CURRENT FILAMENTATION IN PARALLEL FIELD TURBULENT PLASMAS

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IPP 6/124 March 1974

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

Abstract

Observing that the inverse temperature dependence of the anomalous resistivity of an electrostatically turbulent plasma suggests the possibility of current filamentation, we investigate the conditions for the occurrence of instabilities producing current filaments and current sheets in the high frequency domain ($\omega >> \omega_{ci}$) of a parallel current driven turbulent plasma. The effect of such a current filament at high densities ($\omega_{pe} > \omega_{ce}$) is shown to produce a perpendicular acceleration of the electrons which, under conditions where sufficient development of the filament occurs, creates cross-field ion accoustic turbulence. The presence of cross-field ion-acoustic turbulence, which in electrostatic simulation is observed to produce substantial heating, would substantially reduce the existing discrepancies between the parallel turbulent experiments and the corresponding electrostatic simulation.

I. Introduction

In this work we begin with the recent understanding of the effects of Debye-scale electrostatic turbulence that has developed from the extensive experimental and computer simulation studies of ion-acoustic and related electrostatic turbulence.[1)-5)] With this as a basis we have initiated a study of the electromagnetic dynamics of the turbulent plasma. In particular, we have shown in one case that the interaction between the Debye-scale electrostatic turbulence and the c/ω_{pe} - scale electromagnetic dynamics leads to a possible explanation of the strong heating produced in the parallel field heating experiments which are in qualitative disagreement with the corresponding electrostatic simulation experiments.[5)]

In a well developed state of Debye-scale electrostatic turbulence we have an electron fluid possessing an anomalous collision frequency ν such that $\omega_{ci} << \nu << \omega_{ce}$ and where ν exhibits a strong dependence on the electron temperature T_e and the mean drift velocity $u = j/ne$. From the turbulent electron collisions and on space scales large compared to λ_D the plasma exhibits an anomalous resistivity $\eta = m_e \nu / ne^2$, an anomalous cross-field thermal conductivity $\kappa_\perp = \nu \rho_e^{\,2}$, and an anomalous ohmic heating rate ηj^2. In Sec. II we develop the linear electromagnetic stability theory of this turbulent plasma to perturbations representing local concentrations of current and electron temperature. From stationary, uniform equilibrium states instability is shown to occur for $\partial\eta/\partial T_e < 0$ with a maximum growth rate of order $\gamma \simeq \nu \left(\frac{u}{v_e}\right)^2 \left|\frac{d\,\ell n\,\eta}{d\,\ell n\,T_e}\right|$ where $v_e^2 = T_e/m_e$ for perturbations of scale $k_\perp \simeq \omega_{pe}/c$. The effect of the current filament is shown to produce a charge separation which in turn leads to a cross-field acceleration of the electrons.

For the purpose of studying the nonlinear development of the instability and the effects on stability of nonstationary and nonuniform equilibrium conditions we give in Sec. III the equations for the one dimensional dynamics of current sheets and the results from numerical integration of the equations. From the numerical results we find that from an unstable equilibrium there is strong nonlinear growth resulting in conditions requiring the growth of cross-field ion acoustic turbulence. For monotonically decreasing ν versus T_e profiles there appears to be no nonlinear saturation within the original equations.

However, the strong cross-field anomalous transport produced by the cross-field ion acoustic turbulence and other electrostatic instabilities from the strong $T_e(x)$ and $j_{\parallel}(x)$ gradients[6)] can be expected to stabilize the mode after sufficient growth. In addition, studies with nonuniform and nonstationary background profiles show that during a strong secular time dependence of the background instability is prevented. The critical condition for background variation to override the instability is that the rate of change of the background be of the same order as the local growth rate. From the expression for the maximum growth rate compared to the ohmic heating rate we can expect this stabilizing mechanism to occur rather generally during transient periods of energy imbalance.

