On the Dynamics of Toroidal High-ß Stellarators

J. NEUHAUSER

IPP 1/138 Dezember 1973

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Abstract:

The influence of plasma dynamics in toroidal high-beta Stellarators is investigated in an idealized, linear model (incompressible, sharp boundary) with artificial force of gravity instead of the toroidal drift force. The dynamic interference force produced by the coupling of oscillating $\ell = 1$ and $\ell = 2$ stellarator fields is calculated for frequencies near the resonance frequency of the helical modes. A simple interpretation of the frequency response is given and the influence of damping and compressibility is discussed.

The result is used to explain the initial dynamics in the ISAR T 1 experiment and to estimate the effect of plasma heating by helical Alfvén waves on plasma stability.

1. Introduction

Toroidal, net-current-free magnetostatic equilibria with $\beta = 2\mu_0 p_0 / B_0^2 \lesssim 1$ (p_0 = static plasma pressure, B_0 = toroidal magnetic field) can be achieved by superposing spatially periodic fields on a toroidal main field /1-7/. These equilibria are referred to as high-ß stellarator equilibria or else as generalized M & S /1/ equilibria. An essential feature here is that the part of the plasma surface facing the centre of the torus is enlarged by making the plasma contour more strongly corrugated than on the outer side (M & S effect /1/ (Fig. 1). The corresponding magnetic field structure can be achieved in principle /4,5/ by superposing on a strong, toroidal main field (at least) two helical fields with equal field period $\lambda = \frac{2\pi}{h}$ and with multiplicity ℓ differing by one, i.e. equal toroidal wave number and azimuthal wave numbers differing by one. A suitable choice of the magnitude and phase of the helical fields ensures that in equilibrium the plasma is concentric with the helical windings, which generate the helical fields.

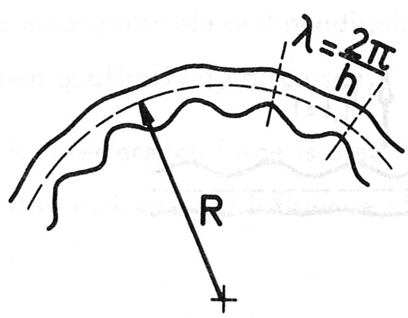

Fig. 1

Toroidal, M & S-like equilibria call for a three-dimensional description with all theoretical and practical consequences such as the possible non-existence of exact equilibria, the compexity and extent of numerical codes etc.. The calculations know at present are therefore mainly based on the surface current model and the ideal MHD equations and the toroidal equilibrium is described by a small parameter expansion around the linear theta pinch (e.g. /4,6,8/). The ratio of helical fields to the main field and the inverse aspect ratio are usually the essential smallness parameters and determine the range of validity of the results. A useful interpretation of the M & S-like equilibria is given by introducing the so-called "interference force" : This is because, if two helical fields with equal field period and multiplicity differing by one, as stated above, are superposed on a linear theta pinch plasma, the marginal, highly symmetric theta pinch changes to a non-equilibrium which is periodic, but not helically symmetric. The plasma column is then subjected to a force $F_{\ell,\ell\pm1}$ perpendicular to the axis in the direction of the maximum corrugation of the surface (Fig.2). This force, whose origin will become clear from the later calculations, is frequenctly referred to in the literature as the "interference force" /5/.

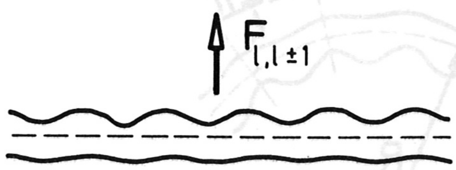

Fig. 2

Toroidal equilibrium is then obtained for not too strong curvature simply by making the toroidal drift force and interference force equal (Fig. 1).

The foregoing magnetostatic equilibria form the basis for the following treatment of dynamic effects. An estimate of dynamic effects has to be made for the following reasons:

1) The production of a high-ß plasma by, for example, fast compression, leads to a dynamic initial phase accompanied by strong inertial forces. It is experimentally observed /9/ that the dynamics has an influence on the interference force.

2) The plasma heating with helical Alfvén waves /10-12/ now under discussion calls for a knowledge of dynamic effects on equilibrium. Inertial effects may also lead to disturbances of equilibrium in other heating methods, e.g. transit time magnetic pumping with periodic fields, etc.

3) The dynamic stabilization of periodic equilibria as described in /13,14,15/, may be regarded in terms of a "dynamic interference force" between the original periodic field and the perturbed fields.

In the following investigation of dynamic effects the surface current model and the validity of the ideal MHD equations are assumed. The time dependent interference force acting on a straight plasma column with time dependent periodic fields and plasma deformations is calculated. The main concern here is the $\ell=1/\ell=2$ combination. First the plasma is treated as incompressible and nondisssipative. The influence of compressibility and damping is then qualitatively discussed.

The resulting dynamic interference force is used to explain dynamic effects in the experiment ISAR T 1 and to estimate the influence of Alfvén wave heating on plasma stability.

II. The $\ell=1/\ell=2$ interference force for arbitrary time dependence of the plasma deformation

1. Description of the equilibrium

Helical $\ell=1$ and $\ell=2$ fields with equal field period $\lambda = 2\pi/h$ are superposed on a linear, cylindrical plasma column with circular cross section in a longitudinal field B_o. A sharp surface and only surface currents are assumed. The plasma density is constant and the plasma is assumed to be incompressible. The plasma pressure, however, contains dynamic components because of inertial forces and so it becomes space and time dependent.

The plasma is treated as an ideal fluid. The flow should be a potential flow. Plasma rotation is not allowed.

The problem of solving the relevant magnetohydrodynamic equations of motion is considerably reduced in this model (e.g. /14/). The magnetic fields inside and outside the plasma can be derived from scalar potentials ϕ_i or ϕ_e, with $\Delta\phi_{i,e} = 0$. Similarly, the local plasma velocity can be derived from a scalar potential χ and it also holds that $\Delta\chi = 0$. The solutions of $\Delta\phi_{i,e} = 0$ and $\Delta\chi = 0$ in cylindrical coordinates (r, θ, z) are known and can be written for the $\ell=1/\ell=2$ system in leading order in the form, for example,

$$\phi \sim z + (c_1 I_1 + c_2 K_1)\sin(\theta - hz) + \\ + (c_3 I_2 + c_4 K_2)\sin(2\theta - hz) + \dots \tag{1}$$

$$\chi \sim c_5 I_1 \cos(\theta - hz) + c_6 I_2 \cos(2\theta - hz) + \dots \tag{2}$$

$I_\nu(hr), K_\nu(hr)$ are the modified Bessel functions. For simplicity we assume $hr_o \ll 1$ (r_o = plasma radius), and so the expansion for small arguments can be used.

