means of a Diamagnetic Coil

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The response of a coil to diamagnetic pulses of exponential shape is calculated. The circuit considered consists of a coil which is terminated in a capacitive and resistive load and at the same time inductively coupled to a shield. Different geometries for compensation are discussed as well as the conditions which lead to integration of the original pulse.

C o n t e n t s

I. FUNDAMENTALS

Microscopic picture (square profile)

During formation inside a magnetic field B_z the plasma particles acquire magnetic momenta

$$\vec{\mu} = \frac{1}{c} \vec{F}_{orbit} \cdot I_{orbit} = \frac{1}{c} \pi \frac{v^2}{\omega^2} q \frac{\omega}{2\pi} = \frac{Mv^2}{2} \frac{1}{B_z} \qquad (1)$$

$$(\omega = q B_z / Mc)$$

which add up to a magnetic moment

$$\vec{M} = n \, \vec{\mu} \, A \qquad (n \text{ part. per cm}^3) \qquad (2)$$

per unit length of the column of cross section A. Supposing cylindrical geometry the elementary currents cancel within the homogeneous column and leave an azimuthal surface current of linear density j_s given by

$$\frac{1}{c} j_s = n \mu = - \frac{n k T_\perp}{B_z} = - \frac{p_\perp}{B_z} \qquad (3)$$

(j_s/c is also called the "magnetization"). It is this surface current which changes the original distribution of the B_z field.

Macroscopic picture (arbitrary profile)

Starting from the radial pressure balance

$$dp_\perp / dr = (j_\varphi / c) \, B_z \quad , \quad p_\perp (R) = 0 \qquad (4)$$

we integrate

$$p_\perp (r) = - \frac{B_z}{c} \int_r^{r_c} j_\varphi (r') \, dr'$$

(assuming only small variations of B_z) and calculate an average pressure

$$\langle p_\perp \rangle = \frac{1}{\pi r_c^2} \int_0^{r_c} p_\perp (r) \, 2\pi r \, dr = - \frac{B_z / c}{\pi r_c^2} \int_0^{r_c} 2\pi r \, dr \int_r^{r_c} j_\varphi (r') \, dr'$$

$$= - \frac{B_z/c}{\pi r_c^2} \int_0^{r_c} \pi r^2 j_\varphi(r)\, dr = - (B_z/c) \underbrace{\int_0^{r_c} \frac{A(r)}{\pi r_c^2}\, j_\varphi(r)\, dr}_{= j_{se}} \quad (5)$$

where j_{se} is now the effective surface current density circulating at R as a consequence of the average pressure $\langle p_\perp \rangle$ which has been produced inside the column during the finite period of creation and heating.

II. GEOMETRY

We have now to choose a geometry which truly allows to measure $\langle p_\perp \rangle$ by means of the effect of j_{se} on the redistribution of the magnetic flux.

The sources for this redistribution are all time varying currents existing inside and outside the plasma. (In Fig. 1 these currents are labelled j_p and j_w respectively.) They may be created by induction (as in the container wall), by a deliberately imposed variation of the external field or be just uncontrolled "noise". During plasma formation the magnetic flux through the coil is changed by an amount

$$\Phi_1 = n_1 \pi r_c^2 \left(H_{ex} + \frac{4\pi}{c} j_{se} \right); \quad H_{ex} = H_w + \tilde{H}_o \quad (6)$$

where $\tilde{H}_o$ is the time varying part of H_o. Therefore, in order to determine j_{se} it will generally not suffice to measure ϕ_1 alone. In particular whenever the details of the time structure of j_p