II. Linear Stability of Filaments

We consider the geometry of parallel field, $E_o \parallel B_o$, turbulent heating experiments of radius a in the slab approximation. During stationary or slowly varying compared to ν^{-1} periods of background evolution such as during collisional diffusion at the rate $\nu(c/\omega_{pe} a)^2 << \nu$, we study the stability of the system to high frequency dynamics $(\omega > \omega_{ci})$ and on intermediate space scales $k_{\perp}^{-1} \sim c/\omega_{pe} << a$ but large compared to the Debye length $\lambda_D = v_e/\omega_{pe}$. The electron dynamics on space scales large compared to λ_D, $\rho_e = v_e/\omega_{ce}$ and time scales large compared to ω_{ce}^{-1}, ω_{pe}^{-1} is given by fluid equations containing the turbulent collision frequency $\nu = \nu(T_e, j)$ $<< \omega_{ce}, \omega_{pe}$ due to the Debye scale electrostatic turbulence.

For equilibrium in the uniform, stationary state (or slowly varying on the space scale $a >> c/\omega_{pe}$ and time scale $\nu(c/a\omega_{pe})^2$) we have the conditions that

$$\eta j_o = E_o \tag{1}$$

$$\eta j_o^2 = \frac{3}{2} \frac{nT_e}{\tau_E} \tag{2}$$

$$\frac{dB_y}{dx} = \frac{4\pi}{c} j_o \tag{3}$$

where $\underset{\sim}{B} = B_o \hat{e}_z + B_y (x) \hat{e}_y$ and j_o and E_o are in the direction of the confining field $B_o \hat{e}_z$, and τ_E = energy confinement time.

We consider perturbations of the equilibrium (1-3) of the form $\delta j_{\parallel} \cdot \exp(i\underset{\sim}{k} \cdot \underset{\sim}{x} + \gamma t)$ that are aligned along the equilibrium magnetic field with $k_{\parallel} = k_z + k_y (B_y/B_o) = 0$. The corresponding magnetic perturbation is $\delta \underset{\sim}{B}$. In this work we consider modes localized in x to regions of order c/ω_{pe} and show in further work that for such modes the effect of shear which forces $k_{\parallel}(x) \neq 0$ is a small effect. Thus, the perturbations represent current filaments that are two dimensional across the field and are aligned with essentially infinite wavelength along the equilibrium magnetic field. In the present case where $k_{\parallel}$ is negligible the parallel electric field is purely an induction field produced by $\delta \underset{\sim}{B}_{\perp}$ and given by $E_{\parallel} = \frac{1}{c} \frac{\partial \psi}{\partial t}$. An electrostatic field $\underset{\sim}{E}_{\perp} = - \nabla_{\perp} \varphi$ is shown to arise from cross-field charge separation.

The equations for the electron dynamics in the regime discussed above are

$$\frac{m_e}{ne^2}\left(\frac{\partial j_\parallel}{\partial t} + \nu j_\parallel\right) = E_\parallel + \frac{1}{en}\nabla_\parallel p_e \tag{4}$$

$$\underset{\sim}{j}_\perp = -\,en\,\underset{\sim}{u}_{\perp e} = -\,enc\,\frac{\underset{\sim}{E}\times\underset{\sim}{B}}{B^2} + \frac{nm_e c^2}{B^2}\frac{\partial \underset{\sim}{E}_\perp}{\partial t} \tag{5}$$

$$\frac{3}{2}\,n\,\frac{dT_e}{dt} + nT_e\,\nabla\cdot\underset{\sim}{u}_e = \eta j^2 + \frac{3}{2}\,n\,\nabla\cdot(\kappa\,\nabla T_e) - \frac{3}{2}\frac{nT_e}{\tau_E} \tag{6}$$

and from Maxwell's equations

$$\hat{n}\cdot(\nabla\times\delta\underset{\sim}{B}) = \nabla_\perp^2\,\psi = \frac{4\pi}{c}\,\delta j_\parallel \tag{7}$$

$$\nabla\cdot\underset{\sim}{j} = 0 \tag{8}$$

where $\underset{\sim}{j} = -en\,\underset{\sim}{ue}$ and $\hat{n} = \underline{B}/|\,\underline{B}\,|$. For modes with negligible $k_\parallel$ Eqs. (4), (6) and (7) decouple from Eqs. (5) and (8) where the latter determine the $\underset{\sim}{E}_\perp$ required for charge neutrality. Linearization of Eqs. (4), (6) and (7) gives

$$\eta \left(\frac{\gamma}{\nu} + 1 + \frac{d \,\ell n\, \eta}{d \,\ell n\, j} \right) \delta j_{\parallel} + j_{\circ} \frac{\partial \eta}{\partial T} \delta T = \frac{\gamma}{c} \psi \tag{9}$$

