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Equilibrium and Stability Investigations in the Belt
Pinch

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Abstract

Simple estimates show that tokamak confinement can be greatly improved by elongating the plasma cross-section in the direction parallel to the torus axis, provided that the safety factor q does not have to be increased beyond the value required for circular cross-sections. The equilibrium and stability of a plasma with elongated cross-section was therefore investigated in a toroidal belt pinch experiment.

It was found that the initial plasma ($kT_e \approx 20$ eV, $n_e \approx 10^{16} \text{ cm}^{-3}$) is not in equilibrium from the outset, but only attains it after $\sim 25 \mu\text{s}$ as a result of contraction in the axial direction. Magnetic field measurements in the plasma showed that the axial contraction does not terminate until the initial radial compression ratio $\kappa_d \approx 10$ has dropped to $\kappa_d \approx 2$, the axis ratio of the plasma cross-section then being ~ 5 .

Measurements of the electron temperature by laser light scattering showed that the plasma already cools to a quasi-stationary final temperature of 2 eV during the axial contraction. It was demonstrated that this fast drop in temperature is due to line radiation of oxygen impurities in the plasma.

Field measurements confirm that this cooling causes the toroidal main field to diffuse completely into the plasma already during the axial contraction. The plasma is then only confined in tokamak fashion by the poloidal magnetic field of the plasma current. The confinement of this relatively cold plasma is terminated after $\sim 100 \mu\text{s}$ by ohmic

damping of the plasma current. With time varying currents in the coil configuration it was possible to delay the decay of the plasma current. The elongation of the plasma cross-section in equilibrium was increased by applying suitable quasi-stationary currents.

In the experiment it was possible to raise the plasma current until a helical $m = 1$ mode with one wavelength on the torus circumference developed. Below this limit there were no instabilities that led to destruction of the confining field configuration. By artificial excitation it was possible to observe stable $m = 1$ oscillations below the stability limit, whereas there was corresponding unstable growth above the stability limit. This allowed the stability limit to be measured very exactly. With an elongation of the plasma cross-section of ≈ 5.5 the stability limit was found to be $q_{\text{crit}} = 2.4$.

1. Introduction and Experimental Objective

In 1968/69 scientists working on the Tokamak T - 3 at the Kurchatov Institute in Moscow were the first to succeed in confining a dense plasma with temperatures in the keV range for longer than 10 ms. This surprisingly good result prompted the building of tokamak experiments in fusion research laboratories all over the world. Today magnetic confinement of a deuterium-tritium plasma in a tokamak reactor is regarded as one of the most promising ways of gaining energy from controlled thermonuclear fusion.

In a tokamak the plasma is contained in a toroidal discharge vessel and ohmically heated by a circular current. The magnetic field of this current is essentially responsible for confining the plasma. The direction of this field is perpendicular to the toroidal plasma current, i.e. poloidal. In addition, the plasma is embedded in a strong toroidal magnetic field, the main field, whose field strength is only limited by technological factors.

Confinement has to be stable so that the plasma does not come into contact with the walls as it increasingly changes shape. A decisive stability condition here is the Kruskal-Shafranov limit, which postulates that the poloidal field may not exceed a certain fraction (about one-tenth being typical) of the main field strength. As the poloidal field is produced by the plasma current, this stipulates a maximum for the plasma current.

This limitation of the plasma current is the major obstacle to progressing from tokamak experiments to a tokamak fusion reactor. A higher plasma current would allow the ohmic heating input to the plasma to be increased and the simultaneous increase of the poloidal

field would lead to a reduction of the energy losses due to neo-classical thermal conduction, these being the most important loss mechanism in present experiments. It would thus be possible to extend the energy confinement time and further raise the plasma temperature.

In a tokamak, moreover, the condition for magnetic confinement that the plasma column be surrounded by closed field lines can only be satisfied if the gas kinetic plasma pressure does not essentially exceed the pressure of the small poloidal field. With realistic assumptions /1/ it is found that the plasma pressure may not be more than only about five per cent of the pressure of the total field. This restriction makes an economically operating fusion reactor practically impossible. If, however, the poloidal field can be increased relative to the main field, a radical improvement could be achieved.

In accordance with the Kruskal-Shafranov limit the maximum poloidal field in the tokamak depends on the geometrical shape of the plasma. In particular, the poloidal field should grow in proportion to the circumference of the plasma cross-section (for the same major torus radius). In conventional tokamaks the cross-section is always circular. For technical reasons their minor radius may not be much larger than about $1/3$ of the torus radius, and so the circumference is fixed. On the other hand, the circumference can, in principle, be extended indefinitely if deviations from the circular cross-section are allowed.

The possibility of improving confinement by deviating from the circular plasma cross-section has not been experimentally investigated very much as yet /2, 3, 4/. In the belt pinch experiment described here the plasma cross-section was therefore strongly deformed by

elongating it in the direction parallel to the torus axis.

The possible increase of the poloidal field due to increasing elongation is then expected to afford rapid improvement of the plasma parameters /5/. A decisive condition, however, for predicting these advantages of the elongated plasma cross-section is that the proportionality factor between the poloidal field and circumference of the cross-section (which is equivalent to the safety factor q in the tokamak literature) at the stability limit does not, in turn, depend on the shape of the plasma.

To check these predictions, first a plasma with elongated cross-section had to be produced and kept in equilibrium. The first part of this report therefore deals with the production and investigation of belt pinch equilibria.

The second part is concerned with checking the basic condition for predicting favourable properties resulting from elongated plasma cross-sections, namely that the safety factor at the stability limit is independent of the elongation. For this purpose the plasma current in a belt pinch equilibrium with strong elongation (axis ratio ~ 6) is raised to the stability limit and the corresponding safety factor measured directly.

2. Magnetic Confinement of the Plasma in the Belt Pinch

2.1 Fields and Currents in the Belt Pinch

Any toroidal vacuum magnetic field exhibits a $1/R$ dependence (R is the distance from the torus axis). The interaction of a diamagnetic plasma with the gradient of this field leads inevitably to an outward torus drift force, which is compensated by introducing additional magnetic fields. In the tokamak and belt pinch these additional fields are mainly produced by a toroidal plasma current. Since such a toroidal current alone would only intensify the drift force owing to the repulsive forces between antiparallel currents, another vertical field has to be superposed parallel to the torus axis. This field is aligned in such a way that it intensifies the field of the plasma current on the outer side of the torus and weakens it on the inner side. This exerts a force which compensates the drift force of the plasma. Figure 1 shows the poloidal magnetic field distribution in the belt pinch.

It can be seen that the plasma is confined in a toroidal coil system with rectangular cross-section. The long sides of the rectangle are formed by cylindrical coils in which toroidal currents flow. In addition, all the limiting surfaces of the torus carry poloidal currents, which produce the toroidal main field, not shown in the figure.

In this equilibrium the vertical field, whose strength is determined by the current in the outer cylindrical coil, is chosen such that the plasma is centred in the middle of the vessel. For this equilibrium the poloidal field on the outer side has to be larger than that on the inner side. This makes the magnetic force flux between the

Fig. 1: Field lines of poloidal magnetic fields for a belt pinch plasma with purely surface current¹⁾

plasma and coil wall greater on the outer side than on the inner side. As a result only some of the field lines can close upon themselves around the plasma, the others closing around the outer cylindrical coil. The boundary between these two

¹⁾ For calculation purposes the currents in the coil and plasma were merged to form discrete current filaments, in accordance with unpublished equilibrium calculations of W. Ochem.

kinds of field lines is called the separatrix.

In the circular area around which the toroidal plasma current is flowing magnetic flux is conserved for times short relative to the decay time of this current. Since all fluxes prior to the start of the belt pinch discharge are zero, the poloidal flux which closes around the plasma must be equal to the flux in the inner cylindrical coil.

This affords the possibility of increasing the poloidal field on the inner side of the plasma by raising the toroidal current in the inner cylindrical coil. To preserve the equilibrium, the vertical field generated by the current in the outer cylindrical coil has to be increased, too. The strength of the toroidal currents in the two cylindrical coils can thus be used to choose the magnitude of the mean poloidal field at the plasma surface and hence the toroidal plasma current. It is thus possible to vary a parameter in the experiment which is decisive for the stability behaviour of the plasma.

