Optimization of
Superconducting Toroidal Magnets
for
Tokamak Fusion Reactors

K.H. Schmitter

IPP 4/108 Januar 1973

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Abstract

Questions concerned with the optimization of toroidal magnets for tokamak fusion reactors are discussed in particular with regard to the conductor costs.

OPTIMIZATION OF SUPERCONDUCTING TOROIDAL MAGNETS FOR TOKAMAK FUSION REACTORS

K. H. Schmitter

Introduction

The electrotechnical problems of stationary and quasi-stationary toroidal fusion reactors are concerned primarily with plasma confinement and ignition. Stationary reactors require a heating energy supply only for starting, in other words, rarely. In quasi-stationary reactors, however, the plasma must be ignited from an external source at the beginning of each new operating cycle. This means that the circulating power in the system of a quasi-stationary fusion reactor, as opposed to a stationary type, will be higher by the mean ignition power taken over one cycle.
In a stationary reactor, for example, the dissipated power of a normalconducting toroidal magnet would already be prohibitively large (comparable to the reactor power output) even disregarding the ignition power requirement. For this reason the confining magnetic field of stationary and quasi-stationary fusion reactors must be produced by superconducting coils. Estimations indicate that the necessary superconducting magnets, together with their auxiliary equipment, are significant contributors to the total cost of a fusion boiler.
In the following, questions concerned with the optimization of such magnets are discussed in particular with regard to the required conductor material. In all calculations, the practical units cm, amp, Gauss, etc., are used if not otherwise indicated.

1. Toroidal Reactor Geometry

The outline principle of a toroidal device with the

major plasma radius	R_o
minor plasma radius	a
radius of the first wall	r_w
radial thickness of the blanket plus nuclear shield	b
inner radius of the superconducting magnet	r_i
radial thickness of the super-conducting magnet	h
radius of the free area at the torus center	R_i
plasma aspect ratio	$A = \frac{R_o}{a}$
magnet aspect ratio	$A_m = \frac{R_o}{r_i}$

is drawn up in Fig. 1.

2. The Tokamak Field

A tokamak magnetic field configuration consists of a toroidal field B_t and a poloidal field B_p. Stability requires that for these fields

$$q = \frac{1}{A} \cdot \frac{B_t}{B_p} > 1 \quad \text{everywhere} \qquad 2.1$$

be satisfied. The field orientations are indicated in the bottom picture of Fig. 1.

The following considerations are based on the assumption that the temperature and density profiles are rectangular and that the plasma pressure ratio β_p related to the poloidal field B_p is equal to the plasma aspect ratio A. We have, for the toroidal field on the axis,

$$B_{t_o} = \frac{n \cdot I}{5 \cdot R_o} \qquad 2.2$$

with n = number of turns.

This field reaches its maximum value at the inside of the torus and the inside of the coil (Fig. 2):

$$B_{t_{max}} = B_{t_o} \cdot \frac{1}{1 - \frac{1}{A_M}} \qquad 2.3$$

The plasma current connected to the poloidal field is

$$I_p = \frac{5 \cdot a^2 \cdot B_{t_o}}{q \cdot R_o} \qquad 2.4$$

The corresponding flux Φ_V through the area inclosed by the torus can be calculated from 2.4 and the inductance(1) of the plasma ring:

$$\Phi_V = 20 \cdot \pi \frac{a^2 \cdot B_{t_o}}{q} \cdot (\ln(8A_p) - 1.75) \cdot 10^{-9} \quad (Vs) \qquad 2.5$$

3. Magnetic Field and Power Output

The thermal power density of a fusion reactor, e.g. the sum of thermonuclear and blanket power with regard to the plasma volume can be determined from the equation:

$$p_{th} = n^2 \cdot Q_T \cdot \langle \sigma v \rangle \cdot 0.4 \cdot 10^{-19} \quad (W/cm^3) \qquad 2.6$$

Q_T (eV) $\qquad$ n (cm^{-3}) $\qquad$ $\langle \sigma v \rangle (cm^3 s^{-1})$

With $T = 20$ keV and $Q_T = 22.4 \cdot 10^6$ eV for the D-T-process, it follows for the total thermal output:

$$P_{th} = \frac{1}{R_o} \cdot \left(\frac{a \cdot B_{t_o}}{q} \right)^4 \cdot 2.93 \cdot 10^{-15} \text{ (W)} \qquad 3.2$$

One of the important limiting factors in the construction of a fusion reactor is the apparent first wall load, p_w. This load determines the magnitude of the required toroidal field B_{t_o} if the geometric parameters of a fusion reactor are given:

$$B_{t_o} = 1.076 \cdot q \cdot \left(\frac{p_w \cdot A^2}{y \cdot a} \right)^{1/4} \cdot 10^4 \qquad 3.3$$

This equation was evaluated for three different major radii and the following parameters: $q = 2.5$, $y = \frac{a}{r_w} = 0.8$, $p_w = 460$ W/cm^2, and $b = 150$ cm. The results are plotted in Fig. 3. The same figure contains as well the corresponding thermal outputs.

Taking for example 5800 MW_{th} as the reference size of a fusion reactor, we find the required field to be about 70 kG for the reactor with $R_o = 10$ m ($A = 4$), about 90 kG for a major radius of 12.4 ($A = 6$), and more than 110 kG for a device with 15 m major radius ($A = 9$). We can conclude that, from the aspect of B_{t_o}-minimization, the torus with the smallest possible major radius would be the proper choice.

The peak magnetic field $B_{t_{max}}$ as a function of the plasma aspect ratio in a toroidal magnet is shown in Fig. 4. There exist minima for the peak fields. They occur when

$$a = \frac{3}{7} \cdot y \cdot (R_o - b) \qquad 3.4$$

or at the corresponding plasma aspect ratio:

$$A = \frac{2.33}{y \cdot (1 - \frac{b}{R_o})} \qquad 3.5$$

Their value:

$$B_{t_{max}} = R_o^{1.5} \cdot p_w^{0.25} \cdot q \cdot y^{-1} \cdot (R_o - b)^{-1.75} \cdot 3.55 \cdot 10^4 \qquad 3.6$$

The equations are defined for

$$A > \frac{1}{y(1 - \frac{b}{R_o})}. \qquad 3.7$$

In order to keep $B_{t_{max}}$ low, one should take aspect ratios close to 3.5. The correct values of the minima and the corresponding magnitudes of the thermal power are as follows:

for R_o = 10 m at an A_p = 3.44 with 7000 MW
for R_o = 12.5 m at an A_p = 3.32 with 11000 MW
for R_o = 15 m at an A_p = 3.25 with 16000 MW

The reference size mentioned above of 5800 MW is very close to the $B_{t_{max}}$-minimum if a 10 m major radius is chosen.

4. Current Densities in Toroidal Magnets

The toroidal magnet will probably be composed of symmetrically arranged cylindrical coils. The number of coils is assumed to be so large that the polygonal shape of the inside contour around the major axis can be replaced by a circle. This results in a total magnet volume of approximately

$$V_c \simeq \pi \cdot R_o \cdot r_m \cdot \frac{B_{t_o}}{j_m} \cdot 10^{-2} \quad (cm^3) \qquad 4.1$$

with the overall current density j_m (kA/cm^2).