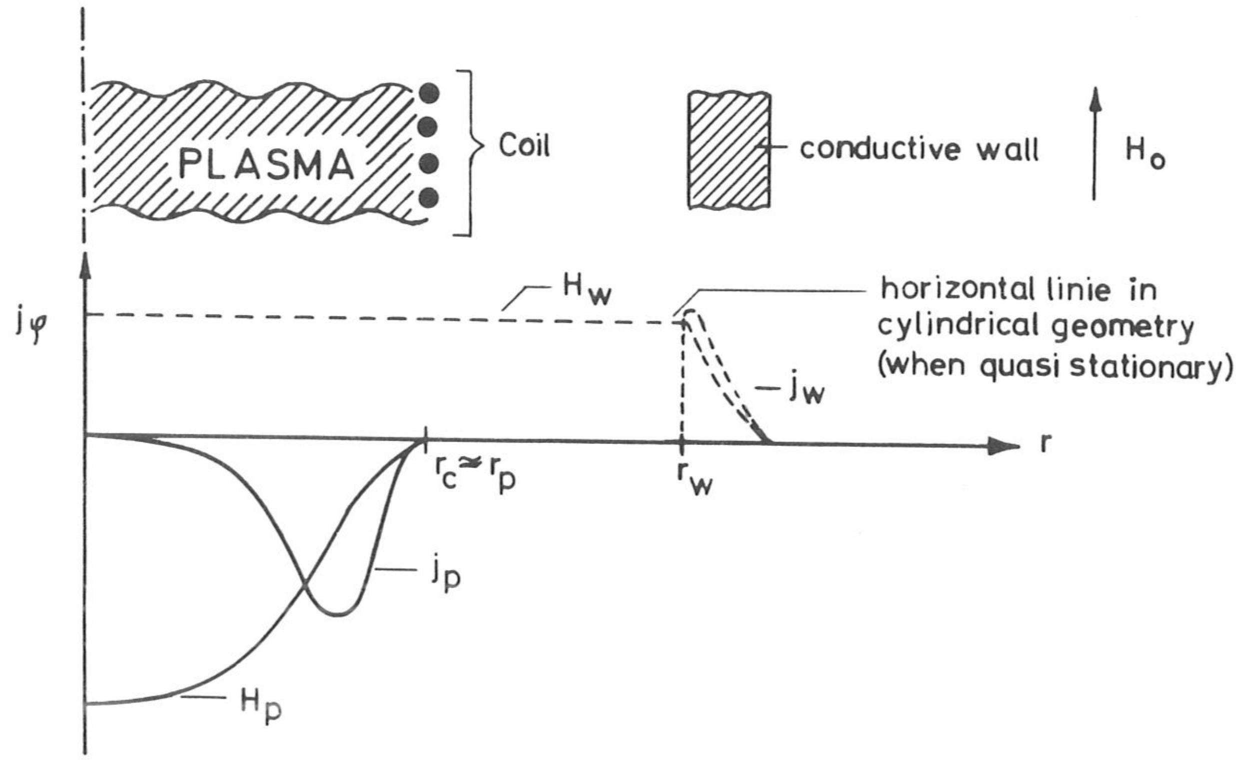


Fig. 1

are of interest, these details could be masked by a completely different time structure of H_{ex}. To solve this difficulty a second coil (n_2A_2 area turns) can be installed outside the plasma (Fig.2).

The anxilary coils accept the flux

$$\Phi_2 = \begin{cases} -n_2 A_2 H_{ex} & \text{in cases a) and b)} \quad (7a) \\ n_2 \left(A_2 H_{ex} + A_1 \frac{4\pi}{c} j_{se}\right) & \text{in cases b) and d)} \quad (7b) \end{cases}$$

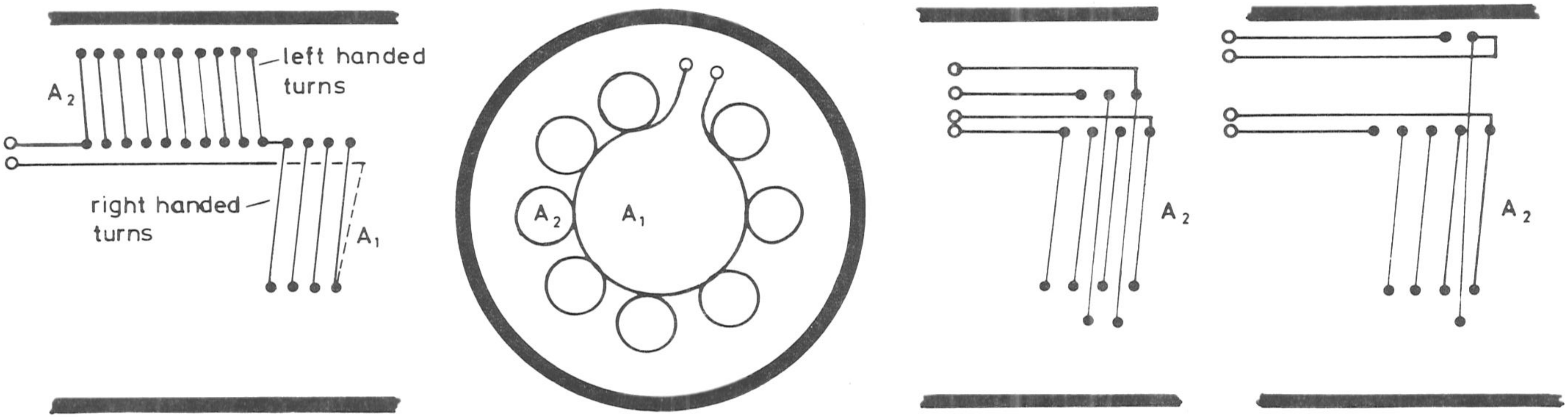
For $n_2A_2 = n_1A_1$ elimination of H_{ex} yields

$$\Phi_1 + \Phi_2 = A_1 n_1 \frac{4\pi}{c} j_{se} \qquad \text{in cases a) and b)} \qquad (8a)$$

$$\Phi_1 - \Phi_2 = A_1 (n_1 - n_2) \frac{4\pi}{c} j_{se} \qquad \text{in cases c) and d)} \qquad (8b)$$

