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Accessibility to the Lower Hybrid Resonance

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A B S T R A C T

Accessibility of a plane electromagnetic wave into an inhomogeneous, magnetized plasma column is studied using the complete cold plasma dielectric tensor. It is found that TM (transverse magnetic) waves incident at small angles with the vacuum-plasma interface are almost completely transmitted into the plasma even though the criterion for accessibility to the lower-hybrid resonance is not satisfied. The significance of this result for coupling rf energy into plasma filled waveguides is discussed.

An electromagnetic wave incident upon an inhomogeneous, magnetized, cold-plasma half-space gradually converts into a slow electrostatic wave, eventually encountering a region of very high refractive index and low group velocity in the vicinity of the lower-hybrid resonance. Some dissipation mechanism such as collisions would then result in efficient absorption of the wave energy.

Before the wave reaches the hybrid layer, it is in general obliged to tunnel through a region of evanescence where part of the wave energy may be reflected. This problem, however, can be avoided if the refractive index n_z along the magnetic field direction satisfies the condition

$$n_z > 2\left(1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}}\right) \qquad (1)$$

This well-known accessibility criterion to the lower-hybrid resonance was formulated by Stix/1/ who also showed using WKB approach that the transmitted wave is completely absorbed at the resonant layer.

Once the accessibility condition of Eq.(1) is fulfilled, n_x the refractive index perpendicular to the magnetic field is real everywhere between the vacuum and the hybrid layer and it is a fair surmise that all the energy indicent on the plasma is transmitted. Although a helpful guideline, this surmise may not be always correct. As is well-known from the waveguide thory, reflections can occur even if the propagation constant is real everywhere between the source and the load. Conversely, it may be possible to find load conditions which will lead to complete transmission even if the load is separated from the source with an evanescent section of waveguide.

Similarly, the adiabaticity condition requiring slow variations of parameters may not be valid in practice and the WKB solution may not reveal the presence of possible reflections /2/.

In order to remove some of the above-mentioned uncertainties regarding the accessibility and absorption at the lower-hybrid resonance, we study in this paper, the reflection and transmission of plane electromagnetic waves from an anisotropic, inhomogeneous, cold-plasma half-space using accurate numerical methods. Unlike the previous works /3,4/ on the subject, the case of oblique incidence is treated employing the complete cold-plasma dispersion.

We use the dielectric tensor $\overleftrightarrow{K}$ with the notation of Ref. 1. The orientation of the axes is shown in Fig. 1a. All field quantities are assumed to possess space and time dependence exp i $(k_x x + k_z z - \omega t)$ with no variation in the y-direction. The absorption mechanism considered is the momentum transfer collisions between the electrons and ions using the Langevin equation so that the modified electron mass m_ν used for calculating the electron plasma and cyclotron frequencies is given by

$$m_\nu = m \left[1 + i \left(\frac{\nu_{ei}}{\omega} \right) \right] \tag{2}$$

where m is the electron mass and ν_{ei} is the electron-ion momentum transfer collision frequency according to Spitzer /5/. We shall specify ν_{ei} by the equivalent temperature T. The mean value of ν_{ei} averaged in the region $o < x < g$ will be denoted by $\langle \nu_{ei} \rangle$. The density profile is taken into account using a stratified plasma model. This model becomes exact as the number of slabs tends to infinity and the thickness of each slab tends to zero. In practice, an acceptable approximation is obtained by making each plasma slab much thinner than the local value of the wavelength. Beyond the inaccuracies inherent in the Langevin collision model and the stratification of the plasma profile, the problem can be treated exactly. For the four waves occuring in each slab, four boundary conditions exist requiring the continuity of the tangential electric and magnetic fields.

The plasma density is assumed to rise linearly from $n_e = o$ to $n_e = n_{max}$ as x increases from $x = o$ to $x = g$ and thereafter stays constant as is shown in Fig. 1a. The region $o < x < g$ is stratified into 99 slabs while $x > g$ forms the 100th slab.