The mean winding radius r_m is

$$r_m = 0.25 \left\{ R_o + 3r_i - d - \left[(r_i + d - R_o)^2 \frac{10^{-2} \cdot B_{t_o} \cdot R_o}{j_m} \right]^{1/2} \right\} \qquad 4.2$$

where d is the radial distance between the coil periphery and the outer wall of the cryostat.

Maximum reliability is of prime importance for power stations. Consequently, the very large superconducting magnets in fusion reactors will very probably have to be fully stable based on steady state stability criteria. This means the superconductors must be imbedded in a sufficient quantity of highly conductive material to prevent propagation of normal-conductive zones on the superconductor.

One of the steady state stability criteria (2), giving a limitation of the current density j_{st} in the stabilizing material after a transition of the superconductor, is

$$j_{st} < \left(p_c \cdot \frac{K}{\rho_{st}}\right)^{2/3} \cdot I_s^{-1/3} \qquad 4.3$$

where p_c is the heat flux into the coolant. ($p_c \sim 0.3$ W/cm^2 for nucleate boiling helium.) The geometric factor K can be assumed[3] to be about 2.

Two different influences on the electrical conductivity of the stabilizing material must be considered:

a. The low temperature effect

The superconducting coil operates at liquid helium temperature (about 4.2°K), where the resistivity of the stabilizing conductor is much lower than at room temperature. The ratio between the two depends on the kind of the metal, its impurity content, and its mechanical and thermal history.

The following resistivity ratios $\rho_{273}/\rho_{4.2}$ were measured for copper of various material qualities:[4]

Quality	$\rho_{273}/\rho_{4.2}$	$\rho_{4.2}$ ($\Omega\cdot$cm)	
Monocristalline zone refined	4400	4×10^{-10}	heat treated
Polycristalline very pure	840	2.1×10^{-9}	" "
Commercial selected	≥ 175	$\leq 10^{-8}$	" "
Commercial normal	~ 110	$\sim 1.6 \times 10^{-8}$	" "

$\rho_{293} = 1.76 \times 10^{-6}$ Ωcm

Compared with copper the resistivity ratio of aluminum is larger. Pure aluminum with an impurity content of the order of 1 ppm can have resistivity ratios of up to about 30,000.

b. The magnetic field effect ("magnetoresistance")

This effect results in an increase of the resistivity with the increase of the magnetic field. At liquid helium temperature the effect is much more severe than at room temperature. The magnitude of the magnetoresistance depends on the kind of metal - for example copper shows a stronger magnetoresistance effect than aluminum - and also on the resistivity ratio mentioned before. The smaller the resistivity ratio, the less the magnetoresistance. Or, in other words, an increasing impurity content results in a decreasing magnitude of the magnetoresistance. Therefore, it may not be worthwhile to use extremely pure metals for stabilization in every case.

Normal commercial copper with a resistance ratio of 110 at a temperature of 4.2°K shows for different field values the following typical resistivities(4):

H (kOe)	ρ_H (Ω x cm) at 4.2°K
0	1.6×10^{-8}
60	3.4×10^{-8}
120	6.7×10^{-8}

If we now introduce these values into the steady state stability criteria already mentioned, we find, under the assumption of a superconductor current of 10 kA, the following upper limits of the current density and the lower limits of the required stabilizing copper cross-section:

H (kOe)	J_{cu} (A/cm^2)	q_{cu} (cm^2)
0	$< 5.2 \times 10^3$	> 1.92
60	$< 3.11 \times 10^3$	> 3.2
120	$< 2 \times 10^3$	> 5

Having in mind that the magnetic field decreases from $B_{t_{max}}$ at the inner winding to almost 0 at the coil periphery (Fig. 2), these values indicate clearly that it will not be worthwhile to use compound superconductors of equal cross-sections for the entire winding. Therefore, coils of superconducting toroidal reactor magnets should be divided into at least two subdivisions with different copper cross-sections.(5) Considering a toroidal magnet with $B_{t_{max}}$ = 120 kG and B_{t_o} = 60 kG, the outer part of the coils (60 kG to almost zero) requires a stabilizing copper cross-section of about 3.5 cm^2; the inner part (60 kG to 120 kG) requires a cross-section of about 5.5 cm^2. This gives a conductor mean current density in the coil of about 2.5 kA/cm^2 ($\leq$ 2 kA/cm^2 only when using a uniform compound conductor). This mean conductor current density and the space required for the steel reinforcement of the windings, the mechanical structure, the cooling channels, and the insulation, lead to an overall current density in the toroidal magnet of about 1.5 kA/cm^2.

Thus far, we have not yet considered the superconductor itself. Because the current carrying capacity of the superconductors decreases as the magnetic field increases, the quantity and kind of the superconducting material imbedded in the stabilizing material should likewise be adapted in steps to the field distribution. This is for economical and technological reasons. The influence on the overall current density is negligible.

5. Cost of the Winding Material

It is appropriate to base the evaluation of the cost of the required compound superconducting material on (A · G · cm) rather than on the conductor volume or weight. Under the condition that one half of the coil is laid out for $B_{t_{max}}$ and the other half for B_{t_o}, the following relation holds for the necessary conductor quantity:

$$M_S = 5\pi \cdot R_o \cdot B_{t_o}^2 \cdot \left(1 + \frac{A_M}{A_M - 1}\right) \cdot r_m \quad (\mathrm{A \cdot G \cdot cm}) \qquad 5.1$$

Oberfläche : $Q_B = 4\pi^2 r_m R_o$

With a specific price c of the conductor material given in DM/A · G · cm, it follows for the total cost of the conductor material necessary for the construction of the toroidal magnet:

$$C = c \cdot M_S \qquad 5.2$$

The relationship between superconductor cost and thermal reactor power is of interest in the optimization of costs:

$$\frac{C}{P_{th}} = \frac{c \cdot M_S}{P_{th}} = 4 \cdot c \, \frac{B_{t_o}^2}{p_w} \cdot \frac{r_m}{r_w} \left(1 + \frac{A_M}{A_M + 1}\right) \cdot 10^5 \qquad 5.3$$

(DM/MW(th))

For further simplification we introduce:

$$y = \frac{a}{r_w} \qquad 5.4$$

and

$$\xi = \frac{A}{A_M} = \frac{1}{y} + \frac{b}{a} \qquad 5.5$$

With the approximation $r_m \simeq 1.1 \cdot r_i$, and allowing a safety margin of 10%, this results in

$$\frac{C}{P_{th}} \simeq \frac{q^2}{\sqrt{p_w}} \cdot \sqrt{\frac{y}{a}} \cdot A \cdot \xi \cdot \left(1 + \frac{1}{1 - \frac{\xi}{A}}\right) \cdot c \cdot 5.4 \cdot 10^{15} \qquad 5.6$$

(DM/MW(th))

The function:

$$f(\xi, A) = \xi \cdot \left(1 + \frac{1}{1 - \frac{\xi}{A}}\right) \qquad 5.7$$

is plotted in Fig. 5.

