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ABSTRACT

The dynamic stabilization of the hydrostatic Rayleigh-Taylor instability and the excitation of the related short wavelength parametric instabilities (resonances) require the application of enforced oscillations vertically to the unstable interface. The use of the resonance between the frequency of these oscillations and the appropriately chosen eigenfrequency of the overall system allows operation in regions of the stability regime which are difficult to reach otherwise. An air cushion between the body of liquid and the vibrator which delivers the enforced oscillations can be used as the spring determining this eigenfrequency instead of the familiar mechanical elements.

INTRODUCTION

The dynamic stabilization of the Rayleigh-Taylor[1] instability of a liquid-gas interface can be achieved by oscillating the body of liquid vertically, i.e. in a direction essentially perpendicular to this interface[2-4]. For a cylindrical container of diameter D, the maximum instantaneous acceleration

$$b_m = b \cos\omega t = a_m \omega^2 \qquad (1)$$

has to remain within the range given by

$$\omega \left(\frac{0.54\, D}{g} \right)^{1/2} \lesssim \frac{b_m}{g} \lesssim \left(\frac{0.5\, \eta^{2/5} \omega}{2\pi} - 30 \right) \qquad (2)$$

The symbols in these equations are: ω = oscillation frequency, a_m = maximum instantaneous oscillation amplitude, g = acceleration of gravity, and η = viscosity in poises (for D and g in cgs units). An extended theoretical analysis on this subject[5] (i) has shown that there exists also a condition on the minimum value of the surface tension lying below that of common liquids and (ii) has given a more rigorous numerical value for the upper stability boundary which is expressed here by the right-hand side of eq. (2). These theoretical results were found to agree with a detailed experimental investigation[6] varying both viscosity and surface tension.

Eq. (2) shows that for the purpose of increasing the diameter D beyond the order of one centimeter and/or reducing the viscosity below the order of one Poise, the values of $\frac{b_m}{g}$ to be applied for dynamic stabilization have to be extended into the regime between 10^2 and 10^3. This is outside the performance of conventional vibrators at their platform. Therefore, in the present paper, we are describing a method for applying values of $\frac{b_m}{g}$ to the body of liquid which overcome those performed by usual vibrators and which are then limited by the external gas pressure providing equilibrium against the hydrodynamic pressure. For this consideration the quality of the approximation as expressed by the right-hand side of eq. (2) is satisfactory.

RESONANCE METHOD

The basic idea of this method is to couple the body of liquid to the vibrator by means of an elastic link acting as a spring. When adjusting the resulting eigenfrequency, ω_o, of the system to the required operational frequency ω of the vibrator determined from eq. (2), the resonance between ω and ω_o can be used to obtain values of the oscillatory acceleration, b_m, at the liquid gas interface which largely exceed those of b_o, which is the maximum acceleration at the vibrator platform. For this case the oscillation amplitude a_m of the liquid gas interface is

$$a_m = \frac{a_o\,\omega_o^2\,M}{\sqrt{M^2(\omega^2-\omega_o^2)^2+k^2\,\omega^2}}\,. \qquad (3)$$

There a_o is the maximum instantaneous oscillation amplitude at the vibrator platform and M is the mass of the liquid (including its container in cases where the container is also coupled to the spring); k is the coefficient of the friction term. The eigenfrequency ω_o of the undamped system is determined by the spring constant s and by M

$$\omega_o = \sqrt{\frac{s}{M}} \qquad (4)$$

Since $b_m = a_m\omega^2$ and $b_o = a_o\omega^2$, eq. (3) can be modified to yield

$$\frac{b_m}{b_o} = \frac{\omega_o^2}{\sqrt{(\omega^2-\omega_o^2)^2+\dfrac{k^2\,\omega^2}{M^2}}}\,. \qquad (5)$$

The maximum value of b_m/b_o is obtained for the case $\omega \approx \omega_o$ to be

$$\frac{b_m}{b_o} \approx \frac{M\,\omega_o}{k} \qquad (6)$$

Among the particular schemes which may be used for a practical application of this method, (i.e. to achieve large values of b_m/b_o according to eq. (6)) we shall not discuss here the obvious solution of an appropriate mechanical spring (e.g. made of rubber or steel) upon which the vessel containing the liquid is mounted, although such an arrangement certainly serves the desired purpose. Instead, we consider a layer of gas being located inside the vessel between its bottom and the body of liquid as shown in Fig. 1. As a consequence of the oscillatory motion the liquid assumes also a plane, dynamically stabilized surface towards this layer of gas which acts like a planar air cushion. According to Fig. 1 the thickness of this air cushion is l while the depth of the (cylindrical) body of liquid is d; note that for eq. (2) to be valid it was assumed $d > D$, otherwise the formula for the lowest possible eigenfrequency, Ω_0, of the liquid surface waves governing the right-hand side of eq. (2) has to be modified.