2.2 Limits on the strength of the plasma current

When the gas kinetic pressure of the plasma is raised, the torus drift force increases, and so the vertical field has also to be raised accordingly. Thus, at a constant plasma current, the separatrix keeps getting nearer the plasma surface. When it reaches the surface, the plasma is no longer magnetically confined. It can be shown /6/ that this imposes an upper limit on the poloidal beta. β_p is the ratio of the plasma pressure nkT to the pressure of the mean poloidal field B_p at the plasma boundary:

$$\beta_p = \frac{nkT}{B_p^2 / 8\pi}$$

The limit for β_p is only slightly dependent on the plasma current distribution and the shape of the plasma /6, 7/ and is always roughly equal to the aspect ratio $A = R/a$, where R is the major torus radius and a is the half-axis of the plasma cross-section in the torus plane:

$$\beta_p^{\max} \approx A$$

For a given plasma this equilibrium limit imposes a lower limit on the mean poloidal field at the plasma surface and hence on the plasma current.

In the belt pinch superposition of the toroidal main field B_t and the poloidal field B_p leads to flux surfaces formed by field lines running helically around the plasma. The field line helix is described by a parameter q , the safety factor. This q indicates how many toroidal turns a field line needs in order to close upon itself around the poloidal circumference once. Thus, $q = 2\pi/\zeta$ is the reciprocal of the rotational transform. If the fields on a flux surface are approximately constant, q can be represented as /8/

$$q = \frac{B_t}{B_p} \times \frac{\ell_p}{\ell_t}$$

where ℓ_t is the length of the minor torus axis and ℓ_p is the length of the poloidal circumference of the flux surface involved.

From experiments and theory (see Sec. 4.1) it is known that sufficiently stable plasma behaviour can only be achieved at q values near the plasma boundary of ≥ 3 .

For a fixed plasma shape and a fixed main field this Kruskal-Shafranov condition thus imposes an upper limit on the poloidal magnetic field and hence on the plasma current.

If the ratio of the plasma pressure to the pressure of the total field at the plasma boundary is defined as beta value

$$\beta = \frac{nkT}{(B_t^2 + B_p^2)/8\pi}$$

which may be one or less, the equilibrium and stability conditions can only be satisfied together if $\beta \leq \beta^{\max}$, where

$$\beta^{\max} \approx \frac{A}{1 + q^2 (\ell_t / \ell_p)^2}$$

For a fusion reactor the beta values have to be as high as possible because with given fields the fusion power increases as the square of beta, and since with beta values of only a few per cent it does not seem possible to keep the energy losses due to electron cyclotron radiation smaller than the heating of the plasma by fusion reactions.

For a circular plasma cross-section it holds that $\ell_t / \ell_p = A$. To obtain a large $\beta^{\max}$, A therefore has to be chosen as small as possible. For technical reasons A is limited to values greater than about three. For $q = 3$ and a circular plasma cross-section this yields a $\beta^{\max}$ of only 3.7 per cent.

This small value can now be appreciably raised by elongating the plasma cross-section. With a plasma cross-section having a large half-axis ratio b/a , the poloidal circumference $\ell_p \approx 4b$ depends almost solely on the large half-axis b , while the aspect ratio $A = R/a$ is governed by the small half-axis alone. The additional freedom in shaping the plasma cross-section thus allows the circumference ratio ℓ_t/ℓ_p to be chosen independently of the aspect ratio.

In the belt pinch described here circumference ratios of $\ell_t/\ell_p \approx 1$ are achieved. This should allow confinement of plasmas with extremely high β values, especially at large aspect ratios ($A \geq 10$). Of course, the safety factor necessary for sufficient stability ought not to be larger than in the case of circular plasmas.

2.3 Experimental Setup

Since plasma heating by fast magnetic compression is at present the only proved way of producing high- β plasmas, this experiment was built as a pinch.

The plasma is produced and confined in a toroidal coil with strongly non-circular cross-section. The dimensions of the coil for this belt pinch experiment are shown in Fig. 2. The coil has a rectangular cross-section with the axis ratio $e = 6.1$.

The coil aspect ratio is $A_s = (r_o + r_i) / (r_o - r_i) = 2.7$, where r_i and r_o are the inner and outer coil radii respectively. The discharge vessel consists of two coaxial "Duran" glass tubes and appropriate cover plates sealed vacuum-tight using a special adhesive.

Fig. 2: Dimensions of belt pinch coil

Electrically, the coil system consists of two separate circuits, the outer coil and the main coil with the corresponding capacitor banks as energy stores. Fig. 3 shows the basic setup.

Fig. 3: Layout and circuitry of the belt pinch

The outer coil produces the vertical field for the equilibrium. It also helps to induce the toroidal plasma current. To allow optical observation of the plasma, the coil consists of individual, two-turn wires. These are connected to a joint vertical collector. This collector acts as a low-inductance electrical contact for the parallel connected wires, and so this coil serves as a flux surface. During the plasma confinement time it may be regarded as impermeable to poloidal field lines.

In the main coil the current from the main bank at first flows on the outer side of the torus vertically upward, i.e. this direction is purely poloidal. The conductors are made of perforated sheet metal to allow observation of the plasma. In the cover segments the current flows radially inwards or outwards respectively and in the torus core it flows through single helical conductors.

This produces in the core both a toroidal and a poloidal current component. The poloidal component (straight down) produces the toroidal main field. The toroidal component induces additional plasma current, thus allowing the build-up of field lines which close poloidally around the plasma (see Sec. 2.1).

To vary the magnitude of the plasma current independently of the main field strength, it has to be possible to set the toroidal component in the core independently of the poloidal component. For this purpose an aluminium cylinder is inserted in the core. The toroidal mirror currents induced in it reduce the

effective toroidal coil current. The plasma can also be influenced by inserts of resistive material or by special shapes of inserts (Sec. 3.10, 3.11, 4.2).

In order to avoid short circuiting of the induced toroidal electric field strength, the main coil is divided by sixteen poloidal insulating slits. The currents in the two circuits are short circuited at the respective current maximum by crowbar spark gaps and then drop with a time constant which is long relative to the plasma lifetime. The technical data of the two circuits are presented in the table below:

Main bank (Isar IV) with main coil

Capacitance	150 μF		
Quarter-cycle	8 μs		
Crowbar time constant	700 μs		
Charging voltage	25	to	40 kV
Energy content	47	to	120 kJ
Toroidal field (on minor torus axis)	6,3	to	10,2 kG
Poloidal electric field strength	110	to	175 V/cm

Supplementary bank with outer coil

Capacitance	24 μF		
Quarter-cycle (with plasma)	6 μs		
Crowbar time constant (without plasma)	600 μs		
Charging voltage	15	to	30 kV
Energy content	2,7	to	10,8 kJ
Toroidal current (with plasma)	170	to	340 kA

3. Belt Pinch Equilibrium

3.1 Equilibrium calculations

Axisymmetric equilibria with toroidal net current and elongated plasma cross-section can be produced in two ways. In the free-boundary case the plasma is in a non-uniform magnetic field which is produced by distant conductors. This field has to be formed by superposing a vertical field and a quadrupole field.

In this case there is no interaction with the field producing conductors when the plasma is in motion, i.e. there is no wall stabilization at all. Mukhovatov and Shafranov /6/ show that pronounced deviations from the circular plasma cross-section call for such strongly non-uniform fields that a $m = 1$, $k = 0$ instability is inevitable.

In the second (fixed-boundary) case an electrically conducting wall which has to be close to the plasma prescribes a suitable outer flux surface for which there is then a corresponding elongated plasma equilibrium. Owing to the magnetic plasma-wall interaction this case is not necessarily unstable even with strong elongations.

Strongly elongated fixed-boundary equilibria require in all cases hitherto calculated a conducting wall which has to be very close to the plasma surface in the direction in which the plasma has its smallest diameter. Figure 4 from /9/ shows for a special equilibrium with purely surface currents the half-axis ratios

b/a of the elongated plasma cross-section which are obtained for various values of the radial compression ratio κ_a . Here $\kappa_a = 2d/2a$ is the ratio of the distance between the coil walls ($2d$) to the plasma diameter ($2a$) in the torus plane.

Fig. 4: Plasma axis ratio b/a as a function of the radial compression ratio κ_a

It can be seen that relatively strongly compressed plasmas ($\kappa_a \gtrsim 3$) are approximately circular ($b/a \approx 1$), while plasmas with weak compression ($\kappa_a \approx 2$) may be strongly elongated.

Although much effort has been expended (notably /9, 10, 11, 12, 13/) it has not yet been possible to find fixed-boundary equilibria in which strong elongation coincides with strong compression in the direction of the smaller half-axis. It is there-

fore improbable that such equilibria exist.