The material costs of the compound conductor for toroidal magnets as a function of the plasma aspect ratio can be taken from Fig. 6. The curves are based on the present specific material price of $c = 2.5 \cdot 10^{-9}$ DM/A · G · cm approximated for large quantities by several authors (5) (6). The curves illustrate the steep increase of the specific conductor costs with the plasma aspect ratio and the influence of the major radii. A reduction of q from 2.5 to 1.25 would lower the conductor cost to about one quarter of that which is plotted here (eq. 5.6).

The magnetic flux related to the plasma current in a tokamak requires, if an iron core transformer is used, sufficient space for the core in the torus center. The lower limit of the plasma aspect is determined from this. The plasma current I_{pl} and the corresponding magnetic flux density swing ΔB_V at the torus center for the production of this current are plotted in Fig. 7. The following lower limits of the plasma aspect ratio have been determined from these curves for ΔB_V = 30 kG:

R_o = 10 m : A = 4
R_o = 12.5 m : A = 3.4
R_o = 15 m : A = 3.1

This limitation is indicated in Fig. 8 where the superconducting material cost (compound conductor) is drawn up as a function of the thermal reactor power. We see that a 5680 MW(th) (2500 MW_e)-reactor with a major radius of R_o = 10 m is optimized with regard to the conductor costs. At R_o = 12.5 the costs have already increased to 1.6 times as much. It would be impossible for a reactor with R_o = 9 m to reach this power because of ΔB_V-limitation. The possible smaller thermal powers result in larger specific superconductor costs. Air-core devices could lead to a further material cost reduction, and if a value of 1.25 of the safety factor q would turn out to be sufficient, the conductor material cost for a 5680 MW(th) tokamak would be as low as 12 DM/KW(th).

6. Stored Energy

Important aspects are the magnetic energy stored in the toroidal field and especially the field economy figure; that is, the quotient of the thermal reactor output and the magnetic energy stored in the toroidal field. These are plotted in Fig. 9 as a function of the plasma radius.

It may be surprising that the stored energy almost decreases if the reactor output at constant major radius becomes larger, (increasing plasma radius is identical to increasing output). This tendency is a consequence when keeping the wall load q_W constant. The reactor, optimized for conductor material costs, with a major radius $R_O = 10$ m and a plasma radius $r_p = 2.5$ m, has a stored energy of $W_M = 75$ GJ in the toroidal field and a $P_{th}/W_M = 7.5 \cdot 10^{-2} s^{-1}$. The corresponding values for a reactor with $R_O = 12.5$ m and the same thermal power, $W_M = 120$ GJ and $P_{th}/W_M = 4.7 \cdot 10^{-2} s^{-1}$, are considerably less favorable.

The B_t-field, winding-cost optimized parameters of a 5680 MW(th) tokamak reactor are listed in Table I. Table II contains the respective data of reactors with larger major radii or q = 1.25 in comparison with the reference reactor.

Table I

Assumed

Total power output:	P_{th} = 5680 MW_{th}/2500 MW_e
Plasma temperature:	$T_i = T_e$ = 20 keV
Total wall loading:	p_w = 4.6 MW/m^2
Plasma size parameter:	$y = a/r_w = 0.8$
Blanket thickness:	b = 1.5 m
Conductor price:	c = 2.5×10^{-9} DM/A·G·cm
Safety factor:	q = 2.5

Calculated

Major torus radius:	R_o = 10 m
Plasma minor radius:	a = 2.5 m
Vacuum wall minor radius:	r_w = 3.125 m
Inner radius of the magnet:	r_i = 4.625 m
Plasma aspect ratio:	A = 4
Magnet aspect ratio:	A_M = 2.16
Toroidal magnetic field:	B_{t_o} = 66.3 kG
Peak magnetic field:	$B_{t_{max}}$ = 123.4 kG
Plasma current:	I_{Pl} = 8.3 MA
Poloidal magnetic field:	B_p = 6.63 kG
Flux through the central area:	Φ_v = 179 Vs
Stored energy of the torus field:	W_M = 75.2 GJ
Field economy figure:	$P_{th}/W_M = 7.5 \cdot 10^{-2} s^{-1}$
Specific conductor cost:	k_c = 47.3 DM/KW_{th}

Table II

R_o	10 m	10 m	12.5 m	15 m
a	2.5 m	2.5 m	3.65 m	4.85 m
A	4	4	3.4	3.1
q	2.5	1.25	2.5	2.5
p_w	4.6 MW/m^2	4.6 MW/m^2	4.6 MW/m^2	4.6 MW/m^2
B_{t_o}	66.3 kG	33.15 kG	~60 kG	~50 kG
$B_{t_{max}}$	123.4 kG	61.7 kG	~110 kG	~100 kG
P_{th}	5680 MW	5680 MW	10700 MW	16600 MW
W_M	75 GJ	19 GJ	~115 GJ	~170 GJ
P_{th}/W_M	$7.5 \cdot 10^{-2}$ (s^{-1})	$3.0 \cdot 10^{-1}$ (s^{-1})	~10^{-1} (s^{-1})	~10^{-1} (s^{-1})
Cost	47.3 (DM/KW_{th})	~12 (DM/KW_{th})	~31 (DM/KW_{th})	~24 (DM/KW_{th})

