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On the Possibility of the Simulation of a Slow-Wave System by a Phased Array of Waveguides

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ABSTRACT

From general considerations it is shown that neither a flush mounted waveguide nor a phased array of waveguides is capable of simulating a transverse-magnetic (TM) slow-wave structure. Their superficial resemblence to slow-wave systems is traced to the paradox of Mach´s "Wave Machine".

In an effort to avoid the mechanical difficulties associated with the construction of a passive slow-wave structure for coupling rf energy to the lower-hybrid resonance in a plasma, several authors have suggested using either a flush mounted[1] waveguide or a phased array[2] of such waveguides in order to obtain the necessary retardation of the electromagnetic waves.

In this note it will be shown that beyond a superficial resemblence (akin to that of Mach's "Wave Machine") such a solution fails to satisfy some of the fundamental requisites of a slow-wave system.

A passive, lossless, slow-wave structure (Fig. 1) operating at frequency ω in the pass band has the following fundamental properties:

1. Refractive index n_z in the longitudinal direction is real with magnitude exceeding unity.

2. $n_x = (1 - n_z^2)^{1/2}$ is imaginary, resulting in pure evanescence in the x-direction.

3. The Poynting flux $P_x = \mathrm{Re}\,\frac{1}{2}\,E\mathrm{x}H^*$ averaged over the length L (see Fig. 1) vanishes identically.

4. A real power flow exists, in general, along the z-direction.

These properties are, of course, interrelated and the last three follow from the first one through a self-consistent solution of Maxwell´s equations. For the Millman-line of Fig. 1 the steady-state fields in the region $x > 0$ may be expressed as,

$$E_z = \sum_{\nu=-\infty}^{\infty} f_\nu \exp i(k_{z\nu} z - \omega t)$$

$$H_y = i \sum h_\nu \exp i(k_{z\nu} z - \omega t)$$

$$E_x = i \sum e_\nu \exp i(k_{z\nu} z - \omega t)$$

where,

$$f_\nu = A \exp(-\varkappa_{x\nu} x)$$

$$h_\nu = \frac{A}{\sigma_{o\nu}} \exp(-\varkappa_{x\nu} x)$$

$$e_\nu = \frac{A}{\omega \varepsilon_o} \frac{k_{z\nu}}{\sigma_{o\nu}} \exp(-\varkappa_{x\nu} x)$$

$$A = \frac{a}{L} E_{zb} \frac{\sin\left(k_{z\nu} \frac{a}{2}\right)}{k_{z\nu} \frac{a}{2}}$$

$$\sigma_{o\nu} = \varkappa_{x\nu} / \omega \varepsilon_o$$

$$\varkappa_{x\nu} = i\, k_{x\nu} > 0$$

$$k_{x\nu}^2 = \frac{\omega^2}{c^2} - k_{z\nu}^2$$

$$k_{z\nu} = k_{zo} + \nu \frac{2\pi}{L} = \frac{2\pi}{\lambda_o} n_{z\nu}$$

where k_{zo} can be obtained from the dispersion relation

$$\frac{2\pi}{\lambda_o} \Sigma \frac{i}{\varkappa_{x\nu}} \left[\frac{\sin(k_{z\nu} \frac{a}{2})}{k_{z\nu} \frac{a}{2}} \right]^2 = \frac{L}{a} \cot\left(\frac{2\pi}{\lambda_o} s \right)$$

From the above equations, we obtain the x and z components of the Poynting vector as,

$$P_x(x,z) = -\frac{1}{2} R_e(E_z H_y^*)$$

$$= \frac{1}{2} \sum_{\nu > \mu} \sum (f_\mu h_\nu - f_\nu h_\mu) \sin\left[\frac{2\pi}{L} (\nu - \mu) z \right]$$

$$P_z(x,z) = \frac{1}{2} \, \mathrm{Re} \, (E_x H_y^*)$$

$$= \frac{1}{2} \sum_{\nu} e_\nu h_\nu$$

$$+ \frac{1}{2} \sum_{\nu>\mu} \sum (e_\nu h_\mu + e_\mu h_\nu) \cos \left[\frac{2\pi}{L} (\nu - \mu) z \right]$$