$$\left[\gamma + k_{\perp}^{2} \varkappa_{\perp} - \frac{2 \eta j_{\circ}^{2}}{3 n_{\circ} T_{\circ}} \frac{d \,\ell n\, \eta}{d \,\ell n\, T} + \frac{\partial}{\partial T} \left(\frac{T}{\tau_{E}} \right) \right] \delta T$$

$$= \frac{4}{3n} j_{\circ} \eta \left(1 + \frac{1}{2} \frac{d \,\ell n\, \eta}{d \,\ell n\, j} \right) \delta j_{\parallel} \tag{10}$$

$$- k_{\perp}^{2} \psi = \frac{4\pi}{c} \delta j_{\parallel} . \tag{11}$$

Imposing the condition for nontrivial solutions we obtain the following quadratic equation for γ

$$\left[\gamma \left(1 + \frac{\omega_{pe}^{2}}{k_{\perp}^{2} c^{2}} \right) + \nu (1 + \eta_{J}) \right] \left[\gamma + k_{\perp}^{2} \varkappa + \gamma_{\circ} + \gamma_{E} \right] - 2\gamma_{\circ} \nu (1 + \frac{1}{2} \eta_{J}) = 0 \tag{12}$$

where

$$\eta_{J} = \left. \frac{d \,\ell n\, \eta}{d \,\ell n\, j} \right|_{j_{\circ}, T_{\circ}} , \quad \eta_{T} = \left. \frac{d \,\ell n\, \eta}{d \,\ell n\, T} \right|_{j_{\circ}, T_{\circ}}$$

$$\gamma_{E} = \left. \frac{\partial}{\partial T} \left(\frac{T}{\tau_{E}} \right) \right|_{T_{\circ}} , \quad \gamma_{\circ} = - \frac{2 \eta j_{\circ}^{2}}{3 n_{\circ} T_{\circ}} \eta_{T} . \tag{13}$$

From Eqs. (9 - 12) we observe that large space scale perturbations occur at nearly constant current and have a thermal stability determined by $\gamma_{k_\perp \to 0} = -\gamma_\circ - \gamma_E$. For $\partial\eta/\partial T < 0$ we have $\gamma_\circ > 0$ and the large scale modes are stable even for $\gamma_E = 0$. (For the unusual case $\partial\eta/\partial T > 0$ thermal stability requires $\gamma_E > |\gamma_\circ|$.) For $\gamma_\circ + \gamma_E < 0$, i.e. stable $k_\perp = 0$ modes, we readily show that the only unstable root of Eq. (12) is

$$\Gamma \equiv \frac{\gamma}{\nu} = \frac{1}{2(1+K_\perp^{-2})} \left[\left\{ \left[1 + \eta_J + (1+K_\perp^{-2})(K_\perp^2 \bar{\varkappa} + \Gamma_\circ + \Gamma_E) \right]^2 \right.\right.$$

$$\left. + 4(1+K_\perp^{-2}) \left[\Gamma_\circ - (1+\eta_J)(K_\perp^2 \bar{\varkappa} + \Gamma_E) \right] \right\}^{1/2}$$

$$\left. - \left[1 + \eta_J + (1+K_\perp^{-2})(K_\perp^2 \bar{\varkappa} + \Gamma_\circ + \Gamma_E) \right] \right] \qquad (14)$$

where $K_\perp = k_\perp c/\omega_{pe}$, $\Gamma_\circ = \gamma_\circ/\nu$, $\Gamma_E = \gamma_E/\nu$ and $\bar{\varkappa} = \varkappa(\omega_{pe}^2/c^2\nu) \simeq \beta = 8\pi p_e/B_\circ^2 << 1$. The condition for instability obtained from Eq. (14) is that

$\Gamma_\circ > (1+\eta_J)(\Gamma_E + K_\perp^2 \bar{\varkappa})$. Thus, for $\Gamma_\circ > \Gamma_{crit} = (1+\eta_J)\,\Gamma_E$ there is growth for $K_\perp < K_{\perp\,crit}$ where

$$K^2_{\perp\,crit} = \frac{\Gamma_\circ - \Gamma_{crit}}{\bar{\varkappa}\,(1+\eta_J)} \quad . \tag{15}$$

