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FINITE TEMPERATURE EFFECTS ON THE LOWER-HYBRID DISPERSION CHARACTERISTICS

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Computations using the full hot-plasma dielectric tensor show that the lower-hybrid wave may be effectively absorbed by electron resistivity even at the thermonuclear temperatures. There is no evidence of coupling to the cyclotron harmonic modes.

Ever since the derivation of the hot-plasma dielectric tensor in an explicit form by Stepanov[1] in 1958, it has been possible, in principal, to compute the dispersion characteristics of linear waves in magnetized, hot, homogeneous plasmas. Such an effort would be rewarding for evaluating the possibility of radio-frequency ignition of thermonuclear plasmas. It is a reflection on the complexity of the undertaking that barring the cases of propagation strictly parallel (Landau waves) and perpendicular (Bernstein modes) to the static magnetic field B_o, little accurate information of the wave behavior is available.

In this letter we study (with an accuracy exceeding one part in a million) the finite temperature effects on the obliquely propagating shear (slow) and compressional (fast) cold-plasma modes using the complete hot-plasma dielectric tensor. The plasma density is allowed to have a gradient perpendicular to B_o. The scale of the gradient is left unspecified such that the "local" dielectric tensor approximation is valid. Anticipating the boundary conditions to be used in future computations, we shall employ the Derfler-Omura[2] dielectric tensor described in Eqs.(5 - 2o) of Ref.3. Since in the derivation of the Derfler-Omura tensor, the total plasma current is subdivided into the polarization and the magnetization currents, the tangential magnetic field at a boundary is continuous. In every other respect, this tensor is identical to the Stepanov tensor.

Electron resistivity contribution $\Delta\varepsilon$ to the dielectric tensor ε is incorporated using the method described in Ref. 4,

$$\Delta\varepsilon = i(\omega\, \nu_{ei}/\omega_{pe}^{2})\ (\varepsilon - 1) \cdot (\varepsilon - 1)$$

where ν_{ei} is the electron-ion momentum transfer collision frequency according to Spitzer[5] (the values of ν_{ei} quoted in this paper are for the average density of $5 \times 10^{13}\ cm^{-3}$). This method of obtaining $\Delta\varepsilon$ is admittedly crude. However, the diepersion curves are insensitive to the precise value of ν_{ei} and the error introduced will not alter the results in a significant manner.

Throughout this paper it is assumed that the ions (deuterium) and the electrons posses isotropic, maxwellian velocity distributions of equal temperatures. The static magnetic field B_o and the gradient of density are along the z and x-directions respectively. All field quantities are assumed to possess space and time dependence $\exp i\ (k_x x + k_z z - \omega t)$ with no variation in the y-direction[6]. The dispersion equation then takes the form

$$a\, n_x^{4} + b\, n_x^{2} + c = o \qquad (1)$$

where,

$$a = \varepsilon_{xx} / \mu_{xx}$$

$$b = n_z^2 (\varepsilon_{xx} \mu_{zz} + \varepsilon_{zz} \mu_{xx})$$
$$- \varepsilon_{xx} \varepsilon_{zz} (\mu_{xx}^2 + \mu_{xy}^2) - \mu_{xx} \mu_{zz} (\varepsilon_{xx}^2 + \varepsilon_{xy}^2)$$

$$c = \varepsilon_{zz} \mu_{zz} \left[n_z^4 + 2 n_z^2 (\varepsilon_{xy} \mu_{xy} - \varepsilon_{xx} \mu_{xx}) \right.$$
$$\left. + (\varepsilon_{xx}^2 + \varepsilon_{xy}^2)(\mu_{xx}^2 + \mu_{xy}^2) \right]$$

$n_x = c\, k_x/\omega$, $n_z = c\, k_z/\omega$ while ε and μ are defined in Ref. 3.

We shall restrict our attention to the roots of Eq.(1) identified as the shear (slow) and the compressional (fast) modes in the cold-plasma limit. For finite values of n_z, these cold-plasma modes exhibit propagation on the low-density side of the lower-hybrid resonance except near the plasma edge where evanescent or complex waves might exist. On the high density side of the resonance, the fast mode continues to propagate while the slow mode becomes evanescent. Introduction of electron resistivity modifies this behaviour somewhat and the resultant cold-plasma characteristics resemble the curves of Fig.1c which are in fact obtained by solving Eq.(1) numerically for the case $T_e = T_i = 10$ eV.