In general, both $P_x(x,z)$ and $P_z(x,z)$ are non-zero. It is readily verified that $P_x(x,z)$ averaged over L vanishes identically. A still stricter requirement that the time-averaged power flux through each slit i.e.

$$\int_{nL - \frac{L}{2}}^{nL + \frac{L}{2}} P_x(x,z) \, dz$$

vanishes follows easily. Equivalently, one might state that the impedance $Z_{z = nL} = - E_z / H_y$ across each slit is purely reactive. This property of the Millman-line may be considered an important characteristic in addition to the four listed already. These observations allow us to visualize the power-flow pattern of the Millman-line and is approximately depicted by the arrows in Fig. 1. The slits are seen to act as "reservoirs" of energy constantly replenished from the left and emptied to to the right. There is no net accumulation of energy in the slits - hence a purely reactive impedance. This action results in a net power flow in the longitudinal direction of the Millman-line.

We now return to the question of a phased array of waveguides (Fig. 2a) fed either with a single generator with subsequent introduction of the desired phase shift or with separate generators controlled by a master oscillator. One notes the simplicity with which the z-variation of the electric and magnetic fields can be accomplished and controlled at will. In the spirit of Refs. 2 and 3 we neglect, at this juncture, any mutual coupling among the waveguides.

A closer look reveals, however, that at the mouth of the waveguides ($x = 0$), E_z and H_y are not in phase quadrature so that P_x is finite averaged over a waveguide aperture as well as when averaged over any other arbitrary length. Thus we note that while reproducing the z-variation of the slow-wave electric field, the waveguide array fails to reproduce either the required x-dependence (pure evanescence) or the phase requisites of the magnetic field.

Apparently confronted by a similar paradox, Mach introduced his "Wave Machine" consisting of a series of uncoupled pendula (Fig. 2b) which may be driven independently with the desired phase sequence. An electrical circuit analogy shown in Fig. 2c, consists of a series of decoupled sections of a slow-wave circuit driven by a generator or a series of generators with the possibility of independent phase control to produce the desired voltage variation in the z-direction.

The common feature of all the three systems described above is an absence of mutual energy coupling between the adjacent elements. This is contrary to the concept of a wave in which mutual coupling (mechanical, collisional, electric, magnetic etc.) and energy storage (kinetic, potential, electromagnetic etc.) are essential to its description. A self-consistent solution of this coupled system finally leads to a dispersion relation which uniquely determines the propagation characteristics of the wave. The Mach "Wave Machine", on the other hand, does not possess a dispersion relation and consequently is unable to define the refractive index n_z. The paradox is created by the temptation to associate a wave number with a non-wave phenomenon.

Although, no further justification seems to be necessary in asserting our contention that a phased array of waveguides does not constitute a slow-wave system, we wish to quote some confirmatory experimental evidence. The initial doubts were stirred from our experimental experience (reconfirmed recently) that open ended waveguides (waveguides with H-plane horns, which do not alter the z-variation of the fields) are almost perfect microwave radiators. This is true even if the z-variation of the fields mimics a slow-wave structure with spectra concentrated predominantly in the region $n_z > 1$. One is at once faced with the contradiction of a slow-wave structure acting as a perfect radiator! Similar considerations and experimental evidence holds true for a single flush mounted waveguide.

We conclude that neither a single waveguide nor a phased array of them constitute a legitimate slow-wave system which could be used for creating a bonafide refractive index $n_z > 1$ for the purpose of obtaining penetration to the lower-hybrid resonance. It is not, however, the intention of this note to discount the possibility of using waveguide feed for coupling energy into a slow-wave structure, or of the existence of slow-waves under conditions of mutual coupling between the waveguides.