Evaluating the eigenfrequency ω_0 of the system consisting of the gas cushion and the body of liquid, we assume that the value of ω_0 is large enough for the gas to be compressed or expanded adiabatically. Then one obtains for the spring constant s

$$s = \frac{\pi D^2}{4} \quad \frac{\gamma p_0}{l} \tag{7}$$

where γ is the adiabatic exponent of the gas, p_0 its static pressure and $\frac{\pi D^2}{4}$ the area of the liquid surface. The mass M of the body of liquid is

$$M = \frac{\pi D^2}{4} \varrho d \tag{8}$$

where ϱ is the mass density of the liquid. Inserting eqs. (7) and (8) into eq. (4) yields

$$\omega_0 = \sqrt{\frac{\gamma p_0}{\varrho dl}} \tag{9}$$

In order to obtain an estimate of the average of the coefficient k to be inserted in eqs (3), (5) and (6), we shall make the rough assumption that at any moment the axial motion of the liquid does not deviate too much from a viscous flow as described by the relation of Hagen-Poiseuille[7]. From that it follows that

$$k \approx 8 \pi \eta d. \tag{10}$$

For the factor $\frac{b_m}{b_0}$ as expressed by eq. (6) one obtains then ($\omega \approx \omega_0$):

$$\frac{b_m}{b_o} \approx \frac{D^2}{32\eta}\sqrt{\frac{\gamma p_o \rho}{dl}} \qquad (11)$$

Finally, in order to obtain an estimate of the values b_m/b_o also in the regions outside the resonance $\omega \approx \omega_o$, the eqs. (7), (8), (9) and (10) have to be inserted into eq. (5). This yields

$$\frac{b_m}{b_o} = \frac{\frac{\gamma p_o}{\rho dl}}{\sqrt{\left(\omega^2 - \frac{\gamma p_o}{\rho dl}\right)^2 + \frac{1024\,\eta^2\omega^2}{D^4\rho^2}}} \qquad (12)$$

Before reporting on a comparison of these estimates with experiments let us inspect the limits of the discussed method. For this purpose we use the relation[2]

$$\eta_c \approx 0.05\; D^{5/4} \qquad (13)$$

for describing a minimum critical value η_c which can be derived from eq. (2). Taking for the vessel diameter D = 3 cm, the value of η_c assumes about o,25 poise as the minimum viscosity to be inserted in eq. (11). Furthermore we choose $\rho = 1$ g/cm^3, d = 3 cm, l = 0.1 cm, $\gamma = 1.4$, and $p_o \approx 10^6$ Dyn/cm^2 (1 atmosphere). This yields for $\frac{b_o}{b_m}$ a factor of about 10^3.

EXPERIMENTAL RESULTS

The experiments where this effect was first observed were carried out by means of a vibrator built as a demonstration model[8] for dynamic stabilization. This vibrator was also used for obtaining the results described here. While usually the arrangement as shown in Fig. 1 was studied, for testing eq. (12) quantitatively it turned out to be easier to use a position where the open side of the vessel pointed upwards - quite as it normally does when a liquid is contained in a cup. By contrast to the everyday situation, however, air was blown carefully at the bottom of the cylindrical vessel from outside by means of a small tube crossing the body of liquid. The air cushion so building up required a steady adjustment of the oscillation parameters in order to maintain always the stabilization condition of the interface between the liquid and the air cushion.

The liquid used was mineral oil with a viscosity of about 2 poises, typical geometric parameters were: d = 2 cm, l = 0,2 cm, D = 1,95 cm. For these a plot of the experimental results is shown in Fig. 2. The acceleration b_o/g quoted there was measured by means of an acceleration probe which had to be calibrated by stabilization experiments without an air cushion. As can be seen in Fig. 2, good agreement was found between the measured values and the drawn curve representing eq. (12). The slight deviations at the point of resonance are mainly attributed to the model used for estimating the damping.