3.2 Onset of plasma equilibrium in the experiment

In the belt pinch the plasma is produced by fast magnetic compression. The processes involved in plasma production and heating take place rapidly because they are essentially governed by the strong toroidal main field ($B_t \approx 6 - 10$ kG). The shape which the plasma initially takes is accordingly also governed only by the shape of the discharge vessel as a boundary condition and by the rise of the main field, which influences the area compression ratio of the plasma.

By contrast, the equilibrium shape of the plasma is mainly produced by the poloidal field ($B_p \approx 0.5 - 2$ kG), whose pressure is one to two orders of magnitude smaller than that of the main field. These processes thus take place much more slowly.

Because of these different time scales it is not to be assumed that the initial shape of the plasma already corresponds to the equilibrium shape. In order to look at the expected slow transition to the equilibrium shape, the plasma was traced by observing it with image converter cameras.

The basic behaviour of the plasma in the belt pinch is recorded in the time resolved pictures in Fig. 5. The top end-on streak picture shows that there is a strongly radially compressed plasma (plasma thickness $2a \approx 2$ cm, radial compression ratio $\kappa_a \approx 9$) at the time of the field maximum ($t = 8 \mu s$).

At this time the height of the plasma belt ($2b \approx 90$ cm) is almost equal to the height of the coil, as the bottom side-on streak picture shows. Accordingly, the half-axis

Fig. 5: Streak pictures of belt pinch discharge in the standard case¹⁾

ratio of the plasma $b/a = \epsilon$ is still extremely large ($\epsilon \approx 50$).

As was shown in Sec. 3.1, it is not likely that a strongly elongated plasma with such a large χ_a is already in equilibrium. As expected, the belt pinch plasma therefore slowly moves into equilibrium by contraction in the axial direction. At the same time ϵ and χ_a get smaller until after about 25 μs the equilibrium state is achieved with $\epsilon \approx 4$ and $\chi_a \approx 2$.

The evaluation of the streak pictures shows that the cross-sectional area of the plasma doubles roughly up to the onset of equilibrium. This suggests that there has been strong

¹⁾ Filling pressure 20 mtorr deuterium; main field 6.3 kG; plasma current 225 kA

diffusion of the plasma across the confining field lines. Simple estimates show that classical field diffusion to this extent is only possible at electron temperatures of very few eV.

Another indication of strong field diffusion and correspondingly low temperatures is given by the diamagnetic signal¹⁾ (Fig. 6), which shows the plasma to be highly diamagnetic during the first few microseconds of the discharge. After about 15 μ s the toroidal flux displaced by the plasma drops to incommensurably small values.

Fig. 6: Diamagnetic signal in the standard case

¹⁾ The diamagnetic signal is a measurement of the toroidal magnetic field flux displaced by the plasma. $\Delta\phi = F_p \times \Delta B_t$, where F_p is the cross-sectional area of the plasma and ΔB_t is the difference between the toroidal field at the plasma boundary and the mean field in the plasma. If ϕ_0 denotes the toroidal flux of the vacuum field at the field maximum, $\Delta\phi / \phi_0$ is a measure of the diamagnetism of the plasma. It is thus possible to estimate the plasma temperature and β value (see /14/).

Once the plasma has attained equilibrium it is possible to use the decay time of the plasma current ($\tau \approx 45 \mu s$) to estimate the plasma resistance and hence the electron temperature. This yields $kT_e \approx 2.4 \text{ eV}$.

The initial plasma temperature that should be achieved by fast magnetic compression is estimated from the snowplough model /15/ to be $kT = 30 - 50 \text{ eV}$. It must thus be presumed that the plasma is already strongly cooled during the axial contraction.

In order to follow and clarify in detail the essential processes occurring during the axial contraction, the supposed cooling and the field diffusion and its effect on equilibrium, it was particularly necessary to measure directly the time development of the plasma temperature and of the magnetic fields in the plasma.

3.3 Measurement of the electron temperature and density by 90° laser light scattering

The principle of laser light scattering from plasmas has been copiously described, e.g. in /16/. The basic layout of the scattering experiment for measuring the electron temperature and density is shown in Fig. 7. The light source was a two-stage ruby laser operated in the Q-switch mode which generated pulses of approx. 300 MW with a length of 10 - 20 ns. The scattered light was analysed with a nine-channel polychromator and detected with multiplier tubes.

Stray light was strongly reduced by guiding the beam in diaphragm tubes terminated by quartz windows at the Brewster angle, carefully blackening the vacuum vessel and placing a beam stopper behind the exit diaphragm tube. Under normal con-

ditions the stray light component on the centre channel at $6943 \text{ \AA}$ was not much greater than the scattered component and so this channel could also be enlisted for evaluation.

Fig. 7: Layout of scattering experiment

The measured time development of the electron temperature for the standard case is shown in Fig. 8 (top curve). It can be seen that the plasma cools in $\sim 15 \mu\text{s}$ from an initial temperature $kT_e = 22 \text{ eV}$ to a final temperature of 2 eV .

It can be assumed that the ion temperature is equal to the electron temperature if the electron-ion relaxation time is short relative to the times in which the plasma

temperature appreciably changes. In this plasma the measured electron density n_e is initially $1.1 \times 10^{16} \text{ cm}^{-3}$ and drops during the cooling process to $5 \times 10^{15} \text{ cm}^{-3}$. From this it can be estimated that the relaxation time is a maximum of a few hundred nanoseconds. The condition $kT_e = kT_i$ can thus be regarded as very well satisfied.

Fig. 8: Time development of temperature in the standard case
Bottom curve: with degree of oxygen impurity doubled

3.4 Cause of plasma cooling

The rapid decrease of the plasma temperature to only 2 eV is most undesirable because this makes high- β confinement of the plasma by the toroidal main field practically impossible. The classical penetration time of the magnetic field over a distance a into a field-free plasma

$$\tau_{\text{diff}} \approx 3 \times 10^{-7} (kT_e)^{3/2} a^2 \quad (\text{s, eV, cm})$$

is only $\sim 3.5 \mu\text{s}$ at $kT_e = 2 \text{ eV}$ and $a = 2 \text{ cm}$. This means that for longer high- β confinement the cooling time has to be appreciably increased. A great deal of effort

was therefore made to clarify and, if possible, avoid the cooling mechanism.

At an electron temperature of 20 eV complete ionization of typical plasma impurities, especially oxygen, is not possible since the ionization times are already too long for medium stages of ionization ($\tau(0V \rightarrow 0VI) \approx 20 \mu s$, $\tau(0VI \rightarrow 0VII) \approx 200 \mu s$). These ions can therefore emit high powers as line radiation due to electron collision excitation.

In order to determine whether this mechanism plays a decisive role in the belt pinch, first the degree of oxygen impurity was determined under various conditions. This was done by measuring the intensity of the oxygen spectral lines while increasing the amount of O_2 added to the filling gas (deuterium) and extrapolating it to zero line intensity. In the standard case the degree of impurity is 1.5 (atomic) per cent. By adding 1.5 % oxygen the degree of impurity of the standard case was doubled to 3 %. The bottom curve in Fig. 8 shows the corresponding time development of the temperature, which was also determined by laser light scattering. It is obvious that the energy losses have strongly increased because the initial temperature is lower and the cooling already leads to the final temperature of 2 eV before the field maximum is reached (at $t = 8 \mu s$).

This suggests that the cooling is in fact caused by the line radiation of oxygen ions. This claim is confirmed by estimates of energy losses due to line radiation, as shown in the next section.

3.5 Energy balance for the cooling phase

The cooling of a plasma with a specific heat capacity $c_p = 5 n_e$ by radiation losses

with the power P_r (per unit volume) is given by

$$\frac{dT_e}{dt} = \frac{P_r}{c_p}$$

Cooling by collisional excitation of line radiation of an impurity ion species with the density n_o can be represented by /17/

$$P_r = F_o(T_e) \times n_o n_e$$

with a factor $F_o(T_e)$, which depends only on T_e and the ion species involved. This yields an inverse cooling time of

$$\frac{1}{T_e} \frac{dT_e}{dt} = n_o \times \frac{F_o(T_e)}{5 T_e}$$

The quantity

$$\frac{1}{T_e} \frac{dT_e}{dt} \times \frac{1}{n_o}$$

is then just a function of the electron temperature and the ion species considered.

In Fig. 9 this quantity is represented for the individual stages of ionization of oxygen, according to Engelhardt and Bateman /18/.

Fig. 9: Plasma cooling due to line radiation of oxygen ions

This representation is based on the radiation rates of DÜchs and Griem /17/, which may be incorrect by a factor of up to about three. An energy balance for the cooling phase can thus only be in the nature of an estimate.