The last two equations are summarized in writing

$$\Phi = A n \frac{4\pi}{c} j_{se} \, . \qquad (9)$$



a) b) c) d)

For a number of reasons the straightforward arrangement of Fig. 2a does not seem to be the best solution especially when a high degree of time resolution is required. We list some of the possible objections

- the compensation coil, due to its bulkiness and its asymmetric position can cause undesirable distrotions of an external E field.
- its distributed capacitance is relatively large which reduces the resonant frequency of the circuit (see "circuit analysis").
- $m \geqslant 1$ modes and irregular oscillations of the plasma column can cause undesired pick-up.
- it is impossible to balance the magnetic dipole moment of the system. The result is larger coupling with the container wall than necessary.
- asymmetric noise and hum of more than one frequency for which the spatial distribution depends on frequency cannot be compensated even if the outputs of the two coils are extracted and weighted separately.

The coil of Fig.2b /5/ offers a high degree of symmetry and good circuit properties. The leads of the coils of Figs.2c,2d are connected externally. The conductors can be arranged to lie closely on the potential surfaces of a radial E field outside the plasma. The leads could be extracted from the vacuum chamber within a metallic diaphragm (a limiter for example). The arrangement of Fig.2d is inadequate if large frequency dependent $m = o$ radial variations of H_{ex} have to be expected.

The reduction in resistivity for cases c) and d) is not a serious one if $n_1 \gtrsim 2\ n_2$.

All the arrangements of Fig.2 can be equipped with electrostatic shields in which induction currents are suppressed either by high resistivity or by slots.

III. CIRCUIT ANALYSIS

In the following a circuit analysis is made on the basis of lumped elements (Fig.3). U. is the voltage at the input of oscilloscope amplifier. L_{11} represents the self of the coil combinations of Fig.2. ϕ and n are explained in Eq.(9). L_{33} can represent alternatively the self of an electrostatic shield without gap or of the metallic vacuum chamber.

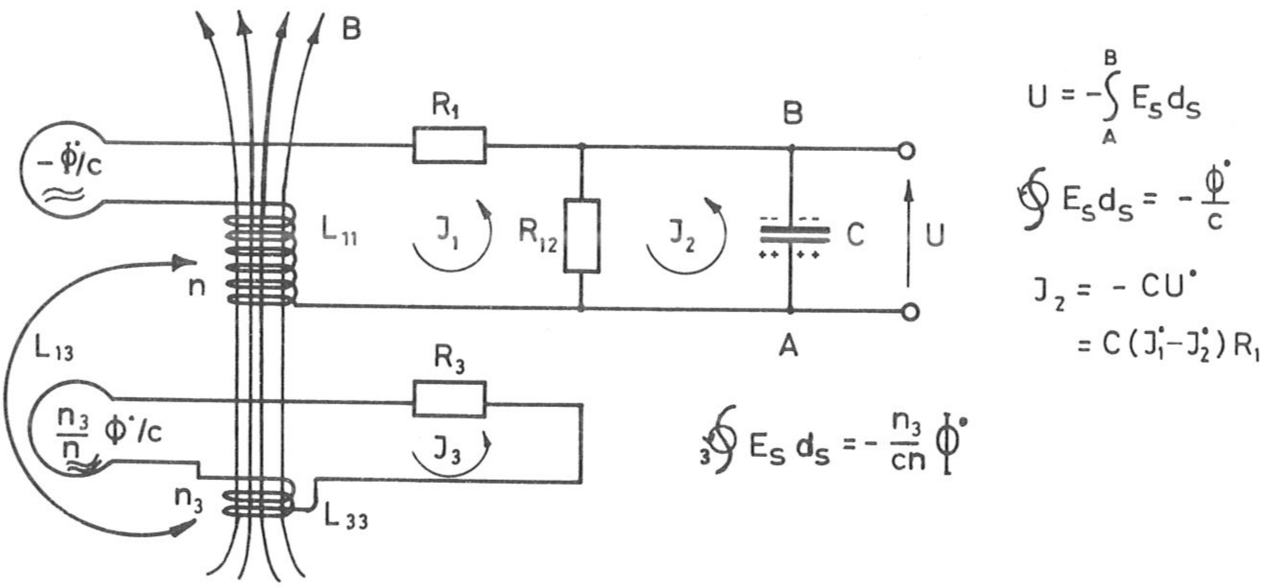


Fig. 3

The circuit behaviour is described by the following set of equations ($R_s = R_{12} + R_1$)