Observe that the slow mode is a backward wave (recognized by the opposite signs of the real and imaginary parts of k_x) which transforms into a forward wave upon passing through the cutoff on the high-density side of the resonance. For temperatures below lo eV, the collisional effects dominate because the wavelength is much too long($|k_x r_{ci}| \ll 1$) for the Larmor radius to play any prominent role. Marked changes, however, are seen to occur at higher temperatures ($T \gtrsim$ loo eV) when the Larmor radius becomes comparable with the wavelength. As might be expected the resonance is flattened and broadened occuring somewhat in advance of the hybrid density. Simultaneously an increased reluctance on the part of the slow wave to change its backward character is observed and beyond $T \sim$ loo eV, the cutoff disappears altogether resulting in a significantly altered dispersion characteristic of Fig.le ($T = 1$ keV). This trend continues up to the thermonuclear ignition temperature of lo keV (Fig.1f) and near the resonance, the hot-plasma effects have altered the appearance of the curves beyond any recognition based on the cold-plasma theory.

Before considering the effects of magnetic field gradients we make two observations pertinent to the problem of plasma heating at the lower-hybrid resonance. Since $|k_x r_{ci}| \ll 1$ near the plasma edge irrespective of the temperature, it is justified to treat the problem of wave accessibility within the framework of the cold-plasma theory [7]. Secondly, even at thermonuclear temperatures $Im(k_x^s)$ is large enough to allow complete absorption of the wave energy within a few Larmor radii.

In an actual thermonuclear device like a torus, the wave of frequency ω is obliged to traverse through harmonics of the ion-cyclotron frequency where a radical departure in the wave behavior can not be apriori ruled out. We have simulated this effect by a fictitious d-c electric current in the y-direction such that B_o decreases [8] linearly with increasing density from the value B_o = 15o kG at the plasma edge to B_o = 1oo kG at the hybrid layer. The resultant dispersion curves (Fig. 2) for both the slow and the fast modes are scarcely affected by the presence of cyclotron harmonics.

The above findings may be summarized as follows:

1. The shear cold-plasma mode despite significant modifications continues to exhibit effective collisional absorption of the wave energy even at thermonuclear temperatures. There is no evidence of the wave turning back towards the plasma edge. The fact that these results conflict with the existing beliefs [9] is entirely plausible because hitherto all the results have been based on finite Larmor radius expansion schemes with dubious prospects of uniform convergence.

2. Neither the shear nor the compressional waves display any tendency to either couple to the cyclotron-harmonic waves or to suffer collisionless Landau damping at the harmonics themselves. This, once again, is in variance with the present theories [9, 1o] based on incomplete models.

Whatever became of the cyclotron-harmonic waves, how indeed is it possible to bypass a harmonic without taking any cognizance of it, what is the validity of the local dielectric tensor and its applicability to boundary value problems, the relation between group velocity and collisional energy absorption, the relative importance of Landau and collisional damping as n_z is varied, are some of the questions being presently investigated.

We express our gratitude to Dr. P. Barberio-Corsetti for providing us with the program [11] for computing the plasma dispersion function.

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References

1 K.N. Stepanov, Sov. Phys. JETP 7, 892 (1958); Zh. Eksp. Teoret. Fiz. 34, 1292 (1958).

2 M. Omura, Electrostatic waves in Bounded Hot Plasmas, Stanford University IPR Report No. 156 (1967).

3 S. Puri, F. Leuterer, and M. Tutter, J. Plasma Phys. 9, 89 (1973); $\sqrt{2}$ in the RHS of Eq.(17) is superfluous while the RHS of Eq.(19) should be divided by 2. Both these errors are typographical.

4 T. H. Stix, Theory of Plasma Waves, McGraw Hill, New York (1962), p. 252.

5 L. Spitzer, Jr., Physics of Fully Ionized Gases, Interscience Publishers, New York (1962).

6 Since in Ref. 3 the signs of space and time dependence are reversed, the signs of ε_{xy} and μ_{xy} should be changed accordingly. Also the integration of z in Eq.(2o) should be performed in the region Im $\zeta < o$.