For $u_\circ \simeq v_e$, $\Gamma_\circ \simeq 1$ then $K_{\perp\,crit} \simeq 1/\beta^{1/2} >> 1$, and the growth rate reaches a maximum in the region $K_\perp > 1$ where

$$\Gamma \simeq \frac{(\Gamma_\circ - \Gamma_{crit})}{1+\eta_J+\Gamma_\circ+\Gamma_E}\left(1 - \frac{\Gamma_\circ - \Gamma_E}{K_\perp^2} - \frac{(1+\eta_J)\,\bar{\varkappa}\,K_\perp^2}{\Gamma_\circ - \Gamma_{crit}}\right) .$$

Thus, the maximum growth rate occurs for

$$K_\perp^2 = K^2_{\perp\,max} = \left[\frac{(\Gamma_\circ - \Gamma_{crit})(\Gamma_\circ + \Gamma_E)}{\bar{\varkappa}\,(1+\eta_J)}\right]^{1/2} \tag{16}$$

and is

$$\Gamma_{max} = \left(\frac{\Gamma_\circ - \Gamma_{crit}}{1+\eta_J+\Gamma_\circ+\Gamma_E}\right)\left[1 - 2\left(\frac{\bar{\varkappa}\,(1+\eta_J)(\Gamma_\circ + \Gamma_E)}{\Gamma_\circ - \Gamma_{crit}}\right)^{1/2}\right] . \tag{17}$$

Thus, for $\bar{\kappa} \simeq \beta << 1$ and $\Gamma_\circ > \Gamma_{crit}$ we have a maximum growth rate of order $\gamma \simeq \nu \left(\frac{u}{v_e}\right)^2 |\eta_T|$ occurring for wavenumbers in the range $1 < k_\perp c/\omega_{pe} < (\kappa)^{-1/4}$. The existence of the modes requires $\partial\eta/\partial T < 0$, and the modes are relatively insensitive to $\partial\eta/\partial j$ although a positive $\partial\eta/\partial j$ is seen from the above equations to have a stabilizing effect.

For a given amplitude of the temperature fluctuation, $\delta T/T$, we obtain from Eqs. (9), (10) and (11) the following relative field fluctuations for $K_\perp^2 \simeq K_{\perp\,max}^2 > 1$:

$$\frac{\delta j_\parallel}{j_\circ} \cong \left(\frac{-\eta_T}{\Delta}\right) \cdot \frac{\delta T}{T}$$

$$\frac{\delta E_\parallel}{E_\circ} \cong \frac{(\Gamma_\circ - \Gamma_c)\ \eta_T}{K_\perp^2\ \Delta} \cdot \frac{\delta T}{T}$$

$$\frac{\delta B_\perp}{B_\circ} \cong \left(\frac{\beta}{2}\right)^{1/2} \left(\frac{u_\circ\ \eta_T}{v_e\ K_\perp\ \Delta}\right) \frac{\delta T}{T} \tag{18}$$

where in the $K_\perp^2 > 1$ region the relative temperature and current variations are large compared to the relative electric and magnetic field variations. Here $\Delta = \Gamma_\circ - \Gamma_{crit} + (1 + \eta_J)(1 + \eta_J + \Gamma_\circ - \Gamma_E) \approx \Gamma_\circ$.

Considering now the cross-field electron dynamics given by Eqs. (5) and (8) we see that the magnetic field from the current filaments produces an $\underset{\sim}{E} \times \underset{\sim}{B}$ drift of the electrons which is uncompensated by ion drifts since $\gamma >> \omega_{ci}$. The electron drift from $E_\circ \times \delta\underset{\sim}{B}$ gives

$$\underset{\sim}{v}_E = \frac{c\, E_\circ\, \hat{e}_z \times (\hat{n} \times \nabla\psi)}{B_\circ^2} \simeq - \frac{c\, E_\circ}{B_\circ^2} \nabla\psi \tag{19}$$

and produces an inward drift of electrons in regions where $\delta j_\parallel > 0$ or a pinch effect. The charge separation produced by this drift builds up an electrostatic field, $\nabla \cdot (\partial \underset{\sim}{E}_\perp/\partial t) = - 4\pi e\, \partial\delta n_e/\partial t$, which in turn produces the additional electron drift $\underset{\sim}{v}_{pe} = - (m_e c^2/eB^2)(\partial \underset{\sim}{E}_\perp/\partial t)$. The ion current during this motion is $\underset{\sim}{j}_i = \frac{ne^2}{M\gamma} \underset{\sim}{E}$ since $\gamma >> \omega_{ci}$. Solving Poisson's equation for $\underset{\sim}{E}_\perp$ we obtain