$$L_{11}\dot J_1 + R_s J_1 - R_{12} J_2 + L_{13}\dot J_3 = -\frac{\dot\Phi}{c} = q_1$$

$$-CR_{12}\dot J_1 + CR_{12}\dot J_2 + J_2 = 0 \qquad (10)$$

$$L_{31}\dot J_1 + L_{33}\dot J_3 + R_3 J_3 = -\frac{n_3}{n}\dot\Phi = q_3$$

For solution we start from the following "Ansatz":

$$J_m(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega t}\tilde J_m(\omega)\,d\omega \quad (11a), \qquad \tilde J_m(\omega) = \int_0^{\infty} e^{i\omega t} J_m(t)\,dt \quad (11b)$$

$m = 1, 2, 3$

$$\Phi(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega t}\tilde{\Phi}(\omega)\,d\omega \quad (12a) \; ; \quad \tilde{\Phi}(\omega) = \int_{0}^{\infty} e^{i\omega t}\Phi(t)\,dt \tag{12b}$$

Solving for $\tilde{J}_m$ we obtain

$$\tilde{J}_m = (-1)^{k+m} q_k \frac{|B_{km}|}{|b_{rs}|} \tag{13}$$

($|b_{rs}|$ = subdeterminant of $|B_{rs}|$). The elements of $|b_{rs}|$ are

$$\begin{array}{lll} b_{11} = -i\omega L_{11} + R & b_{12} = -R_{12} & b_{13} = -i\omega L_{13} \\ b_{21} = i\omega C R_{12} & b_{22} = -i\omega C R_{12} + 1 & b_{23} = 0 \\ b_{31} = -i\omega L_{31} & b_{32} = 0 & b_{33} = -i\omega L_{33} + R_3 \end{array} \tag{14}$$

For the differential current $\tilde{J}_1 - \tilde{J}_2$ we obtain

$$\tilde{J}_1 - \tilde{J}_2 = \frac{i\omega\tilde{\Phi}}{c}\,\frac{b_{33} - n_3 b_{13}/n}{b_{33}(b_{22}b_{11} - b_{12}b_{21}) - b_{13}^2 b_{22}} \tag{15}$$

which yields after evaluation

$$\tilde{U} = -R_{12}(\tilde{J}_1 - \tilde{J}_2) = -i\omega\frac{R_{12}}{R_s}\frac{\tilde{\Phi}}{c}f(\omega)$$

$$f(\omega) = -\frac{(-i\omega + \gamma_3 + i\omega k_{13})\,\omega_R^2}{(-i\omega + \gamma_3)\left\{\left(1 - k^2\frac{-i\omega + \gamma_2}{-i\omega + \gamma_3}\right)\omega^2 + i\omega 2\bar{\gamma} - \omega_R^2\right\}} \tag{16}$$

where we introduced the notations

$$\begin{array}{l} n_3 L_{13}/n L_{33} = k_{13} \; ; \quad n L_{31}/n_3 L_{11} = k_{31} \\ L_{13}^2/L_{11}L_{33} = k_{13}k_{31} = k^2 \\ R_1/L_{11} = \gamma_1 \; ; \quad 1/R_{12}C = \gamma_2 \; , \quad R_3/L_{33} = \gamma_3 \\ R_s/R_{12}CL_{11} = \omega_R^2 \; ; \quad \gamma_1 + \gamma_2 = 2\bar{\gamma} \end{array} \tag{17}$$

We consider the following limits:

α) "open secondary circuit" ($\gamma_3 \to \infty$ or no coupling)

$$f(\omega) = -\frac{\omega_R^2}{\omega^2 - 2i\bar{\gamma}\omega - \omega_R^2} \tag{18}$$

β) "superconducting secondary circuit" ($\gamma_3 \to 0$)

$$f(\omega) = -\frac{1 - k_{13}}{\omega^2 + i\left(\frac{\gamma_1}{1-k^2} + \gamma_2\right)\omega - \frac{\omega_R^2}{1-k^2}} \; \frac{\omega_R^2}{1-k^2} \tag{19}$$

(the resonant frequency has been changed from ω_R to $\omega_R/\sqrt{1-k^2}$)