7 S. Puri and M. Tutter, Nuclear Fusion 14, 93 (1974); also further references on this topic are listed here.

8 A reversal in the sign of B_o leads to similar conclusions.

9 T.H. Stix, Phys. Rev. Letters 15, 878 (1965).

10 V.M. Glagolev, Plasma Phys. 14, 315 (1972).

11 P. Barberio-Corsetti, Calculations of Plasma Dispersion Function, Princeton University Report MATT-773 (1970).

Figure Captions

Fig. 1 Real (solid curves) and imaginary (partially dashed curves) parts of k_x versus density for both the fast and slow waves as a function of temperature. The pair of curves for the slow mode are readily recognized by the resonance characteristics near the lower-hybrid density (shown by a cross on the density axis). The straight dashed lines in (d-f) correspond to the location of the reciprocal ion-Larmor radius $(|k_x \, r_{ci}| = 1)$. For all these curves $n_z = 0.99$, $\omega_{lh}/\omega_{ci} = 13.25$ and $B_o = 100$ kG. The vertical scale is compressed quadratically to facilitate the representation near the axis.

Fig. 2 Effect of magnetic field gradient ($B_o = 150$ kG at the plasma edge, $B_o = 100$ kG at the hybrid layer) on the dispersion curves. Ion-cyclotron harmonic numbers 9 to 15 are shown by the circles on the density axis. For these curves too $n_z = 0.99$ and $\omega_{lh}/\omega^*_{ci} = 13.25$ where ω^*_{ci} is the ion-cyclotron frequency at the hybrid density.