$$1 \frac{\$}{MW} = 3 \cdot 10^{-3} \frac{DM}{kW}$$

$$\cancel{4} \quad 10^4 \frac{\$}{MW} \triangleq 30 \frac{DM}{kW}$$

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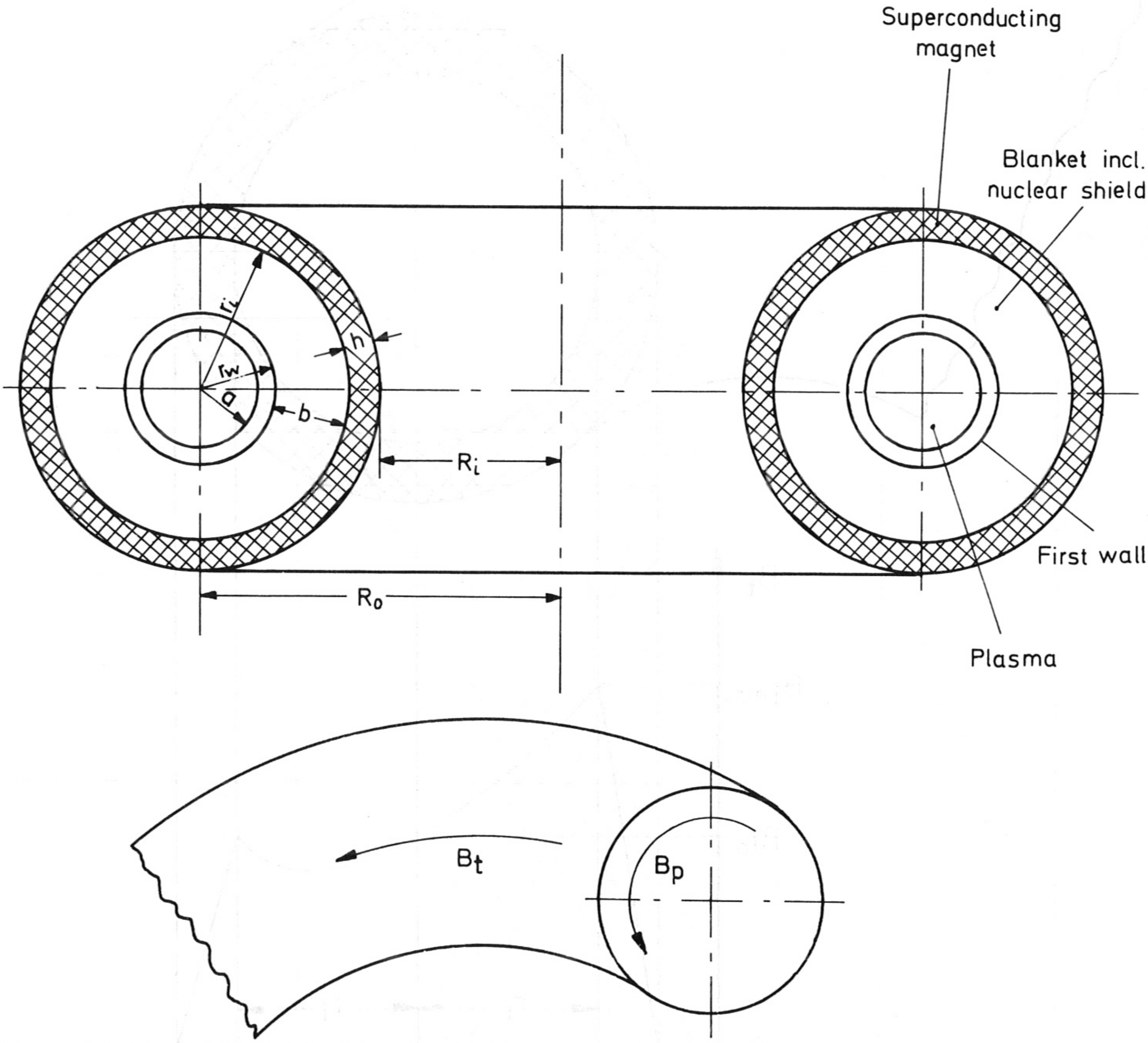


Fig.1 Outline principle of a **tokomak** fusion reactor

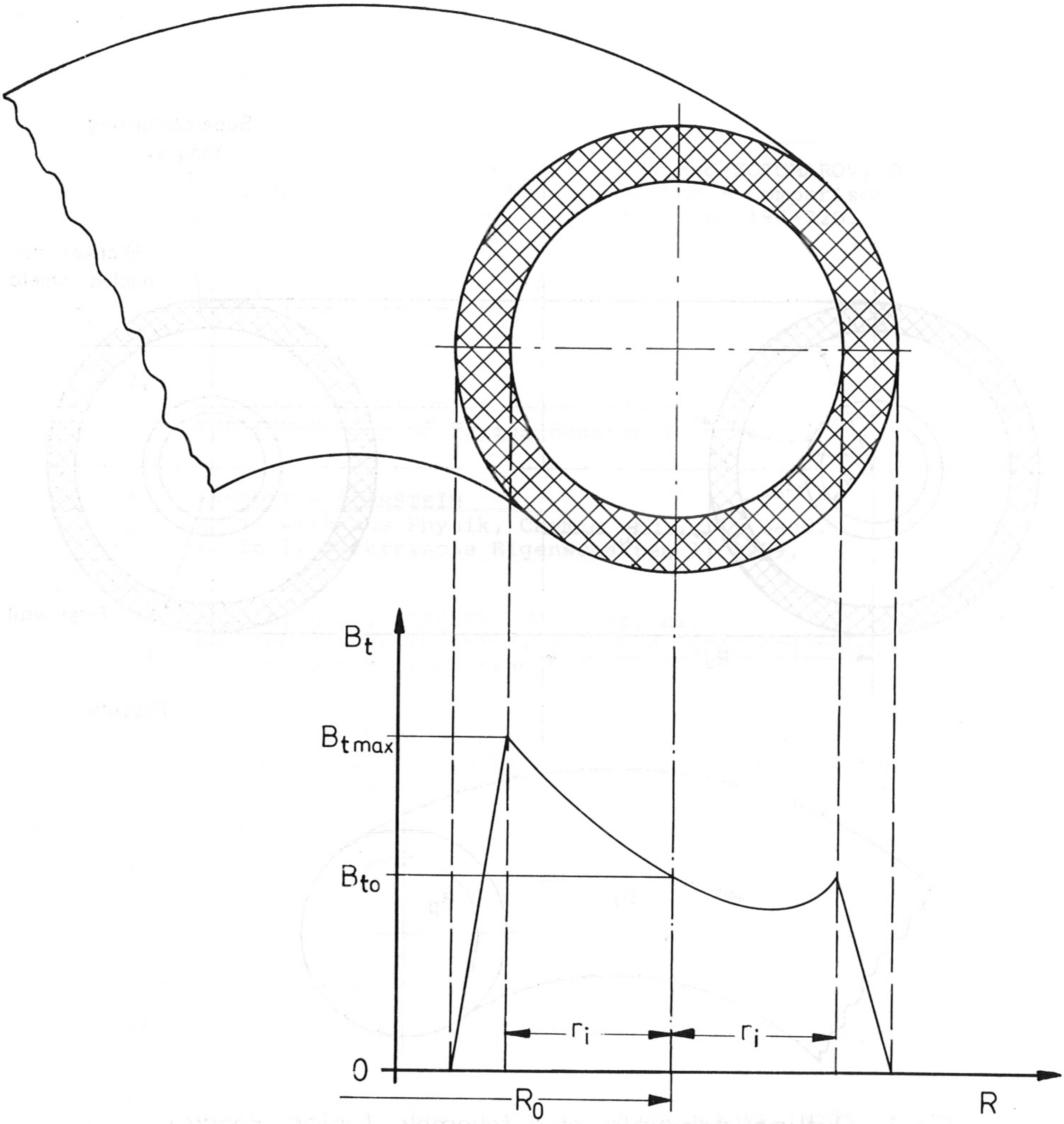
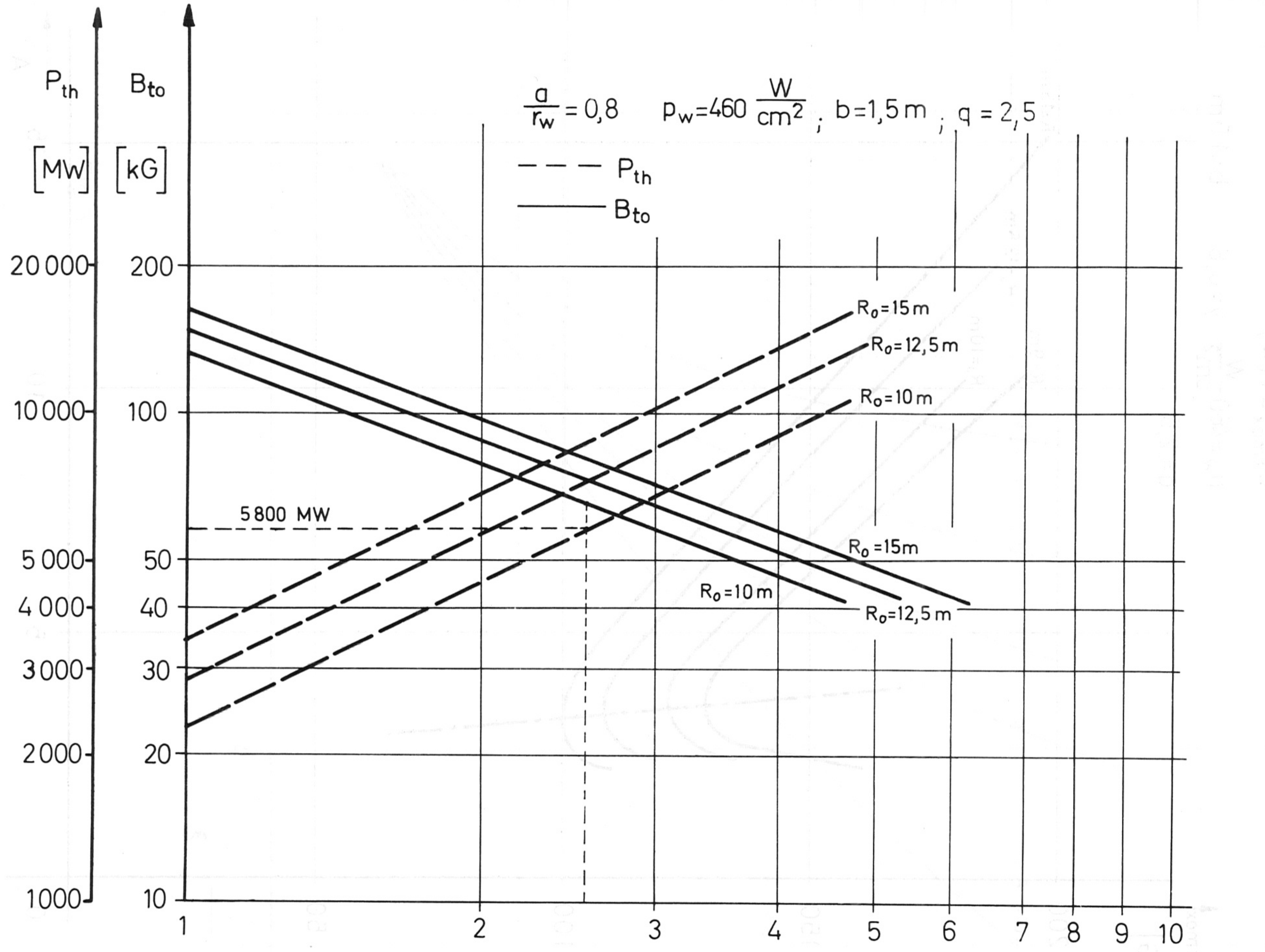


