A Kinetic Model for the Neutral Gas

between Plasma and Wall

S. Rehker, H. Wobig

IPP 2/208 January 1973

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Abstract

Starting from the Boltzmann equation for the neutral gas in a plasma, an integral equation for the density of the neutral gas is derived. We take inelastic collisions into account only as charge exchange and ionisation. The integral equation is solved numerically. It is shown that the reflection mechanism of neutral particles from the wall modifies the density profile. The distribution function of the charge exchange neutrals deviates from a Maxwellian, and for fast particles $(v > v_{th})$ we obtain $f_0 \sim v^{-1} \exp[-(\frac{v}{v_i})^2]$. The total particle and energy fluxes on the wall are also calculated.

I. Introduction

The interaction between ions and neutral particles in a plasma has become a field of increasing interest in the last few years. The neutral particles coming from the wall into a plasma contribute to the particle energy balances of the plasma B. Lehnert,(1972) Y.N. Dnestrovskii, D.P.Kostomarov, N.L. Pavlova,(1972). On the other hand, the hot neutral particles which leave the plasma volume are used for diagnostic purposes W Stodiek et al. (1971). In a thermonuclear plasma it is of special interest to know the flux and distribution functions of hot neutral particles impinging on the walls since a large flux of hot particles can cause severe damage to the first wall H.Vernickel,(1971). The description of the neutral boundary sheath has mostly been made in the frame work of a macroscopic fluid picture B. Lehnert, 1972, S. Yano,(1970). But since the dimension of the neutral sheath it is of the order of the mean free path for ionisation, the macroscopic picture is only an approximate one. This ist especially true if one wants to include details of the reflection mechanism of the neutral particles from the wall.

In the following paper we will give a kinetic description of the neutral hydrogen in a plasma and present numerical solutions of the integral equation for the density of neutral particles. This equation has already been derived and solved in Y.N.Dnestrovskii, D.P. Kostomarov, N.L. Pavlova,(1972). But there the authors approximate the plasma by a delta-distribution function ($f_i = \delta(v+v_i) + \delta(v-v_i)$; v_i = thermal velocity of the ions). In this paper we take into account several reflection mechanisms of neutral particles from the wall and compare the results obtained with a Maxwellian distribution of the ions with those of a delta-like distribution. The effect of the neutral particles on the plasma distribution function is neglected.

II. Kinetic Equation

If the neutral particles penetrate the plasma, they undergo all kinds of interaction with the plasma particles: ionisation, excitation, charge exchange, elastic collisions. In order to derive a kinetic equation which is still mathematically tractable, we shall only take into account the ionisation by electron and ion impact and the resonance charge exchange between like particles. The cross section for the charge exchange process $H + H^+ \rightarrow H^+ + H$ is of the order of $10^{-15}\ cm^2$, which is larger than the cross section of elastic collisions. All the cross sections for ionisation and charge exchange can be found in a paper of A.C. Riviere,(1971).

The loss rate due to ionisation is

$$- \int |\underline{v} - \underline{v}'| [\sigma_{eo} f_e(\underline{v}') + \sigma_{io} f_i(\underline{v}')] \, d^3\underline{v}' \cdot f_o(\underline{v}) \tag{1}$$

with f_e, f_i = electron and ion distribution function

σ_{eo}, σ_{io} = cross section for ionisation by electron and ion impact

$f_o(\underline{v})$ = distribution function of the neutral particles.

The interaction between neutrals and ions due to charge exchange is described by the term

$$\int |\underline{v} - \underline{v}'| \sigma_u [f_o(\underline{v}') f_i(\underline{v}) - f_o(\underline{v}) f_i(\underline{v}')] \, d^3\underline{v}' \tag{2}$$

σ_u is the total charge exchange cross section. It can easily be seen by integration over $\underline{v}$ that the charge exchange term conserves the number of particles.
With (1) and (2) we can write the Boltzmann equation for the neutral particles:

$$\underline{v} \cdot \nabla f_o = - \int | \underline{v} - \underline{v}' | [\sigma_{eo} f_e + \sigma_{io} f_i] d^3 \underline{v}' \cdot f_o(\underline{v}) \tag{3}$$

$$+ \int | \underline{v} - \underline{v}' | \sigma_u \left[f_o(\underline{v}') f_i(\underline{v}) - f_o(\underline{v}) f_i(\underline{v}') \right] d^3 \underline{v}'$$

In order to solve this equation, we must know the distribution of the ions and electrons. We assume that these distribution functions are local Maxwellian with given density and temperature profiles:

$$f_i = N(\underline{x}) C_i \exp[-(\frac{v}{v_i})^2] \; ; \; f_e = N(\underline{x}) C_e \exp[-(\frac{v}{v_e})^2] \tag{4}$$

$$v_i = \sqrt{\frac{2KT_i}{m_i}} , \quad v_e = \sqrt{\frac{2KT_e}{m_e}} \qquad \text{(thermal velocity)}$$

$N(\underline{x})$ = particle density

$$C_{i,e} = \frac{1}{(\sqrt{\pi}\, v_{i,e})^3} \qquad \text{(normalizing constant).}$$

If the neutral particles are slow compared to the electrons and ions, we may neglect $\underline{v}$ in the therm $| \underline{v} - \underline{v}' | \sigma_{e,i}$ (v-v') and the ionisation rate becomes independent of the velocity of the neutral particles:

$$\alpha(\underline{v}, T_i, T_e) = \int | \underline{v} - \underline{v}' | \left[\sigma_{eo} C_e \, e^{-(\frac{v}{v_e})^2} + \sigma_{io} C_i \, e^{-(\frac{v}{v_i})^2} \right] d^3 v' \tag{5}$$

This approximation is valid for the slow neutral particles coming from the wall ($E_o <$ 100 eV). In order to simplify the charge exchange term, we approximate the function $| \underline{v} - \underline{v}' | \sigma_u (| \underline{v} - \underline{v}' |)$ by a constant c. In reality, this function slowly increases in the regime $0 < E_{(|v-v'|)} <$ 30 keV. Therefore, this approximation is also valid for neutral particles and plasma temperatures below 30 keV.