As mentioned, the interference force in linear geometry leads to a non-equilibrium because there is no toroidal curvature and hence no drift force. In the present calculation this is counteracted by introducing an artificial gravitational force which in the time average should be equal to the interference force. There are nevertheless, (e.g. in the case of an oscillating interference force), small, periodic displacements of the plasmacolumn from the initial position. A nearly rigid displacement of the plasma column, however, causes the plasma in the new position to "see" a changed, external magnetic field /7,5/. In addition to an original ℓ-field, one has induced $\ell^* = \ell \pm 1$ side bands, which are roughly by a factor $\ell \cdot \xi / r_0$ smaller. For the $\ell=1/\ell=2$ combination this yields $\ell=0/\ell=2$ and $\ell=1/\ell=3$ fields. The corresponding induced interference forces are proportional to the displacement ξ, causing the well-known unstable behaviour of high beta stellarators (without stabilizing wall). In the following $\frac{\xi}{r_0} \ll 1$ will be assumed and therefore the induced interference froces can be neglected.

In accordance with the foregoing the following magnetic potential is taken:

$$\phi_e = -H_{oe}\left[z + \left(Ar + \frac{B}{r}\right)\sin(\theta - hz) + \left(Cr^2 + \frac{D}{r^2}\right)\sin(2\theta - hz) + \ldots\right]$$

$$\phi_i = -H_{oi}\left[z + Er\sin(\theta - hz) + Fr^2\sin(2\theta - hz) + \ldots\right] \tag{3}$$

All coefficients are time dependent. A and C are determined by currents in the external $\ell = 1$ and $\ell = 2$ windings, E and F by currents in these windings and by their mirror currents in the plasma surface, B and D only by mirror currents.

The plasma surface, is taken as an expansion about the cylinder:

$$r_p = r_0\left[1 + \delta_1\cos(\theta - hz) + \delta_2\cos(2\theta - hz)\right] + \xi\cos\theta + \ldots$$

$$\delta_1(t),\ \delta_2(t),\ \xi(t)/r_0 \ll 1 \tag{4}$$

The next step is to find a form for χ which is consistent with $\phi_{i,e}$ and r_p, i.e. the fluid moves with the boundary. This condition is met in leading order by the ansatz:

$$\chi = -r r_0\dot{\delta}_1\cos(\theta - hz) - r^2\frac{\dot{\delta}_2}{2}\cos(2\theta - hz) - r\dot{\xi}\cos\theta +$$

$$+ r\frac{r_0}{2}\left(\dot{\delta}_1\delta_2 + \dot{\delta}_2\delta_1\right)\cos\theta + \ldots, \qquad \dot{\delta} = d\delta/dt \tag{5}$$

The last term in (5) is of higher order and ensures that there is no nonhelical dipol term with respect to the perturbed plasma surface.

Together with helical terms of the same order and with the boundary conditions at the plasma surface one gets conditions for higher order terms in the magnetic field and the plasma surface. These terms are not needed in the following calculation and have been neglected. One should show, however, that, in principle, a solution can be found in this order.

The velocity field now follows to the required order with $\vec{v} = -\nabla\chi$:

$$v_r = r_0\dot{\delta}_1\cos(\theta - hz) + r\dot{\delta}_2\cos(2\theta - hz) + \dot{\xi}\cos\theta + \ldots$$

$$v_\theta = -r_0\dot{\delta}_1\sin(\theta - hz) - r\dot{\delta}_2\sin(2\theta - hz) - \dot{\xi}\sin\theta + \ldots \tag{6}$$

$$v_z = \ldots$$

The magnetic field follows from (3) with $B = -\nabla\phi$:

$$B_{ez} = B_{eo}\left[1 - h\left(Ar + \frac{B}{r}\right)\cos(\theta - hz) - h\left(Cr^2 + \frac{D}{r^2}\right)\cos(2\theta - hz) + ..\right]$$
$$B_{er} = B_{eo}\left[\left(A - \frac{B}{r^2}\right)\sin(\theta - hz) + \left(2Cr - \frac{2D}{r^3}\right)\sin(2\theta - hz) + ..\right]$$
$$B_{e\theta} = B_{eo}\left[\left(A + \frac{B}{r^2}\right)\cos(\theta - hz) + \left(2Cr + \frac{2D}{r^3}\right)\cos(2\theta - hz) + ...\right] \qquad (7)$$

$$B_{iz} = B_{io}\left[1 - hEr\cos(\theta - hz) - hFr^2\cos(2\theta - hz) + ...\right]$$
$$B_{ir} = B_{io}\left[E\sin(\theta - hz) + 2Fr\sin(2\theta - hz) + ...\right]$$
$$B_{i\theta} = B_{io}\left[E\cos(\theta - hz) + 2Fr\cos(2\theta - hz) + ...\right] \qquad (8)$$

It should be borne in mind that all coefficients are time dependent.

2. Equilibrium conditions and ordering scheme

The coefficients still free are now determined by two conditions:

a) The plasma surface has to be a flux surface.

b) The dynamic pressure balance has to be satisfied everywhere on the plasma surface.

Point a) has only to be satisfied in leading order and one obtains from $\vec{n}\cdot\vec{B}/_{r_p} = 0$ eqs.(7), (8) and ansatz (4):

$$hr_o\delta_1 \approx E \approx (A - B/r_o^2)$$
$$hr_o\delta_2 \approx 2Fr_o = 2(Cr_o - D/r_o^3) \qquad (9)$$

The dynamic pressure balance according to point b) in the idealized model takes the form of a time dependent MHD Bernoulli equation

$$\left[\varrho\frac{\partial\chi}{\partial t} + p_o - \frac{\varrho}{2}v^2 + V\cdot\cos\theta + \frac{B_i^2}{2\mu_o}\right]_{r=r_p} = \left[\frac{B_e^2}{2\mu_o}\right]_{r=r_p}$$

(ϱ = plasma density) (10)

The term $V \cdot \cos\theta$ is assigned to an artificial gravitational field which is used for compensating the interference force. In the foregoing definitions various assumptions on the relative size of individual terms have already been made and many terms have accordingly been neglected.

The order scheme taken as a basis should now be explained in detail.

It was assumed that $hr_0, \delta_1, \delta_2, \xi/r_0 \ll 1$. In the following the relative order of magnitude is characterized by the small number $\varepsilon = hr_0 \ll 1$

$$\delta_1 \approx \delta_2 \approx \varepsilon \ , \quad \xi/r_0 \lesssim \varepsilon$$

From eq.(9) it then follows that

$$\frac{B_{er}}{B_{eo}}, \ \frac{B_{e\theta}}{B_{eo}}, \ \frac{B_{ir}}{B_{io}}, \ \frac{B_{i\theta}}{B_{io}} \approx \varepsilon^2$$

$$\frac{B_{ez}-B_{eo}}{B_{eo}}, \ \frac{B_{iz}-B_{io}}{B_{io}} \approx \varepsilon^3$$

In $B^2/2\mu_0$ there are cross terms between ℓ=1 and ℓ=2 in $[B_{r_1} \cdot B_{r_2}]$ and $[B_0 \cdot B_{z,1,2}(r_p)]$, where $[\ldots]/B_0^2 \approx \varepsilon^4$. The terms $[B_0 \cdot B_{z,1,2}(r_0)]$ are pure helical terms of order ε^3.