$$\epsilon_\perp \frac{\partial \underset{\sim}{E}_\perp}{\partial t} = - 4\pi\, ne \frac{E_\circ}{B_\circ^2} \nabla_\perp \psi \tag{20}$$

where $\epsilon_\perp = 1 + \omega_{pe}^2/\omega_{ce}^2 + \omega_{pi}^2/\gamma^2$. As a consequence of this radial electric field in the current filament a rotational acceleration of the electrons develops according to

$$\frac{\partial \underset{\sim}{u}_{\perp e}}{\partial t} = \frac{c}{B_o^2} \frac{\partial \underset{\sim}{E}_\perp}{\partial t} \times \underset{\sim}{B}_o = \frac{\omega_{pe}^2}{\epsilon_\perp \, \omega_{ce}^2} \cdot \frac{eE_o}{m_e} \cdot \frac{\delta \underset{\sim}{B}_\perp}{B_o} \qquad (21)$$

Thus, particularly in the high density regime where $\omega_{pe} > \omega_{ce}$, under conditions where the nonlinear growth of the modes is sufficient for $u_{\perp e} \approx u_o \int_o^t \frac{\delta B_\perp}{B} \nu \, dt' >> c_s$ conditions in the filaments are produced for the appearance of cross-field ion acoustic turbulence on the microscopic λ_D-scale. In addition, the large temperature and current gradients across the current filaments produces conditions supporting various microscopic electrostatic gradient driven instabilities which can be expected to provide a limit to the growth of the filaments through the stabilizing effect of the anomalous thermal conductivity and viscosity.

III. Nonlinear Dynamics of Current Sheets

Questions such as the nonlinear limit of the growth and the effect of significant nonstationarity of the background on linear stability are very difficult to approach analytically in the original equations (4), (6) and (7). For the purpose of studying such questions we have simplified the problem to the one dimensional case of current sheets and solved the nonlinear equations numerically. It is readily verified that the stability theory of Sec. II for one dimensional perturbations remains unchanged except for the reduction of $k_\perp = k_x$ and $k_y = k_z = 0$. This similarity in the linear theory does not preclude, however, a substantial qualitative difference in the nonlinear dynamics of the one and two dimensional structures. In fact, we do expect such a qualitative difference due to the change in the force law of interaction between the structures and due to the change in the number of modes from $(a\omega_{pe}/c)$ to $(a\omega_{pe}/c)^2$.

We reduce Eqs. (4 - 8) to equations for fields that are only functions of (x, t) and transform the independent variables to $X = x\omega_{pe}/c$ and $\tau = \nu_\circ t$. Scaling the field variables to their initial values by

$$\hat{j} = j_{\|}/j_\circ \,, \qquad \hat{\tau}_E = \tau_E/\tau_E(T_\circ)$$

$$\hat{T} = T/T_\circ \,, \qquad \hat{\nu} = \eta(T,j)/\eta(T_\circ,j_\circ) \,,$$

$$\hat{E} = E_{\|}/E_\circ \,, \tag{22}$$

we obtain

$$\frac{\partial \hat{j}}{\partial \tau} = -\hat{\nu}\,\hat{j} + \hat{E} \tag{23}$$

$$\frac{\partial \hat{T}}{\partial \tau} = \alpha\,\hat{\nu}\,\hat{j}^2 - \beta\,\frac{\hat{T}}{\hat{\tau}_E} \tag{24}$$

$$\frac{\partial^2 \hat{E}}{\partial X^2} = \frac{\partial \hat{j}}{\partial \tau} = -\hat{\nu}\,\hat{j} + \hat{E} \tag{25}$$

where the constants α and β are

$$\alpha = \frac{\eta_\circ j_\circ^2}{\frac{3}{2}\, n_\circ\, T_\circ\, \nu_\circ} = \frac{2}{3}\left(\frac{u_\circ}{v_{eo}}\right)^2$$

$$\beta = \frac{1}{\nu_\circ\, \tau_E(T_\circ)} \,. \tag{26}$$

The boundary conditions for Eqs. (23 - 25) are that the initial ($\tau = 0$) fields $\hat{j}$, $\hat{T}$, $\hat{E}$ be given and that two boundary conditions be given on $\hat{E}$ for $\tau \geq 0$. In the solutions given below we have restricted consideration to the simple case of $E(X = 0, L; \tau \geq 0) = 1.0$ and have taken $L = 2a\omega_{pe}/c = 64$.