With these approximations the kinetic equation for the neutral particles is

$$\underline{v}\cdot\nabla f_o = -N(\underline{x})[\alpha + c]f_o + N(x)C_i e^{-\left(\frac{v}{v_i}\right)^2} c\int f_o(\underline{v}')\, d^3\underline{v}' \tag{6}$$

As can be seen from this equation, the decay length of the neutral particle density is

$$\lambda = \frac{|v|}{N(x)[\alpha + c]} \tag{7}$$

In a dense plasma this length can be smaller than the dimensions of the plasma (i,e the characteristic length of the density and temperature profiles). This is especially true of thermonuclear plasmas, but also of plasmas in large Tokamak and Stellarator devices.

If $\lambda \ll \frac{N}{N'}, \frac{T}{T'}$, we may consider $N\, T_e$, T_i as constants in equation (6). In the region of neutral particles the plasma is homogeneous. If the plasma is impermeable to neutral particles, the structure of the neutral particle layer can be described in the one-dimensional approximation:

$$v_x \frac{\partial f_o}{\partial x} = -N[\alpha + c] f_o + N C_i e^{-\left(\frac{v}{v_i}\right)^2} c\int f_o(v')\, d^3v' \tag{8}$$

The most important quantities which we want to know by solving eq. 8 are the density $n(x) = \int f_o\, d^3v'$ of the neutral particles and the distribution of the outgoing particles $f_-(v_x, v_y, v_z, 0)$; $v_x < 0$ on the wall.

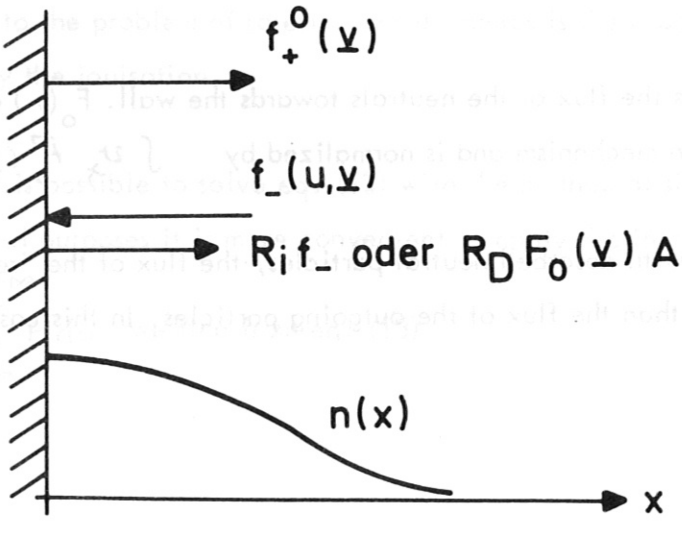

Fig. I

Boundary Conditions

The distribution of the ingoing particles $f_+(v_x, v_y, v_z, 0)$ consists of two parts. One type of particle is those which are emitted from the wall by thermal desorption, recycling plasma particles etc. The other part consists of reflected neutral particles, which in the simplest case is a linear combination of the neutral particles impinging on the wall.

$$f_+(0,\underline{v}) = f_+^0(\underline{v}) + \int R(\underline{v},\underline{v}')f_- \, d^3\underline{v}' \qquad (9)$$

Two limiting cases are

1) ideal case

$$f_+(0,\underline{v}) = f_+^0 + R_i f_-(0,\underline{v}) \qquad (9)$$

2) diffuse reflection

$$f_+(0,\underline{v}) = f_+^0 + R_D F_0(\underline{v}) \int\!\!\int\!\!\int_0^\infty v_x' f_- \, d^3\underline{v}' \qquad (10)$$

$R_{i,D}$ = reflection coefficient. The case $R_{i,D} < 1$ may describe the actual situation when a fraction of the outgoing particles is pumped away.

$A_- = \int\!\!\int\!\!\int_0^\infty v_x f_- \, d^3\underline{v}$ is the flux of the neutrals towards the wall. $F_0(\underline{v})$ depends on the details of the desorption mechanism and is normalized by $\int v_x F_0(\underline{v}) \, d^3\underline{v} = 1$

If the wall is covered with absorbed neutral particles, the flux of the "reflected" particles can be larger than the flux of the outgoing particles. In this case we have diffuse reflection with $R_D > 1$. The problem described here is similar to the albedo problem of astrophysics S. Chandrasekhar, 1960. In addition to the simple albedo problem we here have to deal with internal reflections.

