

MULTIPOL TOKAMAK EQUILIBRIA

W. Feneberg, K. Lackner

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**MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK**

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### Abstract

Solutions are given for axisymmetric MHD equilibria obtained by superposing the magnetic field of the plasma currents and the field of current carrying conductors situated outside the plasma. These equilibria were numerically calculated by several different iteration methods for solving the nonlinear free boundary problem. They can describe experiments in which the discharge time is long compared with the diffusion time of the magnetic field through the external conducting walls. Equilibria were obtained both for approximately circular plasma cross sections and strongly elongated (elliptical) ones. Such calculations are of interest for constructing axisymmetric Tokamak divertors, as well as for producing elongated cross sections, which have been postulated to be stable for higher values of plasma pressure.

The results presented here are restricted to the case of zero plasma pressure and a particular current distribution in order to reduce the number of free parameters and allow more detailed discussion of the dependence of the equilibrium configurations on the arrangement of the external conductors and the currents.

The calculations presented here are restricted to force-free equilibria with a single multipole distribution of volume currents. These restrictions were chosen to reduce

## INTRODUCTION

The vertical magnetic field needed for equilibrium in a TOKAMAK device can be produced either by the mirror currents induced by the toroidal discharge in a conducting casing or by additionally applied currents in external conductors. For experiments of sufficiently long duration, the diffusion time of the magnetic field through the conducting walls becomes too short to make the first method practicable.

The present paper is therefore concerned with the computation of axially symmetric ideal MHD equilibria in which the total poloidal magnetic field is obtained by superposing the vacuum field of currents in axially symmetric conductors on the field of the toroidal plasma currents. The superposition of these two fields can result in the appearance of a magnetic separatrix, which can be used for constructing an axially symmetric divertor [1]. Such multipole currents can be used, moreover, to deform the magnetic surfaces so as to produce configurations having better stability properties than TOKAMAKS with circular plasma cross section [2,3].

The equilibrium problem which we have treated numerically in this paper is a free boundary value problem, with boundary conditions at the axis and at infinity. Previously SHAFRANOV [4] and JOHNSON [5] have already used approximation methods to study the equilibrium position of a plasma column with circular cross section in an external vertical field. A problem analogous to the present one also occurs in the theory of the geomagnetic ring current, where it was solved by a technique similar to that used here [6].

The calculations presented here are restricted to force-free equilibria with a simple sharp boundary distribution of volume currents. These restrictions were chosen to reduce

the numbers of free parameters during the first tests so as to allow discussion of a number of geometrically different configurations. Calculations with varying plasma pressure and current profiles will be reported later for a reduced number of plasma cross sections. Tests have already shown the numerical methods developed in this paper to be equally applicable to a much wider class of current distributions.

### 1. MATHEMATICAL FORMULATION AND METHOD OF SOLUTION

Ideal magneto hydrodynamic equilibria with axial symmetry are described by solutions of the equation [7]

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} = - \left( 4\pi r^2 \frac{dp}{d\psi} + F \frac{dF}{d\psi} \right) \dots (1)$$

for the flux function  $\psi$ , where  $p$  is the plasma pressure and  $\psi$  and  $F$  are defined in terms of the magnetic field by

$$\vec{B} = \frac{1}{r} F \vec{e}_\varphi - \frac{1}{r} \vec{e}_\varphi \times \nabla \psi.$$

$F$  and the pressure  $p$  can be shown to be functions of  $\psi$  only. In the following we shall abbreviate the differential operator on the left of equ. (1) by  $L$  and the function on the right by  $f(\psi, r)$ .

It is convenient to split the flux function  $\psi$  into two parts

$$\psi = \psi_0 + \psi_p$$

where  $\psi_0$  is due to the applied currents in the external conductors and  $\psi_p$  due to the currents in the plasma

and any mirror currents induced by them in conducting walls. In the case of a multipole TOKAMAK without conducting casing,  $\psi_p$  has to satisfy

$$L \psi_p = f((\psi_0 + \psi_p), r)$$

and the boundary condition  $\psi_p = 0$  at the axis and at infinity. The contribution  $\psi_0$  of the multipole conductors can easily be computed using the standard expressions for the field of circular wire loops.

For the general case of  $f(\psi, r)$  the above problem is nonlinear and has to be solved by iterative techniques. Lackner [6], Fisher [8] and Marder and Weitzner [9] have previously shown that the most straightforward scheme for the above equation

$$L \psi_p^{n+1} = f((\psi_0 + \psi_p^n), r) \dots \dots \dots (2)$$

has only a limited range of convergence. They found that if the formulation of the problem allows of more than one solution, this scheme is able to give only one, generally the "shallow" and often physically uninteresting one. If, in particular, the problem allows of the trivial solution  $\psi_p = 0$ , as do many of the cases to be reported in this paper, the above scheme (2) will converge to it only.

The range of convergence can be significantly extended by using a more general iteration scheme of the form

$$L \psi_p^{n+1} = f_n((\psi_0 + \psi_p^n), r) \dots \dots \dots (3)$$

in which also the functional form of the right-hand side of equ. (1) is changed from iteration to iteration [10].

This change, which in practice implies a change of one or two parameters of the function  $f$ , is governed by the requirement that an equal number of other parameters  $b_i$  which depend on the solution of (1) are kept constant during the iterations, i.e.

$$b_i^n = F_i(\psi^n) = b_i = F_i(\psi). \dots (4)$$

A scheme of this type was used in [6] for computing equilibrium configurations of the geomagnetic ring current, where keeping constant the dipole moment of the plasma currents allowed to obtain equilibria far beyond the branch point.

A mathematically more detailed explanation of the success of the scheme and a comparison with the alternative method of Marder and Weitzner ( a three-point iteration scheme) is given elsewhere [10]; purely by intuition alone one would already expect a scheme of the type (3) to be successful, provided the  $b_i$  can be chosen so as to determine uniquely a solution. In particular, it is, of course, very easy to find restrictions of the form (4) which discriminate against the trivial solution  $\psi = 0$ .

As outlined in [10], a scheme of the type (3) has some further advantages concerning the rate of convergence of the iterations and the selection of parameter intervals corresponding to the range of existing solutions. Furthermore, the parameters  $b_i$  of formula (4) often turn out to be physically important and transparent quantities (such as the total current or height of the plasma column) which can thus be chosen prior to the calculations.

The results presented in this paper are restricted to a force-free, sharp boundary plasma with distribution of toroidal currents given by

and 
$$i_\psi = c \frac{FF'}{4\pi r} \dots (5)$$

$$FF' = A \text{ for } \psi \geq \psi_c$$

$$FF' = 0 \text{ for } \psi < \psi_c$$

Formulated in this way, the problem is nonlinear since the position of the plasma-vacuum boundary  $\psi = \psi_c$  depends on the solution itself. In particular, it allows for the trivial solution  $\psi_p \equiv 0$  in all cases where  $\psi_c \geq \max \psi_0$ , e.g. when  $\psi_c > 0$  and  $\psi_0$  is produced only by currents with a sign opposite to that of the plasma currents.

In implementing iteration scheme (3), A was either kept constant or varied so as to conserve the total plasma current  $I_p$ . The value of  $\psi_c^n$  was adjusted during iterations so as to ensure that the plasma boundary would either always intersect the symmetry plane  $z = 0$  at the same point at the inner (fig. 1 a) or outer side of the torus (fig. 1 b) or always touch either a double cone going through the center point (fig. 1 c) or a plane  $z = h$  parallel to the symmetry plane (fig. 1 d). The combinations found to be most successful here were:

- I. keeping A constant and varying  $\psi_c$  as in fig. 1 b (for plasma columns of approximately circular cross section) and
- II. keeping  $I_p$  constant and varying  $\psi_c$  as in fig. 1 c (for plasma columns elongated in the vertical direction).

Other combinations tested and giving convergence are: constant  $I_p$  and variation of  $\psi_c$  as in fig. 1 a (mainly used, however, with current distributions other than eqs. (5)) and constant  $I_p$  and variation of  $\psi_c$  as in fig. 1 b (used for elongated plasma columns, but giving in many cases worse convergence and over a smaller parameter range than II). In no case did we get convergence when keeping  $I_p$  constant and varying  $\psi_c$  as in fig. 1 b.

The linearized equation (3) for  $\psi_p^{n+1}$  was solved by a method similar to that used in [6] which consisted in numerical evaluation of its formal solution in spherical coordinates  $\rho$  and  $x = \sin \theta$ :



$$\psi_p^{n+1} = \sum_{v=0}^{\infty} \frac{1}{2v+3} \left[ \int_0^1 \int_1^{\rho} g_v d\rho + \int_1^{\rho} \int_0^1 g_v d\rho \right] (1-x^2) P_v^{(1,1)}(x) \quad (6)$$

where

$$g_v = -\frac{1}{h_v} \int_{-1}^1 f'(\psi_0 + \psi_p^n, \rho \sqrt{1-x^2}) P_v^{(1,1)}(x) dx \quad (7)$$

$P_v^{(\alpha, \beta)}(x)$  are the Jacobi polynomials and

$$h_v = \int_{-1}^1 (1-x^2) (P_v^{(1,1)}(x))^2 dx$$

their normalization factor. The numerical scheme for the evaluation of (6) and (7) can be reduced essentially to two matrix multiplications corresponding to a computational effort involving  $2 \cdot N \cdot M \cdot (J/2)$  multiplications per iteration cycle, where  $N$  and  $M$  are the number of radial and azimuthal grid points respectively and  $J$  the highest order of the expansion into Jacobi polynomials. This method automatically incorporates the correct boundary condition and is quite satisfactory except in the case of very small aspect ratio ( $\leq 0.06$ ), for which the expansion into Jacobi polynomials converges slowly. The computer time for a typical equilibrium calculation (excluding the time required for computing the field of the external conductors) with  $N = 100$ ,  $M = 50$ ,  $J = 50$  and requiring 10-20 iteration cycles is less than 10 sec on a 360/91.

