GRAZING INCIDENCE COUPLING TO THE LOWER-HYBRID RESONANCE

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Abstract

The boundary value problem of "grazing incidence" coupling to the lower-hybrid resonance is studied under idealized conditions. It is found that in addition to the $n_z \simeq 1$ (n_z is the longitudinal refractive index) mode, a significant fraction of the incident energy is coupled to a slow wave, thereby greatly enhancing the plasma penetration by the radio frequency wave. The slow mode exists as an eigensolution of the waveguide formed between the metallic wall and the reactive surface impedance of the plasma. The slow wave is sufficiently retarded to permit accessibility to the lower-hybrid resonance for any foreseeable low-β experiment. The "grazing incidence" approach thus combines the advantages of accessibility as well as matching without having to construct any extensive structure within the machine walls.

I. INTRODUCTION

For optimum transmission of rf energy into a plasma, it is desirable that n_x, the refractive index in the direction of the density gradient, be real everywhere between the plasma edge and the resonant layer. For the case of the lower-hybrid resonance, this is essentially true provided that the longitudinal refractive index n_z satisfies the requirement (STIX, 1962; PARKER, 1971; GOLANT, 1971; GLAGOLEV, 1972; PESIC, 1972; PURI and TUTTER, 1973; ARTICO and SPIGLER, 1974)

$$n_z^2 \geqslant 1 + (\omega_{pe}/\omega_{ce})^2 \tag{1}$$

or equivalently

$$\nu_x \geqslant \omega_{pe}/\omega_{ce} \tag{2}$$

where $\nu_x = in_x$ in vacuum. Since $n_z > 1$, the longitudinal phase velocity is less than the light velocity in vacua and the waves are referred to as slow or retarded waves.

An obvious means of obtaining the required longitudinal retardation is to construct a Millman line (GOLANT, 1971) inside the torus walls. This is technologically complicated in a thermonuclear environment. Moreover, if the separation between the plasma and the slow-wave structure exceeds a fraction of a wavelength, the rf fields "stick" to the Millman line leading to problems of electrical breakdown and poor efficiency.

Another alternative variously called the "phased array" (KARNEY, BERS and KULP, 1973; BERS and KARNEY, 1974) or a "grill" (LALLIA, 1974) consists of a number of waveguides flush mounted on the torus wall with phasing appropriate for producing the required retardation and are more sophisticated variants of PARKER's (1971) single flush-mounted waveguide used in the MIT Alcator. This system suffers the drawback of possible mismatch as the plasma parameters and position vary with attendant problems of dynamic tuning at high power levels. Also doubt remains whether such a configuration possesses slow-wave properties at all (TUTTER and PURI, 1974).

Yet another approach exploits the fact that for typical thermonuclear parameters $n_e = 10^{14}$ cm^{-3}, $B_o = 60$ kG and $(\omega_{pe}/\omega_{ce})^2 \sim 0.25$, the accessibility condition (1) is not severely violated if one launches "grazing incidence" (Fig.1a) waves ($n_z \sim 1$) and counts on the possibility of tunnelling through the thin evanescent layer near the plasma edge specially in thermonuclear plasmas where steep density profiles are predicted (PURI and TUTTER, 1973).

In this paper it would be shown that the credibility of the "grazing incidence" coupling is further enhanced by its ability to excite retarded waves capable of penetrating any foreseeable low-β plasma without having to construct additional slow-wave structures. It is found that the reactive surface impedance of the plasma successfully imitates the properties of a Millman line without suffering the drawbacks of technical complexity or poor efficiency.

II. NORMAL MODES OF THE PLASMA WAVEGUIDE

The geometry of Fig.1a may be idealized to admit of analytical tractability without loss of essential physics. The cylindrical geometry will be replaced by a slab model (Fig.1b) and variations in the y-direction will be ignored.

The only plasma property entering the boundary value problem is its surface impedance σ_s. Since σ_s is dominantly reactive (PURI and TUTTER, 1974) it is proper to characterize the plasma by a surface reactance $\sigma_s = iX$.

The final, relatively innocuous, simplification consists in assuming that only the dominant mode is present in both the incident and the reflected waves in the coupling waveguide. However, the complete set of modes of the plasma waveguide will be retained.