Defining $\tilde{S} = S - n_z^2$, the dispersion relation may be written as

$$n_x^2 = \frac{S\tilde{S} - D^2 + P\tilde{S}}{2S} \pm \left[\left(\frac{S\tilde{S} - D^2 + P\tilde{S}}{2S} \right)^2 + \frac{P}{S}\left(D^2 - \tilde{S}^2\right) \right]^{1/2} \tag{2}$$

Corresponding to the four roots of this equation, the x-components of the electric fields associated with the four waves in a given plasma slab are

$$E_{x\pm}^{f,s} = \hat{E}_{x\pm}^{f,s} \exp i\left(k_{x\pm}^{f,s}\, x + k_z z - \omega t\right) \tag{3}$$

while E_y, E_z, H_x, H_y and H_z may be expressed in terms of E_x using the dielectric tensor and the Maxwell´s equations. The indices f (fast) and s (slow) correspond respectively to the smaller and the larger roots of n_x^2. For each of the roots n_x^2, the two waves associated with $\pm n_x$ are labled plus and minus respectively so that $n_{x+} = -n_{x-}$. The slow wave is the quasi-extraordinary mode which encounters the resonance at the lower-hybrid layer as $S \rightarrow o$. In the plasma slab to the right of $x = g$, there are only two waves, one fast and one slow, each of which decays for increasing x, there being no sources at infinity. In the vacuum region, there is an incident wave (either TM or TE) of unit amplitude and two reflected waves one TM and one TE.

One may readily verify that the number of undetermined quantities exactly equals the boundary conditions. The resultant set of 400 complex, linear, algebraic equations is solved using standard computational methods. In order to obtain the maximum computational accuracy, the slab widths Δx are adjusted so that the phase change $\Delta\phi = |k_x^s \ \Delta x|$ in each slab is approximately equal. Typically for the case $g = 1$ cm, $\Delta\phi/2\pi < 1/30$ in each of the slabs while in the slabs lying in the immediate vicinity of the hybrid layer $\Delta\phi/2\pi < 1/300$.

If R_{TM} and R_{TE} are the amplitudes of the reflected TM and TE waves, the total reflected energy is given by,

$$R^2 = |R_{TM}|^2 + |R_{TE}|^2 \quad ,$$

so that the fractional transmitted energy is given by $(1-R^2)$.

The plasma parameters used in the computations are: static magnetic field, 100 kG; ion-cyclotron frequency, 76 MHz; rf frequency 760 MHz, maximum plasma density, 3.5×10^{13} cm^{-3} which is 1.3 times the plasma density corresponding to the lower hybrid resonance.

The relative energy coupled to the fast and slow modes has a large variation as the parameters are changed, but for the interesting case of TM waves for $n_z \lesssim 1$, typically over 80% of the energy is coupled into the slow wave and is absorbed principally near the hybrid layer.

Fig. 1 shows the transmitted energy as a function of for several values of temperature T for an incident TM wave. For the sake of comparison, the energy transmission coefficient into stainless steel is also shown in Fig. 1b (dashed line). While the incident energy is almost totally transmitted for small φ, the transmission coefficient drops rapidly with increasing φ. The effect of decreasing ν_{ei} or increasing T ($T = 10^6$ corresponds to $\langle \nu_{ei} \rangle / \omega \approx 10^{-4}$), is to introduce fluctuations in the transmission coefficient (plotted as a function of φ) although there is no significant change in the general trend.

For $\varphi \sim 90^\circ$ i.e. for nearly normal incidence, the electric field is almost parallel to the static magnetic field. Since the plasma possesses a large conductivity in the longitudinal direction, it is not surprising that for this case the wave is reflected as if from a metallic wall. For the case of small φ, on the other hand, the small longitudinal component E_z provides the necessary electron flow to produce a normal electric field component inside the plasma edge which inturn establishes efficient coupling between the large normal component of the incident electric field and the plasma. This is precisely the coupling mechanism suggested by the authors for coupling to TEM like co-axial waveguide modes. /6/.

Similar curves for an incident TE wave are shown in Fig. 2. In this case there is poor transmission for both small and large φ with a rather broad maxima in the middle. The effects caused by changing T are akin to the TM case already described.

Both for the TM and TE cases the effect of increasing "g" with a consequently less steep profile results in decreased transmission (Figs. 3 and 4) especially for large φ, presumably due to an increase in the width of the evanescent region.