The examples of application of this method that are given in the following sections refer to two sets of configurations:

- (1) equilibria with approximately circular plasma cross section and magnetic divertors
- and (2) equilibria with strongly noncircular plasma cross section.

The results are given in dimensionless quantities based on a characteristic distance  $R_0$  (indicated in the figures) and a current  $I_0$ . The corresponding c.g.s. quantities are given by

$$\bar{I}_p = I_0 I_p^* \dots \dots \dots \text{(total plasma current)}$$

$$\vec{B} = \frac{4\bar{I}_0}{R_0 c} \vec{B}^* \dots \dots \dots \text{(magnetic field)}$$

$$\psi = \frac{4I_0 R_0}{c} \psi^* \dots \dots \dots \text{(flux function)}$$

$$i_\psi = \frac{\bar{I}_0}{R_0^2} i_\psi^* = \frac{\pi I_0}{R_0^2} \frac{A^*}{r^*} \dots \dots \dots \text{(toroidal plasma current density)}$$

and  $A = \frac{4\pi^2}{c} \frac{I_0}{R_0} A^*$  from the dimensionless

values  $I_p^*$ ,  $\vec{B}^*$  etc. The asterisk for the dimensionless quantities is dropped again in the following.

## 2. TOKAMAK CONFIGURATIONS WITH MAGNETIC DIVERTORS

Fig. 2 shows an equilibrium configuration produced by the vacuum field of four conductors, with currents flowing in the direction opposite to the plasma currents. This configuration originally suggested by LEHNER [11], can be used to construct a divertor with a separatrix passing through a stagnation point on the inner side of the plasma torus. The four conductors are situated on a circular torus with an aspect ratio of 1 : 3, the current in each of the inner conductors is  $-0.725 \cdot I_p$  in each of the outer ones:  $-0.362 \cdot I_p$ . This configuration leads to a strong coupling

between the plasma and the multipole currents: any change with time of the multipole currents during the duration of the discharge induces an electric field in the toroidal direction, driving additional plasma currents.

To minimize this coupling between the plasma and multipole currents, we chose for fig. 3 an arrangement in which the sum of the currents in the five external conductors vanishes. For this configuration we compared the numerically computed equilibrium positions with those resulting from SHAFRANOV's formula [4], which for our case and in our dimensionless variables reads:

$$B_z = \frac{I_p}{4R_c} \left( \ln \frac{8R_c}{r_p} - \frac{5}{4} \right) \dots \dots \dots (8)$$

Here  $B_z$  is the vertical magnetic field (assumed to be homogeneous for Shafranov's formula) required to keep a circular plasma column with homogeneously distributed current  $I_p$  and a small radius  $r_p$  in equilibrium at a distance  $R_c$  between the axis and center line of the plasma. Fig. 4 gives a comparison between the equilibrium positions resulting from the above formula with those of our calculations for cases with constant values of A and of the external currents, but varying total plasma current  $I_p$ . As the actual field produced by the external conductors is inhomogeneous, we substituted  $B_z$  in equ. (8) by its value at  $r = R_c$ . The agreement with Shafranov's formula is very good except for the larger values of  $I_p$ , where the plasma column approaches the stagnation point and starts to deviate significantly from the postulated circular shape.

An additional problem is the stability of the plasma column to horizontal and vertical displacements [12]. For approximately circular plasma cross sections we found it possible to obtain solutions which are stable to such disturbances by adding suitable external conductors.

### 3. NON-CIRCULAR PLASMA CROSS SECTION

In addition to balancing the hoop force and forming magnetic divertors, external conductors can also deform the shape of the plasma column. According to a semi-empirical argument by ARTSIMOVICH and SHAFRANOV [3], plasma cross sections elongated in the axial direction should be stable to helical deformations for plasma pressures up to an order of magnitude larger than those permitted for circular columns. Flute type instabilities would then have to be stabilized by giving the elongated plasma column the shape of a segment [3]. As elliptic deformations are produced by external fields which destabilize the column with respect to vertical displacements [13], plasma of this type will probably require feedback stabilization of its vertical position. In the following we discuss three ways of producing elongated equilibrium configurations.

#### 3.1. CONDUCTORS ON A TOROIDAL SURFACE

For the case of a straight geometry, MUKHOVATOV and SHAFRANOV [13] have given an expression for the ellipticity

$$\epsilon = \frac{\rho_z^2 - \rho_x^2}{\rho_z^2 + \rho_x^2} \quad \text{of a plasma column with homogeneous}$$

current density in the quadrupole field of surface currents distributed over a concentric cylinder of radius  $d$  with a surface current density  $i = i_c \cos 2\omega$

$$\epsilon = -\frac{\pi}{2} \frac{i_c d}{I_p} \frac{(\rho_z + \rho_x)^2}{d^2} \dots \dots \dots (9)$$

To check the validity of equ. (9) in the toroidal case, we substituted the currents on the surface of the cylinder by sixteen discrete conductors arranged equidistantly on the surface of a circular torus of aspect ratio 0.491. To balance the hoop force, four conductors were added on the

surface of the torus (see fig. 5), with currents equal to  $+ 0.2 \cdot I_p$  in the inner pair and  $- 0.2 \cdot I_p$  in the outer pair. No effort was made to compensate for the fact that owing to the finite aspect ratio increasing the currents of the quadrupole conductors shifts the whole plasma column.

Results of our calculations for five different values of  $D = i_e d$ , together with the predictions of equ. (9) for these values of  $D$ , are given in fig. 6, showing remarkable agreement. The limit of the usefulness of equ. (9) is given by the fact that it treats  $D$  and the ratio  $(l_z + l_x)/d$  as independent parameters. While it is possible to find for a given  $D$  equilibria with plasma columns of various height  $l_z$  (and hence various values of  $(l_z + l_x)/d$  as well), this variation is limited by the existence of a stagnation point between the top of the plasma column and the torus formed by the external conductors (fig. 5). The position of this stagnation point depends strongly on  $D$  and weakly on  $l_z$ , so that for a given value of  $D$  there exists a maximum value of  $l_z$  for which the plasma surface coincides with the separatrix (fig. 7). These values of  $l_z$  and the resulting values of  $l_x$  also define a maximum of ellipticity  $\epsilon_m$  for a given  $D$ . The opposing effects of increasing  $D$  (reduction of  $l_{z_{max}}$ , but increase of  $\epsilon$  for a fixed  $l_z$ ) result in the existence of an absolute maximum in  $\epsilon$ , corresponding to a ratio of the semiaxes of approximately 5.9. From the experimental point of view this conductor arrangement has the advantage of great flexibility (the same conductors could be used to produce a hexapole field, thus resulting in triangular deformation of the column), but uses the available volume rather inefficiently.

### 3.2. CONDUCTORS ON TWO COAXIAL CYLINDERS

In order to improve the use of the volume, the conductors should be brought closer to the plasma column. We therefore studied a configuration in which the conductors are arranged along concentric cylinder surfaces of finite height with equal currents in all conductors of one wall (fig. 8a). Such an arrangement corresponds, at least in geometry, to that of a belt pinch.

The results for this configuration showed that for given currents in the inner and outer conductors the radial position of the magnetic axis depends practically only on the total plasma current  $I_p$  and not on the height of the plasma column or the parameter  $A$  (which for elongated equilibria with nearly constant radial distance is just proportional to the current density and inversely proportional to the plasma volume). For a given total plasma current  $I_p$ ,  $A$  determines the height of the plasma column and the ratio of its half-axes but not in a unique fashion (fig. 9): for values of  $A < A_1$  there exists no solution; for values  $A_1 < A < A_2$  there are two: a strongly elongated and a near-circular one.  $A_2$  is determined by the coincidence of the plasma boundary and the separatrix, giving the maximum possible elongation of the plasma column. Fig. 8 and 10 show that with increasing elongation of the plasma, the stagnation point at the top is pushed upward but recedes much more slowly than the plasma column advances, so that the latter ultimately catches up.

This maximum height of the plasma column was found empirically to correspond to approximately half the height of the conductor. The obvious suggestion of extending the row of conductors is unpracticable, however, since increasing currents are required in the inner conductors.

In this geometry the plasma volume is on the outside of the coil formed by the inner conductors and therefore makes use only of the stray fields, which naturally decrease with increasing length of the coil. Already for the case of figs. 8 - 10, the sum of the currents in the inner conductors is six times that in the outer conductors and 11.5 times the total plasma current.