Assuming an exp i $(k_x x + k_z z - \omega t)$ dependence of the field quantities, Maxwell's equations in vacuum may be written as

$$k \times H = -\omega \varepsilon_o E \quad (3)$$

and

$$k \times E = \omega/\mu_o H \quad (4)$$

which combine to give

$$k_x^2 + k_z^2 = \omega^2/c^2 = k_o^2 \quad (5)$$

For the TM waves (in the subsequent analysis the time dependence will be dropped),

$$H = H_y = \hat{H}_y \cos(k_x x) \exp i\,(k_z z) \cdot \qquad (6)$$

From (3) and (6)

$$E_x = \hat{H}_y k_z/\omega \varepsilon_o \cos(k_x x) \exp i\,(k_z z) \qquad (7)$$

$$E_z = -\hat{H}_y ik_x/\omega \varepsilon_o \sin(k_x x) \exp i\,(k_z z) . \qquad (8)$$

The form (6) for H_y was selected in anticipation of the boundary condition $E_z = o$ at $x = o$. The second boundary condition $-E_z/H_y = \sigma_s = iX$ at $x = b$ yields the dispersion relation

$$k_x \tan(k_x b) = \omega \varepsilon_o X . \qquad (9)$$

In addition to an infinite number of real roots $k_{x\nu}$, (9) possesses an imaginary solution $\varkappa = i\,k_x$ given by

$$\varkappa \tanh(\varkappa b) = -\omega \varepsilon_o X \qquad (1o)$$

provided that $X < o$, i.e. if the plasma presents a capacitive surface impedance, which is indeed the case for the transverse magnetic (TM) waves. The actual value of the surface impedance normalized with respect to the permittivity η of free space is shown as a function of n_z (or ν_x) in Fig.2a which is reproduced from an earlier publication (PURI and TUTTER, 1974). Although not shown in this figure, the values of σ_s/η for the case $n_z < 1$ are found to be still dominantly capacitive with an approximate value of 4 Ω.

Rewriting (5), (9), and (1o) in terms of the refractive indices $n_x = k_x/k_o$, $\nu_x = in_x = \varkappa/k_o$ and $n_z = k_z/k_o$ we get

$$n_x^2 + n_z^2 = 1 \tag{11}$$

$$n_x \tan(n_x 2\pi b/\lambda) = \chi \tag{12}$$

and

$$\nu_x \tanh(\nu_x 2\pi b/\lambda) = -\chi \tag{13}$$

where $\chi = X/\eta$. Since $\chi \rightarrow o$ for $n_z \sim 1$ $(n_x \rightarrow o)$, one of the solutions of (12) is

$$n_x^2 = \varkappa \lambda/2\pi b. \tag{14}$$

Since typically $\chi \sim 4/377$, choosing $b \sim \lambda$ gives

$$n_x^2 \sim o.oo2 \tag{15}$$

$$n_z \sim o.999 \tag{16}$$

This is in fact the so-called TEM mode observed in the plasma parallel-plate waveguide treatment (PURI and TUTTER, 1973). The value of $n_x \sim 4 \times 1o^{-2}$ corresponds roughly to an incidence angle of 2.5 ^{o} where the most optimum coupling between the plasma and the fast waves ($n_z < 1$) is expected (PURI and TUTTER, 1973). For $\chi \to o$ and $b = \lambda$ the next higher root of (12) occurs for $n_x \sim 1$ so that $n_z^2 \sim o$ and the wave is barely propagating. Choosing b only slightly lower than λ, this and all other solutions of (12) will be henceforth assumed to be cut-off modes.

Hitherto we have considered only the real values of n_x which for typical parameters sustain one propagating ($n_z^2 > o$) TEM-like mode with $n_z \sim 1$. There is yet another propagating mode for imaginary n_x given by (13). Since tanh (ν_x $2\pi b/\lambda$) is nearly unity we get

$$\nu_x \simeq -\chi. \tag{17}$$

From Fig.2a (it is necessary to extrapolate the curves for this estimate), this condition is satisfied for the TM excitation when

$$\nu_x \sim 3.5 \tag{18}$$

corresponding to

$$n_z \sim 3.6. \tag{19}$$

The above value of n_z is sufficiently large to permit accessibility to any foreseeable low-β plasma. Discovery of this slow mode occuring naturally in the waveguide formed between the plasma and the metallic walls of the machine is the principal contribution of this paper. This finding frees us from the need to construct mechanical structures inside the machine. The existence of such slow waves might be suspected from the analogy of the reactive surface impedance presented both by the plasma and the Millman line. Such waves were inadvertently observed in our early studies of the plasma parallel-plate waveguide (PURI and TUTTER, 1973), although their potential importance escaped our notice at the time. It may be verified from (11) - (13) that the product of phase and group velocities $v_{ph} \; v_{gr} \gtrless c^2$ accordingly as $n_z \lessgtr 1$, unlike the case of a metallic waveguide in which such a product always equals c^2.