γ) "weak coupling" (neglect of the k^2 term in the denominator of Eq.(16))

$$f(\omega) = -\frac{(-i\omega + \gamma_3 + i\omega k_{13})\,\omega_R^2}{(-i\omega + \gamma_3)(\omega^2 + 2i\omega\bar{\gamma} - \omega_R^2)} \tag{20}$$

δ) "extreme coupling" ($k_{13} \approx k \approx 1$)

$$f(\omega) = \frac{\omega_R^2/(1 + \gamma_1/\gamma_3)}{\omega^2 + 2i\omega\bar{\gamma}\,\frac{\gamma_3 + \omega_R^2/2\bar{\gamma}}{\gamma_1 + \gamma_3} - \omega_R^2/(1+\gamma_1/\gamma_3)} \tag{21}$$

(the resonant frequency has been changed from ω_R to $\omega_R/\sqrt{1+\gamma_1/\gamma_3}$).

Back transformation.
Before applying the back transformation (11b) we have to specify the excitation function $\Phi(t)$. We choose

$$\Phi(t) = \begin{cases} 0 & \text{for } t<0 \\ \Phi_0 e^{-\gamma_0 t} & \text{"} \; t>0 \end{cases} \quad \text{with} \quad \frac{\dot{\Phi}(t)}{\Phi_0} = \begin{cases} 0 & \text{for } t<0 \\ -\gamma_0 e^{-\gamma_0 t} & \text{"} \; t>0 \end{cases} \tag{22}$$

i.e. a sudden rise followed by an exponential decay. (A sum of two functions of this type allows to describe with good approximation a large variety of practical situations.) The Fourier transforms of (22) are simply

$$\tilde{\Phi} = \frac{\Phi_0}{-i\omega + \gamma_0} \; ; \quad \frac{\tilde{\dot{\Phi}}}{\Phi_0} = \frac{\omega}{\omega - i\gamma_0} \tag{23}$$

For U(t) we now obtain

$$U = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega t}\,\tilde{U}\,d\omega = \frac{1}{2\pi}\frac{R_{12}}{R_s}\frac{1}{c}\int_{-\infty}^{\infty} e^{-i\omega t} f\,\tilde{\Phi}^{\circ}\,d\omega$$

$$= \frac{R_{12}}{R_s}\frac{\Phi_0}{c}\frac{1}{2\pi}\oint_{L.H.P.}$$

where the last symbol indicates integration along a contour around the lower ω- halfplane of the integrand

$$g \equiv g(\omega, t) = \frac{e^{-i\omega t}\,\omega \cdot f}{\omega - \omega_1}\,, \quad \omega_1 = -i\gamma_0 \tag{24}$$

which can be contracted around the poles ω_n with residues G_n according to

$$\frac{R_s}{R_{12}}\frac{c}{\Phi_0}U = \frac{1}{2\pi}\int_{-\infty}^{\infty} g\,d\omega = i^{-1}\sum_n G_n\;;\; G_n = \frac{g}{\omega-\omega_n}\Big|_{\omega\to\omega_n} \tag{25}$$

α) "open secondary circuit"

We have

$$g = \frac{\omega\, e^{-i\omega t}}{(\omega-\omega_1)(\omega-\omega_2)(\omega-\omega_3)}\,\omega_R^2 \tag{26}$$

with

$$\omega_1 = -i\gamma_0\;;\quad \omega_{2,3} = -i\bar{\gamma} \pm \Omega\,,\quad \Omega = \sqrt{\omega_R^2 - \bar{\gamma}^2}$$

and

$$G_1 = \frac{i\gamma_0\, e^{-\gamma_0 t}}{\omega_R^2 + \gamma_0^2 - 2\bar{\gamma}\gamma_0}$$

$$G_2 + G_3 = \frac{i\omega_{12}^2}{\Omega}\, e^{-\bar{\gamma} t}\,\frac{(\omega_R^2 - \bar{\gamma}\gamma_0)\sin\Omega t + \gamma_0\Omega\cos\Omega t}{\omega_R^2 + \gamma_0^2 - 2\bar{\gamma}\gamma_0} \tag{27}$$