Certainly the current distribution assumed in this case does not constitute an optimum and allowing a variation in the currents along the cylindrical surfaces will reduce the external currents required. Since with the present method such optimization has to be accomplished by trial and error adjustment of all external currents, it would be desirable to have available as a guide the solution of the corresponding boundary value problem with  $\psi = 0$  along a torus with rectangular cross section coincident with the surface on which the conductors are to be placed [10]. This solution would give the external current distribution required to make this surface a flux surface and to maintain the desired plasma equilibrium. The procedure corresponds essentially to the virtual casing principle of SHAFRANOV and ZAKHAROV [14].

### 3.3. CONDUCTORS ON ONE CYLINDER WALL

As the conductors on the inner cylinder wall of the examples in the last section are obviously very ineffective it is of interest to study the effect of the outer conductors alone. Since the hoop force will tend to press the plasma against these conductors, some elongation of the plasma column can be expected in this case as well. The conductor

arrangement and some typical equilibria are illustrated in fig. 11, showing an elongated and sector-like plasma column with half-axis ratio of up to  $\sim 2.7$  and an aspect ratio (defined as  $\frac{1}{2}$  height of plasma column/radius of magnetic axis) of up to 1.8. As the plasma is inside the coil formed by the outer conductors, only comparatively small currents are needed in the latter, for the example of case 3 in fig. 11 45 % of the plasma currents.

Contrary to the cases in sect. 3.2, fixing the plasma current  $I_p$  for given external currents does not fix the radial position of the magnetic axis. For a given value of  $I_p$ , there exists a series of equilibria corresponding to a narrow range of column heights (fig. 12), but a larger range of radial distances of the magnetic axis (fig. 13), and different values of the parameter A. This variation is limited by the plasma column touching the conductors for large A and getting very close to the axis for small A. As there are no stagnation points at the top and bottom of the column, the latter is not limited by a separatrix, although one could easily be created by an additional conductor pair with currents parallel to the plasma current above and below the plasma.

#### 4. CONCLUSION

We have developed a fast and a accurate general method for computing equilibrium configurations of multipole tokamak devices. The results presented in this paper are restricted to force free fields and a simple current distribution, which nevertheless lead to a nonlinear problem. Other tests have, however, shown the range of



applicability of this method to be much wider.

The results of such calculations are important for the construction of magnetic divertors and for the design of configurations in which the cross section of the plasma column has to be significantly non-circular. Furthermore they also automatically show the effect of the discreteness of the external conductors [1], which has also been discussed recently by YOSHIKAWA [15]. Future work with this method will concern equilibria with finite  $\beta$  and different current distributions, as well as design studies for plasma experiments.

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Fig. 1 Restriction on the variation of the plasma cross section between successive iteration cycles, as imposed by the four methods for determining  $\psi_c^n$  described in the text.

Fig. 2 Equilibrium configuration with circular cross section and stagnation point on the inside.  $I_p = 1$ ,  $I_1 = -0.725$ ,  $I_2 = -0.362$ ,  $A = 46$ .

Fig. 3 Equilibrium configuration with circular cross section and stagnation point on the inside.  $I_p = 0.926$ ,  $I_1 = 0.982$ ,  $I_2 = -0.341$ ,  $I_3 = -0.15$ ,  $A = 35$ .

Fig. 4 Equilibrium positions of a plasma column with  $A = 35$  and varying  $I_p$  in the field of the external conductors of fig. 3, as resulting from SHAFRANOV's formula (solid line) and from numerical calculations.

Fig. 5 Conductor arrangement used for the calculation in section 3.1. The plasma column in this figure corresponds to the limiting case, where plasma surface and separatrix coincide up to the stagnation point ( $I_p = 1$ ,  $D = -1$ ,  $A = 0.4683$ ).

Fig. 6 Comparison of the formular of MURKHOVATOV and SHAFRANOV (solid lines) for the ellipticity of a plasma column in a quadrapole field with results of numerical calculations in toroidal geometry, for  $D/I_p = -12$ ,  $-6$ ,  $-3$ ,  $-1$ ,  $-0.5$  (from left to right).

- Fig. 7 Limiting values of  $\epsilon$  and  $\ell_z/d$ , determined by the coincidence of plasma boundary and separatrix, as function of  $D/I_p$ .
- Fig. 8 Conductor arrangement, plasma boundary and position of stagnation points for the runs described in section 3.2. The current in each inner conductor is 0.21, in each outer conductor 0.035;  $I_p = 0.164$  (in all cases). (a)  $A = 4.71$ , (b)  $A = 3.33$ , (c)  $A = 3.226$ , (d)  $A = 3.423$ , (e)  $A = 3.63$ , (f)  $A = 3.75$ .
- Fig. 9 Axis ratio  $\alpha$  for the plasma columns shown in fig. 8 as a function of  $A$ .  $A_1$  corresponds to a branch point,  $A_2$  is determined by the coincidence of plasma boundary and separatrix.
- Fig. 10 Height of the plasma column and vertical distance between stagnation points as function of axis ratio  $\alpha$ , for the conditions of figs. 8 a - 8 f.
- Fig. 11 Conductor arrangement used in section 3.3. Currents in each of the external conductors : -0.05. Plasma contours (1) and (2) correspond to the same plasma current  $I_p = 2.888$ , illustrating the variation in shape and radial position with varying  $A$  ( $A = 3.155$  for (1),  $A = 4.41$  for (2)). Contour (3) corresponds to a higher plasma current ( $I_p = 3.33$ ,  $A = 3.144$ ).
- Fig. 12 Half height of the plasma column as function of  $A$ , for different values of plasma current.
- Fig. 13 Radius of the magnetic axis of the plasma column as function of  $A$ , for different values of plasma current.