APPLICATIONS

With respect to practical applications of the effects connected with the dynamic stabilization of the Rayleigh-Taylor instability, the method described here obviously extends the achievable parameter range. Moreover, the resonant character of the b_m/b_o curve allows the mass M of the oscillated material to influence the value of b_m, as follows from eqs. (5) and (12). This leads to the interesting result that, for instance, a system can be so adjusted that the amount of liquid in an inverted container does not exceed a certain upper limit; i.e. the growing total mass M causes the value of b_m/g to increase until the upper stability boundary (right-hand side of eq. (2)) is surpassed and the resulting "rain effect" stabilizes the value of M at a maximum level M_m. This means that without changing the parameters of the oscillatory circuit (i.e. b_o and ω), the quantity of fluid which is "rained down" depends solely on the quantity of fluid refilled into the inverted container, e.g. by means of pumps and pipes. The application of this effect requires, of course, that another type of spring has to be used than the air cushion described here, since otherwise the parametric instabilities causing the "rain" are also excited in the interface between the liquid and the air cushion.

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FIGURE CAPTIONS

Fig. 1 Schematic arrangement of the resonance system. The upper layer of gas with thickness L acts as the spring determining the eigenfrequency of the oscillatory system.

Fig. 2 Comparison of experimental results with an air cushion as a spring and the theoretical model according to eq. (12). The experimental parameters were: $d = 2{,}0$ cm; $l = 0{,}2$ cm; $D = 1{,}95$ cm; $p_o \approx 10^6$ dyn/cm^2; $\varrho \approx 1$ g/cm^3; $\eta \approx 2$ poises.

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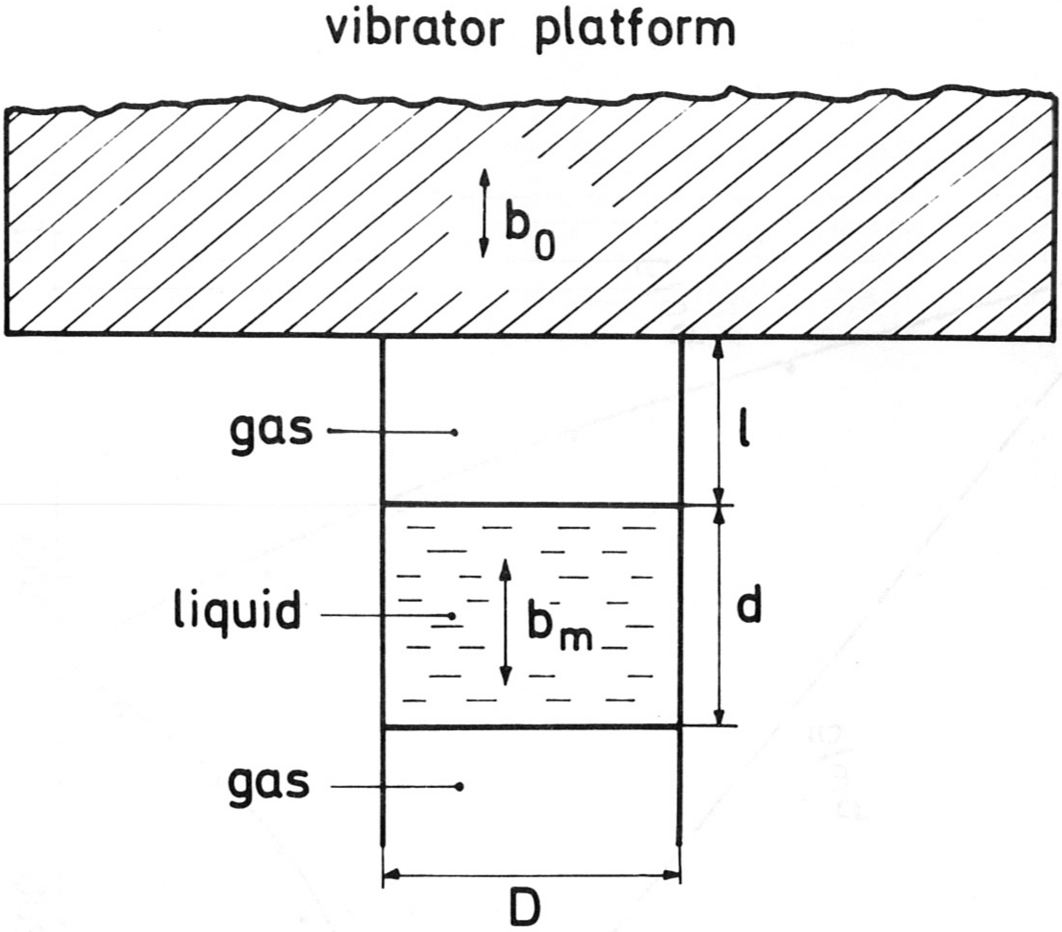

Fig. I

Schematic arrangement of the resonance system. The upper layer of gas with thickness L acts as the spring determining the eigenfrequency of the oscillatory system.