To calculate the interference force in leading order, only terms of the relative magnitude ε^4 need be taken in the pressure balance.

The relative magnitude of the plasma pressure is essentially determined by the frequency. An essential influence of the inertial force is to be expected in the region of the resonance frequency of the helical oscillations. With

$$\delta_1 = \bar{\delta}_1 \sin\omega t, \quad \omega \approx h^2 v_A^2, \quad v_A^2 = B_{eo}^2/\mu_0 \rho$$

one obtains for a typical term:

$$\frac{\rho \frac{\partial \chi}{\partial t}}{B_{oe}^2/2\mu_0} \rightarrow .. \, h^2 r_0^2 \delta_1 \cos(\theta - hz) \cdot [1 + \delta_1 \cos(\theta - hz) + \delta_2 \cos(2\theta - hz) + ..] \approx$$
$$\approx \mathcal{O}(\varepsilon^3, \varepsilon^4)$$

As in the magnetic field pressure one thus obtains pure terms $\mathcal{O}(\varepsilon^3)$ and cross terms or anharmonic terms $\mathcal{O}(\varepsilon^4)$ when ω is near the resonance.

$(\varrho v_2^2/2)/(B_o^2/2\mu_o)$ is of the order ε^4 and also contains cross terms. The order scheme thus fits exactly in the region of interest surrounding the resonance.

The dynamic pressure balance (10) now looks as follows:

a) The main field pressure and static plasma pressure are $\mathcal{O}(1)$ and yield the theta pinch pressure balance $p_o + \frac{B_{oi}^2}{2\mu_o} = \frac{B_{oe}^2}{2\mu_o}$, where the plasma beta can now be introduced

$$\beta = p_o / (B_{oe}^2/2\mu_o) = (B_{oe}^2 - B_{oi}^2)/B_{oe}^2 \tag{11}$$

b) Terms $\mathcal{O}(\varepsilon^3)$ are pure helical terms and yield the equation of motion for the helical eigenmodes. With A or C = 0 one obtains the natural frequency of the (m=1, k=h) and (m=2, k=h) modes.

c) The terms of the order ε^4 contain

α) constant and helical terms

β) cross terms of the form sin (θ-hz)$\cdot$ sin (2θ-hz) and cos(θ-hz) $\cdot$ cos(2θ-hz) which can be split into helical terms and non-helical terms proportional to cosθ. The latter have to be calculated explicitly.

3. Equation of motion of the helical modes

In the pressure balance (10) all terms up to $\mathcal{O}(\varepsilon^3)$ are taken:

$$\begin{aligned} &-\varrho r_o^2 \ddot{\delta}_1 \cos(\theta - hz) - \varrho r_o^2 \frac{\ddot{\delta}_2}{2} \cos(2\theta - hz) + p_o + \\ &+ \frac{B_{io}^2}{2\mu_o}\left[1 - 2hEr_o\cos(\theta-hz) - 2hFr_o^2\cos(2\theta - hz)\right] = \\ &= \frac{B_{eo}^2}{2\mu_o}\left[1 - 2h\left(Ar_o + \frac{B}{r_o}\right)\cos(\theta - hz) - 2h\left(Cr_o^2 + \frac{D}{r_o^2}\right)\cos(2\theta - hz)\right] \end{aligned} \tag{12}$$

The constant terms yield the theta pinch pressure balance (11). The time and space dependent terms in conjunction with the flux surface condition (9) give the equations of motion of the helical modes (in general forced oscillation, i.e. $A \neq 0$, $C \neq 0$).

$$\ell=1: \quad \rho r_o^2 \ddot{\delta}_1 + \frac{B_{eo}^2}{\mu_o} h^2 r_o^2 (2-\beta)\delta_1 = 2hr_o A \frac{B_{eo}^2}{\mu_o} ; \tag{13a}$$

$$A=0,\ \delta_1 = \tilde{\delta}_1 \sin \omega_{01} t \rightarrow \omega_{01}^2 = (2-\beta) h^2 \frac{B_{eo}^2}{\rho \mu_o} = (2-\beta) h^2 v_A^2$$

$$\ell=2: \quad \rho r_o^2 \ddot{\delta}_2 + \frac{B_{eo}^2}{\mu_o} (2-\beta) h^2 r_o^2 \delta_2 = 4 h r_o^2 C \frac{B_{eo}^2}{\mu_o} ; \tag{13b}$$

$$C=0,\ \delta_2 = \tilde{\delta}_2 \sin \omega_{02} t \rightarrow \omega_{02}^2 = (2-\beta) h^2 v_A^2 = \omega_{01}^2 \doteq \omega_o^2$$

For constant $\delta_{1,2} = \bar{\delta}_{1,2}$ we get the well known result /5/

$$\bar{\delta}_1 = \frac{2A}{(2-\beta) h r_o} = \frac{2}{(2-\beta)} \cdot \frac{\hat{B}_{r1e}}{B_{oe}} \cdot \frac{1}{h r_o} \tag{13c}$$

$$\bar{\delta}_2 = \frac{4 C r_o}{(2-\beta) h r_o} = \frac{2}{(2-\beta)} \cdot \frac{\hat{B}_{r2e}}{B_{oe}} \cdot \frac{1}{h r_o}$$

For $\ell=1$ and $\ell=2$ of equal field period $\lambda = 2\pi/h$ the resonance frequencies are equal. With finite damping there would also be a $\dot{\delta}$- term. The influence of which on the interference force as well as the effect of the compressibility will be discussed later.

4. Calculation of the interference force

Contributions to the directed force are made in eq. (10) only by terms with the angular dependence $\cos\theta$. These are obtained from the cross terms according to the relations

$$\sin(2\theta - hz)\sin(\theta - hz) = \frac{1}{2}[\cos\theta - \cos(3\theta - 2hz)],$$
$$\cos(2\theta - hz)\cos(\theta - hz) = \frac{1}{2}[\cos\theta + \cos(3\theta - 2hz)]. \tag{14}$$

At this point it becomes clear that cos θ-terms are obtained just when the longitudinal wave numbers are equal and the azimuthal wave numbers differ by one. At the same time, however, one also obtains helical coupling terms (and also pure anharmonic terms) of the same order, which cause a slight additional plasma deformation and may lead to parametric effects.