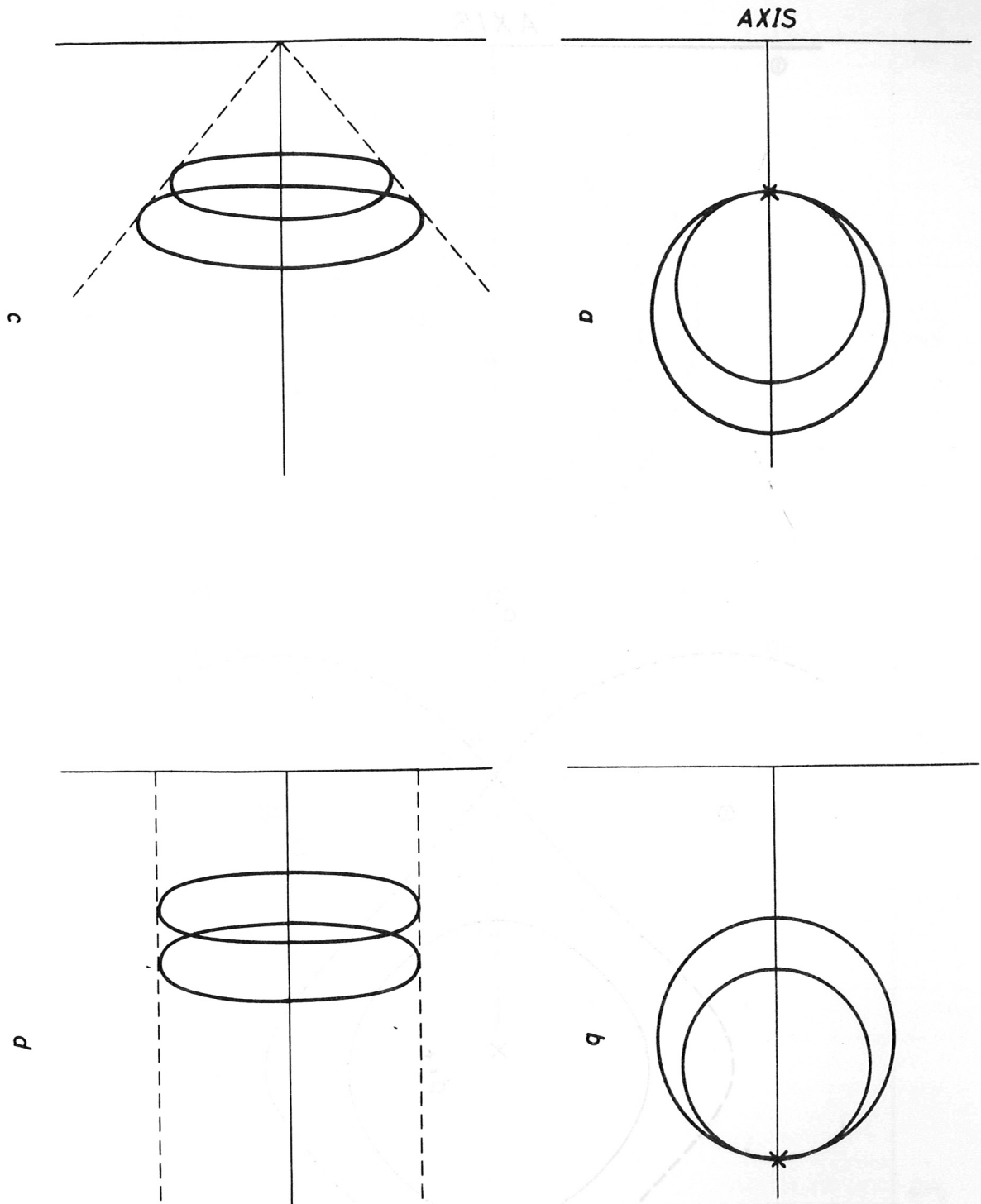


Fig. 1

Fig. 2

AXIS

Fig. 3

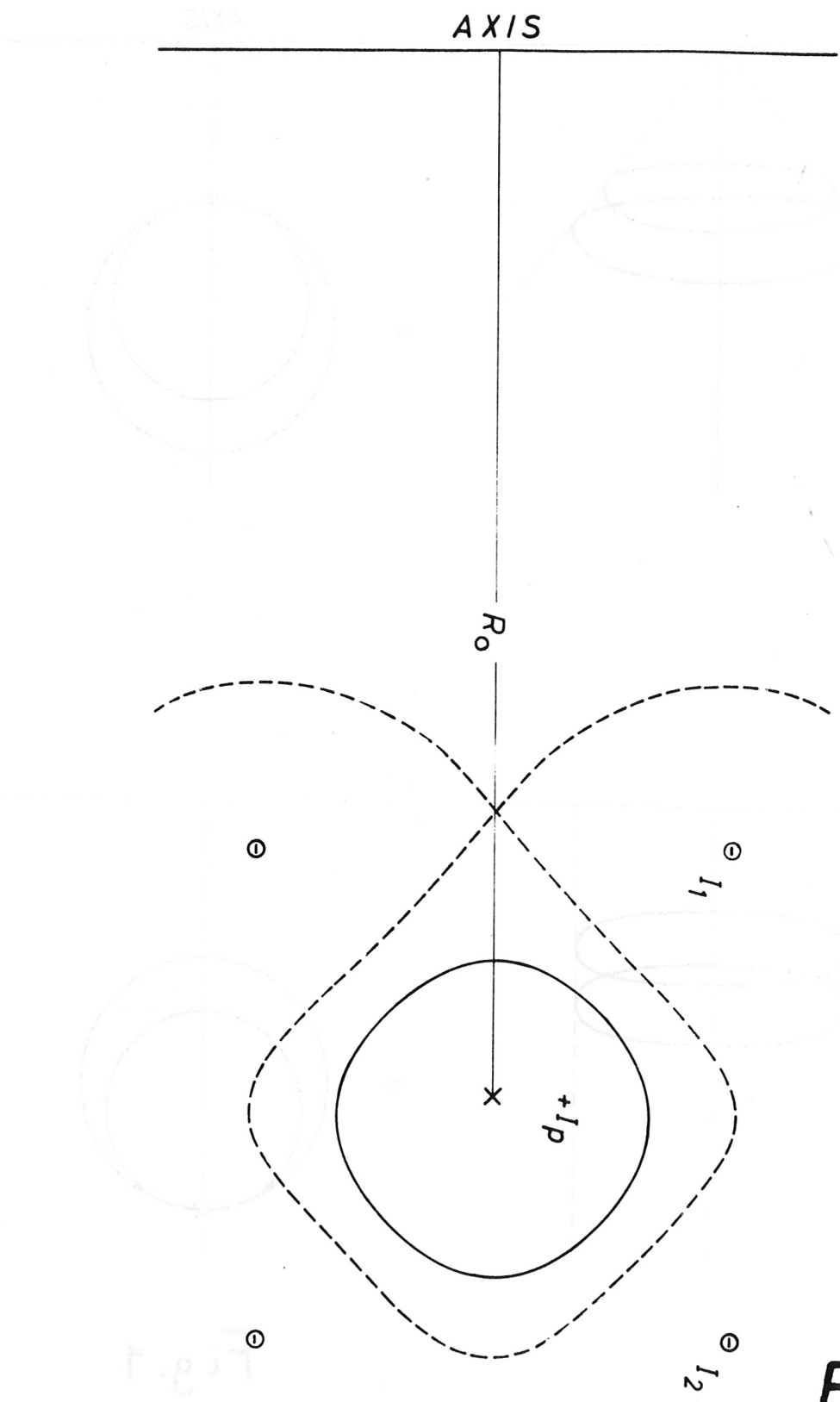


Fig.2