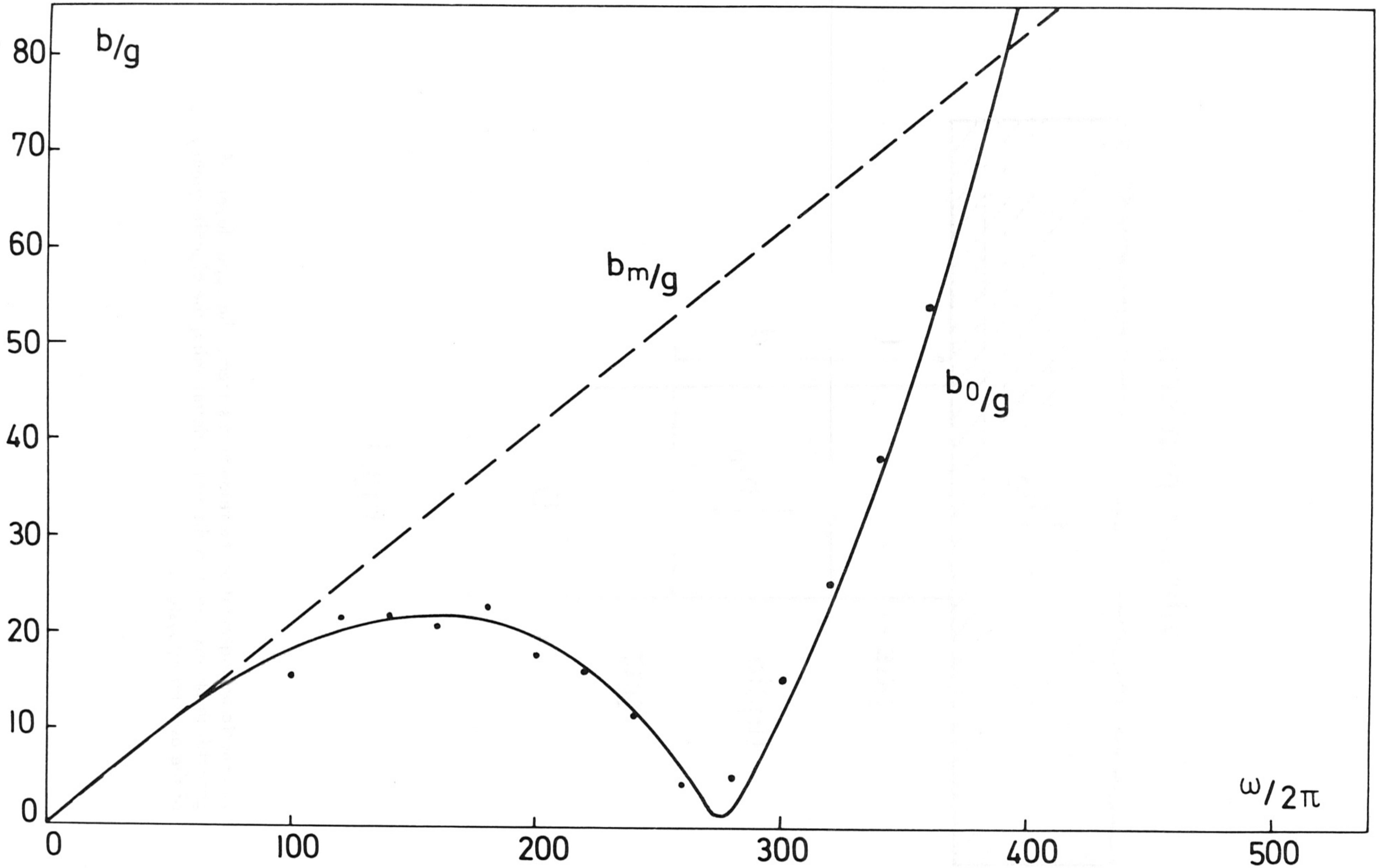

Fig. 2

Comparison of experimental results with an air cushion as a spring and the theoretical model acc. to eq. $(12)_3$. The experimental parameters were: d = 2.0 cm; l = 0.2 cm; D = 1.95 cm; $p_o \approx 10^6$ dyn/cm^2; $\varrho \approx 1$ g/cm^3; $\eta \approx 2$ poises.