Formal Solution of the Kinetic Equation

Because of the homogeneous plasma background, eq. (8) can be simplified. Since α and c are constants, eq. (8) can be integrated over v_y and v_z. $F(v_x) = \int f_o \, dv_y dv_z$. We measure the velocity in units of the thermal velocity of the ions:

$$\underline{u} = \frac{\underline{v}}{v_i} \tag{11}$$

and introduce a new space coordinate z by

$$z = \int \frac{N(\alpha + c)}{v_i} dx \tag{12}$$

With these simplifications we find

$$u \frac{\partial F}{\partial z} + F = \beta \frac{e^{-u^2}}{\sqrt{\pi}} \int_{-\infty}^{+\infty} F \, du \tag{13}$$

u is the x component of $\underline{u}$ and $\beta = \frac{c}{\alpha + c}$

This equation is well known in the theory of rarefied gas dynamics (M.M.R.Williams, 1971). The difference to the problem of rarefied gas dynamics is the absorption mechanism which is introduced by the ionisation.

In principle, it is possible to solve eq. (13) with the method of singular eigenmodes, but for numerical purposes it is more convenient to apply the integral equation method.

With $n(z) = \int_{-\infty}^{+\infty} F \, du$ we find from eq. (13)

$$u > 0 \rightarrow F_+ = \beta \int_0^z \frac{e^{-u^2}}{\sqrt{\pi}} n(z') \frac{e^{+(z'-z)/u}}{u} \, du\, dz' + F_+(0,u) e^{-\frac{z}{u}}$$

$$u < 0 \rightarrow F_- = \beta \int_\infty^z \frac{e^{-u^2}}{\sqrt{\pi}} n(z') \frac{e^{\frac{z'-z}{u}}}{u} dz' \tag{14}$$

The distribution of the outgoing particles is

$$F_-(0,u) = \beta \int_0^\infty \frac{e^{-u^2}}{\sqrt{\pi}} n(z') \frac{e^{-\frac{z'}{|u|}}}{|u|} dz' ; \tag{15}$$

With n (z) = $\int_0^\infty F_+ \, du + \int_{-\infty}^0 F_- \, du$

we obtain

$$n(z) = \beta \int_0^\infty \int_0^\infty \frac{e^{-u^2 - \frac{|z-z'|}{u}}}{\sqrt{\pi}\; u} du\, n(z')dz' + \int_0^\infty F_+(0,u) e^{-\frac{z}{u}} du \tag{16}$$

The second term on the right-hand side describes the particles which come from the wall and arrive at z without any ionisation or charge exchange process. The first term describes those charge exchange neutrals which originate at z′ and reach the point z without any further interaction.

The general behavior of the solution is determined by the properties of the kernel. The kernel $K(|z-z'|)$ is positive and has a logarithmic singularity at z′ = z. It can be shown that

$$\int_0^\infty K(|z-z'|)dz' \leq 1 \quad ; \quad K = \int_0^\infty \frac{e^{-u^2 - \frac{|z-z'|}{u}}}{\sqrt{\pi}\; u} du \tag{17}$$

With this result one finds by iteration of eq. (16)

$$n(z) \leq [1 + \beta + \beta^2 + \ldots] \max \int_0^\infty F_+ e^{-\frac{z}{u}} du \tag{18}$$

and

$$\int_0^\infty n(z)\,dz \leq \frac{1}{1-\beta}\int_0^\infty F_+(0,u)\,u\,du \qquad (19)$$

This means that for $\beta < 1$ and finite input flow of neutral particles ($\int_0^\infty F_+ u\,du < \infty$) a solution of eq. (16) exists and can be found by a Neumann series. Furthermore, inequality (19) says that n (z) $\to$ 0 if z$\to +\infty$.

From $\beta < 1$ we see that the presence of ionisation guarantees the existence of a unique solution.

By differntiation of eq. (16) and partial integration we find

$$\lim_{z\to 0} n'(0) = K(|z|)\,n(0) + \int_0^\infty K(|z'|)\,n'(z')\,dz' - \int_0^\infty \frac{F_+}{u}\,du \qquad (20)$$

If there are no arbitrarity slow particles from the wall (i.e. $F^0_+ = 0$ for $u < u_0$), the last term in (20) is bounded ($\int_0^\infty F_+ \frac{du}{u} < \infty$). In this case n' (o)$\to\infty$ because of the logarithmic singularity of K ($|z|$) at z=0. The general shape of n (z) is shown in fig. 2.

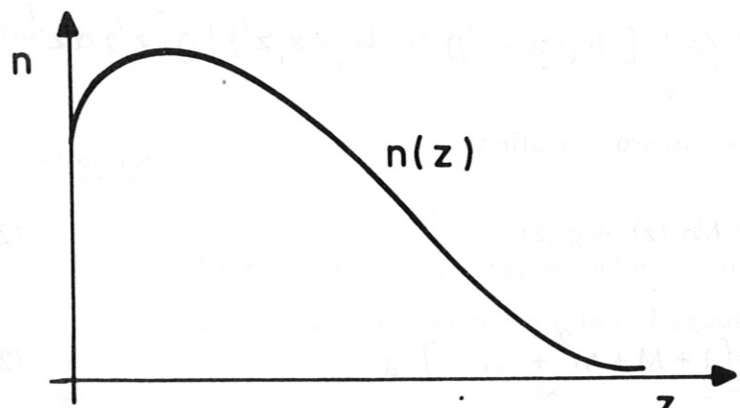

Fig. 2

In the foregoing considerations we have made an assumption about $F_+ (o,u)$ ($\int_0^\infty F_+ u\,du < \infty$). But since $F_+ (o,u)$ depends on $F_- (o,u)$, we have to consider this in more detail.