Depending on the relative magnitude of $\gamma_0, \omega_R, \bar{\gamma}$ the quantity approaches the following limits

case	$\frac{R_S}{R_{12}} \frac{c}{\Phi_0} U(t)$:
A) $\gamma_0 \ll \omega_R, \frac{\omega_R^2}{\bar{\gamma}}$	$-\gamma_0 e^{-\gamma_0 t} + \frac{\omega_R^2}{\Omega} e^{-\bar{\gamma} t} \sin \Omega t$
B) $\gamma_0 \gg \omega_R, \frac{\omega_R^2}{\bar{\gamma}}$	$-\frac{\omega_R^2}{\gamma_0} e^{-\gamma_0 t} + \frac{\omega_R^2}{\gamma_0} e^{-\bar{\gamma} t} (\cos \Omega t - \frac{\bar{\gamma}}{\Omega} \sin \Omega t)$
C) $\bar{\gamma} \gg \gamma_0, \frac{\omega_R^2}{\gamma_0}$	$\frac{\omega_R^2}{2\bar{\gamma}} e^{-\gamma_0 t} - \frac{\omega_R^2}{2\bar{\gamma}} e^{-\bar{\gamma} t} (\cos \Omega t - \frac{\bar{\gamma}}{\Omega} \sin \Omega t)$
D) $\bar{\gamma} = \omega_R$	$-\frac{\omega_R^2 \gamma_0}{(\omega_R - \gamma_0)^2} e^{-\gamma_0 t} + \left[\frac{\omega_R^2 \gamma_0}{(\omega_R - \gamma_0)^2} + \frac{\omega_R^3 t}{\omega_R - \gamma_0}\right] e^{-\omega_R t}$
E) $\bar{\gamma} = \omega_R = \gamma_0$	$\omega_R^2 t e^{-\omega_R t}$

(28)

The first terms correspond to the forced oscillation, the second terms to the damped "eigen" oscillations of the L-C-circuit. Case D) describes the aperiodic limit. In case E) both types of oscillations have merged to one aperiodic pulse.

The excitation pulse of Eq.(22) - owing to its discontinuity - cannot describe a physical event since the magnetic field needs time for build-up. The addition of two such pulses

$$\Phi = \begin{cases} 0 & \text{for } t < 0 \\ \Phi_0 \left(e^{-\gamma_{01} t} - e^{-\gamma_{02} t} \right) & \text{"} \quad t > 0 \end{cases} \tag{29}$$

however, produces a pulse of finite rise time which is well suited to describe various plasma histories. By superposition we obtain

case :	$\frac{R_s}{R_{12}} \frac{c}{\Phi_0} U(t)$:	
A) $\gamma_{01}, \gamma_{02} \ll \omega_R, \omega_R^2/\bar{\gamma}$	$-\gamma_{01} e^{-\gamma_{01} t} + \gamma_{02} e^{-\gamma_{02} t}$	(3o)
C) $\bar{\gamma} \gg \gamma_{01}, \gamma_{02}, \omega_R^2/\gamma_{01}, \omega_R^2/\gamma_{02}$	$\frac{\omega_R^2}{2\bar{\gamma}} \left(e^{-\gamma_{01} t} - e^{-\gamma_{02} t} \right)$	(31)

We note that the "eigen" oscillations of the L-C have not been excited in these two cases, the second terms becoming independent of γ_0 . This is not true in the general case (taking for instance $\gamma_0 \geqslant \omega_R, \omega_R^2/\bar{\gamma}$).

In case A) the response is proportional to $\dot{\Phi}(t)$. In case C) it is proportional to $\Phi(t)$ itself. The factor

$$\frac{\omega_R^2}{2\bar{\gamma}} = \frac{\omega_R^2}{2\gamma_{0i}} \frac{\gamma_{0i}}{\bar{\gamma}} \ll \frac{\gamma_{0i}}{2} \qquad (32)$$

indicates a reduction of sensitivity which seems to be a dis-advantage at the first sight. The "ringing" terms, however, of Eq.(28) never cancel exactly and their relative magnitude is much higher in case A) than in case C).

β) superconducting secondary circuit
This case is similar to case α).

γ) weak coupling
Starting from Eq.(2o) we note that g as given by Eq.(26) has now to be corrected by an amount

$$\Delta g = k_{13} \frac{\omega^2 e^{-i\omega t}}{(\omega-\omega_1)(\omega-\omega_2)(\omega-\omega_3)(\omega-\omega_4)} \qquad (33)$$

in which the four poles ω_n play a completely symmetric role. The same correction would therefore result from an increment

$$\Delta(\tilde{\Phi}^{\circ}/\Phi_0) = k_{13} \frac{\omega^2}{(\omega-\omega_1)(\omega-\omega_4)} \tag{34}$$

to the Fourier amplitude of the excitation $\tilde{\Phi}^{\circ}/\Phi_0$ as given by Eq.(23). For this reason the back transformation of the r.h. side of Eq.(34) yields the correction which has to be applied to Eq.(22) to account for the secondary circuit. This correction amounts to

$$\Delta(\Phi^{\circ}/\Phi_0) = k_{13} \frac{\gamma_0^2 e^{-\gamma_0 t} - \gamma_3^2 e^{-\gamma_3 t}}{\gamma_0 - \gamma_3} \tag{35}$$

It vanishes for $\gamma_3 \to \infty$ and becomes $k_{13}\gamma_0 e^{-\gamma_0 t}$ for $\gamma_3 = 0$ in accordance with Eq.(2o). In this case the response is simply reduced by the factor $1-k_{13}$, its shape, however, is conserved. (This also agrees with Eq.(19) for $k^2 \ll 1$.)