The derivation of the individual terms of order ε^4 will be briefly indicated:

$$\left[\rho \frac{\partial \chi}{\partial t} - \rho \frac{v^2}{2} \right]_{r_p} \rightarrow -\rho r_0 \ddot{\xi} \cos\theta \tag{15}$$

(This is the only ε^4 term containing the displacement ξ)

$$\left.\frac{B_{eo}^2 - B_{io}^2}{2\mu_o}\right|_{r=r_p} \rightarrow \frac{B_{eo}^2}{2\mu_o} \cos\theta \left\{ 2\left(A - \frac{B}{r_o^2}\right)\left(Cr_o - \frac{D}{r_o^3}\right) - 2(1-\beta)EFr_o - \right.$$

$$- \delta_2 h r_o \left(A - \frac{B}{r_o^2}\right) - 2\delta_1 h r_o \left(Cr_o - \frac{D}{r_o^3}\right) + \frac{2}{r_o^2}\left(Ar_o + \frac{B}{r_o^2}\right)\left(Cr_o + \frac{D}{r_o^2}\right) +$$

$$\left. + (1-\beta)\left[h r_o \delta_2 E + 2 h r_o \delta_1 F r_o - 2EFr_o \right] \right\} \tag{16}$$

Together with the equations of motion (12) and (13) and the flux surface condition (9) this finally yields in order ε^4:

$$\left.\frac{B_{eo}^2 - B_{io}^2}{2\mu_o}\right|_{r_p} \rightarrow \frac{B_{eo}^2}{2\mu_o} \cos\theta \left[-(2-\beta)\beta h^2 r_o^2 \delta_1 \delta_2 + \right. \tag{17}$$

$$\left. + \frac{2 p_1 p_2 \mu_o^2}{h^2 r_o^2 B_{oe}^4} - (1-\beta)\frac{2\mu_o}{B_{eo}^2}\left(\delta_1 p_2 + \frac{\delta_2 p_1}{2}\right) \right]$$

$$p_1 = -\rho r_o^2 \ddot{\delta}_1 \quad , \quad p_2 = -\rho r_o^2 \ddot{\delta}_2 / 2 \tag{18}$$

According to eq.(10) this produces the following equation of motion for the transverse displacement ξ of the plasma:

$$V \cdot \cos\theta - \varrho r_0 \ddot{\xi} \cos\theta = \frac{B_{oe}^2}{2\mu_0} \cos\theta \Big[-(2-\beta)\beta h^2 r_0^2 \delta_1 \delta_2 + $$

$$+ \frac{2 p_1 p_2 \mu_0}{h^2 r_0^2 B_{oe}^4} - (1-\beta) \frac{2\mu_0}{B_{oe}^2} \left(\delta_1 p_2 + \delta_2 p_1 / 2 \right) \Big]$$

or with $v_A^2 = B_{oe}^2 / \mu_0 \varrho$, $\omega_0^2 = h^2 v_A^2 \cdot (2-\beta)$ (cf. eq. (13))

$$V - \varrho r_0 \ddot{\xi} = \frac{B_{oe}^2}{2\mu_0} (2-\beta) h^2 r_0^2 \left[-\beta \delta_1 \delta_2 + (2-\beta) \frac{\ddot{\delta}_1 \ddot{\delta}_2}{\omega_0^4} + (1-\beta) \frac{\delta_1 \ddot{\delta}_2 + \delta_2 \ddot{\delta}_1}{\omega_0^4} \right] \qquad (19)$$

The dynamic interference force F_{12} per unit length is obtained by setting $V = 0$ and $F_{12} = \varrho \pi r_0^2 \ddot{\xi}$, or:

$$\boxed{F_{12} = \frac{B_{oe}^2}{2\mu_0} (2-\beta) \pi h^2 r_0^3 \left[\beta \delta_1 \delta_2 - (2-\beta) \frac{\ddot{\delta}_1 \ddot{\delta}_2}{\omega_0^4} - (1-\beta) \frac{\delta_1 \ddot{\delta}_2 + \delta_2 \ddot{\delta}_1}{\omega_0^2} \right]} \qquad (20)$$

As a check, the static interference force ($\delta_1 = \bar{\delta}_1$, $\delta_2 = \bar{\delta}_2$) per unit length agrees with known calculations /5/.

$$F_{12,stat} = \frac{B_{oe}^2}{2\mu_0} (2-\beta) \pi h^2 r_0^3 \beta \cdot \bar{\delta}_1 \bar{\delta}_2$$

In the formulas above $\delta_1(t)$, $\delta_2(t)$ are assumed to be given. If, on the other hand, A(t) and C(t) are prescribed, then $\delta_1(t)$, $\delta_2(t)$ have to be calculated from the equations of motion (13).

Let A(t) and C(t) be periodic in time:

$$A(t) = A_0 \sin\omega_1 t$$

$$C(t) = C_0 \sin\omega_2 t$$

$$\delta_1(t) = \frac{2A_0}{(2-\beta)hr_0} \cdot \frac{\omega_0^2}{\omega_0^2-\omega_1^2} \sin\omega_1 t \doteqdot \tilde{\delta}_1 \sin\omega_1 t$$

$$\delta_2(t) = \frac{4Cr_0}{(2-\beta)hr_0} \cdot \frac{\omega_0^2}{\omega_0^2-\omega_2^2} \sin\omega_2 t \doteqdot \tilde{\delta}_2 \sin\omega_2 t \qquad (21)$$

One then obtains:

$$V - r_0\varrho\ddot{\xi} = \frac{B_{0e}^2}{2\mu_0}(2-\beta)h^2r_0^2\,\tilde{\delta}_1\cdot\tilde{\delta}_2\cdot\sin\omega_1 t\cdot\sin\omega_2 t\,\cdot \qquad (22)$$

$$\cdot\left[-\beta+(2-\beta)\frac{\omega_1^2\omega_2^2}{\omega_0^4}-(1-\beta)\frac{\omega_1^2+\omega_2^2}{\omega_0^2}\right], \text{ or}$$

$$F_{12}(t) = \varrho\pi r_0^3\cdot\frac{\omega_0^2}{2}\cdot\tilde{\delta}_1\tilde{\delta}_2\,\sin\omega_1 t\,\sin\omega_2 t\,\cdot \qquad (23\text{a})$$

$$\cdot\left[\beta-(2-\beta)\frac{\omega_1^2\omega_2^2}{\omega_0^4}+(1-\beta)\frac{\omega_1^2+\omega_2^2}{\omega_0^2}\right], \text{ or}$$

$$F_{12}(t) = \pi r_0^2 \frac{B_{oe}^2}{\mu_0} \cdot \frac{4 A_0 C_0}{(2-\beta)} \cdot \sin\omega_1 t \cdot \sin\omega_2 t \cdot \tag{23b}$$

$$\cdot \frac{\omega_0^4}{(\omega_0^2-\omega_1^2)(\omega_0^2-\omega_2^2)} \cdot \left[\beta - (2-\beta)\frac{\omega_1^2\omega_2^2}{\omega_0^4} + (1-\beta)\frac{\omega_1^2+\omega_2^2}{\omega_0^2} \right]$$

As an example we discuss the special case $\omega_1 = \omega_2 = \omega$.
We define

$$f_\beta = \left[\beta - (2-\beta)\frac{\omega^4}{\omega_0^4} + (2-2\beta)\frac{\omega^2}{\omega_0^2} \right]$$

Plots of the function f_β and $f_\beta \cdot \omega_0^4/(\omega_0^2-\omega^2)^2$ are shown in Fig.3. From the curves the following behaviour of the time mean value is obtained for oscillating $\ell=1$ and $\ell=2$ fields with the same frequency (for $\omega_1 \neq \omega_2$ the situation is more complicated):

a) For $\omega < \omega_0$ the interference force acts in the direction $\theta = 0$, i.e. in the direction of the more pronounced corrugation as in the static case.

b) At the resonance $\omega = \omega_0$ the force goes to zero when the plasma amplitude is kept constant (Fig.3a). As the external fields are zero there, no force can be exerted from outside. With constant external fields the amplitude would tend to infinity because there is no damping; the order scheme is then only valid at a sufficiently large distance from the resonance (Fig.3b).

c) For $\omega > \omega_0$ the interference force acts in the opposite direction, i.e. in the direction of less corrugation.