I Ideal Reflection

Following eq. (9) $\quad [\, F_+ = F_+^o + R_i F_- \,]$

and eq. (15), we find

$$n(z) = \beta \int_0^\infty K(|z-z'|)\,n(z')\,dz' + \frac{\beta R_i}{\sqrt{\pi}} \iint_0^\infty \frac{e^{-u^2}}{u}\, e^{-\frac{z+z'}{u}}\,du\,n(z')\,dz' + \int_0^\infty F_+^o(u)\,e^{-\frac{z}{u}}\,du \tag{21}$$

The presence of ideal reflection thus means that the kernel $K\,(|z-z'|)$ has to be replaced by $\bar{K} = K\,(|z-z'|) + K_1(z,z')$, where

$$K_1 = \frac{R}{\sqrt{\pi}} \int_0^\infty e^{-u^2} \frac{e^{-\frac{z+z'}{u}}}{u}\,du \tag{22}$$

This kernel is positive but without singularities.

With

$$g(z) =: \int_0^\infty F_+^o\, e^{-\frac{z}{u}}\,du$$

and

$$Mg =: \beta \int_0^\infty [\, K(|z-z'|) + K_1(z,z')\,]\, g(z')\,dz'$$

the formal solution of the integral equation

$$n(z) = Mn(z) + g(z) \tag{23}$$

is

$$n(z) = [\,1 + M + M^2 + \dots\,]\; g \tag{24}$$

All terms in (24) are positive.

By estimating Mg we find

$$Mg \leq \beta \int_0^\infty [K(|z-z'|) + K_1(z,z')]dz' \cdot \max g$$

With

$$\int_0^\infty K(|z_i - z'|)\, dz' \leq 1$$

and

$$\int_0^\infty K_1(z, z')\, dz' \leq \frac{R_i}{2}$$

we obtain the result

$$Mg \leq \beta \left(1 + \frac{R_i}{2}\right) \max g \tag{25}$$

and from (24)

$$n(z) \leq \sum_0^\infty \left[\beta\left(1 + \frac{R_i}{2}\right)\right]^m \cdot \max g \tag{26}$$

max $g = \int_0^\infty F_+^0\, du$ is the density of the inflowing particles and therefore finite (max $g < \infty$). We therefore conclude the existence of a solution under the condition

$$\beta\left(1 + \frac{R_i}{2}\right) < 1 \tag{27}$$

This condition is only sufficient for the existence of a solution, but not necessary.

II Diffuse Reflection

To prove the existence of a solution in this case, we take into account the degeneracy of the Kernel. In the case of diffuse reflexion the integral equation (16) becomes

$$n(z) = \beta \int_0^\infty K(|z-z'|)\, n(z')\, dz' + R_D \int_0^\infty F_0(u)\, e^{-\frac{z}{u}}\, du \cdot A_- + g(z) \tag{28}$$

$$A_- = \frac{\beta}{\sqrt{\pi}} \iint_0^\infty e^{-u^2 - \frac{z'}{u}} \, du \; n(z') \, dz' \tag{29}$$

A_- is the flux of the outgoing neutrals. The solution procedure is as follows: Eq. (28) will be solved with a given $A_- < \infty$. The solution u(z) is inserted into the right-hand side of (29), and (29) will be solved for $A_- > 0$.

Let us consider the special case $R_D = 1$. We integrate (28) over z and obtain

$$\int_0^\infty n(z)\, dz = \beta \iint_0^\infty K(|z-z'|)\, dz \, n(z')\, dz' + A_- + A_+^0 \tag{30}$$

$$A_+^0 = \int_0^\infty g(z)\, dz = \int_0^\infty F_+^0 \, u \, du$$ is the input flow.

From the explicit form of $K(|z-z'|)$ we find

$$\int_0^\infty K(|z-z'|)\, dz' = 1 - \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-u^2 - \frac{z}{u}} \, du$$

This formula together with (29) and (30) yields

$$(1-\beta) \int_0^\infty n(z)\, dz = A_+^0 \tag{31}$$

The solution of (28) is

$$n(z) = [1 - M_0]^{-1} \left\{ A_- \int_0^\infty F_0(u)\, e^{-\frac{z}{u}} + g(z) \right\} \tag{32}$$

Def. $\quad M_0 \equiv: \beta \int_0^\infty K(|z-z'|) \ldots dz'$

From (31) and (32) follows

$$(1-\beta)A_{-}\left\{\int_0^\infty [1-M_o]^{-1}\int_0^\infty F_o(u)e^{-\frac{z}{u}}\,du\,dz\right. \qquad (33)$$

$$= A_{+}^{o} - (1-\beta)\int_0^\infty [(1-M_o)^{-1}g(z)]\,dz$$

If the right-hand side of (33) is positive, we have found the solution. In order to show this, we start from the equation

$$f(z) = M_o f(z) + g(z) \qquad (34)$$

With the formal solution

$$f(z) = (1 - M_o)^{-1} g(z) \qquad (35)$$

By integrating eq. (34) over z we find the estimate (see eq. (19)):

$$\int_0^\infty f(z)\,dz \leq \frac{1}{1-\beta}\int_0^\infty g(z)\,dz$$

or

$$\int_0^\infty [(1-M_o)^{-1}g(z)]\,dz \leq \frac{1}{1-\beta}A_{+}^{o} \qquad (36)$$

This inequality shows that the solution of eq. (33) is positive and finite for $\beta < 1$.