During the cooling phase ($t = 5$ to $13 \mu s$) the plasma emits energy to the outside. In the same time interval its internal energy decreases owing to the temperature drop. In addition, there is energy input to the plasma as a result of several heating mechanisms. Energy conservation then requires that the algebraic sum of the input energies and change of the internal energy be equal to the energy output to the outside.

The following individual energy terms appear:

1. Cooling: In the cooling process an energy of $3/2 k \Delta T$ per particle is liberated.

With the temperatures known from laser light scattering an energy of 24 eV/particle is obtained.

2. Adiabatic heating from $t = 5 \mu s$ to the field maximum at $t = 8 \mu s$: It holds that $\frac{T_2}{T_1} = \left(\frac{B_2}{B_1}\right)^{4/5}$ for adiabatic compression of a field-free plasma in three degrees of freedom. The zero field condition is not satisfied in this plasma, but the result of 5 eV/particle should be sufficiently exact for an estimate.

3. Ohmic heating by the toroidal plasma current: The total energy of the plasma current $W = 1/2 L_p I_p^2$ is initially equivalent to ~ 50 eV/particle. With a decay time of $50 \mu s$ for the plasma current about 8 eV/particle falls to the defined cooling interval of $8 \mu s$.

4. Diffusion of the toroidal magnetic field into the plasma: When an external field diffuses into a previously field-free electrically conductive body, an

amount of heat equal to the energy of the penetrating field is liberated. In an initially field-free plasma with $kT = 22$ eV the complete penetration of the field into the plasma thus liberates energy of 22 eV/particle.

As the field in the plasma already accounts for $\sim 75\%$ of the external field prior to the start of the defined cooling interval (see Sec. 3.7), the energy density of the penetrating field is already equivalent to about half the energy density of the external field. As a result of the subsequent complete field diffusion during the cooling phase it is thus only possible for about half of the 22 eV/particle to be liberated. The resulting energy is therefore ~ 11 eV/particle.

The sum of the energy yielding terms 1 to 4 is ~ 48 eV/particle. On the other hand, radiation cooling acts as a loss mechanism: Since

$$W_{\text{rad}} = \int_5^{13 \mu\text{s}} kT_e \times n_o \left(\frac{1}{T_e} \frac{dT_e}{dt} \frac{1}{n_o} \right) dt$$

the inverse cooling times according to Fig. 9, which were globally evaluated for OIV, the oxygen species probably dominant for the plasma parameters here, yield a total emitted energy W_{rad} equivalent to ~ 95 eV/particle, using the measured electron temperatures and the measured oxygen impurities.

Despite the errors that have to be assumed for the radiation rates of oxygen ions, it is thus found that the line radiation of oxygen impurities is a loss mechanism

sufficient to account for the observed fast cooling of the plasma.

From Fig. 9 it is seen that the radiation cooling for the helium and hydrogen-like oxygen ions (OVII and OVIII) is a few orders of magnitude less than for the lower stages of ionization. Engelhardt and Bateman /19/ have calculated from several models that with the degree of impurity present here an initial temperature $k(T_e + T_i) \approx 120$ eV would be needed to ionize the oxygen to OVII before the plasma would be cooled by the radiation cooling of the lower stages of ionization. For technical reasons this condition could not be satisfied in the given experiment.

3.6 Effects of cooling on plasma confinement

After the plasma is cooled a quasi-stationary temperature of 2 eV sets in (Fig. 8), at which the energy losses of the plasma are covered by the ohmic heating of the toroidal plasma current. As was demonstrated (Sec. 3.4), such a plasma can be confined by the main field for a few microseconds at most. Confinement is then achieved in tokamak-like fashion just by the poloidal field of the plasma current. This situation should be maintained as long as the slowly decaying plasma current is sufficiently high to produce closed field lines around the plasma.

From experience it is known that outside the dense plasma column there is a thin plasma with high electrical conductivity. As this outer plasma has no appreciable gas kinetic pressure, currents in this region have to be force-free, i.e. flow parallel to field lines. In the belt pinch force-free currents in the region outside the separatrix are expected to decay quickly since all field lines intersect

here with the insulating wall of the vacuum vessel, thus interrupting the current path. Furthermore, thermal conduction parallel to the field lines should lead to strong cooling of this plasma and the electrical conductivity will be correspondingly low.

In order to obtain confirmation, it was necessary to make direct space and time resolved measurements of the magnetic fields in the belt pinch.

3.7 Magnetic field measurements in the plasma

The fields in the belt pinch were measured in the midplane of the torus with a probe containing two induction coils for simultaneous measurement of the toroidal and poloidal field components. Spatial resolution was achieved by radially shifting the probe from shot to shot.

Figure 10 gives the radial profile of the toroidal field B_t for various times. It can be seen that a small fraction of the external main field has already penetrated into the plasma after the first implosion ($t = 1.7 \mu s$). As the plasma continues to cool ($t \approx 10 \mu s$) there is then increased field diffusion and at $t = 15 \mu s$ the field profile is practically no longer any different from the $1/R$ decay of the vacuum field.

Figure 11 shows the time development of the main field in the centre of the plasma (B_t^{int}) and at the plasma boundary (B_t^{ext}) and also the poloidal field averaged over the value at the inner and outer plasma boundaries (B_p).

Fig. 10: Toroidal field B_t in the torus plane (standard case)

Fig. 11: Time behaviour of magnetic fields

It is again observed that the penetration of the main field into the plasma is very fast and the decay of the poloidal field is relatively slow.

Owing to the diffusion of the main field the β value on the magnetic axis drops in approx. 15 μs from initially 100 % to 2 % (Fig. 12).

Fig. 12: Time variation of β value in the standard case

The poloidal beta

$$\beta_p = \frac{nkT}{B_p^2 / 8\pi}$$

describes how high the plasma pressure is relative to the pressure of the poloidal field.

For reasons of pressure equilibrium it then holds for $\beta_p > 1$ that $B_t^{\text{int}} < B_t^{\text{ext}}$, and accordingly for $\beta_p < 1$ that $B_t^{\text{int}} > B_t^{\text{ext}}$.

In the standard case the gas kinetic pressure is very well known from the results of the laser light scattering, and the poloidal field from the probe measurements. It is thus possible to determine β_p . Once equilibrium is attained ($t = 25 \mu s$) it holds that $\beta_p = 1/2$. During the next $20 \mu s$ β_p increases to almost unity. This means that the toroidal magnetic field inside the plasma must be 70 G higher than at the plasma boundary at certain times. This field excess is too small to be detected directly by probe measurements.

The current density in the outer plasma in the region outside the separatrix can be determined from the space resolved field measurements. The toroidal current density is shown in Fig. 13. It can be seen that currents in this region are quickly damped, as expected.

Fig. 13: Time behaviour of the toroidal current density component in the outer plasma outside the separatrix

3.8 Field distribution after onset of equilibrium

At the end of the axial contraction the plasma attains its equilibrium shape, which it retains until the plasma current is fully damped after approx. $100 \mu s$. The measured profile of the poloidal field and the resulting plasma current distribution are shown in Fig. 14 for the start of this phase ($t = 30 \mu s$).

Fig. 14: Poloidal field and toroidal plasma current density in the torus plane after onset of equilibrium

It can be seen that the plasma current flows with a flat profile from the inner vessel wall to the separatrix, i.e. only in the region of closed field lines. In the outer region the currents have already decayed (see Fig. 13).

The radial compression ratio $\kappa_a = 2d/2a$ is 1.9. Here $2d$ is the radial distance between the coil walls and the plasma diameter $2a \approx 9$ cm is given by the distance

between the inner vacuum vessel wall and the separatrix. A highly elongated equilibrium with such a small radial compression is in agreement with calculated equilibria.

It is thus found in the experiment that a plasma initially highly elongated and strongly compressed in the radial direction contracts axially until a radial compression ratio of ~ 2 has been reached.

The fact that the current density at the separatrix steeply goes to zero at the outside shows that the separatrix acts like a non-material limiter which prevents currents flowing in the outer region. This is also seen from end-on streak pictures (see Fig. 5) in which the visible plasma is sharply limited outwards at the location of the separatrix.