The total field in the plasma waveguide may now be expressed as a linear superposition of the eigenfunctions

$$v_1 = \cosh (\varkappa x) \exp i (k_{z1} z) \tag{2o}$$

and

$$v_\nu = \cos (k_x x) \exp i (k_{z\nu} z), \quad \nu \geqslant 2 \tag{21}$$

where the subscripts 1 and 2 will denote the two propagating modes, namely the slow mode and the quasi TEM mode, respectively. All higher modes corresponding to $\nu > 2$ will be assumed to be cut-off solutions for the parameters under consideration. Further analysis is facilitated by expressing the fields in terms of the normalized, orthogonal functions φ_ν constructed from v_ν as follows:

$$\varphi_1 = \frac{v_1}{\sqrt{\langle v_1 v_1^* \rangle}}, \quad \langle v_1 v_1^* \rangle = \frac{1}{b} \int_0^b v_1 v_1^* \, dx \tag{22}$$

and

$$\varphi_{\nu \geqslant 2} = \frac{f_\nu}{\sqrt{\langle f_\nu f_\nu^* \rangle}}, \quad f_\nu = v_\nu - \sum_{n=1}^{\nu-1} \langle v_\nu \varphi_n^* \rangle \varphi_n . \tag{23}$$

III. THE BOUNDARY VALUE SOLUTION

The field components E_x^c and H_y^c in the coupling waveguide may be written as the sum of the incident and reflected fundamental modes at z = o as

$$E_x^c = (E_{xi} + E_{xr}) \cos (\pi/a\ x) \tag{24}$$

and

$$H_y^c = 1/\eta\ (E_{xi} - E_{xr}) \cos (\pi/a\ x) \tag{25}$$

whereas in the plasma waveguide at z = o

$$E_x^p = E_o \sum_{\nu=1}^{\infty} a_\nu \varphi_{o\nu} \tag{26}$$

and

$$H_y^p = \frac{E_o}{\eta} \sum_{\nu=1}^{\infty} \frac{1}{n_{z\nu}} a_\nu \varphi_{o\nu} \tag{27}$$

where $\varphi_{o\nu} = [\varphi_\nu]_{z=o}$ while a_ν is so adjusted that $E_o = (E_{xi} + E_{xr})$. Matching the tangential electric fields in the coupling and plasma waveguides at z = o gives

$$\cos (\tfrac{\pi}{a}x) = \sum_{\nu=1}^{\infty} a_\nu \varphi_{o\nu} . \tag{28}$$

Multiplying (28) by $\varphi_{o\mu}$ with subsequent integration from x = o to x = b gives

$$a_\nu = \frac{1}{b} \int_o^a \varphi_{o\nu} \cos (\tfrac{\pi}{a}x)\ dx \tag{29}$$

Finally equating the Poynting flux in the two waveguides, we get

$$P_z = \frac{1}{2\eta} E_o^2 \sum_{\nu=1}^{\infty} a_\nu a_\nu^* \int_0^b \frac{1}{n_{z\nu}} \varphi_\nu \varphi_\nu^* dx = \frac{a}{4\eta} (E_{oi} + E_{or})(E_{oi} - E_{or}). \quad (3o)$$

Using $E_o = (E_{oi} + E_{or})$, one obtains from (3o)

$$\frac{1+\rho}{1-\rho} = a \Big/ 2 \sum_{\nu=1}^{\infty} a_\nu a_\nu^* \int_0^b \frac{1}{n_{z\nu}} \varphi_\nu \varphi_\nu^* dx . \quad (31)$$