For $R_D < 1$ the proof of existence is similar to the procedure above. The case of diffuse reflection is mathematically more convenient than the case of ideal reflection. The reason is that we have to deal with a Wiener-Hopf-type integral equation for which analytical methods of solution are available (W.L. Smirnov, 1963).

Special Choice for Ion Distribution Function

We approximate the ion distribution $f_i(u)$ by

$$f_i(u) = \frac{1}{2}[\delta(u+1) + \delta(u-1)] \tag{37}$$

This distribution function was used by Dnestrovskii in the case of an inhomogeneous plasma. With this distribution, the kernel of the integral equation becomes

$$K = \frac{1}{2} e^{-|z-z'|} \tag{38}$$

There is no singularity in K. The solution of this integral equation is given in W.L. Smirnov,(1963). If the particles coming from the wall can be considered as monoenergetic

$$\begin{aligned} F_+^{\,0}(u) &\sim \delta(u-u_0) \\ F_0(u) &\sim \delta(u-u_0) \end{aligned} \tag{39}$$

Eq. (28) becomes

$$n(z) = \frac{\beta}{2}\int_0^\infty e^{-|z-z'|}\, n(z')\, dz' + a\, e^{-\frac{z}{u_0}} \tag{40}$$

a = const.

The solution is (W.L. Smirnov, 1963)

$$n(z) = A\, e^{-\frac{z}{u_0}} + B\, e^{-\sqrt{1-\beta'}\, z} \tag{41}$$

$$A = a \frac{1 - u_o^2}{1 - u_o^2 (1-\beta)}$$

with

$$B = \frac{a}{2\sqrt{1-\beta}} \left\{ \frac{\beta u_o}{1 - u_o\sqrt{1-\beta}} - \frac{u_o[1-\sqrt{1-\beta}]^2}{1 + u_o\sqrt{1-\beta}} \right\} \quad (42)$$

The density profile can be described as the superposition of two exponential functions.

Another approximation of the distribution function is

$$f_i(u) = \left\{ \begin{array}{ll} \frac{1}{2} & -1 < u < +1 \\ 0 & u < -1 \, , \, u > +1 \end{array} \right\} \quad (43)$$

The kernel of (28) is

$$K(|z-z'|) = \frac{1}{2} \int_0^1 \frac{e^{-\frac{|z-z'|}{u}}}{u} \, du \quad (44)$$

and the integral equation (28) becomes the Milne integral equation of the radiative transfer problem (S. Chandrasekhar, 1960).

III. Numerical Calculations

To solve the integral equations (21) and (23), we first transformed the domain of integration from $(0,\infty)$ to the finite region $(0, 1)$. The transformed equations were converted to a set of algebraic equations by replacement of the integration by a finite summation. It has already been mentioned in the discussion of the kernel that a singularity exists at $z = z'$, but that the integral $\int_0^\infty K(|z-z'|)\, dz'$ is finite. We therefore replaced the singular values $K(z_i, z_i)$ in the diagonal elements of our equations by the interpolated value $\frac{1}{2}\left(K(z_i\, z_i + \frac{1}{2}) + K(z_i\, , z_i - \frac{1}{2}\right)$. The system so obtained is solved numerically. In the special case (37) the numerical solution could be checked by comparison with the

the analytical solution. Very good agreement was found for nearly all values of parameters u_o and ß. For $u_o = 10$ and $ß = 0.9$ the difference reached 10 % at small values of z.

We compared three different cases:

a) without reflection on the wall

b) ideal reflection $(R_i = 1; R_D = 0)$

c) diffuse reflection $(R_i = 0; R_D = 1)$

As distribution function of the incoming particles we choose

$$F_+^o = \frac{\delta(u-u_o)}{u_o} \quad \text{and } F_+^o = \frac{2}{u_o^2}\, e^{-(\frac{u}{u_o})^2}$$

The normalizing constants are selected in order to give the flux 1:

$$A_+^o =: \int_0^\infty u F_+^o \, du = 1$$

In the case of diffuse reflection the distribution of the reflected particles was taken to be

$$F_o(u) = \frac{2}{u_k^2}\, e^{-(\frac{u}{u_k})^2}$$

If we consider a plasma with temperatures of several 100 eV, the energy of the recycling and reflected neutrals is about a factor of 100 less than the energy of the plasma. In order to include this fact, we made the numerical calculations with $u_o = 0.1$, $u_k = 0,1$.

1) <u>Density Profiles</u>

Figs. 3 and 4 show the density profiles of "cold"neutrals ($u_o = 0.1$). Close to the wall there is a fast decrease of the fraction of cold neutrals and then one obtains a slow decrease of the number of particles which are heated by charge exchange. The reflection mechanisms considerably modify the profiles. Diffuse reflection has an accumulating effect on the density of cold neutrals close to the wall, whereas with ideal reflection the particles can penetrate further into the plasma. The details of the distribution F_+^o of incoming particles have no great effect on the profiles. Fig. 5 shows a comparison between the density profiles n (z) with two plasma distribution functions:

$$f_i \sim e^{-u^2} \text{ and } f_i \sim \frac{1}{2}[\delta(u+1) + \delta(u-1)]$$

It shows that the analytical solution which can be found with the δ-like distribution function is a goood approximation for the density profile. Figs. 6 and 7 show the details of the density profiles close to the wall.

2) Distribution Function

The distribution function $F_-(o,u)$ of the outgoing particles is of importance for two purposes:

a) It describes the energy loss due to charge exchange;
b) it gives information about the temperature of the plasma.