3.9 Disturbance of the plasma fields by the probe

Owing to the size of the belt pinch plasma and the low temperatures the field and current distributions in the plasma do not seem to be disturbed by the presence of a probe, even after long times. There are several reasons supporting this:

1. There is no detectable influence of the probe on integral parameters measured such as the plasma current and the diamagnetic signal.
2. The position of the separatrix is calculated from the measured $B_p(R)$ profile (Fig. 14) by equating the fluxes left and right of the magnetic axis. The current density distribution can be obtained independently by differentiating the $B_p(R)$ curve. At practically the same point it goes sharply to zero.

This agreement indicates a certain consistency of the experimental values. In addition, streak pictures show a sharp boundary for the plasma glow at the same place.

3. There is agreement between the gas kinetic plasma pressure $p_{\text{gas}} = 2n_e kT_e$, where n_e and T_e were measured by laser light scattering, and the effective magnetic field pressure (Fig. 15):

$$p_B = \frac{1}{8\pi} (B_p^2 + (B_t^{\text{ext}})^2 - (B_t^{\text{int}})^2)$$

Fig. 15: Gas kinetic plasma pressure and magnetic field pressure

The deviation of the two pressures in the range 20 - 30 μs by a factor of up to two is within the error bounds for p_B . The deviation can be explained in principle by the fact that for all times $t \geq 15 \mu\text{s}$ it was decided to set $B_t^{\text{int}} = B_t^{\text{ext}}$ since probe

measurements fail to yield any difference between the two fields. In the experiment the poloidal beta β_p in this range will be smaller than unity (Sec. 3.7). It then holds that $B_{\theta}^{\text{int}} > B_{\theta}^{\text{ext}}$, where the difference between the two fields is smaller than the error bounds for the field measurement. With these fields the two curves would then coincide.

3.10 Elongation of the plasma cross-section in equilibrium

Side-on streak pictures show that the belt pinch plasma initially contracts in the axial direction. In order to demonstrate that the contraction involves not only the (optically visible) plasma mass but also the plasma current distribution, a space and time resolved measurement of the magnetic field component B_z alongside and in the direction of an axis z (sketched in Fig. 16) was made. Figure 16 shows the local dependence of B_z for various times.

It can be seen that the sign of B_z changes near the ends of the coils. The point where $B_z = 0$ corresponds to the point at which the separatrix penetrates the coil wall (see Fig. 1). Equilibrium calculations with diffuse current profile for the belt pinch configuration /20/ show that the upper and lower plasma boundaries coincide fairly well with the points of inflection of the $B_z(z)$ curve. With this condition it is possible to determine the magnetic height of the plasma as a function of time (Fig. 17).

The experimental points marked show that the height determined by field measurements agrees well with the height obtained from streak pictures.

Fig. 16: B_z at the inner coil wall

Fig. 17: Plasma height as a function of time

After equilibrium is attained the plasma height $2b$ is about 37 cm. Since the (minor) plasma diameter $2a$ in Sec. 3.8 has already been found to be ~ 9 cm, a half-axis ratio $b/a \approx 4$ is obtained for the equilibrium state.

3.11 Influencing the equilibrium shape of the plasma

As was shown in Sec. 3.7, in the belt pinch the initial highly radially compressed and highly elongated plasma contracts until the compression ratio becomes $\kappa_a \approx 2$. If the cross-sectional area is assumed to be constant during contraction, the initial plasma of the standard case with $2b \approx 90$ cm and $2a \approx 2$ cm would then contract to an axis ratio $b/a \approx 1.9$. The final axis ratio $b/a = 4$ is, however, much higher owing to the increase of the cross-sectional area due to field diffusion.

The field diffusion is a result of the rapid cooling of the plasma due to oxygen impurities, and so it can be expected that an increase of the cooling time will lead to stronger axial contraction.

Measurements have shown that the oxygen impurities enter the plasma from the vessel wall during preionization. It was therefore possible to reduce the degree of impurity of 1.5 % in the standard case by optimizing the preionization to 0.4 %.

This should roughly triple the cooling time of the plasma. Figure 18 shows the diamagnetic signal for this and the standard case.

It can be seen that the high- β confinement by the main field is maintained for $\sim 30 \mu s$. It is thus concluded that for this range of time the temperatures are higher and the field diffusion correspondingly smaller.

Fig. 18: Diamagnetic signal in the standard case and with reduced degree of oxygen impurity

As the side-on streak picture (Fig. 19) shows, the plasma contracts much more strongly than in the standard case (Fig. 5), as expected. It can be seen that the minor diameter $2a$ of the plasma hardly increases during contraction (end-on streak picture in Fig. 19).

Owing to the strong contraction some stability limit whose nature is not understood is exceeded and the plasma "bursts" apart. After this unstable phase the individual plasma strands merge together and form a low- β plasma subjected to tokamak-like confinement.

In order not to achieve highly elongated equilibria by field diffusion alone, numerous efforts were made to increase the axis ratio of the plasma cross-section by means of suitable additional fields. For this purpose additional currents were applied in

the coil configuration to exert forces on the toroidal plasma current as a mean of pulling the plasma apart in the axial direction.

First these currents were produced by inserting two short metal cylinders into the core of the torus coil (Fig. 20). As the belt pinch fields build up toroidal mirror currents are induced in these cylinders (~ 37 kA/cylinder) which flow parallel to the plasma current and therefore attract it. It can be seen from the streak pictures in Fig. 20 that these forces lead to an axial $m = 1$, $k = 0$ instability. The cross-section is not noticeably more strongly elongated, but the plasma as a whole moves towards one of the two attracting currents. The transition from the top to the bottom picture was achieved by shifting the two metal cylinders only 10 mm upwards. This instability can be interpreted as the predicted unstable mode (Sec. 3.1) of the free-boundary equilibrium.

The instability can be avoided if the two cylinders are not placed too far from the midplane of the torus (Fig. 21). In this case the elongation of the plasma cross-section gradually increases after termination of the axial contraction until the plasma occupies the entire vacuum vessel in the axial direction.

In Figs. 20 and 21 the attracting forces exerted by the metal cylinders are not sufficient to influence the axial contraction noticeably. In order to avoid the contraction itself, it is apparently necessary to have much higher external currents at the ends of the torus core. To produce these currents, a new core was developed with which toroidal currents could be actively fed to suitable coils.

Fig. 19: A plasma with excessive axial contraction becomes unstable and "bursts".
Top: end-on streak picture; bottom: side-on

Fig. 20: Attractive forces are exerted on the plasma by metal cylinders in the torus core. Here the plasma moves in an unstable manner towards one of these centres of force

Fig. 21: The instability is avoided by bunching the metal cylinders. The forces now cause a gradual increase in the height of the plasma belt.

Fig. 22: Prevention of axial contraction by actively fed additional currents. Top: without additional currents; bottom: 2×230 kA

The position of these windings is sketched in Fig. 22. From the streak pictures it can be seen that the axial contraction was avoided. The current in each winding (230 kA) is thereby practically just as high as the plasma current. With such a large disturbance of the original belt pinch fields, however, the plasma is not expected to be still surrounded by closed field lines. A new separatrix extending to the plasma will be created.

3.12 Prolongation of plasma confinement by subsequent induction of plasma current

The plasma current is induced while the fields are being built up and therefore attains its maximum at the crowbar time ($t = 8 \mu s$). In the toroidally short-circuited plasma there is flux conservation for times that are small relative to the decay time of the current: $L_p I_p = \oint_p = \text{const.}$ As a result of the axial contraction L_p increases, and so I_p quickly drops at first. After axial equilibrium has been attained the plasma current decays more slowly as a result of ohmic damping with a time constant $\tau \approx 45 \mu s$.

As the decay of the plasma current puts an end to confinement, it is most desirable that it be maintained for longer times. In the belt pinch this is made possible by a technically very simple procedure¹⁾. This consists in replacing the aluminium cylinder in the core of the torus with one of insulating material whose surface is given a defined conductivity by winding resistive wire around it. In the experiment the surface resistance was chosen such that the initially induced mirror currents are damped with an L/R time constant of $50 \mu s$. While these currents are decaying, they produce a peripheral voltage $\dot{\oint}$ in the direction of the plasma current. This

¹⁾ A similar circuit has already been used in the TEE experiment at Jülich /21/.

slows down the natural decay of the plasma current.

Fig. 23 shows the measured time development of the peripheral voltage $\dot{\phi}$ on the resistive core and the plasma currents with aluminium and resistive cores.

Fig. 23: Slowing down the decay of the plasma current with a resistive core

The coil configuration used here was not optimized for operation with a resistive core. In principle, the plasma current can be maintained much longer in the same way.