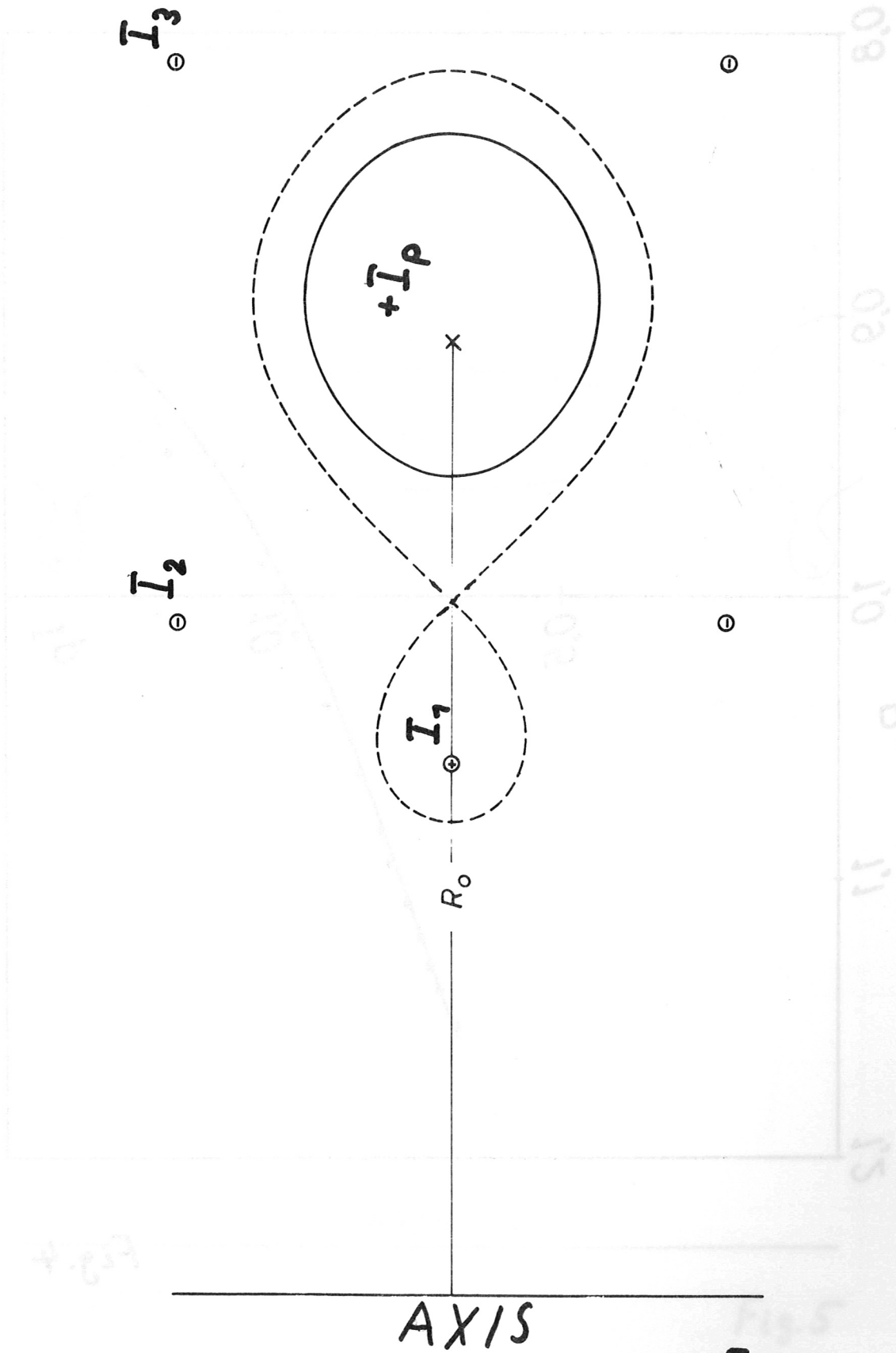


Fig. 3

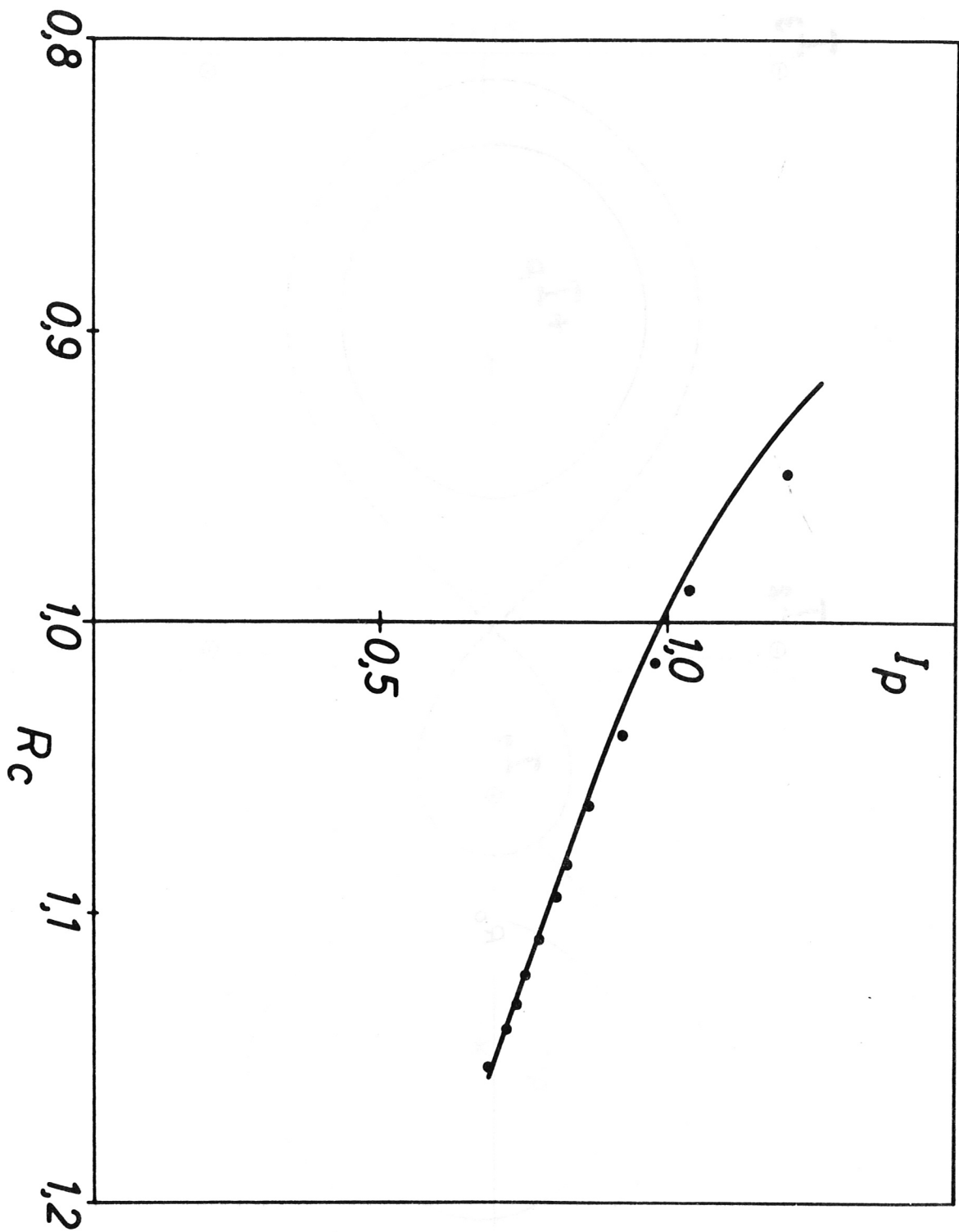
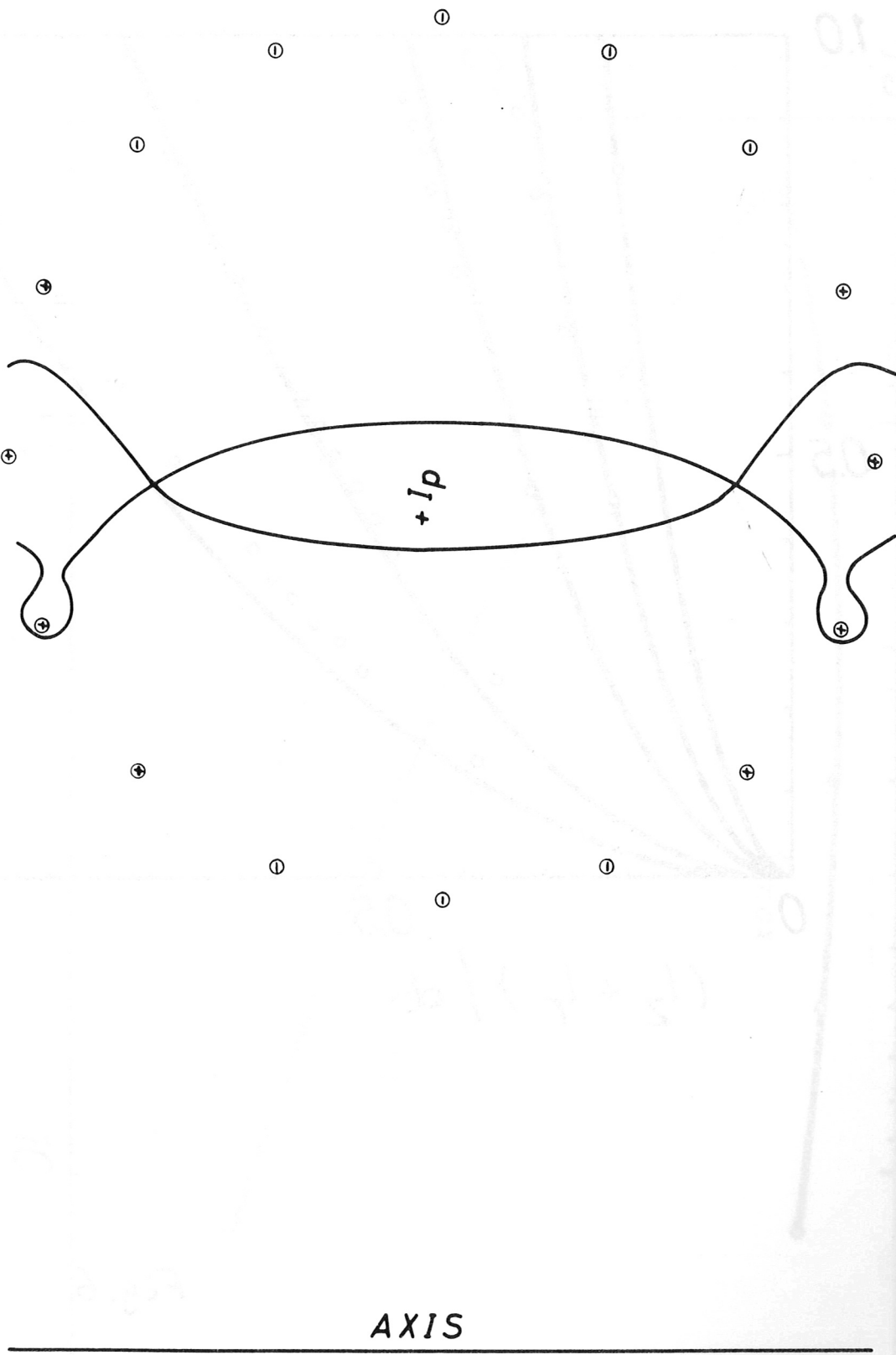