The function $F_-(o,u)$ is given by formula (15). In the case of a homogeneous neutral density(n (z) = const) which we find in a permeable plasma, F_- is proportional to a Maxwellian

$$F_-(o,u) \sim e^{-u^2} \tag{45}$$

But because of the inhomogeneity of n(z), we obtain deviations from the Maxwellian.

With the approximation (41) for n(z) we calculate

$$F_-(o,u) = \beta \frac{e^{-u^2}}{\sqrt{\pi}} \left\{ \frac{A}{1+\frac{u}{u_o}} + \frac{B}{1+u\sqrt{1-\beta}} \right\} \tag{46}$$

A comparison between approximation (46) and the results of numerical calculations is shown in Figs. 8.

Fig. 9 shows the effect of the different reflection mechanisms of the distribution function. This effect is not as great as it is in the density profiles. Since the distribution F_- (o, u) is calculated by integration over n (z), the different reflection mechanisms only add a constant to ln F_-. The dependence on the velocity u is not affected very much.

3) Mass Flow and Energy Flow Towards the Wall

The particle flow of neutral particles is calculated from

$$A_- = \int_0^\infty u F_-(o,u)\, du$$

The relation between A_- and A_+^o is given in Fig. 10. Without reflection this flux is always smaller than the input flux A_+^o. But if the charge exchange mechanisms dominate over ionisation ($|(\beta - 1)| \ll 1$), the flux A_- can be larger than A_+^o owing to the reflection at the wall.

Since in the case of ideal reflection there is no energy loss, we calculated the energy flow S_- on the wall for $R_D = 0$ and $R_D = 1$':

$$S_- \sim \iiint (v)^2\, v_x f_-(o, \underline{v})\, d^3\underline{v}$$

$f_-(o, \underline{v})$ is calculated from eq. (8). Fig. 11 shows

$$S_- = \frac{\int (\underline{v})^2 v_x f_-\, d^3\underline{v}}{A_+^o \int f_i\, \underline{v}^2 d^3\underline{v}} \tag{48}$$

Diffuse reflection on the wall increases the energy loss due to charge exchange by a maximum factor 2 - 3.

IV. Discussion and Conclusion

The results obtained by numerical calculations show that the δ-function-approximation of the ion distribution is sufficient for calculating the density of the neutral particles.

The reflection from the wall modifies the density profile of neutrals by a factor of 2-4. Especially diffuse reflection enhances the density of neutrals close to the wall. The distribution function of the outgoing neutral particles deviates from the ion distribution function. This fact has to be taken into account if this distribution is measured for diagnostic purposes. Because of the factor $\frac{1}{u}$ for large values of the velocity, there are fewer fast particles in the distribution function $F_-(o,u)$ than in the Maxwellian distribution. With respect to the damage which fast neutral particles can cause on the wall, this is a favourable result.

In order to give an example, we consider a plasma between $T = 10^2$ eV and $T = 10^3$ eV. In this regime we only have ionisation by electron impact and the ionisation rate for hydrogen is nearly independent of the temperature. $\alpha_e \approx 3 \,.\, 10^{-8}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$. The charge exchange rate (averaged with a Maxwellian) is also nearly independent of the plasma temperature and the velocity of the neutrals ($E_o\ (v) \leq 500$ eV) (S. Rehker, E. Speth, 1972.) The numerical value is $< \sigma_u |\underline{v} - \underline{v}'| > = 5 - 6 \,.\, 10^{-8}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$. This yields values of ß between ß = 0.62 and ß = 0.66. In the regime below T = 100 eV the ionisation rate decreases rapidly and we obtain for a 10 eV plasma:

$\alpha_e \approx 5 \,.\, 10^{-9}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$ and $< \sigma_u |\underline{v} - \underline{v}'| > \approx 2.5 \,.\, 10^{-8}\ \mathrm{s}^{-1}$. This yields ß = 0.83.

In a thermonuclear plasma ($T = 2.10^4$ eV) the ionisation rate (including ionisation by ions) and the charge exchange rate are about equal $\alpha_e + \alpha_i \approx < \sigma_u |\underline{v} - \underline{v}'| >$ and ß becomes 0.5. These values are independent of the energy of the neutrals if this energy is smaller than the thermal energy of the plasma particles.

Experiments in Tokamaks (W. Stodiek 1971) show a distribution of the charge exchange neutrals which is similar to the results shown in Fig. 8 and 9.

But a quantitative comparison between these calculations and the experimental data is not yet possible, because of the inhomogeneity of the plasma density and temperature. The inhomogeneity of the plasma also modifies the distribution of the charge exchange neutrals.

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Figure Captions

Fig. 1 Schematic picture of the interaction zone between plasma and wall. $n(x)$ = density of neutral particles.

Fig. 2 General form of the density profile of the neutral particles.

Fig. 3 Density profile of neutral particles with different reflection mechanisms. The incoming particles are monoenergetic.

Fig. 4 Density profile of neutral particles. The distribution of incoming particles is Maxwellian.

Fig. 5 Comparison of density profiles of neutrals calculated with different plasma distribution functions.

Fig. 6,7 Density of neutral particles close to the wall.

Fig. 8 Distribution function $F_{-}(o,u)$ of the escaping neutral particles. Effect of different plasma distributions function and different reflection mechanisms.

Fig. 9 Distribution of outgoing neutrals as a function of energy. The incoming distribution of neutrals is Maxwellian.

Fig. 10 Particle flux on the wall normalized to the flux of incoming particles.