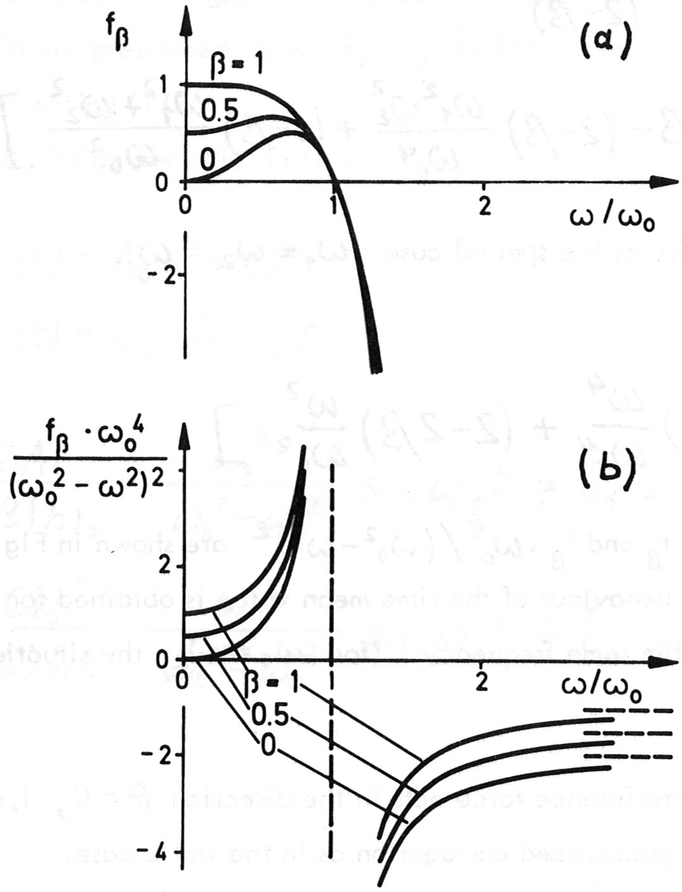

Fig. 3a,b

The explanation for this frequency response is the following: In the static and low frequency case ($\omega < \omega_0$) the plasma advances into regions of low magnetic pressure and recedes at regions of high magnetic pressure. In the region of stronger corrugation the mean pressure at the plasma surface is thus also more strongly reduced. The deformed column is therefore subjected in the static and low-frequency case to a force in the direction of stronger corrugation.

This behaviour changes drastically at the resonance frequency $\omega_0 = \sqrt{2-\beta}\, h\, v_A$ because there is a jump in the phase relation between the plasma oscillation and the driving field from zero to π . Thus above the resonance frequency ω_0 the plasma is driven in regions of high magnetic pressure by inertial forces and vice versa. The mean pressure averaged along the surface is therefore higher at the side of stronger corrugation and the plasma column is pushed in the direction to lower corrugation.

At very high frequency ($\omega \gg h\, v_A$) the plasma surface can no longer follow the field oscillations and becomes more and more cylindrical (if there is no static part of the helical fields). Again the mean magnetic pressure is stronger at the side of stronger field modulation and the force is in the direction to lower field modulation. For constant magnetic field amplitude the interference force becomes independent of the frequency in this simple model. In Fig. 4 a corrugated asymmetric plasma column and the direction of the interference force is shown schematically for different frequency.

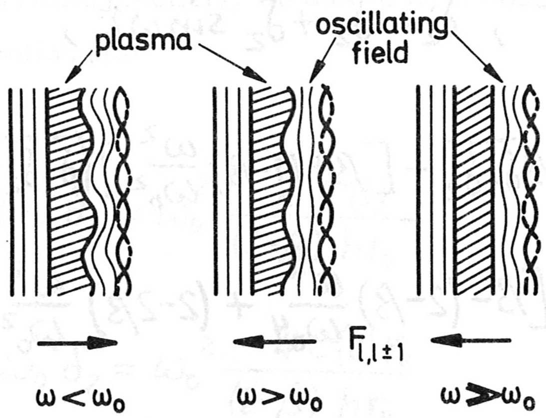

Fig. 4

The latter case corresponds to that of a rigid, conductive cylinder (e.g. Cu pipe), where inertia is replaced by intrinsic strength. That is to say a Cu pipe will always be forced in the direction of weak corrugation, irrespective of the frequency (ideal conductivity being assumed). This special case can be exactly calculated and leads to the same result as the approximation calculation.

From the foregoing it is also clear that the frequency response of the interference force is not only valid for the $\ell=1/\ell=2$ system, but probably for every M-and-S structure. Nevertheless in a real plasma the phase relation between the field and plasma oscillations is more complicated, possibly resulting in a qualitative change of the frequency response at very high frequency $(\omega \gg \omega_0)$.

As many MHD instabilities in M & S equilibria are produced by interference forces, it is anticipated that a reversal of such interference forces above the resonance would cause a change from instability to stability, i.e. these model would be dynamically stabilized.

Finally, ξ (t) is given for the case of static fields with oscillating component $(\omega_1=\omega_2=\omega)$:

$$\delta_1=\bar{\delta}_1+\tilde{\delta}_1 \sin\omega t \;, \quad \delta_2=\bar{\delta}_2+\tilde{\delta}_2 \sin\omega t \;,$$

$$\ddot{\xi}=\frac{V}{\varrho r_0}+\frac{\omega_0^2}{2} r_0 \left\{ \beta\bar{\delta}_1\bar{\delta}_2 + \left[\beta+(1-\beta)\frac{\omega^2}{\omega_0^2}\right]\left(\bar{\delta}_1\tilde{\delta}_2+\bar{\delta}_2\tilde{\delta}_1\right)\sin\omega t + \right.$$
$$\left. + \tilde{\delta}_1\tilde{\delta}_2 \sin^2\omega t \left[\beta-(2-\beta)\frac{\omega^4}{\omega_0^4}+(2-2\beta)\frac{\omega^2}{\omega_0^2}\right] \right\} \tag{24}$$

V is adapted so that no constant term occurs, i.e. equilibrium is maintained in the time mean:

$$\frac{V}{\varrho r_0} = -\frac{\omega_0^2}{2} r_0 \left\{ \beta\bar{\delta}_1\bar{\delta}_2 + \right.$$
$$\left. + \frac{1}{2}\tilde{\delta}_2\tilde{\delta}_1 \left[\beta-(2-\beta)\frac{\omega^4}{\omega_0^4}+(2-2\beta)\frac{\omega^2}{\omega_0^2}\right] \right\} \tag{25}$$