Acknowledgement

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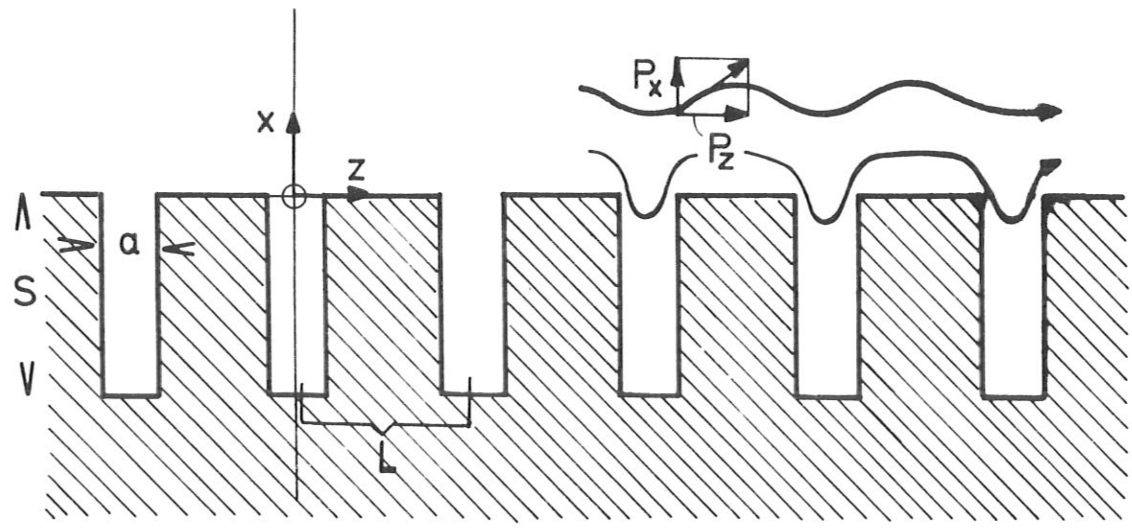

Fig. 1 The Millman line as an example of a slow-wave structure. The Poynting vector depicts the path of power flow.

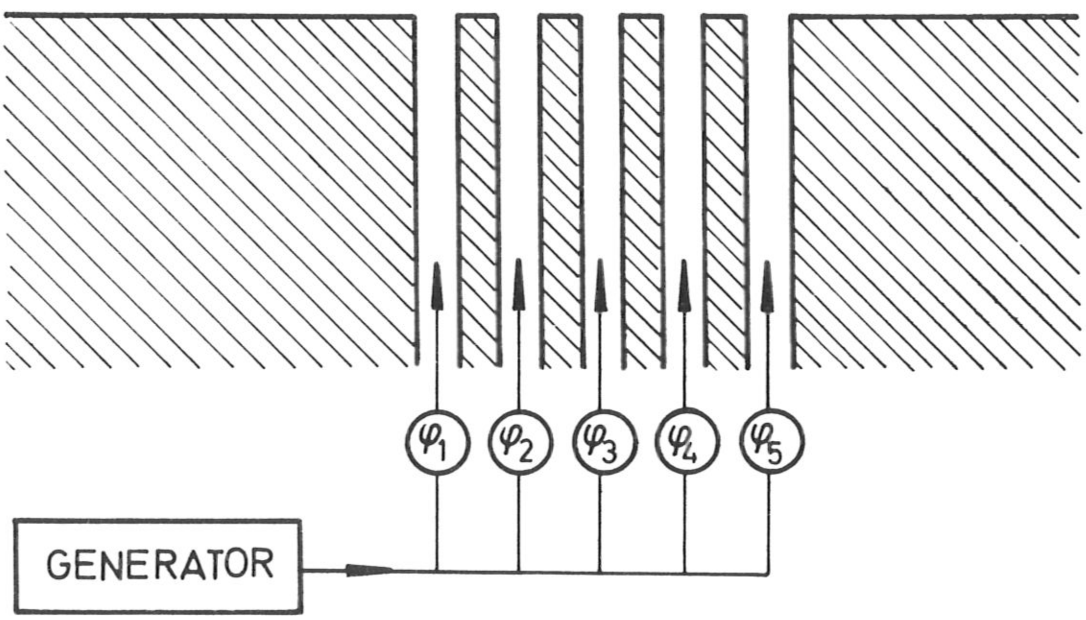

Fig. 2a Phased array of waveguides.