Fig. 11 Energy flux on the wall normalized to the flux of incoming particles times the thermal energy of ions.

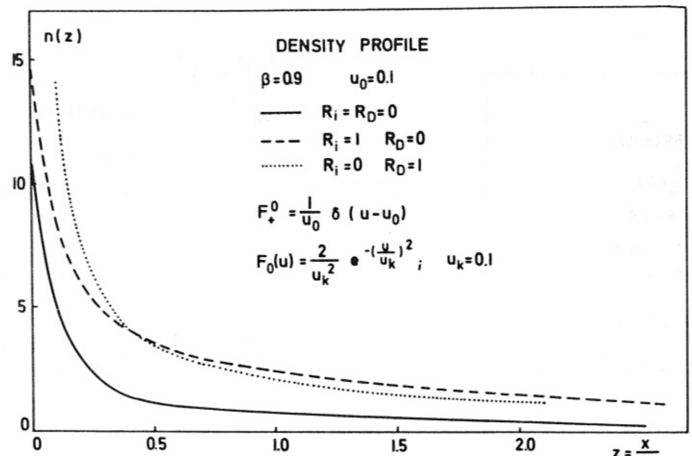


Fig. 3

Fig. 3

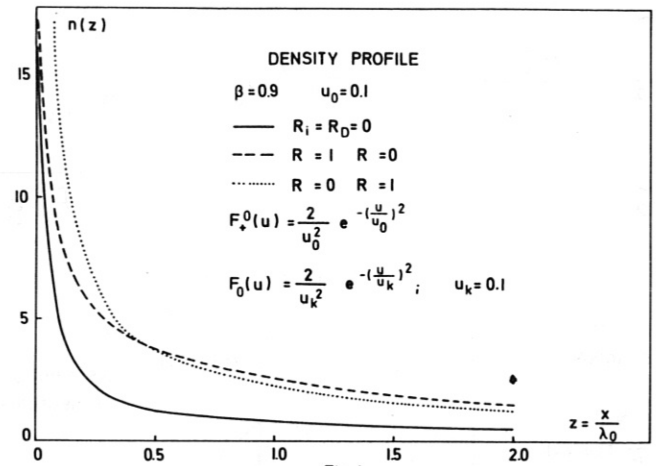


Fig. 4

Fig. 4

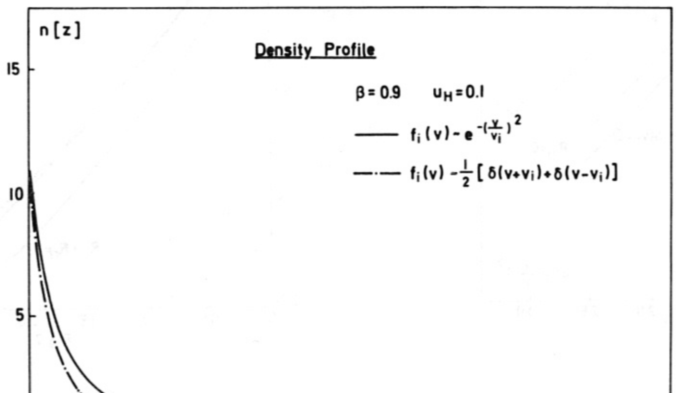
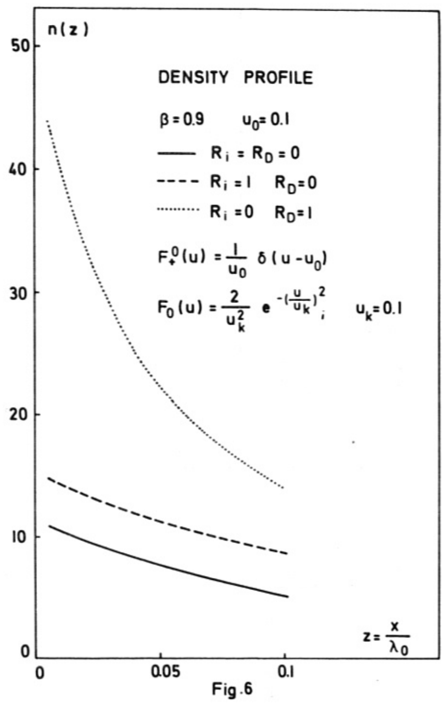


Fig. 5

Fig. 5



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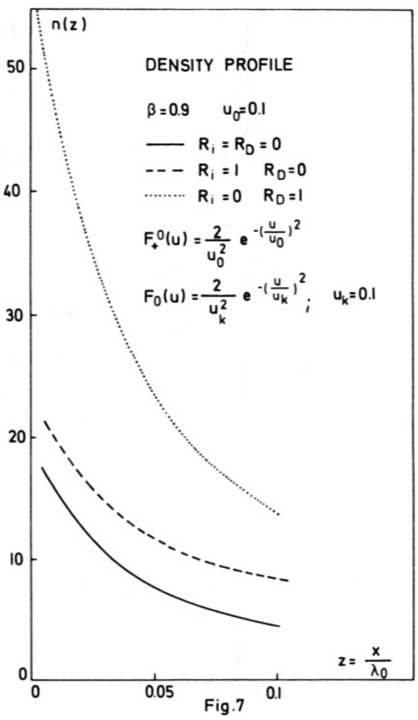