Fig. 2

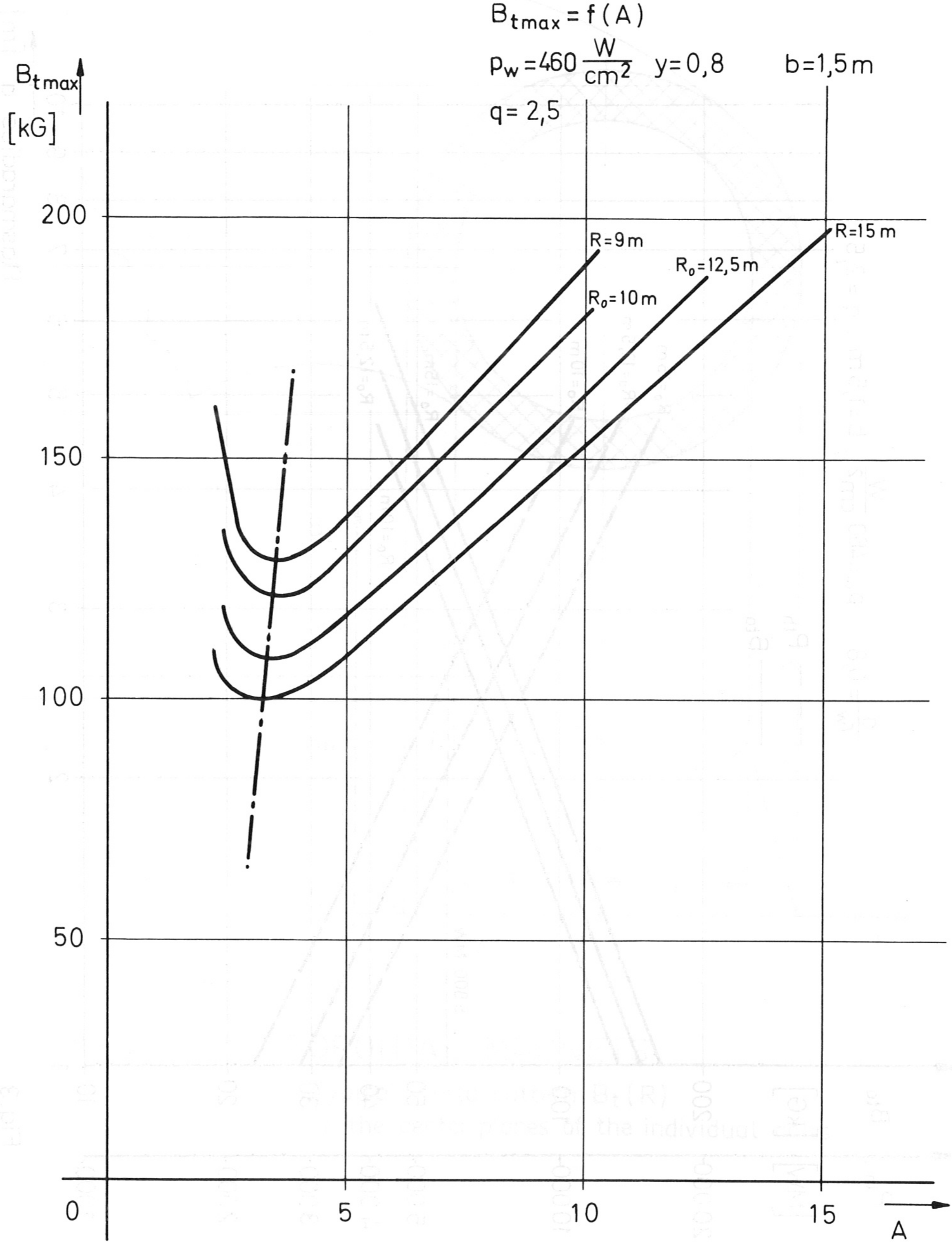
TOROIDAL MAGNETS

Typical field pattern $B_t(R)$
in the center planes of the individual coils



P_{th}
B_{to}
[MW]
[kG]
$\frac{a}{r_w} = 0,8$ $p_w = 460 \frac{W}{cm^2}$; $b = 1,5\,m$; $q = 2,5$
P_{th}
B_{to}
20 000
10 000
5 000
4 000
3 000
2 000
1000
200
100
50
40
30
20
10
R_o=15 m
R_o=12,5 m
R_o=10 m
R_o=15m
R_o=10 m
R_o=12,5 m
5 800 MW
1
2
3
4
5
6
7
8
9
10
Plasmaradius a [m]