The most significant result obtained is that for small angles ($n_z = \cos \lesssim 1$), the TM vacuum wave is almost completely transmitted into the plasma. Another important result is that the transmission properties of TM and TE waves are quite different. While the first of these two results stands in contradiction to the accessibility condition of Eq. (1), the second is certainly not apparent from the WKB approach.

These results have important bearing on the problem of coupling rf energy into a plasma column contained in a waveguide. We shall consider two cases, namely i) when the vacuum wavelength greatly exceeds the transverse waveguide dimensions and ii) when the vacuum wavelength is less than the transverse dimensions of the waveguide. In either case propagating modes exist provided the lower-hybrid resonance lies somewhere in the plasma column /7/. While the first case is relevant to the laboratory scale experiments, the second pertains to fusion reactors.

Case (i): It was pointed out earlier in this paper that the efficient coupling of the TM vacuum wave for small φ to the plasma is reminiscent of a TEM like field configuration in a co-axial waveguide. Thus a current loop oriented to produce an azimuthal magnetic field will launch waves which would easily couple into the plasma.

Case (ii): Likewise, in a fusion reactor, the optimum antenna would launch TM waves at grazing incidence on the plasma surface. The part of the energy not at once transmitted into the plasma will bounce back and forth between the plasma and the metal wall, because due to the large machine dimensions involved, propagating modes could exist in the vacuum region between the plasma and the metal wall. Wall irregularities will gradually scatter these waves both in orientation and polarization so that they will eventually be admitted into the plasma. Thus in a large machine, the antenna problem may be indeed uncritical.

These considerations together with the findings of Ref. 7 make a strong case for coupling rf energy directly into the plasma filled waveguides. This is to be contrasted with the existing coupling schemes which are dictated principally by the accessibility requirements of Eq. (1) or its variants. In these schemes the accessibility criterion is met either by using the "Stix coil" for which n_z can be chosen arbitrarily /4/ or by considering "gap excitation" which presumably couples most of the spectral energy in parallel wavenumbers which fulfill the accessibility condition /8/. Instead of considering the plasma and metal walls as forming a waveguide system, Ref. 8 concentrates on a waveguide feed which due to its small dimensions (the "gap") in the z-direction is instrumental in launching waves with a large n_z.

Inclusion of finite temperature effects, collisionless absorption mechanisms, parametric and non-linear instabilities will undoubtedly alter these results quantitatively; but not the qualitative considerations which continue to play an important role in the coupling and absorption of rf energy into the plasma.

References

1. T.H. Stix, Theory of Plasma Waves (McGraw Hill, New York, 1962)
2. Ref. 1, p. 243
3. S. S. Pešić, Nuclear Fusion 11, 461 (1971)
4. V. Agnello, D. Colombant, P. Fieffe-Prevost, G. Ichtchenko and S. Kulinski, IV ECCFPD, Rome, 131 (1971)
5. L. Spitzer, Jr., Physics of Fully ionized Gases (Interscience, New York, 1962)
6. S. Puri and M. Tutter, APS Bulletin 1259 (1971); also Report IPP IV/30, MPI für Plasmaphysik, 8046 Garching, Germany (1971)
7. S. Puri and M. Tutter, APS Bulletin 772 (1972); also to be published in "Nuclear Fusion"
8. R. Parker, QPR No. 102, Res. Lab. of Electronics, MIT, Cambridge, Mass. 97 (1971)