Since cut-off modes carry no net Poynting flux the summation in (31) is required for the propagating (assumed to be only two in this case) modes only. The fractions of the total incident energy coupled to the slow and the quasi TEM waves, $p_{z1} = P_{z1}/P_{zo}$ and $p_{z2} = P_{z2}/P_{zo}$, respectively, are given by

$$p_{z1} = 4 \frac{n_{zo}}{n_{z1}} \frac{a}{b} \left[\frac{\varkappa a}{\pi^2 + (\varkappa a)^2} \right]^2 \frac{2\varkappa b \sinh^2 \varkappa a}{2\varkappa b + \sinh 2\varkappa b} \quad (32)$$

and

$$p_{z2} = \frac{16}{\gamma^3} \frac{n_{zo}}{n_{z1}} \frac{a}{b} \left[\alpha \frac{\varkappa a \sinh \varkappa a}{\pi^2 + (\varkappa a)^2} + \frac{k_{x2} a \sin k_{x2} a}{\pi^2 - (k_{x2} a)^2} \right]^2 \quad (33)$$

where

$$n_{zo} = \left[1 - (\lambda/2b)^2 \right]^{1/2} \quad (34)$$

$$\gamma = 1 + \alpha^2 - 2\alpha\beta + \frac{\sin 2k_{x2} b}{2k_{x2} b} + \alpha^2 \frac{\sinh 2\varkappa b}{2\varkappa b} \quad (35)$$

$$\alpha = (4\varkappa b)^2 \frac{k_{x2}b \sin k_{x2}b \cosh \varkappa b + \varkappa b \cos k_{x2}b \sinh \varkappa b}{\left[(k_{x2}b)^2 + (\varkappa b)^2\right]\left[2\varkappa b + \sinh 2\varkappa b\right]^2} \tag{36}$$

$$\beta = \frac{k_{x2}b \sin k_{x2}b \cosh \varkappa b + \varkappa b \cos k_{x2}b \sinh \varkappa b}{(k_{x2}b)^2 + (\varkappa b)^2} . \tag{37}$$

Assuming that $a = b = \lambda$ and $n_z = 3$, one may obtain from (32) to (36) that of the total incident energy about 6 % is coupled to the slow wave and the bulk of the rest to the quasi TEM mode.

IV. DISCUSSION

Analysis similar to that of Section II but for TE excitation reveals the existence of slow waves, too, provided that the plasma presents an inductive surface impedance as was indeed found to be the case in an earlier work (PURI and TUTTER, 1974) from which the curves of Fig.2b are reproduced. The dispersion relation for the slow wave in this case is given by

$$\nu_x \coth(\nu_x 2\pi/B) = 1/\chi . \tag{38}$$

Since $\coth(\nu_x 2\pi/B) \sim 1$ and $\chi^{-1} \sim 3.5$, ν_x once again has the value 3.5 as was also the case for the TM excitation (18). By all appearances, the plasma exhibits an uncanny propensity to be penetrated by the rf energy by encouraging the existence of slow waves both for the TM or TE excitation.

The fact that only about 6 % of the incident energy is coupled to the slow waves need not be too discouraging. While any energy coupled to the slow waves is quickly and irrevocably admitted into the plasma, continuous generation of the slow-wave component is to be foreseen by discontinuities, obstacles and the wall curvature itself inevitably present under actual experimental conditions. Also in the more realistic case of three-dimensional geometry the factor n_{zo} in (32) and (33) effectively becomes unity and considerably more efficient coupling to the slow waves is possible by reducing a and b. This reduction would in any case be necessary for the present generation machines where often very little space is available between the plasma surface and the torus walls.

Considerations very similar to the ones in this paper will tend to enhance the performance of TM loops (PURI and TUTTER, 1973) which are at present being used in the Garching LIWEREX and the Garching-Grenoble WEGA machines for coupling to the lower-hybrid resonance.

The surface impedance values of Fig.2 were computed using plasmas of relatively small dimensions. Caution must be exercised in projecting these results to plasmas of larger sizes. Such computations are under way and we hope to present the results in the future. Since ν_x varies logarithmically with χ, we expect that the conclusions of this paper to be modified without being fundamentally altered when plasmas of larger dimensions are used.