This last result can have practical consequences in the construction of diamagnetic coils. We must, however, be aware of the fact that this property is lost as soon as the coupling becomes appreciable (see below).

Changes in shape occur when γ_3 approaches γ_0. We then obtain

$$\Delta(-\Phi^{\circ}/\Phi_0) = -k_{13}\gamma_0(2-\gamma_0 t)\, e^{-\gamma_0 t}, \qquad \gamma_3 = \gamma_0 \tag{36}$$

δ) Extreme coupling

Again this case is similar to case α). For $\gamma_3 \ll \gamma_1$ (which usually applies for a coil wrapped on a metallic tube) we can write Eq.(21) in the form

$$f(\omega) = \frac{\omega_R'^2}{\omega^2 + 2i\omega\bar{\gamma}' - \omega_R'^2} \tag{37}$$

with

$$\omega_R'^2 = \gamma_3 \omega_R^2/\gamma_1, \qquad 2\bar{\gamma}' = \omega_R^2/\gamma_1$$

Depending on whether the ordering is

$$\omega_R' \gg \gamma_3 \gg \gamma_0 \qquad \text{or} \qquad \omega_R' \gg \gamma_0 \gg \gamma_3$$

we have either realized case A) or case C) of Eq.(28) yielding signals either proportional to $\ddot{\phi}$ or to ϕ itself. For a plasma history with a relatively sharp rise and a decay in the milli-second range we have usually a mixture of both cases. In addition, the skin effect which has been neglected throughout this analysis will cause γ_3 to become dependent on ω. Nevertheless, under favourable conditions, it may be possible to obtain a response which is proportional to the integrated signal from a coil which is wrapped on the outside of a metallic vacuum container.

IV. INTERPRETATION

It is easy to interprete the limiting cases A) and C) of Eqs.(3o) and (31) in physical terms. In A) the leakage impedance parallel to $\vec{U}$ is kept high enough to ensure that U essentially represents the voltage induced in the coil. In C) the time integral is formed by some storage mechanism.

Two extreme alternatives are included in the latter case which are respectively electrostatic and magnetic in nature. In the former a capacitor is charged by a current which is made proportional to $\dot{\phi}$ by choosing R_1 sufficiently large and leakage is restricted by making R_{12} as large as possible. In the magnetic alternative the capacitor becomes redundant since (apart from a backlash due to losses) the induced current in the short circuited coil is already proportional to ϕ . The current is measured from its voltage drop along the resistor R_{12}. It is now necessary to keep R_1 and R_{12} as small as possible in order to restrict their influence on the current in the coil.

It may be illustrative to quote the mechanical analogues of the above situations. Case A) corresponds to a mass on a spring, the resonant frequency of this combination being well above the frequency of the exciting force exerted directly on the mass. The elongation and tension of the spring will then be proportional to the force. For the electrostatic version of case C) the mass is surrounded by a liquid of high viscosity which makes its velocity (almost) proportional to the exciting force. The elongation of the spring will now be proportional to the time integral of the force. In the magnetic version of this case the mass is heavy, the viscosity is low and the spring is extremely soft, just to keep the mass floating. The integral of the exciting force now equals the momentum acquired by the mass and is proportional to its velocity. The (slight) viscous friction must be used to measure the latter.

V. SENSITIVITY

Disregarding coupling effects we only consider the limits given respectively by Eqs.(3o) and (31).

A) $\quad \gamma_0 \ll \omega_R, \omega_R^2/\bar{\gamma} \; : \qquad U(t) = \frac{R_{12}}{R_s} \frac{1}{c} \frac{d\Phi}{dt}$

C) $\quad \bar{\gamma} \gg \gamma_0, \omega_R^2/\gamma_0 \; : \qquad U(t) = -\frac{R_{12}}{R_s} \frac{\omega_R^2}{2\bar{\gamma}} \frac{1}{c} \left[\Phi(t) - \Phi(t=0)\right]$