We begin by showing results obtained from applying perturbations to the uniform, stationary equilibrium which requires $\alpha = \beta$ in Eq. (24). The initial fields are $\hat{j} = \hat{E} = \hat{T} = 1$. In Fig.1 a gaussian perturbation with a 10% maximum amplitude is applied to the system with a model resistivity $\hat{\nu} = \hat{j}^2/\hat{T}$ and confinement time $\hat{\tau}_E = \hat{T}$. The system is linearly unstable and remains unstable at large amplitudes with no saturation mechanism arising within the system. We observe that no dominant Fourier mode develops from the many modes in the initial perturbations presumably due to the broad maximum of $\gamma(k)$ in the linear theory. The results of applying a sinusoidal perturbation are shown in Fig. 2 where $k_\perp c/\omega_{pe} = 1.0$. Fourier analysis in the nonlinear regime shows a weak first harmonic component $2k_\perp$ and agrees with a finite amplitude perturbation solution of Eqs. (23 - 25).

The effect of a significant nonstationary background in the system results when $|\alpha - \beta| \gtrsim \alpha$. In this case the background temperature rises or falls at the rate ν, and the result is found always to stabilize the system. In Figure 3 the same perturbation as in Fig. 1 is imposed on an "equilibrium" with insufficient heat transport by choosing the coefficients $\alpha = 2/3$, $\beta = 1/2$. Since we impose a constant induction field $\hat{E}$ at the plasma boundary, the global plasma heating is concentrated near the edges where a skin current of increasing magnitude is produced. The growth of the perturbation at the center of the system is seen to be slower than in the case of a stationary background, Fig. 1. The effect of a nonstationary background appears to limit substantially the possible occurrence of the mode in a short pulsed experiment where energy balance on periods long compared to ν^{-1} is not achieved.

As predicted by the linear theory we also observe that the numerical solutions are sensitive to the T-dependence of the energy sink and that an anomalous confinement law of the form $\tau_E^{-1} \propto T$ has a substantial stabilizing effect. The condition on the negative $\partial\eta/\partial T$ - gradient follows from combining the equilibrium condition (2) and the instability condition $\gamma_O > \gamma_E$, Eq. (17), to give

$$-\frac{d\ \ell n \nu(T)}{d\ \ell n\ (T/\tau_E)} > 1 \qquad (27)$$

for instability.

IV. Conclusions

We have shown that for certain forms of the anomalous collision frequency $\nu(T,j)$ produced by microscopic turbulence and energy confinement laws $\tau_E = \tau_E(T)$ the high frequency electromagnetic dynamics of the plasma is unstable to the formation of current filaments of scale c/ω_{pe}. Conditions for the growth of filaments are given and appear more difficult to satisfy if there is a strong. anomalous energy confinement law $\tau_E^{-1} \propto T_e$, Eq. (27). Also, during a short, pulsed system in which energy balance is not obtained on times long compared to ν^{-1} the secular change in time of the electron temperature overrides the instability.

The suggestion is, however, that during longer lived turbulent experiments the turbulent heating in parallel field experiments involves the interaction of electromagnetic current filament-thermal instabilities on the scales c/ω_{pe}, ν^{-1} and electrostatic instabilities on the Debye scale $\lambda_D = v_e/\omega_{pe}$ and $\gamma \sim \omega_{pi}, \frac{du}{dx}$. The growth of current filaments is shown to produce conditions for electrostatic instabilities driven by the strong cross field gradients in

$u_{\parallel}(x)$ and $T_e(x)$ and due to the cross-field acceleration of the electrons given by Eq. (21). This theory of turbulent heating could remove the qualitative discrepancy in the purely electrostatic simulation of the $E_o \parallel B_o$ experiments. The simulations show a limit of the heating to a factor of 2 or 3 times the initial temperature. The emergence of cross-field ion-acoustic turbulence from the growth of current filaments would prevent the saturation and predict a much higher increase in electron temperature. The magnitude of the relative magnetic field fluctuations are given in Eq. (18). In addition, a coherent set of such current filaments in a rotating plasma would produce a multipole radiator with frequency $\omega \cong k_\perp v_\theta$ where $v_\theta = cE_r/B$.