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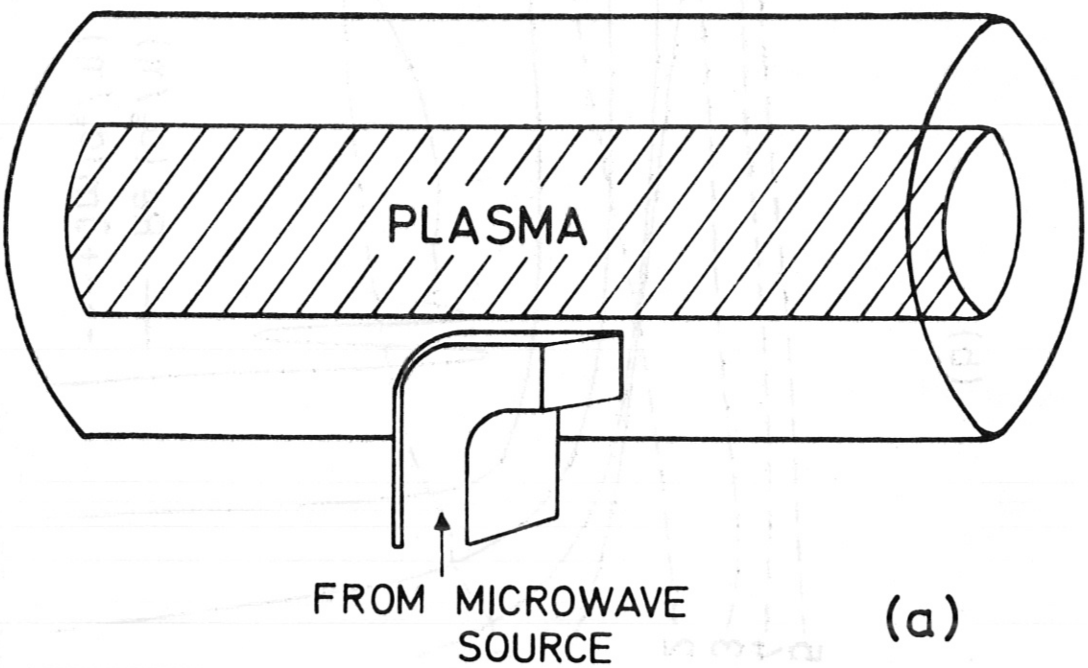
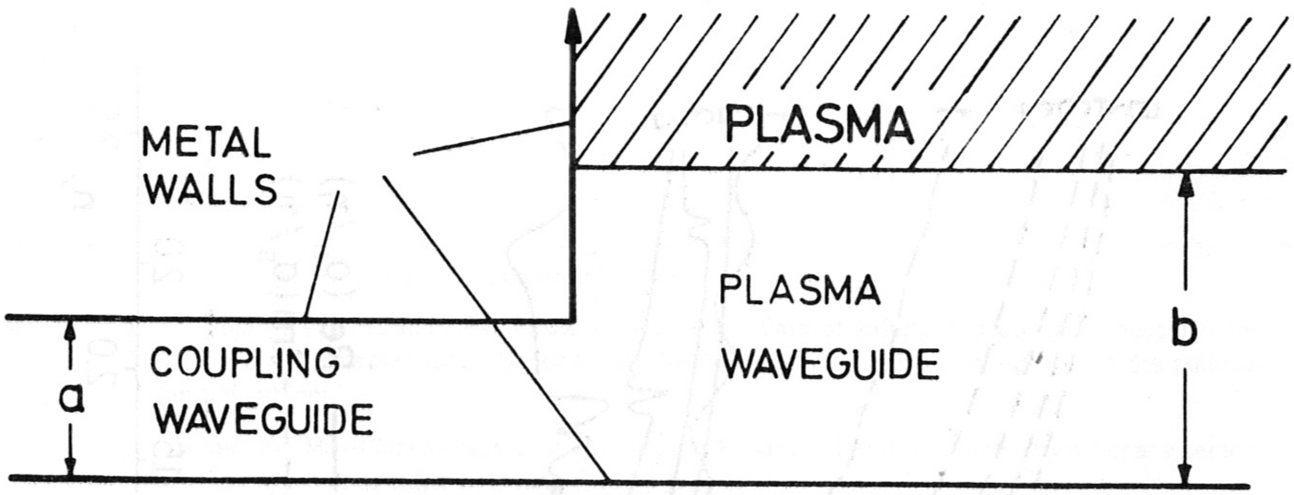
FIGURE CAPTIONS

Fig. 1 The realistic cylindrical geometry (a) and the idealized slab model (b).

Fig. 2 Real and imaginary parts of the normalized surface impedance σ_s/η as a function of n_z (or ν_x). The parameters used in the computations are B_o = 100 kG, linear density profile with the lower-hybrid layer 0.8 cm away from the plasma edge, ω_{pe}/ω_{ce} = 0.17, 0.35, 0.57, 0.88, and 1.46 for the curves numbered 1 to 5, respectively. Figures (a) and (b) are for the two cases of TM and TE excitations, respectively.