Fig. 4





**Fig.5**

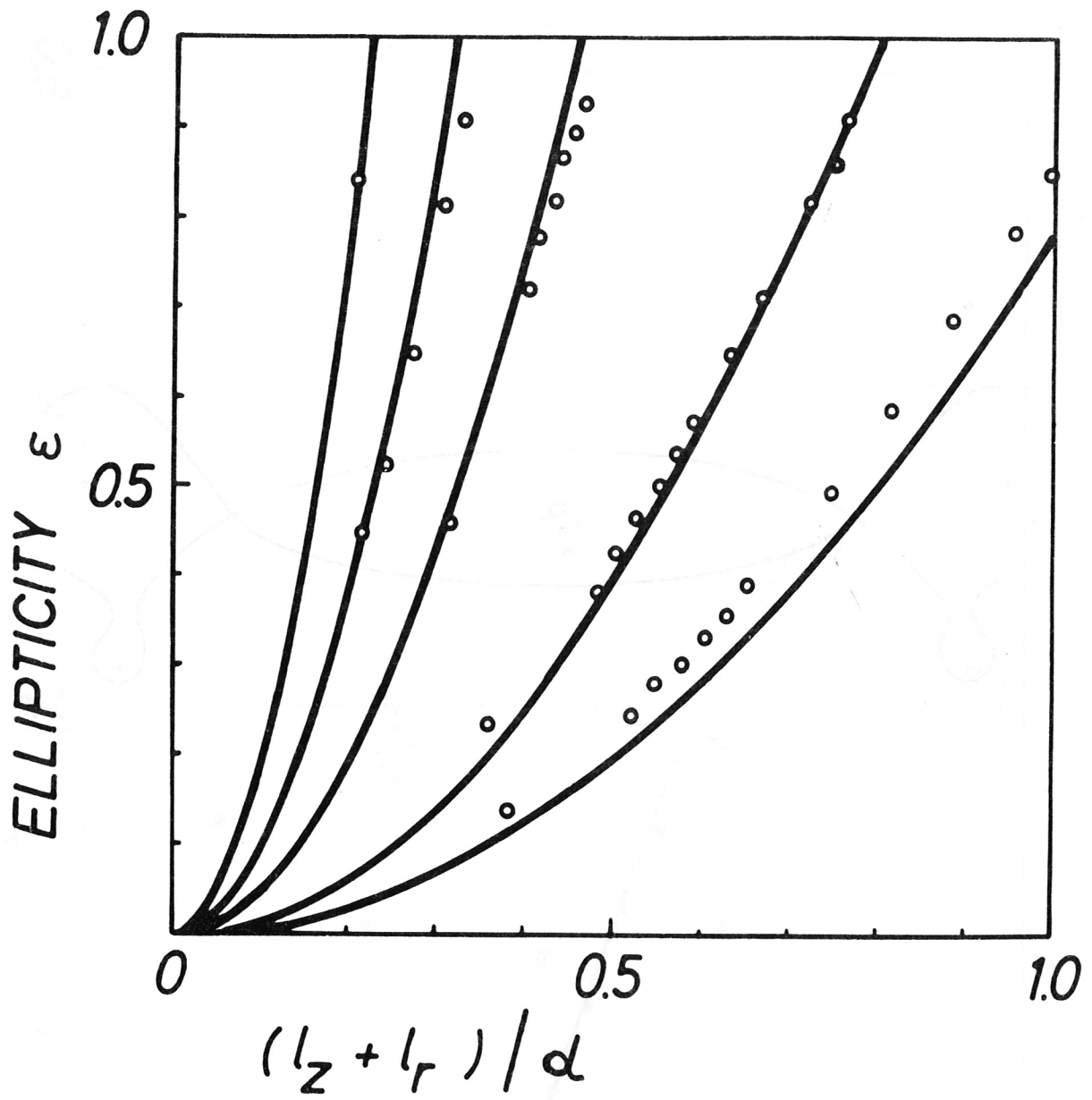


Fig. 6

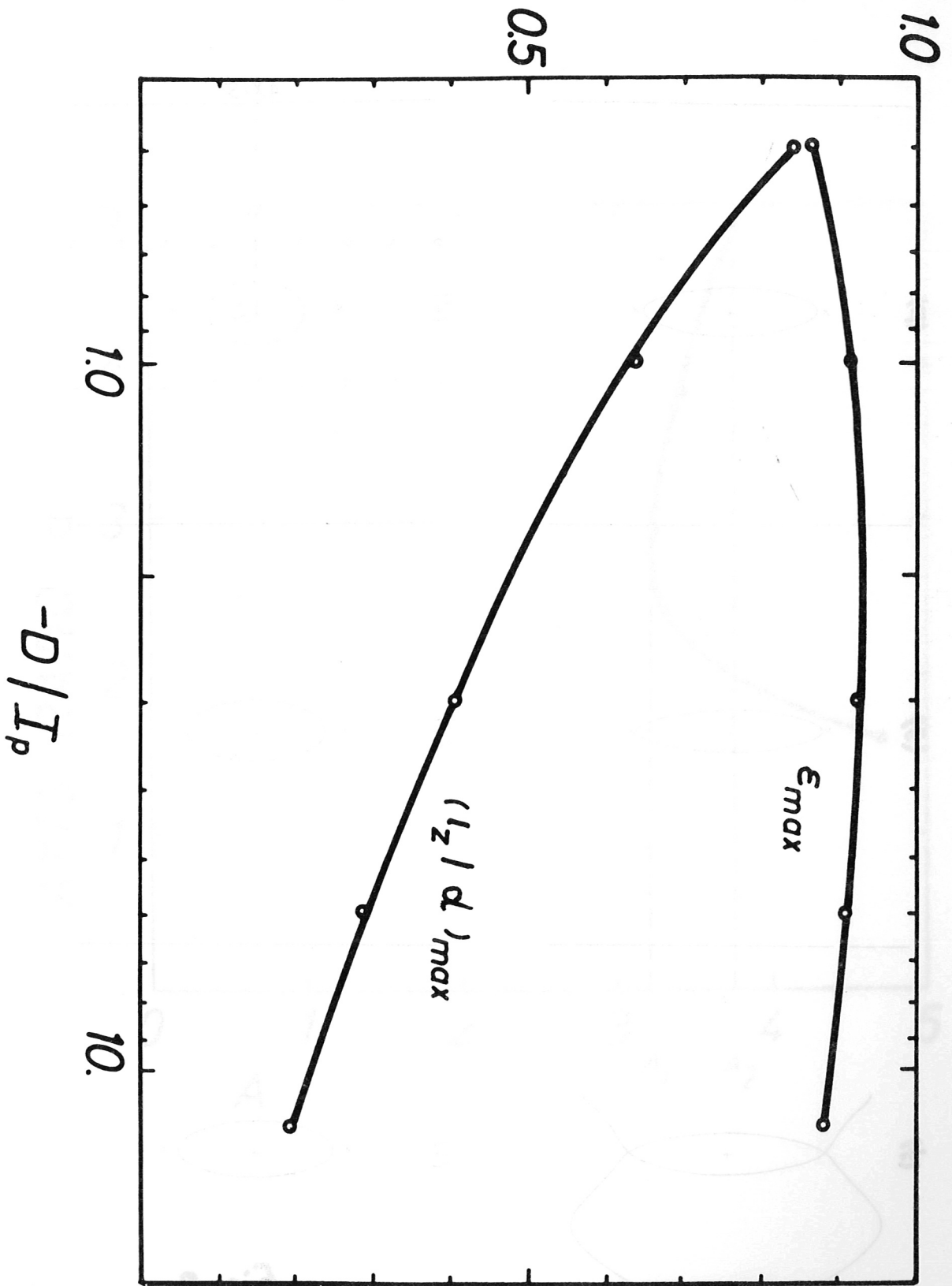


Fig. 7