Integration then yields:

$$\frac{\xi}{r_0} = -\sin\omega t \cdot \frac{\omega_0^2}{2\omega^2}\left(\bar{\delta}_1\tilde{\delta}_2 + \bar{\delta}_2\tilde{\delta}_1\right)\left[\beta + (1-\beta)\frac{\omega^2}{\omega_0^2}\right] -$$
$$-\cos 2\omega t \; \frac{\omega_0^2}{16\omega^2}\tilde{\delta}_1\tilde{\delta}_2\left[\beta - (2-\beta)\frac{\omega^4}{\omega_0^4} + (2-2\beta)\frac{\omega^2}{\omega_0^2}\right] \tag{26}$$

As it holds that $\tilde{\delta}_1, \bar{\delta}_1, \tilde{\delta}_2, \bar{\delta}_2 \approx \varepsilon$, the relative deflection for $\omega \approx \omega_0$ is even $\xi/r_0 \approx \varepsilon^2$ and so the assumption made previously is justified.

5. Influence of damping on the dynamic interference force

Damping of the helical oscillations can be caused by classical effects. Much more important at high temperatures in real plasmas, however, seems to be the decay of modes as a result of phase mixing /10/, mode coupling, excitation of parametric resonances etc. followed by classical damping of the secondary modes . $m \gtrsim 2$ modes may also be subjected to strong damping by finite gyroradii. Without defining the damping mechanism more precisely, we arbitrarily introduce a damping term into the equations of motion (13):

$$\ddot{\delta}_1 + 2\gamma_1\dot{\delta}_1 + \omega_0^2\delta_1 = \omega_0^2\frac{2A(t)}{(2-\beta)hr_0}$$
$$\ddot{\delta}_2 + 2\gamma_2\dot{\delta}_2 + \omega_0^2\delta_2 = \omega_0^2\frac{4C(t)r_0}{(2-\beta)hr_0} \tag{27}$$

This results in a changed helical dynamic pressure

$$P_{1D} = -\rho r_0^2\left(\ddot{\delta}_1 + 2\gamma_1\dot{\delta}_1\right) = \rho r_0^2\omega_0^2\left(\delta_1 - 2A/((2-\beta)\cdot hr_0)\right)$$
$$P_{2D} = -\rho\frac{r_0^2}{2}\left(\ddot{\delta}_2 + 2\gamma_2\dot{\delta}_2\right) = \rho\frac{r_0^2}{2}\omega_0^2\left(\delta_2 - 4Cr_0/((2-\beta)hr_0)\right)$$

It then follows from eq. (17 - 19) that

$$\frac{\ddot{\xi}}{r_0} = \frac{V}{\rho r_0^2} + \frac{\omega_0^2}{2}\Big\{\beta\delta_1\delta_2 - \frac{2-\beta}{\omega_0^4}\left(\ddot{\delta}_1 + 2\gamma_1\dot{\delta}_1\right)\left(\ddot{\delta}_2 + 2\gamma_2\dot{\delta}_2\right) -$$
$$-(1-\beta)\frac{1}{\omega_0^2}\left[\delta_1\left(\ddot{\delta}_2 + 2\gamma_2\dot{\delta}_2\right) + \delta_2\left(\ddot{\delta}_1 + 2\gamma_1\dot{\delta}_1\right)\right]\Big\} \tag{28}$$

In the freely oscillating, damped case ($A=0, C=0, \omega_{1,2}^2=\omega_0^2-\gamma_{1,2}^2$) the interference force term is zero.

For a stationary, oscillating case we get the well known solution of (27):

$$A(t)=A_0 \sin \omega_1 t, \quad C(t)=C_0 \sin \omega_2 t .$$

$$\delta_1(t)=\frac{2}{(2-\beta)}\frac{A_0}{hr_0}\frac{\omega_0^2}{\sqrt{(\omega_1^2-\omega_0^2)^2+4\gamma_1^2\omega_1^2}}\sin(\omega_1 t-\varphi_1),$$

$$\delta_2(t)=\frac{4}{(2-\beta)}\frac{C_0 r_0}{hr_0}\frac{\omega_0^2}{\sqrt{(\omega_2^2-\omega_0^2)^2+4\gamma_2^2\omega_2^2}}\sin(\omega_2 t-\varphi_2), \tag{29}$$

$$\operatorname{tg}\varphi_{1,2}=2\gamma_{1,2}\cdot\omega_{1,2}/(\omega_0^2-\omega_{1,2}^2) .$$

For $\gamma_1/\omega_1, \gamma_2/\omega_2 \ll 1$ the interference force is not essentially changed. For strong damping, however, the combined effect of phase shift and reduction of plasma amplitude can cause a reversal of the interference force well below the resonance frequency. For $\gamma_{1,2} \gg \omega_0$, for instance, the plasma cannot follow the magnetic field oscillations. Thus we get the case of a nearly cylindrical plasma column also for low frequency, while without damping it occurs only for $\omega \gg \omega_0$, as found in the preceding section.

6. Influence of compressibility on the interference force

So far incompressibility, i.e. velocity of sound $c_s \to \infty$ has been assumed. In experiment, however, one has $c_s^2 = v_A^2\cdot\gamma_K\cdot\beta/2$, i.e. $c_s^2 \lesssim v_A^2$ (γ_K = adiabatic exponent). It has therefore to be expected that there will be additional effects near the magnetoacoustic resonance. With finite compressibility appropriately modified expressions P_{1K}, P_{2K} have to be substituted for P_1, P_2 in the oscillation equations and correspondingly in the interference force (19).

P_{1K} is obtained from the paper of G.Berge /13/ on dynamic stabilization for

$$\delta_1=\tilde{\delta}_1 \sin \omega_1 t, \quad \gamma_1=0:$$

$$P_{1K} = -r_0^2 \tilde{\delta}_1 \varrho \omega_1^2 \cdot K_1(v_A, c_s, \beta, \omega_1) \,,$$

$$K_1 = \left[1 - \frac{h^2 v_A^2 (1-\beta)}{\omega_1^2}\right] \Big/ \left[\frac{v_A^2(1-\beta)}{c_s^2} + 1 - \frac{h^2 v_A^2 (1-\beta)}{\omega_1^2}\right] \qquad (30)$$

A similar form is probably valied for P_{2K}:

$$\delta_2 = \tilde{\delta}_2 \sin \omega_2 t \,, \quad \gamma_2 = 0 :$$

$$P_{2K} = -r_0^2 \frac{\tilde{\delta}_2}{2} \varrho \omega_2^2 \cdot K_2(v_A, c_s, \beta, \omega_2) \,. \qquad (31)$$

Instead of (23) the interference force is now qualitatively ($\omega_1 = \omega_2 = \omega$) :

$$F_{12,K} = \frac{B_{oe}^2}{2\mu_0} (2-\beta)\pi h^2 r_0^3 \tilde{\delta}_1 \tilde{\delta}_2 \sin^2 \omega t \cdot f_{\beta,K} \,,$$

$$f_{\beta,K} = \left[\beta - (2-\beta)\frac{\omega^4}{\omega_0^4} K_1 K_2 + (1-\beta)\frac{\omega^2}{\omega_0^2}(K_1 + K_2)\right] \qquad (32)$$

$f_{\beta,k}$ is represented in the following sketch for $\gamma_k = 5/3$, $\beta = 0.6$ on the assumption that $K_1 = K_2$, with the incompressible case for comparison (Fig.5).