Fig. 3



$B_{tmax} = f(A)$
$p_w = 460 \frac{W}{cm^2}$ $y = 0,8$ $b = 1,5\,m$
$q = 2,5$
B_{tmax}
[kG]
200
150
100
50
0
5
10
15
A
R=9m
R_o=10m
R_o=12,5m
R=15m

Fig.4

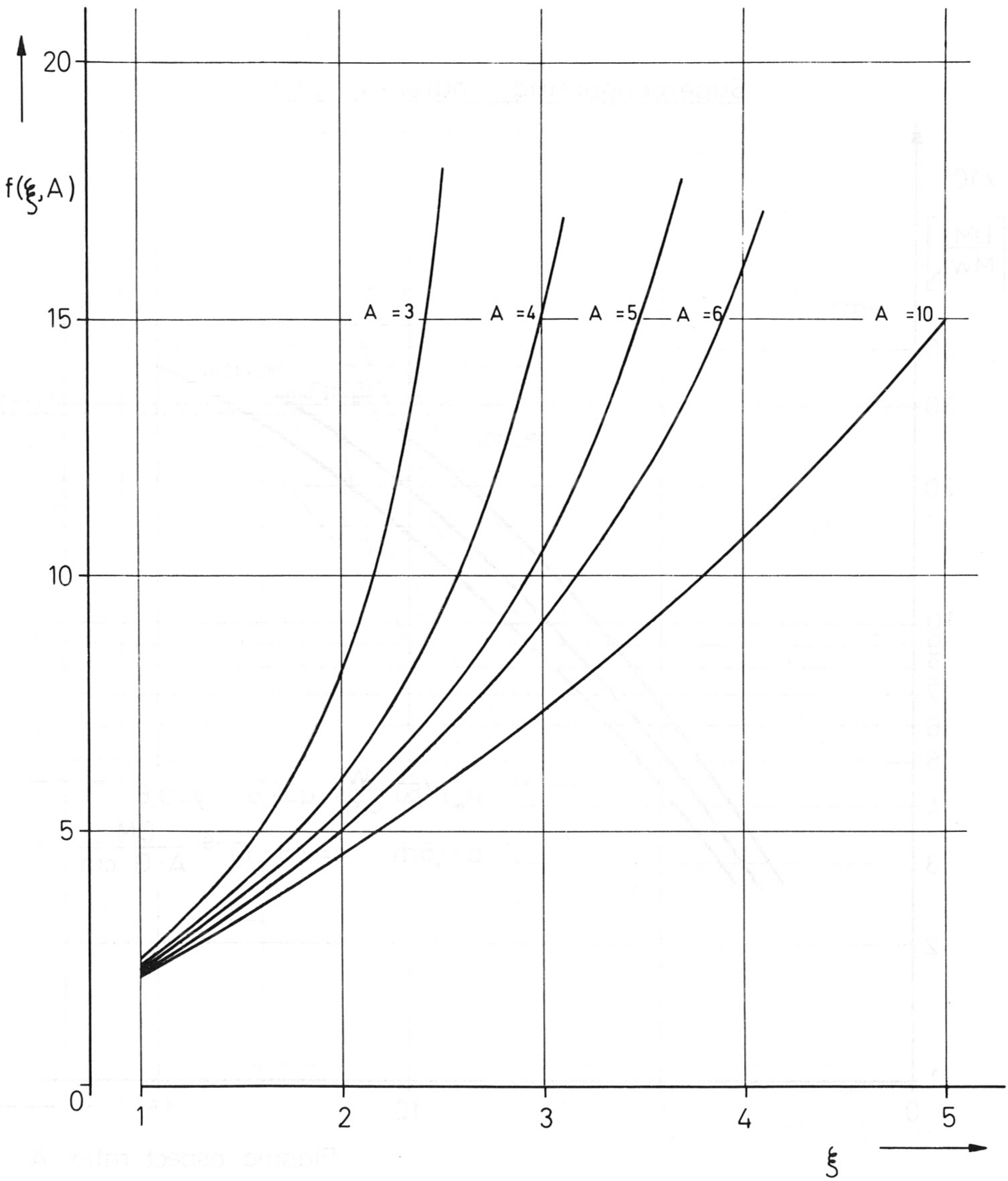


Fig.5

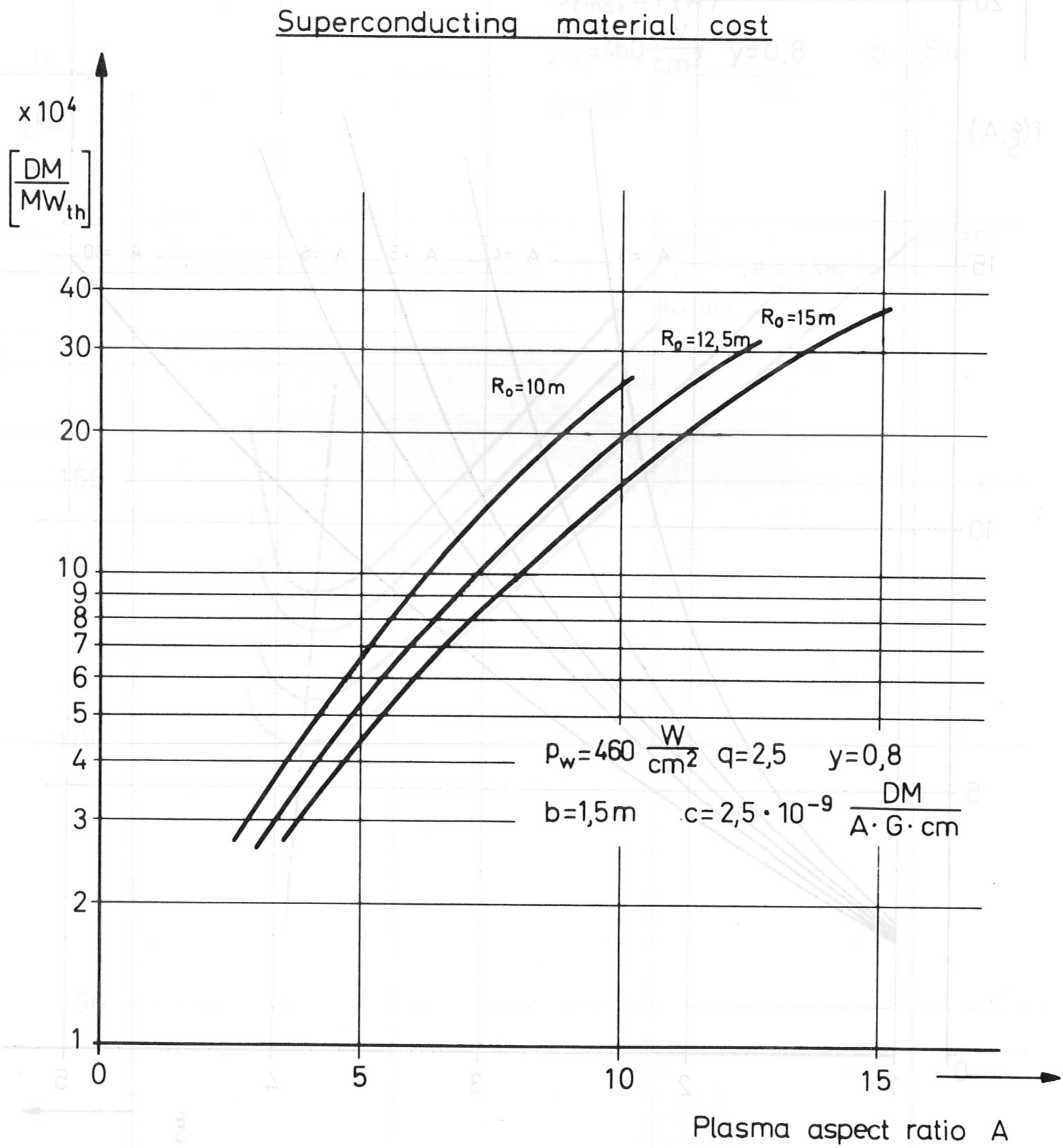
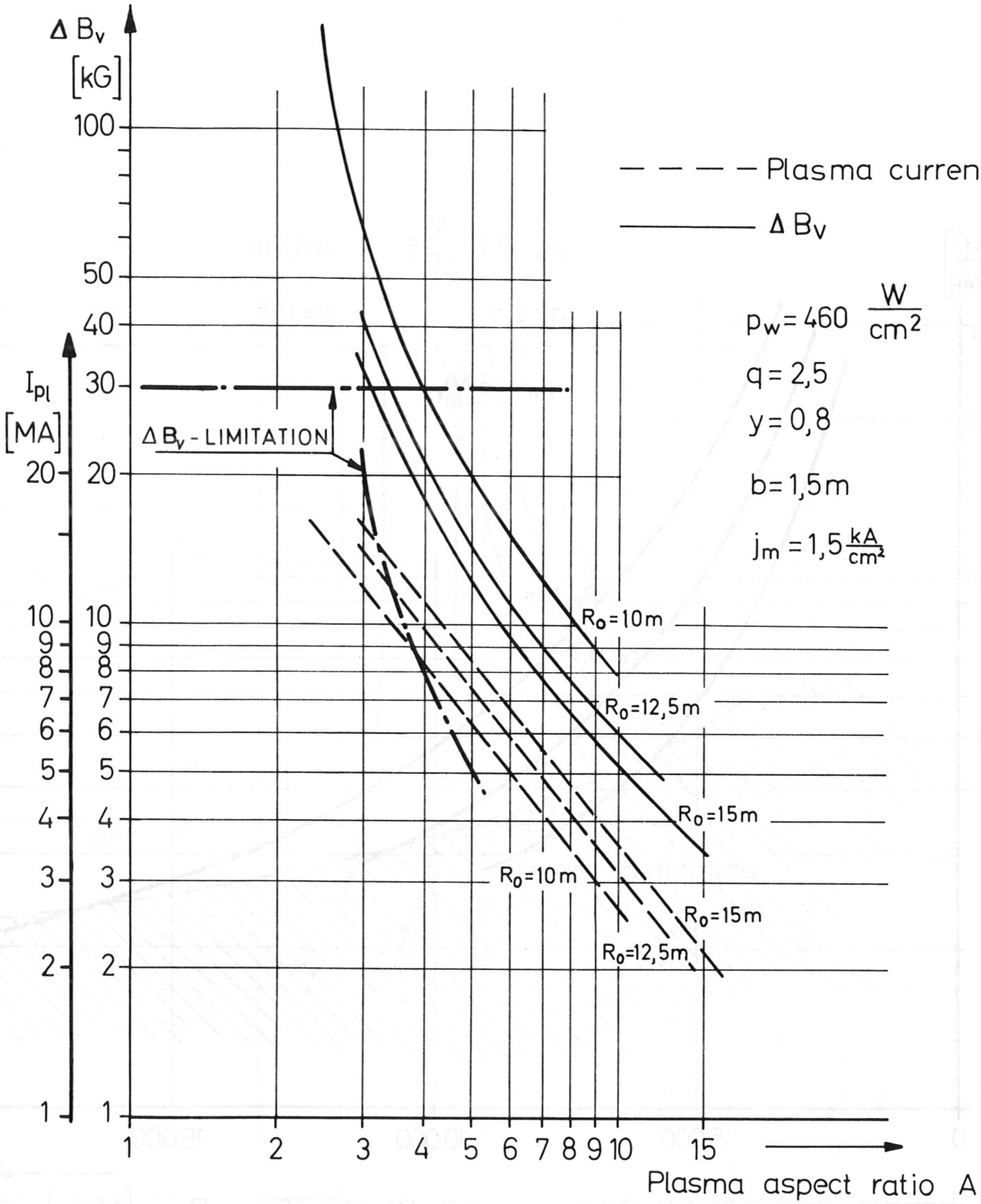
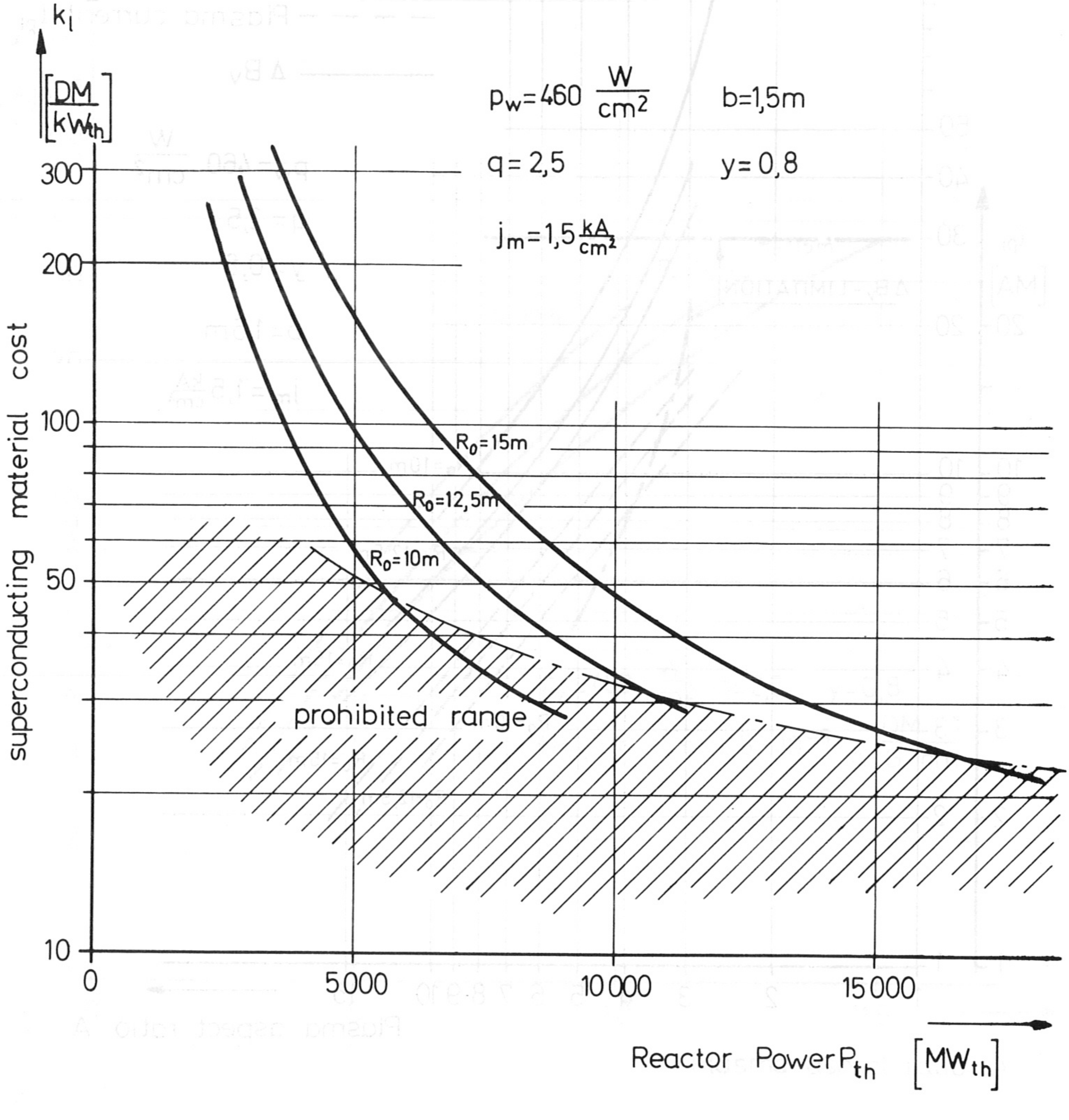