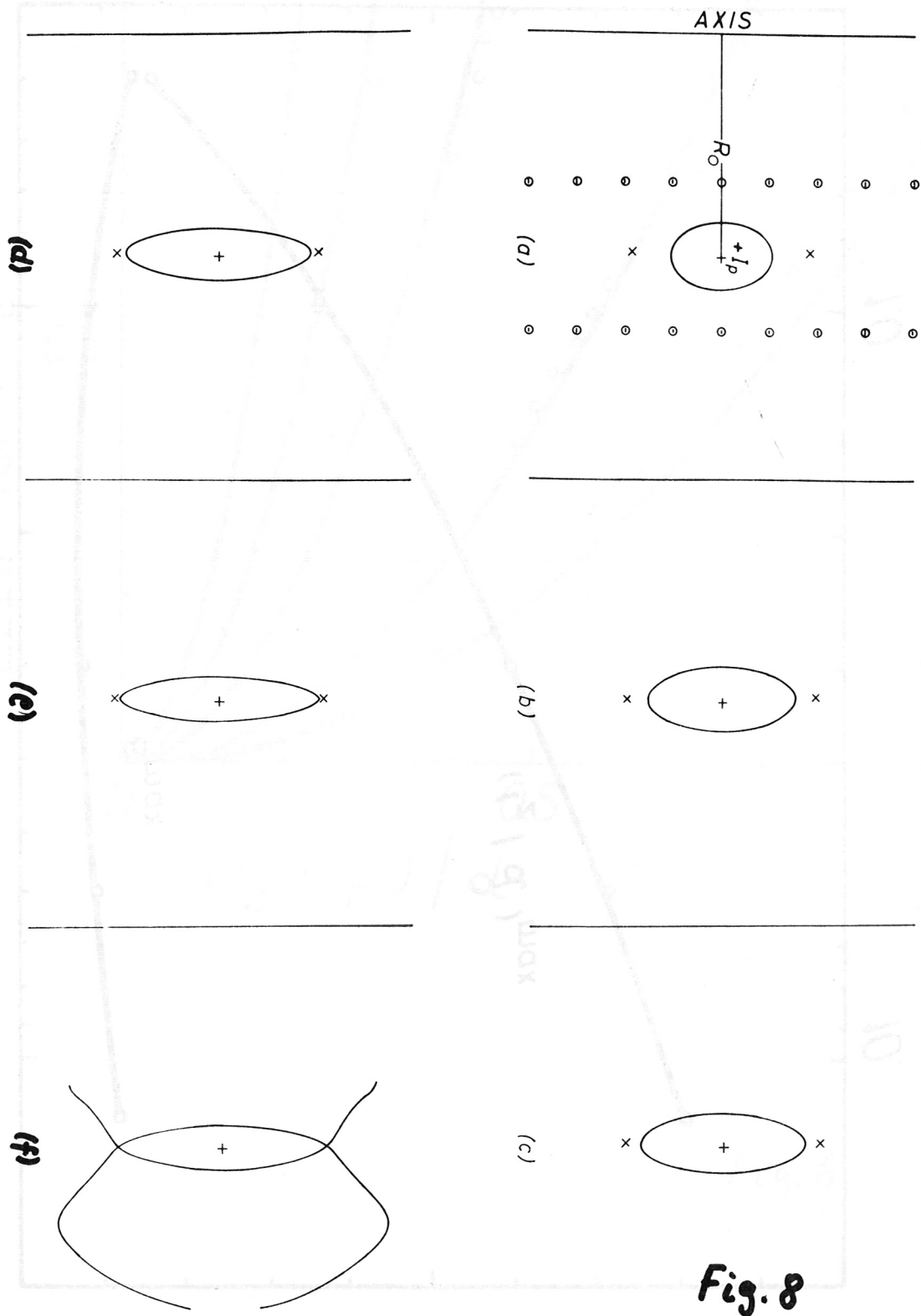


Fig. 8

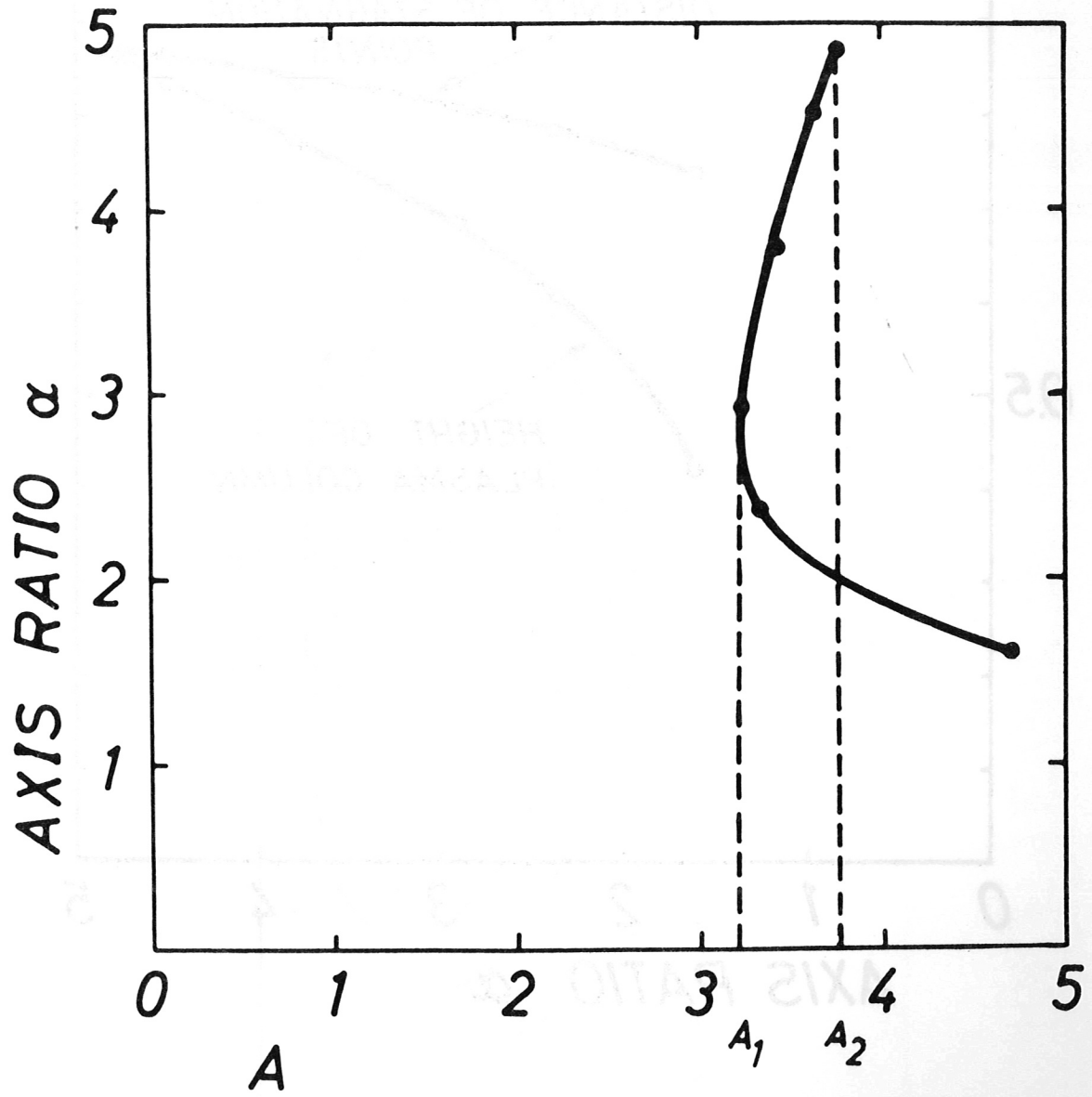


Fig. 9

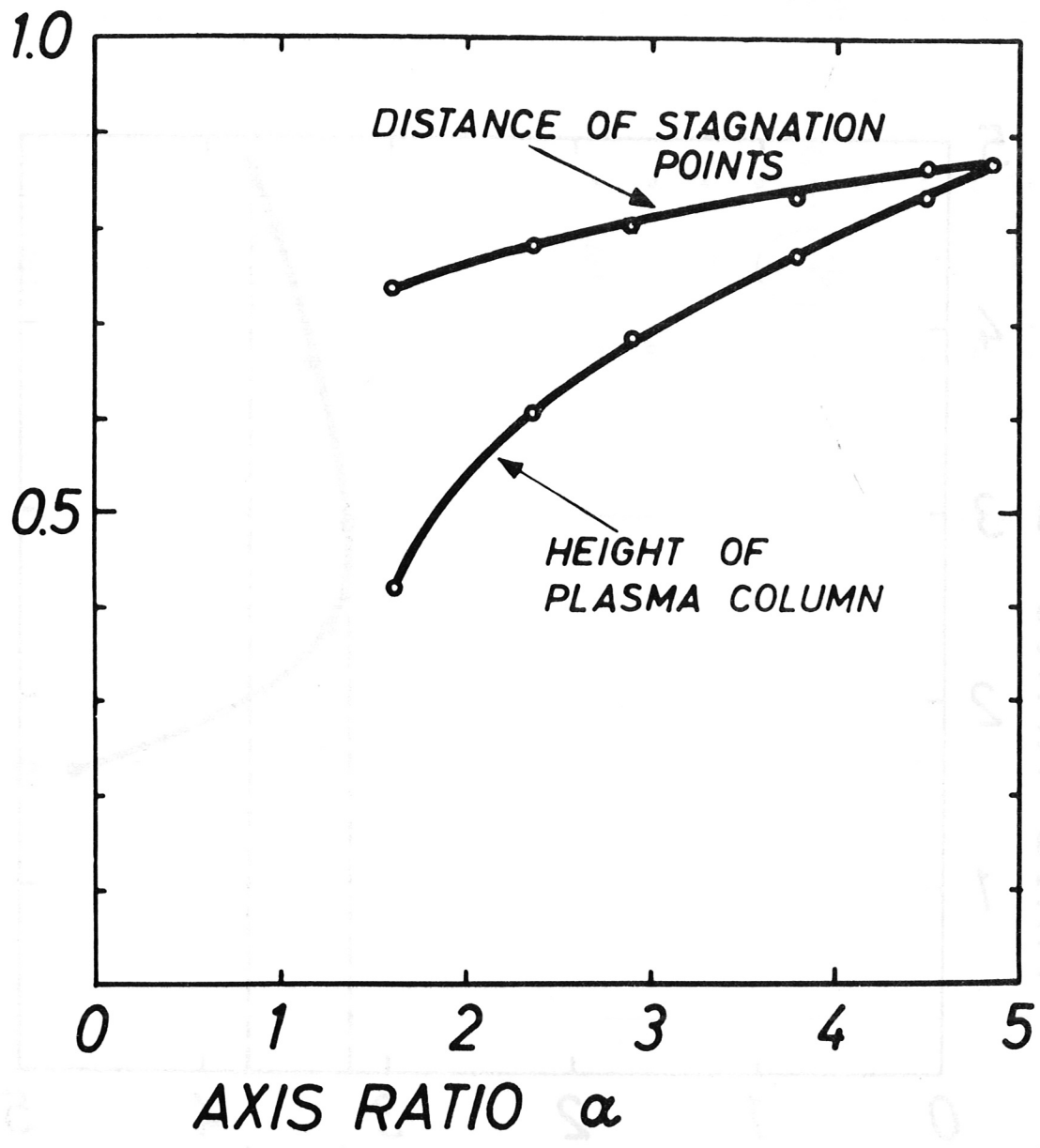


Fig. 10