Acknowledgements:

The authors are grateful to Mrs. M. Walter for carrying through the numerical computations.

" This work was performed in part under the terms of the agreement on association between the Max-Planck-Institut für Plasmaphysik and EURATOM".

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Fig. 1

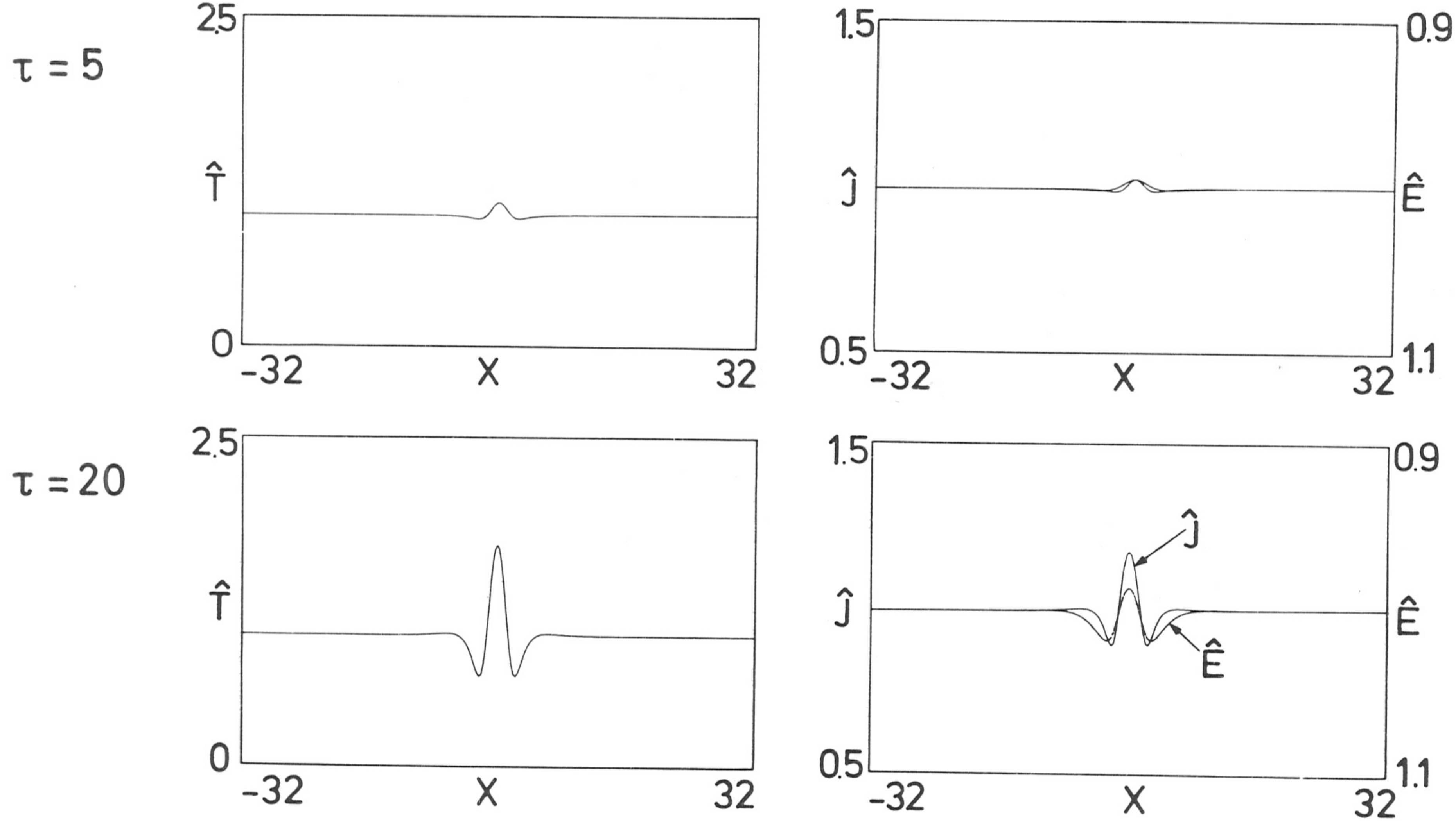

Fig. 2

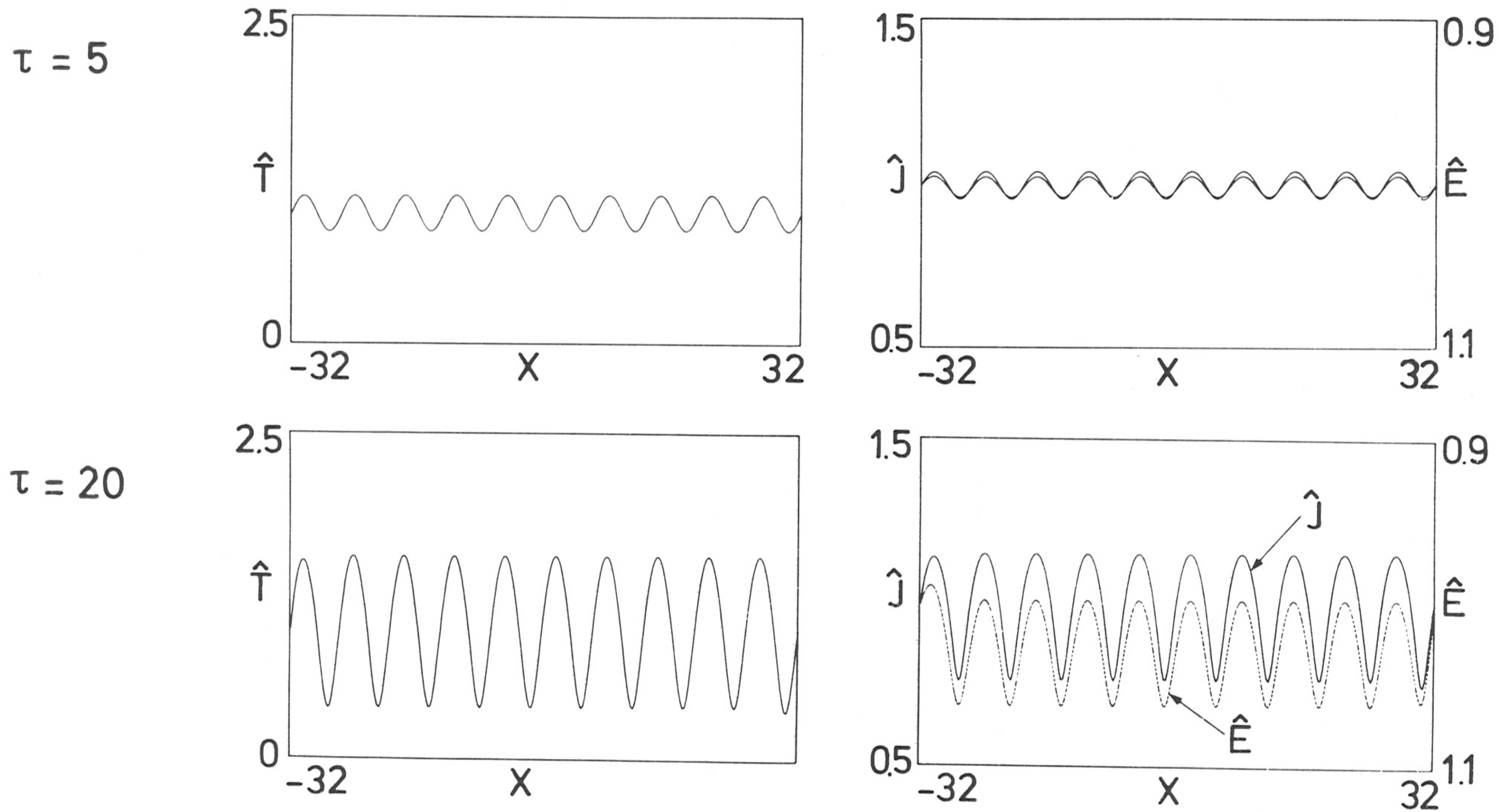

Fig. 3