4. Stability of Belt Pinch Equilibrium

4.1 Stability limits

As stability calculations for the belt pinch are still in their infancy, they do not yet yield such a complete picture as the theories for tokamaks with circular cross-sections. As is well known, the stability behaviour in this configuration is described by the safety factor $q = 2\pi/\iota$, the reciprocal of the rotational transform. The plasma is stable for modes with the poloidal wave number m if $q > m$ (Kruskal-Shafranov limit [22, 23]).

The modes $m \geq 2$ are unstable in a range $q < m$, whose size is highly dependent on β and the plasma current distribution. The unstable range may even vanish. Tokamaks work in a sufficiently stable manner with a typical value of $q = 3$ at the plasma boundary.

The simplest assumption for predicting the stability behaviour of the belt pinch plasma is to use the same critical q values as stability limits. As is to be expected, for the belt pinch as well detailed determination of stability is strongly dependent on the assumed current profiles. Laval et al. [24], for example, again obtain for a flat current profile with elliptical cross-section for $m = 1, 2$ and 3 the upper stability limit $q = m$, but the unstable regimes $q < m$ are very wide and overlap with increasing ellipticity. With a bell-shaped current profile [25], on the other hand, the unstable domains are narrow and remain separate.

Stability calculations for pure surface currents in the case of elliptical [26, 27/

and arbitrary /28/ plasma cross-sections have been made. It was thereby found that the critical q value q_{crit} for complete stability to all kink modes in the tokamak limit ($B_t^{\text{int}} = B_t^{\text{ext}}$) has to be strongly increased with increasing ellipticity /27/:

$$(q_{\text{crit}} - 1) \propto \left(\frac{b}{a} - 1 \right)^{5/3}$$

The unstable deformation of the plasma at the stability limit is formed by superposing numerous m modes. With increasing ellipticity there is a marked increase of the harmonic content. This fine structure of the plasma surface should, however, react sensitively to profile effects and possible stabilization by finite gyroradii. As yet there are no estimates available.

Freidberg and Haas /26/ obtain as an optimum a maximum of $\beta = 0.37$ (for $q \rightarrow \infty$) for an axis ratio of $b/a = 2.2$. This is a factor of about two better than for the circular cross-section. As $q = \infty$ is not possible in the torus for reasons of equilibrium, the gain for β compared with the circular case is reduced to about 30 % for $b/a = 2$ /27/. With increasing ellipticity these values again deteriorate. This deterioration can be avoided by means of the racetrack plasma shape¹⁾ /28/. Wall stabilization does not become effective until unrealistically low compression ratios ($\kappa \approx 1.1$) are reached /27/.

For a rectangular, linear fixed-boundary equilibrium with a current profile gently rising towards the magnetic axis G. Bateman et al. /29/ calculate growth rates

¹⁾ In the racetrack shape the cross-sectional area of the plasma consists of a rectangle (width $2a$) closed at the top and bottom by semi-circles (radius a).

for the individual m modes. It is found that for each mode considered separately the critical q value is almost independent of the axis ratio b/a ²⁾. Accordingly, β and j increase on the axis. However, with increasing b/a for the same q previously stable modes with higher m numbers become unstable. If stability to all m modes is postulated, the result is that an increase of j beyond the value of the case with $b/a = 1$ is not possible.

It is an open question which can probably only be finally clarified in an experiment whether unstable higher m modes have the same disturbing effect on confinement as the $m = 1$ mode, or whether these modes are sufficiently well stabilized by non-ideal MHD effects. If this is the case, then according to this calculation q_{crit} need not be increased with increasing elongation. It is then possible by elongating the plasma cross-section to achieve a major improvement in confinement, as is shown in Appendix A.

A simplifying summary of the most important stability theories considered here is presented in Fig. 24 according to /38/.

In a surface current model with elliptical cross-section in which full allowance is made for toroidal effects Freidberg et al. /26/ take into account all kink modes and obtain a q value (curve F_1) rising steeply with the axis ration under the condition that $b/a \approx 1$ for $\beta = 0$. With increasing beta ($F_2 : \beta = 10\%$, $F_3 : \beta = 12\%$)

²⁾ It appears that this statement is also dependent on the type of plasma current distribution. For a rectangular current profile the critical current density for the presence of the $m = 1$ mode remains constant with increasing b/a . This means that q_{crit} increases with b/a /30/.

Fig. 24: Critical q value as a function of the axis ratio b/a for various stability calculations

$F_1 - F_3$ acc. to J.P. Freidberg et al. /26/, for $A = 3$ and elliptical plasmas,

$F_1 : \beta = 0$, $F_2 : \beta = 10 \%$, $F_3 : \beta = 12 \%$

$M_1 - M_4$ acc. to B.M. Marder /28/, for elliptical (M_1 and M_2) and racetrack

plasmas (M_3 and M_4), M_1 and $M_3 : \beta = 0$, M_2 and $M_4 : \beta = 10 \%$

$B_1 - B_3$ acc. to G. Bateman et al. /29/;

$B_1 : A = 3$, only $m = 1$ instability; $B_2 : A = 1$, only $m = 1$ instability;

$B_3 : A = 1$, $m = 1, 2, 3$ instability

the range of b/a in which stability is possible at all quickly narrows. There is a maximum β value which at $A = 3$ is $\beta^{\max} = 12 \%$. At this β value the plasma is

only stable for an optimum axis ratio $b/a = 2.2$ for $q \approx 3$ (F_3). In a linear surface current model in which toroidal effects are only roughly taken into account "ad hoc" B. M. Marder /28/ investigates different plasma shapes. In this model there is no restriction on b/a . For elliptical cross-sections there is agreement with F_1 for $\beta = 0$ (curve M_1). Because of the different conditions prevailing this agreement with Freidberg's calculations is no longer obtained at higher β values, as can be seen from curve M_2 ($\beta = 10\%$) in contrast to F_2 . With racetrack plasma shapes ($M_3 : \beta = 0$, $M_4 : \beta = 10\%$) the q values for large axis ratios are well below those of the comparable elliptical plasma shapes (M_1 and M_2). This shows that, at least for this model, details of the plasma shape may have a decisive influence on the stability behaviour.

In the volume current model of G. Bateman et al. /29/ the critical q value is almost independent of the axis ratio for every single m mode. Curve B_1 shows the stability limit for $m = 1$ at $A = 3$. For smaller aspect ratios the stability limit shifts to somewhat higher q values, as curve B_2 shows for $A = 1$. At the same aspect ratio there are also higher m modes. If $m = 1, 2, 3$ is taken into account, curve B_3 is obtained as stability limit.

Various authors determine the stability of an elongated plasma to localized modes by applying the Mercier criterion. For complete stability this criterion is only necessary. For special analytical equilibria Herrnegger /31/ shows the Mercier criterion to be satisfied in the entire plasma volume. Laval et al. /32/ show that in the vicinity of the magnetic axis the stability limit for β and j can be extended beyond that for the circular case by slight triangular deformation. By applying

an additional sufficient criterion Lortz and Nührenberg /33/ prove that the stability limit in the vicinity of the magnetic axis is highly dependent on the current profile. For a diamagnetic plasma they calculate a q limit of ~ 0.7 ($b/a \rightarrow \infty$). This value near the axis is thus below the limit which is prescribed at the plasma boundary by the stability to kink modes. It thus appears that with a suitable plasma shape and plasma current distribution local modes do not impose any severe limit to belt pinch confinement.

4.2 Stability behaviour of the belt pinch plasma in the experiment

1. Axial $m = 1$, $k = 0$ mode

The toroidal equilibrium of the belt pinch plasma is created by the interaction of the plasma current with the vertical field B_z , which is produced by the outer coil. Because of the finite length of this cylindrical coil the vertical field is non-uniform and the field lines curve away from the torus axis. The field index $n = - (R/B_z) (\partial B_z / \partial R)$ is therefore negative. The stability condition $0 < n < 3/2$ (/6, 34/) is thus violated. The plasma must therefore be expected to be unstable with respect to displacement in the axial direction ($m = 1$, $k = 0$). The side-on streak picture of the standard case (Fig. 5) shows that this instability does in fact occur. By placing a suitable short-circuit ring near the end of the belt pinch from which the plasma moved away it was always possible to keep the initial disturbance of this instability so small that this slowly growing mode did not lead to wall contact during the lifetime of the plasma. The growth rate is increased when the unfavourable curvature of the vertical field lines is intensified by additional currents in the core (Fig. 20, Sec. 3.11).