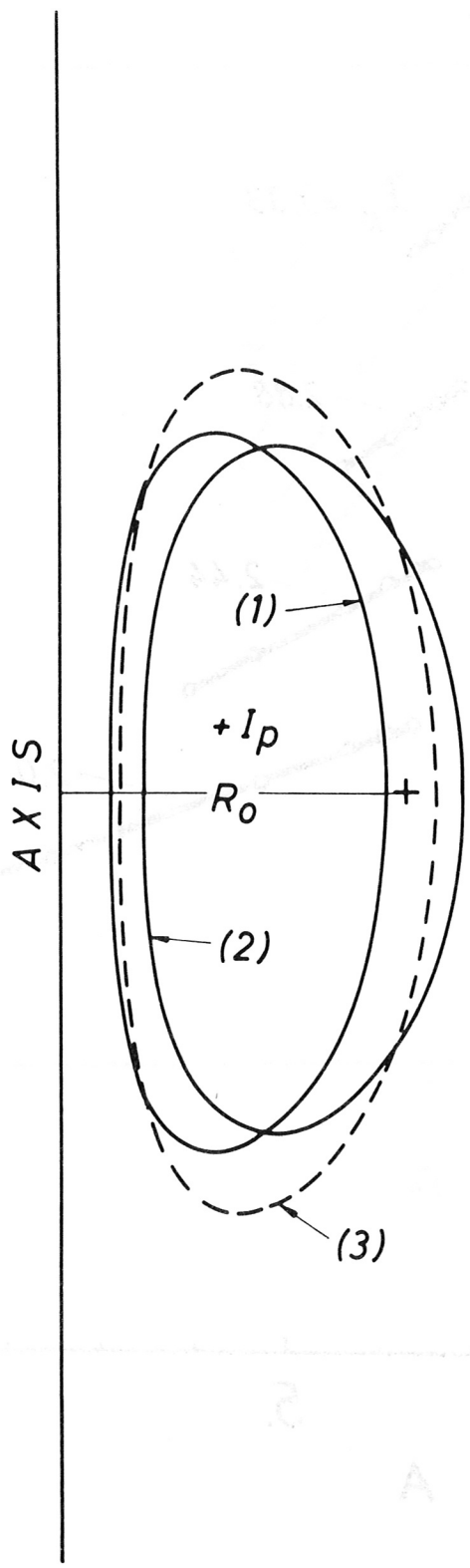


Fig.11

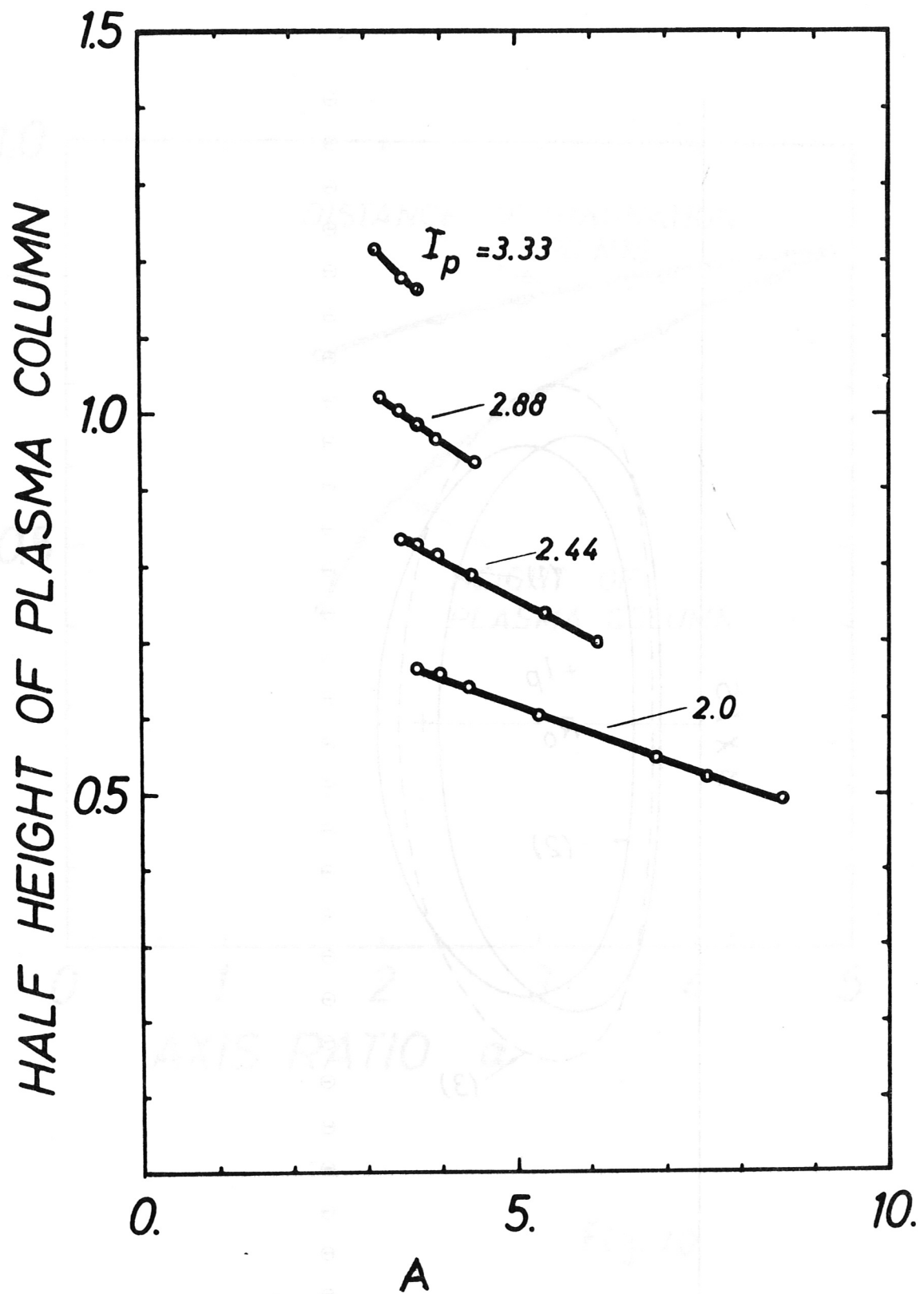


Fig. 12



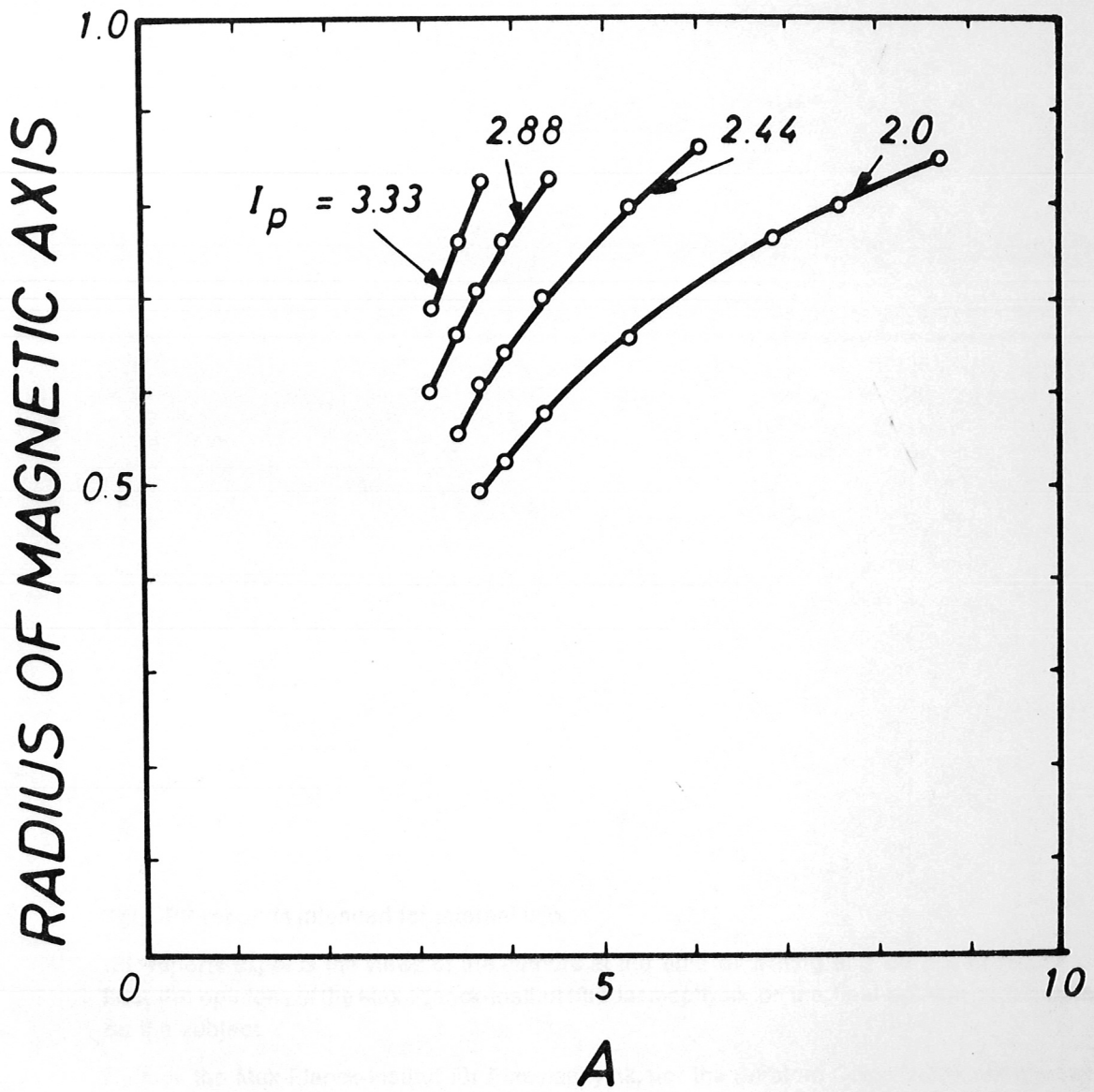


Fig. 13