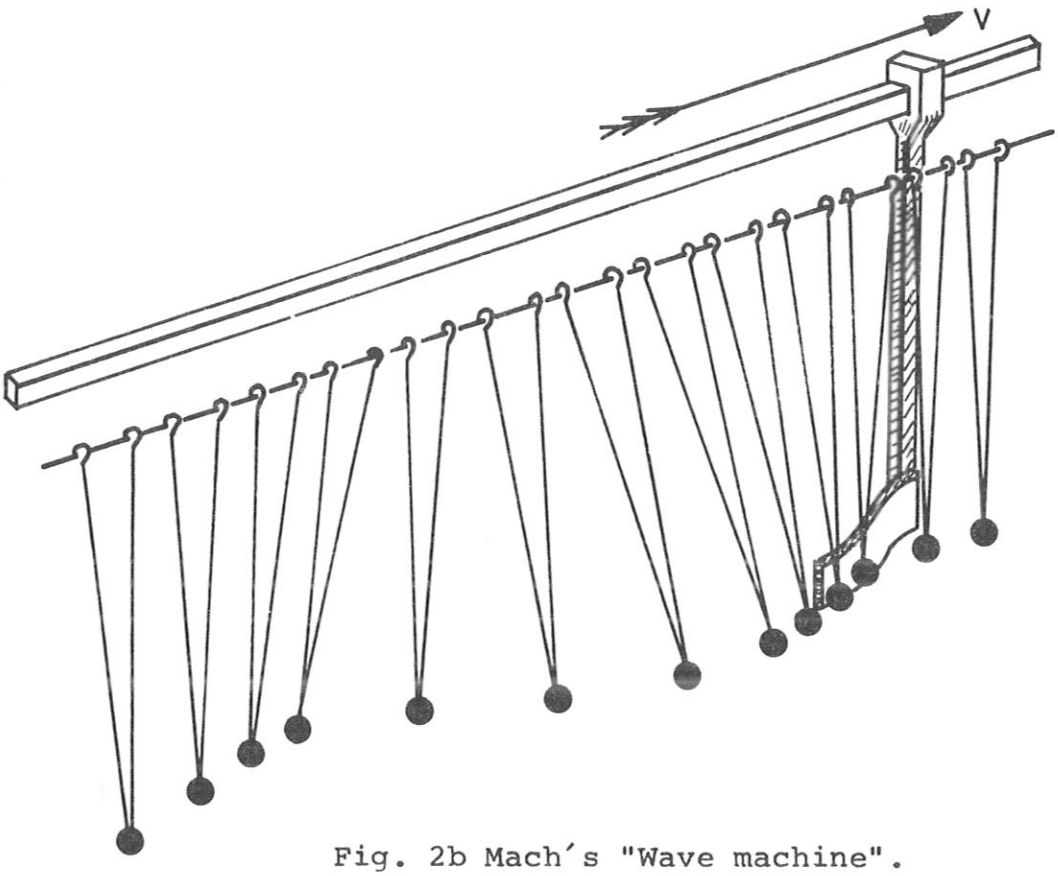

Fig. 2b Mach´s "Wave machine".

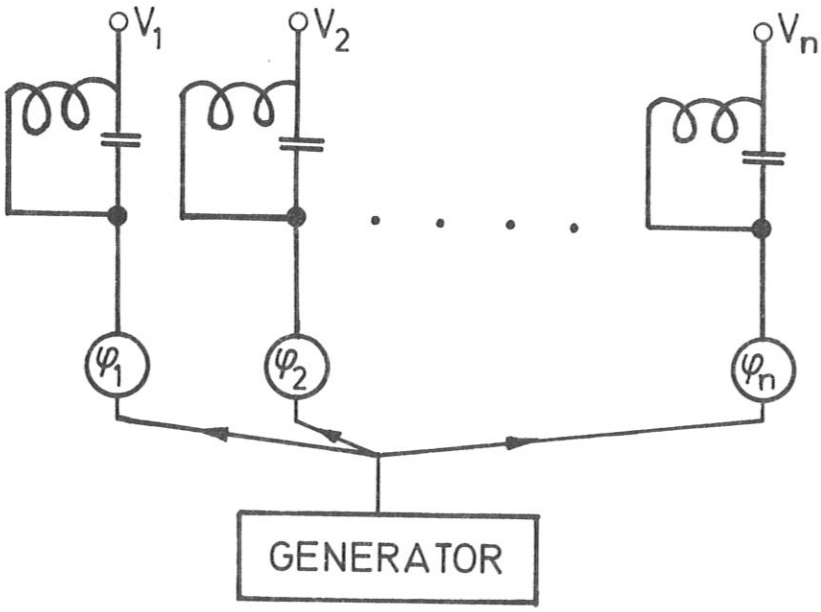

Fig. 2c Phased array of decoupled sections of a slow wave circuit.