Fig. 25: Excitation of an axial $m = 1$, $k = 0$ oscillation

The mode can be stabilized if additional forces are exerted on the plasma to pull it back into the torus plane. Such attractive forces can be produced by having a toroidal current flow parallel to the plasma current in the middle of the coil core. In the experiment this current is simply produced by inserting in the core a short metal cylinder (Fig. 25) in which toroidal surface currents are induced while the fields are being build up. A slight displacement of the cylinder from the torus plane even excites this mode to oscillate. Stability is thus positively proved.

2. Helical filaments

Particularly in discharges with high filling pressure (30 - 50 mtorr deuterium) there are helical structures present in the plasma after the field maximum (Fig. 26).

These filaments again disappear during the axial contraction and the accompanying increase of the radial plasma dimension. The pitch of the filaments is equivalent to that of the field lines on the outer side of the plasma at the time of their formation. There is no indication that the filaments are also present on the inner side of the plasma with a correspondingly different pitch. The development of the filaments is independent of the magnitude of the plasma current. As the filaments run parallel

to the field lines, their toroidal wave number corresponds to the q value ($q \approx 3 - 5$).

Fig. 26: Helical filaments in the belt pinch, side-on framing pictures

The nature of this instability is not yet finally clarified. As the filaments only appear on the outside, in the region of unfavourable field line curvature (the field strength decreases with increasing distance from the plasma) and only at large density gradients, it is probably local surface modes of the flute type that are involved.

Similar structures have already been observed by Düchs et al. /35/ and connected with disturbances in the preionization phase. This is improbable in the case of the belt pinch since optical investigation shows the preionization to be completely uniform in the axial direction.

3. $m = 1$ mode with $k = 1/R$

Long-wave length $m = 1$ modes are particularly dangerous for plasma confinement since these are not expected to be stabilized by finite ion gyro radii. At high

plasma currents these modes also occur in the belt pinch. An example is given in Fig. 27. The helical filaments described can again be seen in the second picture, after which there is increasing tilting of the plasma belt. This is equivalent to a helical $m = 1$ mode with one wavelength on the torus circumference, i. e. $k = 1/R$.

Fig. 27: $m = 1, k = 1/R$ instability at a plasma current of $I_p^{\max} = 540$ kA, side-on framing pictures

This mode is of particular interest since with a gradual increase of the plasma current in the experiment it is the first mode whose growth evidently leads to destruction of the field configuration and hence to termination of plasma confinement. Under the conditions prevailing the general stability limit for an increase of the plasma current is thus equal to the stability limit for this special mode.

In the following the plasma is therefore experimentally investigated near the stability limit, and the safety factor q at the stability limit is determined.

4.3 Artificial excitation of the helical $m = 1$ mode

To determine the stability limit, it is necessary to recognize the transition from the stable to the unstable regime when the plasma current is increased. Instability can be detected by the growth of a deformation. If, however, the growth rate becomes comparable with the available observation time, it is no longer possible to draw a clear distinction between instability and stability.

In order to obtain direct proof of not only instability but also stability, one can induce artificial deformation of the plasma and observe whether this disturbance oscillates indicating stability or grows in an unstable manner. In the experiment the helical $m = 1$, $k = 1/R$ mode was therefore excited to produce a defined initial disturbance.

Fig. 28: Position of winding for producing the initial disturbance and its current $I_a(t)$

Figure 28 shows the position of the auxiliary coils through which a current pulse $I_a(t)$ is passed during the preionization phase. The diamagnetic preionization plasma is repelled by the non-uniform field of this current. In the main discharge

Fig. 29: Development of the helical $m = 1$ mode in the stable case (A) and unstable case (C), with and without artificial excitation

the density distribution is therefore disturbed. This initial disturbance, which contains the desired $m = 1$ component, is independent of the polarity of the excitation current.

Figure 29 presents side-on framing pictures for a case A in which the plasma current is just below the limit where instability sets in and a case C in which the limit is just exceeded.

It is obvious that in the stable case A the excited mode undergoes damped oscillation, while in the unstable case C it first continues to grow. In the late phases of the unstable discharge C the magnitude of the $m = 1$ disturbance becomes stationary and even declines slightly. This suggests that in case C the plasma changes after some time from the initially unstable state to the stable state.

In order to find out more about the time development of the stability behaviour, a quantitative measure of the magnitude of the $m = 1$ elongation was introduced and the time behaviour of this magnitude was investigated.

4.4 Time development of the helical $m = 1$ disturbance

The magnitude of the $m = 1$ disturbance is characterized by means of the tilt angle ξ (see Fig. 30), which was determined directly from framing pictures.

Figure 31 represents the time behaviour of the tilt angle $\xi(t)$ for three cases A, B and C. The three cases differ in the magnitude of the plasma current, which is

Fig. 30: Definition of the tilt angle ξ

lowest in the stable case A. In the transition case B the current was raised by 8 % and in the unstable case C by 17.5 %.

Figure 31 shows that the three cases differ a great deal from one another in their stability behaviour despite the slight change in the plasam current. It can be seen that the presence of an artificial $m = 1$ disturbance leads in all cases at the time $t = 10 \mu s$ to an inital tilt angle $\xi \approx 10^\circ$. In the stable case A there then starts a damped oscillation about the equilibrium position at $\xi = 0^\circ$. The force producing the acceleration ξ is always directed to the equilibrium position. This means that the plasma is always stable in case A.

In case C the accelerating force is initially directed away from the equilibrium position, i.e. the plasma is unstable. At the time $t = 38 \mu s$ the $\xi(t)$ -curve has a point of inflection (arrow), which indicates a change of sign on acceleration. For times $t > 38 \mu s$ the plasma is thus stable in case C.

Fig. 31: Time development of the helical $m = 1$ disturbance at various plasma currents

In case B with artificial excitation of the $m = 1$ disturbance the curve corresponds qualitatively to case C. The unstable acceleration, however, is less at the beginning and the inflection point is reached earlier, at the time $t = 28 \mu s$.

For case B it was found in the experiment that by chance there is also distinct tilting without artificial excitation. The relevant $\xi(t)$ -curve is given in Fig. 31. Here, too, there is an inflection point (arrow), which indicates the transition to the stable regime. The time of the transition agrees with that in the case of artificial excitation.

To compare the observed stability behaviour with the theory, it is necessary to know the plasma parameters, especially the plasma shape, safety factor and β value at the respective stability limit. These values were therefore determined.

4.5 Measurement of the most important plasma parameters at the stability limit

The cases A, B and C investigated differ from the standard case only in a higher filling pressure ($p = 35$ mtorr instead of 20 mtorr) and a higher plasma current ($I_p^{\max} = 380$ kA in case A instead of 225 kA), and so the most essential results of the investigation of the standard case can be utilized. In particular, it may be assumed that in these cases, too, the plasma cools during the axial contraction to a few eV, that the toroidal field therefore completely diffuses into the plasma, and that after axial contraction the radial current distribution extends with a flat profile from the inner vessel wall to the separatrix.

The increase of the plasma current on transition from case A to case B or C was achieved in the experiment just by slightly raising the toroidal current in the

outer coil. This causes the position of the separatrix to shift each time a few millimetres in the direction of the torus axis. The exact position of the separatrix was determined from flux and field measurements outside the vacuum vessel. The diameter of the cross-sectional area of the plasma in the torus plane after axial contraction (a typical value being $2a = 9.5$ cm) is then obtained from the distance between the inner vessel wall and the separatrix.

The height of the plasma belt (after contraction $2b \approx 55$ cm) was determined in case A by measuring $B_z(z)$ (see Sec. 3.10). The optical determination of the height is very inaccurate in the case of these diffuse plasmas, but within the error bounds it agrees with the result of the field measurements. In cases B and C the plasma contracts slightly more strongly, this being concluded from the fact that the measured poloidal field in the torus plane at the coil boundary (+ 13 % (B) and 26 % (C)) grows more strongly than the plasma current. As the plasma thickness $2a$ and the plasma height $2b$ decrease by practically the same factor (~ 9 %), there is no change in the axis ratio $b/a = \epsilon$. The time development of ϵ is represented in Fig. 32.

The q values at the plasma boundary were determined according to

$$q = \frac{B_t}{B_p} \times \frac{\ell_p}{\ell_t}$$

The length of the poloidal circumference can be approximated by $\ell_p = 2(2a + 2b)$, the toroidal circumference $\ell_t = 2\pi R$ is given by the major plasma radius R and the main field B_t by the value of the vacuum field on the radius R .

The poloidal field B_p was measured directly at the coil boundary in the torus plane. It is only at times $t \leq 15 - 20 \mu s$ that the experimental values for B_p at the inner and outer coil boundaries differ appreciably. In these cases both values were averaged. This yields the time development of the q value at the plasma boundary. It is represented in Fig. 32 for cases A, B and C.