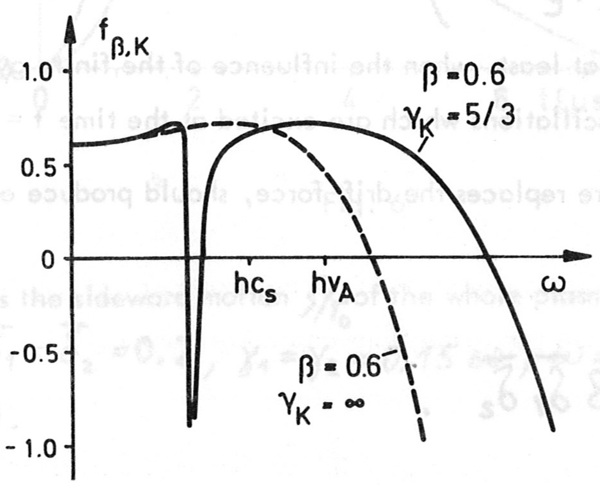

Fig. 5

The interference force is now negative in two regions, namely in the immediate vicinity of the magnetoacoustic resonance and above the slightly shifted Alfvén resonance. With strong damping the first region may disappear. In the case of long mean free path a new calculation has to be made, because the velocity of sound then losses its significance and new effects occur.

One should also notice that for $\omega \gg \omega_0$ radial plasma oscillations become important. In this region the phase relation between plasma and field oscillations may be a complicated function of the frequency and the result of the incompressible model may be completely misleading for very high frequency.

III. Some practical applications

1. Dynamics in the ISAR T 1 high-beta stellarator /9/

In the ISAR T 1 torus the $\ell=1/\ell=2$ interference force (and partly the $\ell=1/\ell=0$) is used to compensate the drift force. The plasma is heated by fast compression in a simple quartz torus, thus the plasma deformations are initially almost zero. The helical fields are switched on almost simultaneously with the main field, forcing the plasma column into a helical equilibrium position and therefore causing helical oscillations. The observed oscillation of the m = 1 mode is approximately of the form

$$\delta_1 \approx \bar{\delta}_1 \left(1 - \cos\omega_1 t \cdot e^{-\gamma_1 t}\right), \quad \gamma_1 \approx 0.1 \ldots 0.2\, \omega_1$$

The m = 2 oscillation is more difficult to measure. For $\delta_2(t)$ the same form is used here:

$$\delta_2 \approx \bar{\delta}_2 \left(1 - \cos\omega_2 t \cdot e^{-\gamma_2 t}\right)$$

Again $\gamma_2 \approx 0.1 \ldots 0.2\,\omega_2$ holds, at least, when the influence of the finite gyroradius is weak. One thus has damped oscillations which are excited at the time t = 0.

The potential V, which here replaces the drift force, should produce equilibrium for $t \to \infty$:

$$\frac{V}{\rho r_0^2} = - \frac{\omega_0^2}{2} \beta \bar{\delta}_1 \bar{\delta}_2 \; .$$

Integration of eq. (28) with $\xi(t=0)=\dot{\xi}(t=0)=0$ yields

$$\xi(t) = -\frac{r_0\bar{\delta}_1\bar{\delta}_2}{2}\Big[(\gamma_1+\gamma_2)t + \frac{\omega_1^2+\omega_2^2}{\omega_0^2} + \frac{\gamma_1^2-\omega_1^2}{\omega_0^2}e^{-\gamma_1 t}\cdot\cos\omega_1 t - 2\frac{\omega_1\gamma_1}{\omega_0^2}e^{-\gamma_1 t}\cdot\sin\omega_1 t + \frac{\gamma_2^2-\omega_2^2}{\omega_0^2}e^{-\gamma_2 t}\cdot\cos\omega_2 t - 2\frac{\omega_2\gamma_2}{\omega_0^2}e^{-\gamma_2 t}\cdot\sin\omega_2 t\Big], \tag{33}$$

$$\xi_{t\to\infty} = -\frac{r_0\bar{\delta}_1\bar{\delta}_2}{2}\Big[(\gamma_1+\gamma_2)t + \frac{\omega_1^2+\omega_2^2}{\omega_0^2}\Big].$$

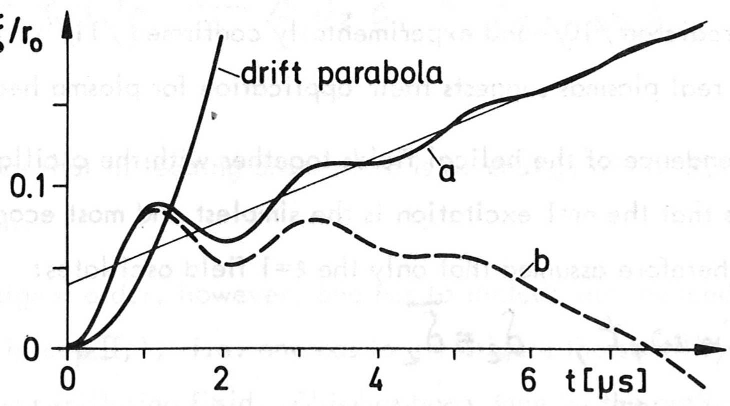

Fig. 6

Fig.6a shows the sideward motion ξ/r_0 of the whole plasma column compared to the drift parabola ($\bar{\delta}_1=\bar{\delta}_2=0.2$, $\gamma_1=\gamma_2=0.15\,\omega$, $\omega=\omega_1=\omega_2=3\cdot 10^6\,s^{-1}$, $\beta=0.5$).

One finds that with helical fields in the first half-cycle the plasma moves in the drift direction faster than in the case of pure toroidal drift. The asymtotic behaviour is characterized by an initial displacement $S_0 \approx -r_0 \overline{\delta}_1 \overline{\delta}_2$ and a constant velocity $v_0 \approx -r_0 \overline{\delta}_1 \overline{\delta}_2 \cdot \gamma$. By increasing the helical fields (here 3.5 % each) for some time, the plasma can be forced back to the axis again ("field programming") (Fig. 6b).

This behaviour is also observed in the experiment /9/. One should keep in mind, however, that experimentally the dynamic phase is much more complicated and the scaling used here, in principle, is not valid for ISAR T 1 ($\delta_{1,exp} \gtrsim 1$). On the other hand, calculations of the static interference force have hitherto yielded the same results in most scalings, and therefore the explanation in terms of a dynamic interference force should be at least qualitatively valid. On the basis of the above calculations one can now consider ways of avoiding such a displacement in the dynamic phase of high beta stellarators (shaped discharge vessel, programmed $\ell=1/\ell=2$ fields, etc).