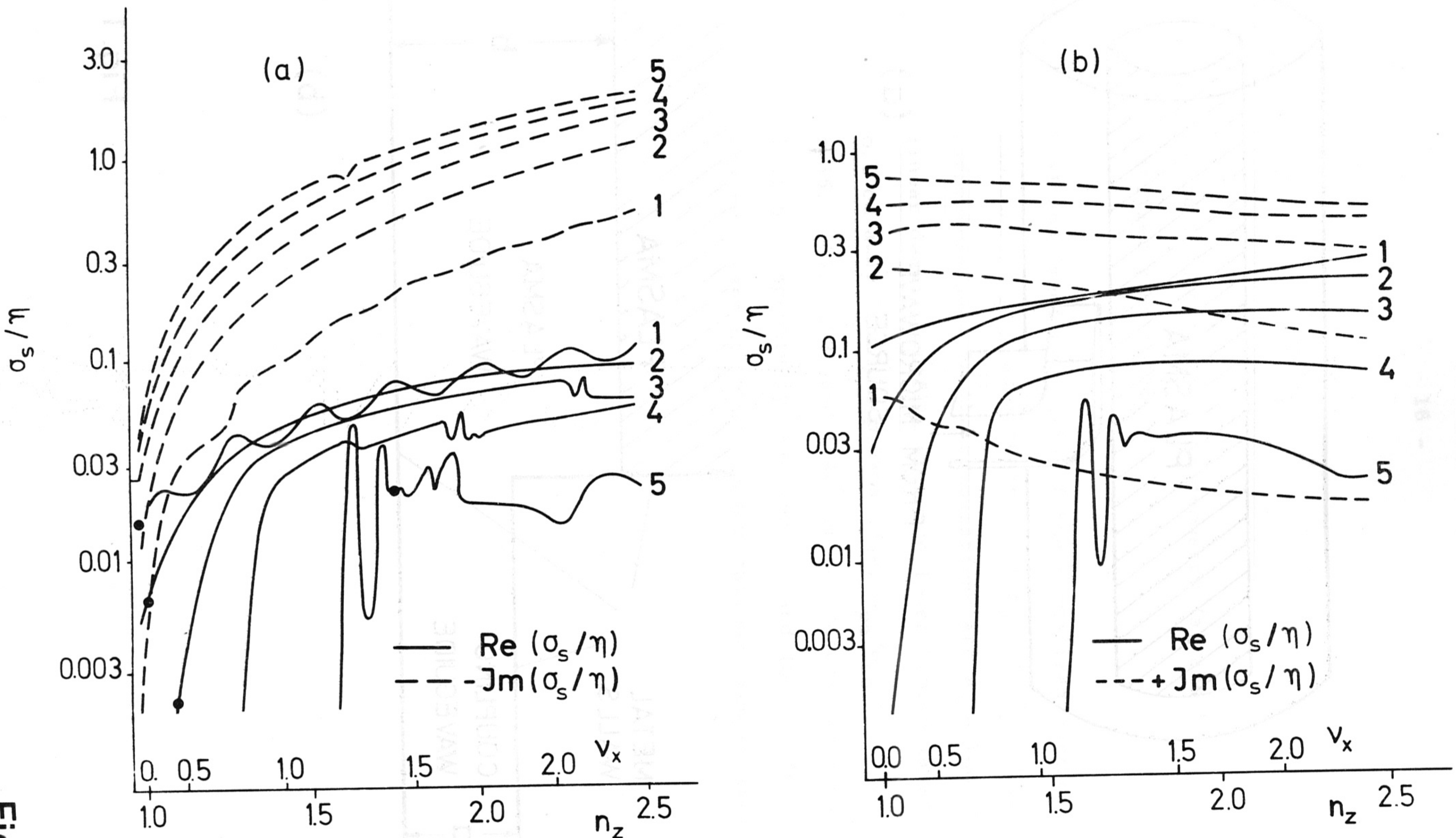

PLASMA
FROM MICROWAVE SOURCE

(a)


METAL WALLS
PLASMA
PLASMA WAVEGUIDE
COUPLING WAVEGUIDE
a
b

(b)

Fig. 1



(a)
(b)
3.0
1.0
0.3
0.1
0.03
0.01
0.003
σ_s / η
1
2
3
4
5
Re (σ_s/η)
-Jm (σ_s/η)
+Jm (σ_s/η)
ν_x
n_z
0.0
0.5
1.0
1.5
2.0
2.5

Fig. 2