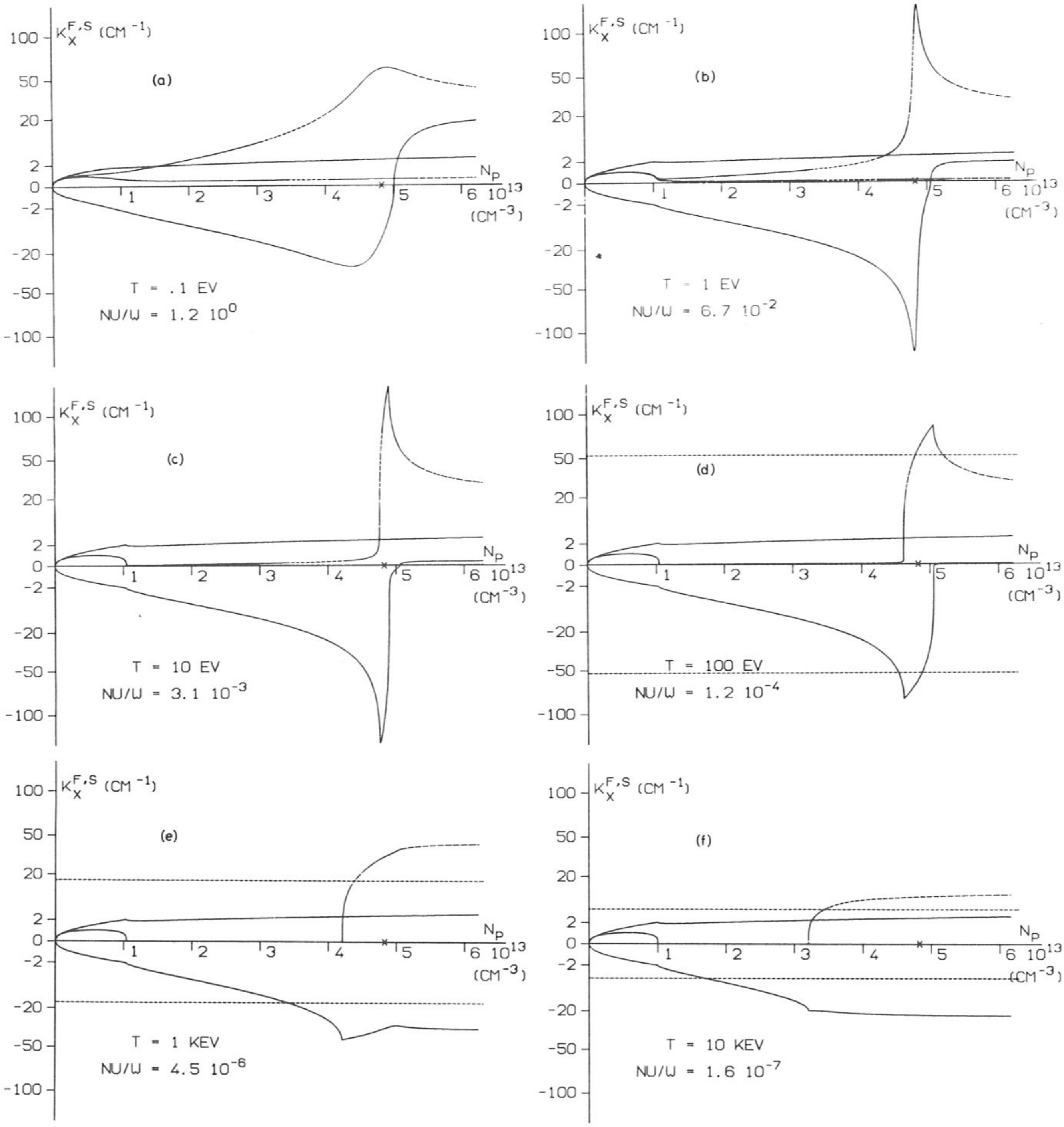
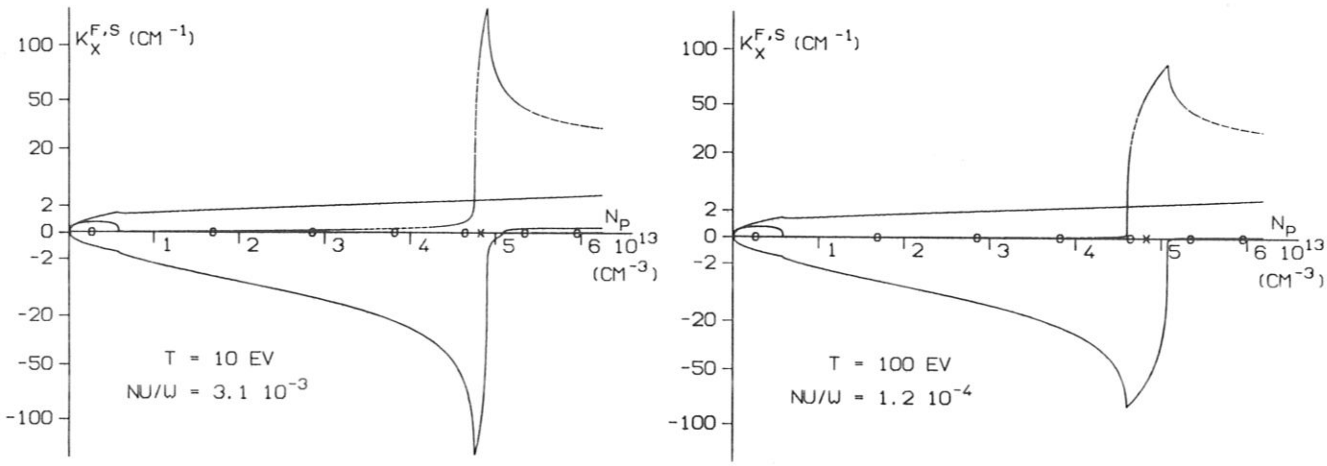

(a)
T = .1 EV
NU/W = 1.2 10^0
(b)
T = 1 EV
NU/W = 6.7 10^-2
(c)
T = 10 EV
NU/W = 3.1 10^-3
(d)
T = 100 EV
NU/W = 1.2 10^-4
(e)
T = 1 KEV
NU/W = 4.5 10^-6
(f)
T = 10 KEV
NU/W = 1.6 10^-7

Fig. 1



K_X^{F,S} (CM -1)
100
50
20
2
0
-2
-20
-50
-100
N_P
1
2
3
4
5
6 10^13
(CM^-3)
T = 10 EV
NU/W = 3.1 10^-3
T = 100 EV
NU/W = 1.2 10^-4

Fig. 2