From Eqs.(5) and (9) we find for the voltage as delivered by the coil

$$U(t) = \frac{R_{12}}{R_s} \cdot n \cdot A \cdot \frac{4\pi}{c} \begin{cases} -d\langle p_\perp \rangle / dt & \text{for } \gamma_0 \ll \omega_R, \omega_R^2/\gamma_0 \\ (\omega_R^2/2\bar{\gamma}) \langle p_\perp \rangle & \text{"} \; \bar{\gamma} \gg \gamma_0, \omega_R^2/\gamma_0 \end{cases} \tag{38}$$

Numerically, for a plasma column $(R, B_z, \langle p_\perp \rangle)$

$$\frac{U(t)}{\mu\,\text{volts}} = 2\,\frac{R_{12}}{R_s}\,\frac{nA}{\text{cm}^2}\,\frac{10^4\,\text{Gauss}}{B_z}\begin{cases} -\dfrac{d\langle p_\perp\rangle/dt}{10^{14}\,\text{ev}/\text{msec cm}^3} & \gamma_0 \ll \omega_R,\ \dfrac{\omega_R^2}{\bar{\gamma}} \\ \dfrac{\omega_R^2}{2\bar{\gamma}}\,\dfrac{\langle p_\perp\rangle}{10^{14}\,\text{ev}/\text{msec cm}^3} & \bar{\gamma} \gg \gamma_0,\ \dfrac{\omega_R^2}{\gamma_0} \end{cases} \tag{39}$$

It has been pointed out already that conditions which provide automatic integration will reduce the signal as well as the circuit's disposition for ringing a pick-up of noise.

Let R_{12} represent the input of an amplifier. Keeping R_1 fixed the absorbed power will be

$$P = U\,J = \frac{R_{12}}{(R_1 + R_{12})^2}\left(\frac{n\,d\Phi}{c\,dt}\right)^2 \tag{40}$$

and is largest for $R_{12} = R_1 = R$. The resistance of the coil is given by

$$R_c = 2\pi n^2 \rho\, r_c / \alpha F_c \tag{41}$$

(r_c = coil radius, αF_c = conducting cross section, F_c = total cross section, ρ = resistivity).

For a given cross section F_c

$$P \propto \frac{\alpha}{\rho}\,\frac{F_c}{r_c} \tag{42}$$

which is independent of the number of turns, n. (The skin effect which can cause α to depend on frequency is ignored.) Thus, apart from the purpose of improved matching to the high impedance of an amplifier, there is no motivation for the use of many turn coils.

VI. APPENDIX: SOME FORMULAS

The inductance of a coil the conductive layer of which does not deviate too much from a cylindrical current sheet of radius r and length l is approximately given by /6/

$$\frac{L}{H} = \frac{4 \cdot 10^{-8}}{cm} \frac{r^2}{l + 0.9r} \cdot n^2 \quad ; \quad l \gtrsim r/2 \tag{44}$$

The inductance of a loop (r = loop radius, a = wire radius) is approximately given by

$$\frac{L}{H} = 1.3 \cdot 10^{-8} \frac{r}{cm} \ln \frac{r}{a} \quad ; \quad a \ll r \tag{45}$$

The time constant L/R of a tube of wall thickness δ and radius r, neglecting skin effect, is given by

$$\frac{\tau}{sec} = 3.6 \; 10^{-3} \frac{r\delta}{cm^2} \quad \text{for copper}$$

$$= .8 \cdot 10^{-4} \frac{r\delta}{cm^2} \quad \text{for stainless steel} \tag{46}$$

The coupling of a longer outer coil with a shorter inner coil in coaxial and concentric position is approximately given by /6/

$$\frac{L_{12}}{\sqrt{L_{11} L_{22}}} = k = \frac{r_1^2 l_1}{r_2^2 l_2} = \frac{V_1}{V_2} \tag{47}$$

where index $_1$ applies to the inner coil and V denotes the coil volume.

The skin depth is given by

$$\frac{\delta}{cm} \approx \frac{7}{(f/Hz)^{1/2}} \quad \text{for copper}$$

$$\approx \frac{46}{(f/Hz)^{1/2}} \quad \text{for stainless steel} \tag{48}$$

The skin depth given by Eq.(48) is the e-folding length of a stationary wave impinging on the metallic halfspace.

The propagation time of an electric field pulse through a metal slab of thickness δ is different from τ/f as given by Eq.(47). According to Ref./4/ it is smaller by a factor $(2/\pi)^2\ 1/\pi$ for the lowest (and slowest) mode i.e.

$$\frac{\tau_p}{\mathrm{msec}} = 2.6\left(\frac{\delta}{\mathrm{cm}}\right)^2 \quad \text{for copper}$$

$$\frac{\tau_p}{\mu\mathrm{sec}} \approx 60\left(\frac{\delta}{\mathrm{cm}}\right)^2 \quad \text{for stainless steel} \qquad (49)$$

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