2) Alfvén wave heating and its effect on equilibrium and stability

The theoretically predicted /10/ and experimentally confirmed /11/ strong damping of the helical modes in real plasmas suggests their application for plasma heating.

From the radial dependence of the helical fields together with the oscillation equations (13) it follows that the m=1 excitation is the simplest and most economic proposition /12/. It is therefore assumed that only the $\ell=1$ field oscillates:

$$\delta_1 = \overline{\delta}_1 + \tilde{\delta}_1 \sin \omega_1 t , \quad \delta_2 = \overline{\delta}_2$$

From the equation of motion (27) we calculate the heating power per unit length of the helical plasma column:

$$P = \tilde{\delta}_1^{\,2} \cdot \omega_1^2 \cdot \gamma_1 \cdot \pi \rho r_0^4 \tag{34}$$

In order to drive this oscillation we need an oscillating external $\ell=1$ field $\tilde{B}_{r1,Vacuum}$ at the plasma surface

$$\frac{\tilde{B}_{r1,Vacuum}}{B_{oe}} = \tilde{\delta}_1 \frac{(2-\beta)}{2} h r_0 \frac{\sqrt{(\omega_0^2-\omega_1^2)^2+4\gamma_1^2\omega_1^2}}{\omega_0^2} .$$

From theory /10/ as well as from experiment /11/ one finds $\gamma_1 \approx 0.1 \ldots 0.2\,\omega_1$. For economic reasons (optimum ratio of heating power to external losses /12/) one wants to work near the resonance, and it is very important to know, if there is any influence on the equilibrium. Therefore we calculate the interference force and its time average :

$$F_{12} = \rho\pi r_0^3 \frac{\omega_0^2}{2}\Big[\beta\bar{\delta}_2(\bar{\delta}_1+\tilde{\delta}_1 \sin\omega_1 t) -$$

$$-(1-\beta)\frac{\bar{\delta}_2}{\omega_0^2}\tilde{\delta}_1(2\gamma_1\omega\cos\omega t - \omega^2\sin\omega t)\Big] ,$$

$$\langle F_{12}\rangle = \rho\pi r_0^3 \frac{\omega_0^2}{2}\beta\bar{\delta}_2\bar{\delta}_1 = F_{12,stat}. \tag{35}$$

It turns out that in leading order there is no change of the equilibrium forces in the time mean.

In next higher order, however, one has to include the induced interference force as described in cap.II,1, i.e. one has to investigate the stability of the system in the presence of an oscillating field. This has been done by the author in a preliminary way showing the possiblity of dynamic stabilization of an m=1 displacement above the Alfvén resonance ($\omega^2 > (2-\beta)\, h^2 v_A^2$). With damping stabilization occurs also at lower frequency. The explanation of this fact is quite similar to that given in section II,4, but now corresponding to the induced interference force. A similar result was obtained in /15/ for the case of a travelling bumpy field structure superposed on a linear theta pinch and, in a more general way, in /13,14/.

It should thus be possible in, for example, an $\ell=1/\ell=2$ system to choose the static and oscillating field components such that equilibrium is preserved, that it is stable to long-wave modes, and that sufficient heating is provided at the same time. This state could be very interesting in the heating phase of a stationary high-ß stellarator reactor since external high-frequency losses in the initial phase are insignificant when the burning time is long.

V. Summary of results

The influence of the plasma dynamics on high-beta stellarator equilibria was investigated in an idealized, linear model with artificial force of gravity instead of the toroidal drift force.

Time dependent $\ell=1$ and $\ell=2$ fields with equal magnetic field period were superposed on a circularly cylindrical plasma column in a homogeneous longitudinal main field. The equations of motion of the plasma including anharmonic effects and mode coupling of the $\ell=1$ and $\ell=2$ fields was explicitly calculated for frequencies in the region near the resonance frequency of the helical modes. It is frequency dependent and changes direction above the resonance frequency. A simple interpretation was given for this frequency response.

The influence of damping and compressibility was discussed. Essentially, the two effects only lead to quantitative corrections without changing the basic behaviour. With strong damping reversal of the interference force is possible also for low frequency.

Judging by known results, those obtained here are probably relatively independent of the scaling chosen, and should thus be valid in other scalings. In similar form these results should be valid for all theta pinch like equilibria with periodic additional fields.

The calculated frequency dependent interference force was enlisted to interpret the plasma motion in the dynamic phase of the ISAR T 1 high-beta stellarator.

Finally, it was found that with sufficiently strong damping of the helical modes (e.g. by phase mixing and the like) it might be possible to combine plasma heating and dynamic stabilization to advantage. This phenomenon could be interesting in the heating phase of a stationary stellarator reactor with high ß.

References:

/1/ Meyer, F., H.U. Schmidt; Z. Naturforsch. 13a, 1005 (1958)

/2/ Blank, A, A., H. Grad, H. Weitzner; Plasma Physics and Contr. Nucl. Fusion Research, IAEA, Vienna, 1969, Vol. II, p.607

/3/ Herrnegger, F., J. Nührenberg; Proc. 2nd Top. Conf. Pulsed High-Beta Plasmas, Garching 1972, IPP 1/127, B 2 (W. Lotz, Ed.)

/4/ Ribe, F.L.; Report LA-4098, Los Alamos (1969)

/5/ Ribe, F.L., M.N. Rosenbluth; Phys. Fluids 13, 2572 (1970)

/6/ Weitzner, H.; Plasma Physics and Contr. Nucl. Fusion Research, IAEA Vienna, 1971, Vol. III, CN 28/J-6

/7/ Rosenbluth, M.N., J.L. Johnson, J.M. Green, K.E. Weimer; Phys. Fluids 12, 726 (1969)

/8/ Freidberg, J.P.; Plasma Physics and Contr. Nucl. Fusion Research, IAEA, Vienna, 1971, Vol. III, CN 28/J-5

/9/ Fünfer, E., M. Kaufmann, W. Lotz, J. Neuhauser, G. Schramm; VI. Europ. Conf. on Contr. Fusion and Plasma Physics, Moscow (1973), Vol. I, p.109 and Vol. II (Suppl).

/10/ Grossmann, W., J. Tataronis; Z. Physik 261, 217 (1973)

/11/ Grossmann, W., M. Kaufmann, J. Neuhauser; Nucl. Fusion 13, 462 (1973)

/12/ Grossmann, W., M. Kaufmann, J. Neuhauser; 3rd Int. Symp. on Tor. Plasma Confinement, Garching (1973), paper E 3

/13/ Berge, G; Phys. Fluids 15, 150 (1972)

/14/ Berge, G.; Plasma Physics and Contr. Nucl. Fusion Research, IAEA, Vienna, 1969, Vol. II, p.483.

/15/ Haas, F.A., J.A. Wesson; Phys. Rev. Lett. 19, 833 (1967)