Fig. 7

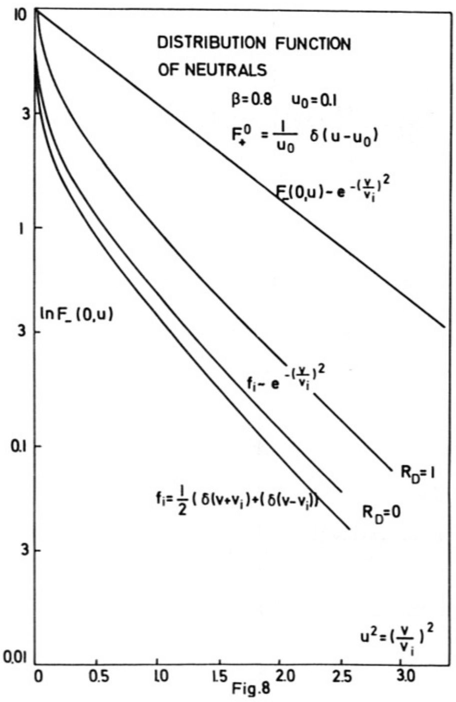

Fig. 8

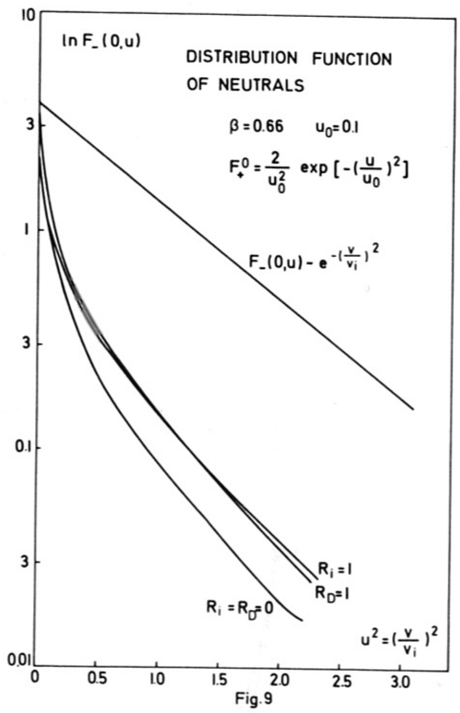

Fig. 9

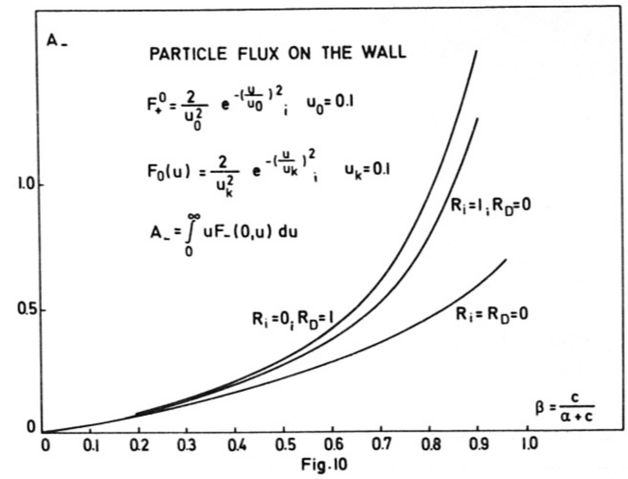
A₋
PARTICLE FLUX ON THE WALL
$F_+^0 = \frac{2}{u_0^2} e^{-(\frac{u}{u_0})^2}$; $u_0 = 0.1$
$F_0(u) = \frac{2}{u_k^2} e^{-(\frac{u}{u_k})^2}$; $u_k = 0.1$
$A_- = \int_0^\infty u F_-(0,u)\, du$
$R_i = 1;\ R_D = 0$
$R_i = 0;\ R_D = 1$
$R_i = R_D = 0$
$\beta = \frac{c}{\alpha + c}$
1.0
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fig. 10

Fig. 10

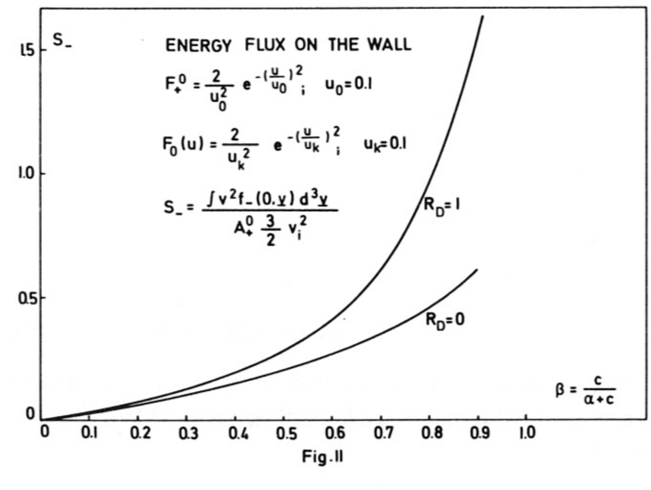
S₋
ENERGY FLUX ON THE WALL
$F_+^0 = \frac{2}{u_0^2} e^{-(\frac{u}{u_0})^2}$; $u_0 = 0.1$
$F_0(u) = \frac{2}{u_k^2} e^{-(\frac{u}{u_k})^2}$; $u_k = 0.1$
$S_- = \frac{\int v^2 f_-(0,\underline{v})\, d^3\underline{v}}{A_+^0 \frac{3}{2} v_i^2}$
$R_D = 1$
$R_D = 0$
$\beta = \frac{c}{\alpha + c}$
1.5
1.0
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fig. 11

Fig. 11