Superconducting material cost
x 10^4
[DM/MW_th]
40
30
20
10
9
8
7
6
5
4
3
2
1
0
5
10
15
R_0=10m
R_0=12,5m
R_0=15m
P_w=460 W/cm^2 q=2,5 y=0,8
b=1,5m c=2,5 · 10^-9 DM/A·G·cm
Plasma aspect ratio A

Fig. 6



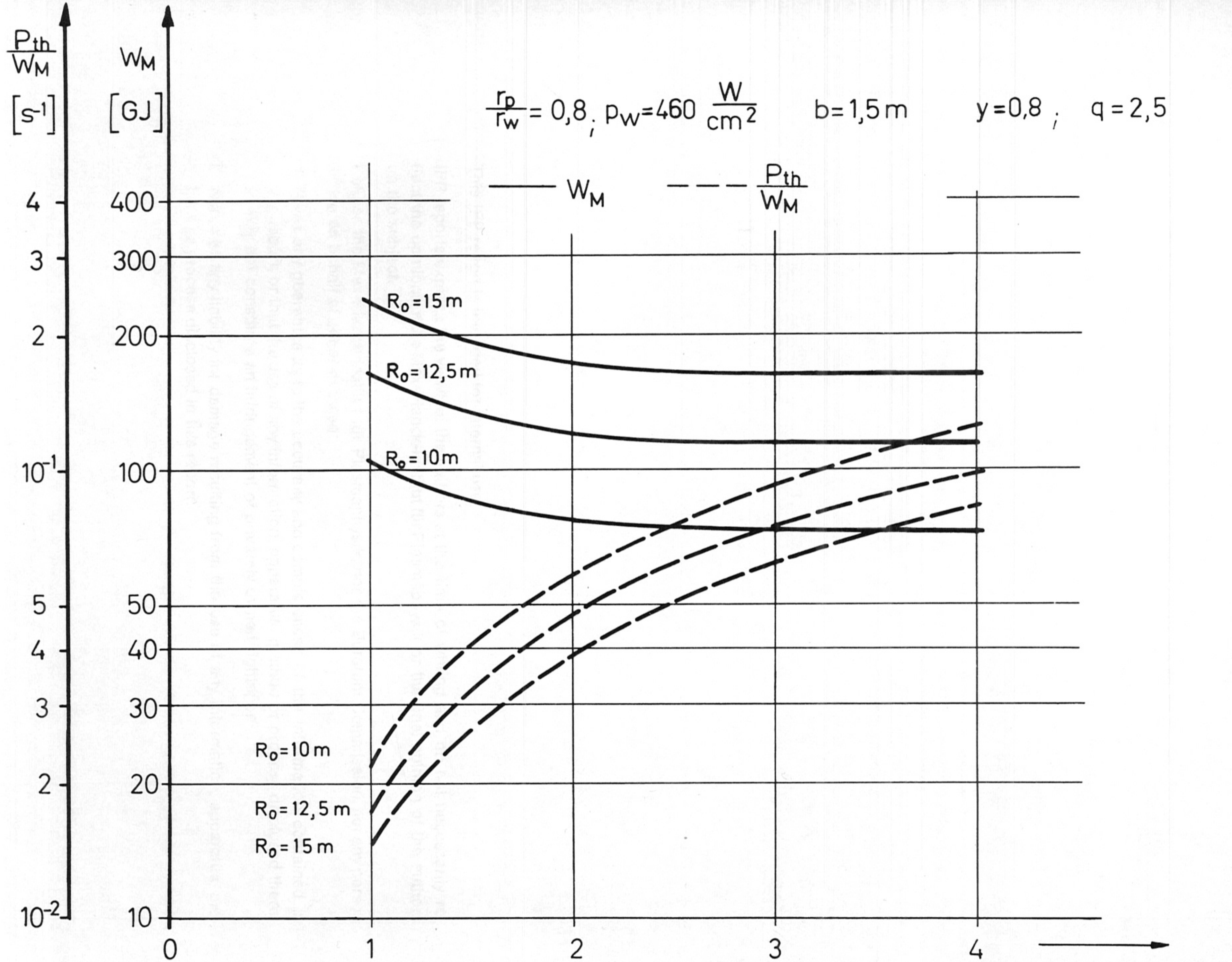
Δ Bv
[kG]
100
50
40
30
20
10
9
8
7
6
5
4
3
2
1
Ipl
[MA]
- - - - Plasma current Ipl
——— Δ Bv
pw = 460 W/cm²
q = 2,5
y = 0,8
b = 1,5m
jm = 1,5 kA/cm²
Δ Bv - LIMITATION
R0 = 10m
R0 = 12,5m
R0 = 15m
R0 = 10m
R0 = 15m
R0 = 12,5m
1
2
3
4
5
6
7
8
9
10
15
Plasma aspect ratio A

Fig. 7



k_l
[DM / kW_th]
superconducting material cost
300
200
100
50
10
$p_w = 460 \frac{W}{cm^2}$
b=1,5m
q= 2,5
y= 0,8
$j_m = 1,5 \frac{kA}{cm^2}$
R_0=15m
R_0=12,5m
R_0=10m
prohibited range
0
5000
10000
15000
Reactor Power P_{th} [MW_{th}]

Fig. 8



$\frac{P_{th}}{W_M}$ $[s^{-1}]$
W_M $[GJ]$
$\frac{r_p}{r_w} = 0,8$; $p_W = 460 \frac{W}{cm^2}$ b=1,5 m y=0,8 ; q=2,5
—— W_M – – – $\frac{P_{th}}{W_M}$
R_o=15 m
R_o=12,5 m
R_o=10 m
R_o=10 m
R_o=12,5 m
R_o=15 m
4
3
2
10^{-1}
5
4
3
2
10^{-2}
400
300
200
100
50
40
30
20
10
0
1
2
3
4
Plasmaradius a[m]

Fig.9