Fig. 32: Axis ratio ϵ of cross-sectional area of plasma and q values at the plasma boundary

In Sec. 4.4 the times are determined at which the plasma in cases B and C changes from the unstable to the stable regime. These times are marked in Fig. 32 by arrows. It can be seen that in both cases the transition occurs at the same q value. This stability limit $q_{crit} = 2.4$ was marked.

Since

$$\beta = \frac{\beta_p}{1 + (B_t/B_p)^2}$$

the β value of the plasma can be determined from field measurements at the coil boundary. Under the assumption $\beta_p = 1$ one obtains for $t = 35 \mu s$

in case A $\beta = 7.5 \%$

in case B $\beta = 9.6 \%$

in case C $\beta = 11.3 \%$.

As the plasma current is slowly damped, these values decrease accordingly. The condition $\beta_p = 1$, however, is not satisfied equally well in the three cases. From the diamagnetic signal it is possible to determine a mean elevation of B_t in the plasma region and calculate β_p on the axis if a rectangular profile is assumed. For $t = 30 - 40 \mu s$ one obtains

in case A $\beta_p \approx 1$

in case B $\beta_p \approx 0.7$

in case C $\beta_p \approx 0.5$.

As the magnitude of β_p is determined by the equilibrium of ohmic heating and energy losses /36/, the change of β_p that occurs when the stability limit for the helical $m = 1$ mode is approached and exceeded is perhaps an indication that the plasma energy losses rise sharply at the same limit.

4.6 Possible measuring errors in determining the stability limit

The determination of the q values is appreciably influenced by the inaccurate measurement of the plasma height and by calibration errors in the field measurements. Owing to the flat $q(t)$ curve the uncertainty in determining the time at

which the plasma changes from the unstable to the stable regime is not critical. These obvious experimental errors should not lead to an inaccuracy of the q_{crit} value of more than about $\pm 20\%$.

It is uncertain whether the same q_{crit} value remains valid for plasmas with higher temperatures as well. In principle, it is possible that some of the disturbed current components occurring are damped in times comparable with the growth rate of the instability. Such an effect, however, should influence mainly the growth rate of the instability, and less the marginal point which was determined here. Furthermore, the fact that genuine oscillation (and not a case of aperiodic creeping) was observed in case A indicates that the plasma can move relatively undamped.

4.7 Comparison with theoretical predictions

In the surface current model of Marder /28/ it is expected for the axis ratio $b/a = 5 - 6$ involved here and $\beta \approx 5\%$ that

$$q_{crit} = 12 \text{ (elliptical cross-section)}$$

$$q_{crit} = 3.4 \text{ (racetrack plasma shape).}$$

The experimental $q_{crit} = 2.4$ is well below these values. It seems, however, that the observed $m = 1$ mode does not necessarily correspond to the instability which is calculated in /28/ because the latter should consist mainly of a superposition of higher m modes and only contain $m = 1$ as one component. Discrepancies between this theory and the experiment are not surprising because the assumption made for the current profile is not satisfied in the belt pinch and the exact shape of the flux surfaces is unknown.

In the volume current model of Bateman et al. /29/, the conditions there being very close to those in this experiment, the observed mode corresponds to an $m = 1$, $ka = 1/4$ case, for which a q_{crit} of a little over one has to be extrapolated. The postulated equilibrium is linear and free of poloidal currents, i.e. $\beta_p = 1$, while the real plasma is toroidal and it holds here that $\beta_p < 1$. As such a strong toroidal influence would be surprising at a plasma aspect ratio of four, this is an indication that the paramagnetism of the plasma has a destabilizing effect in the experiment. For reasons of field economy, however, it will be attempted to achieve $\beta_p > 1$ for a fusion reactor, which is always possible, in principle, by applying additional heating methods /36/. As a result, the stability behaviour may be even more favourable.

The measured q_{crit} is equivalent to the value at which it also becomes possible to achieve practical operation of tokamaks with circular cross-section.

It is therefore interesting to investigate what influence the elongation of the plasma cross-section has on tokamak confinement, under the condition that the q value does not have to be raised appreciably with increasing axis ratio.

Appendix A

Possible Advantages of Elongated Plasma Cross-sections for Tokamak Confinement

The confinement properties of a tokamak become more and more favourable the smaller the aspect ratio A and the stronger the main field are made. Both parameters are only limited for technical reasons. To compare tokamaks with circular and elongated plasma cross-sections, one thus has to postulate the same B_t and A . In addition, comparison should be made for the same plasma volume V since the magnetic field costs and fusion power output in a reactor are then the same.

It is then possible to investigate various plasma parameters to determine whether they are more favourable for a fusion reactor when the plasma cross-section is elongated and A , B_t and V are constant.

a) Maximum beta

In Sec. 2.2 it has already been shown that the equilibrium limit

$$\beta_p < A$$

and the Kruskal-Shafranov stability limit can only be satisfied simultaneously if

$$\beta < \beta^{\max} \approx \frac{A}{1 + q^2 (\ell_t / \ell_p)^2}$$

If the circumference of an ellipse is approximated by

$$\ell_p = \pi \left(\frac{3}{2} (a + b) - \sqrt{ab} \right)$$

and cases with elliptical cross-section are denoted by the index E and circular cases by K, one obtains

$$\frac{\beta_E^{\max}}{\beta_K^{\max}} = \frac{q_K^2}{q_E^2} \frac{A_K}{A_E} \times \frac{(\frac{3}{2}(1+\epsilon) - \sqrt{\epsilon})^2}{4}$$

In the limiting case of large axis ratios $b/a = \epsilon \rightarrow \infty$ elongation is expected to have a positive effect if the critical q value increases with ϵ less than linearly.

b) Power density of ohmic heating

The ohmic heating power density Q_Ω at constant conductivity is proportional to j^2 , where j is the toroidal current density. According to Kadomtsev /5/ it then follows that

$$\frac{(Q_\Omega)_E}{(Q_\Omega)_K} = \frac{q_K^2}{q_E^2} \times \left(\frac{1}{4} \epsilon^{-4/3} + \frac{1}{2} \epsilon^{2/3} + \frac{1}{4} \epsilon^{8/3} \right)$$

c) Plasma temperature

Energy balances for elliptical plasma cross-sections are not yet very well developed. With ohmic heating and neo-classical thermal conduction as the only mechanisms the authors of /37/ confirm for the plateau regime by means of numerical calculations the formula:

$$T_i \propto (I_p^2 R_t n_e \frac{1}{F(\epsilon)})^{1/3}$$

$F(\epsilon)$ is a geometry dependent correction factor for the coefficient of thermal conductivity. For mean elongations it holds that $F(\epsilon) \approx \epsilon^{-1.5}$.

Since the plasma current rises quickly with increasing axis ratio according to /5/:

$I_p \propto B_t (a^2 + b^2) / (Rq)$ it follows that

$$T_i \propto \left(\frac{V B_t^2 n_o}{q} \right)^{1/3} \left(\frac{1 + \epsilon^2}{\epsilon F(\epsilon)} \right)^{1/3} \propto \epsilon$$

On transition to higher temperatures the plateau regime is departed from. In the banana regime the ion temperature scales with $I_p^2 / 7$, and therefore the favourable influence of elongation should be even stronger here.

d) Energy confinement time

Estimates of the energy confinement time τ_e /5/ show that in all of the models discussed it depends only slightly on the elongation of the plasma cross-section. On the assumption of thermal conduction losses in the plateau regime it follows, for example, that

$$\frac{\tau_e^E}{\tau_e^K} \approx \epsilon^{4/3}$$

If other dominant loss mechanisms are assumed, one obtains scalings in which τ_e^E decreases weakly with increasing ϵ .

Conclusions

It is obvious that practically all essential confinement properties of a tokamak become more favourable with increasing elongation of the plasma cross-section, pro-

vided that the q value does not have to be strongly increased.

This experiment has shown that even with strong elongation it is only necessary for sufficient stability to have the same q value at the plasma boundary as must also be met in tokamaks with circular cross-section.

This argument is only valid at first for the relatively cold plasma under consideration here. It is not yet possible to state whether new instabilities that raise the critical q value occur on transition to fusion temperatures and correspondingly collisionless plasmas.

For clarification of these questions it will be necessary to enlist bigger and more elaborate belt pinch and belt tokamak experiments. Such experiments are being built in various places or have recently been put into operation /39, 40, 41/.

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