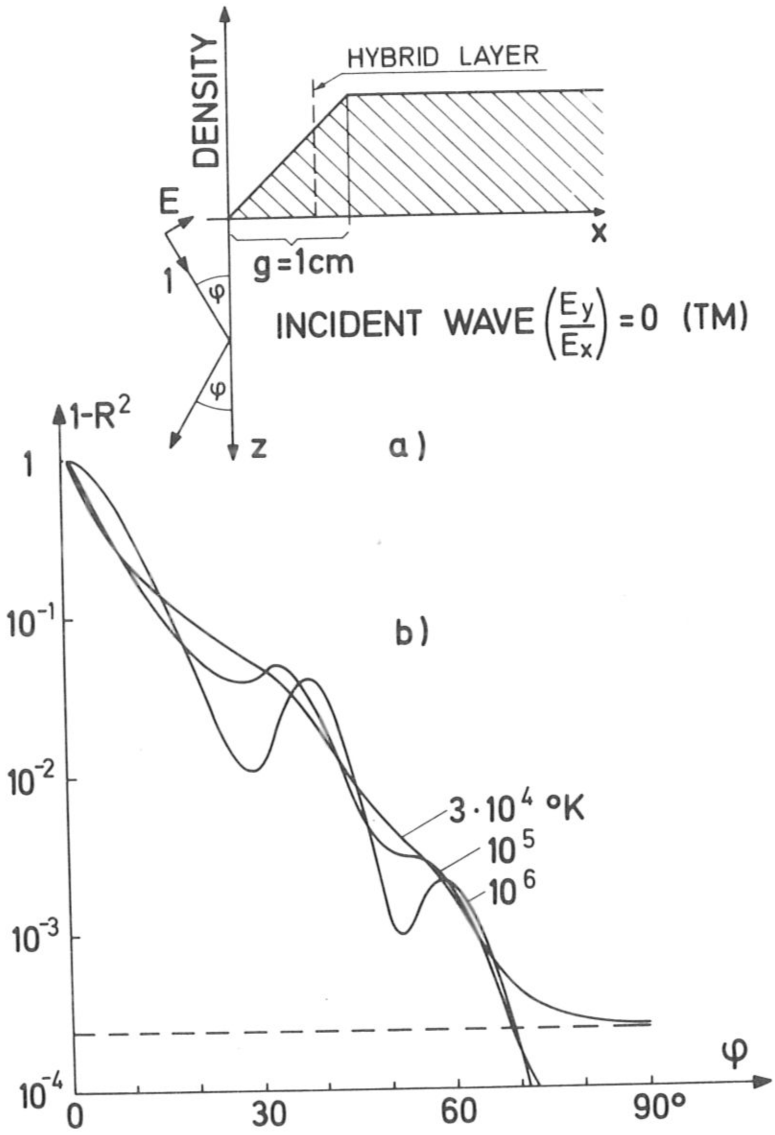

Fig. 1 Transmitted energy $(1-R^2)$ as a function of the angle of the incident TM wave with the vacuum-plasma interface for several values of tempera-ture T.

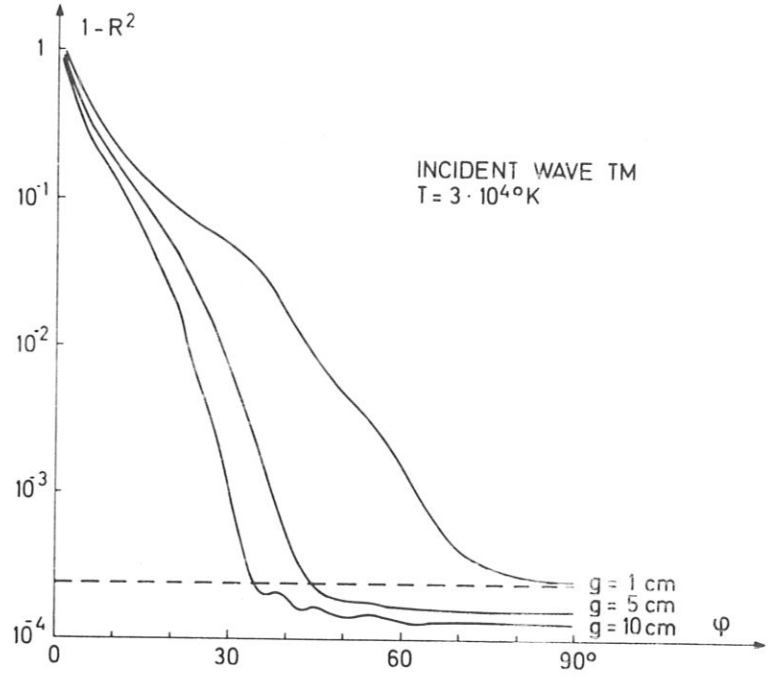

Fig. 2 Transmitted energy as a function of and T for an incident TE wave.

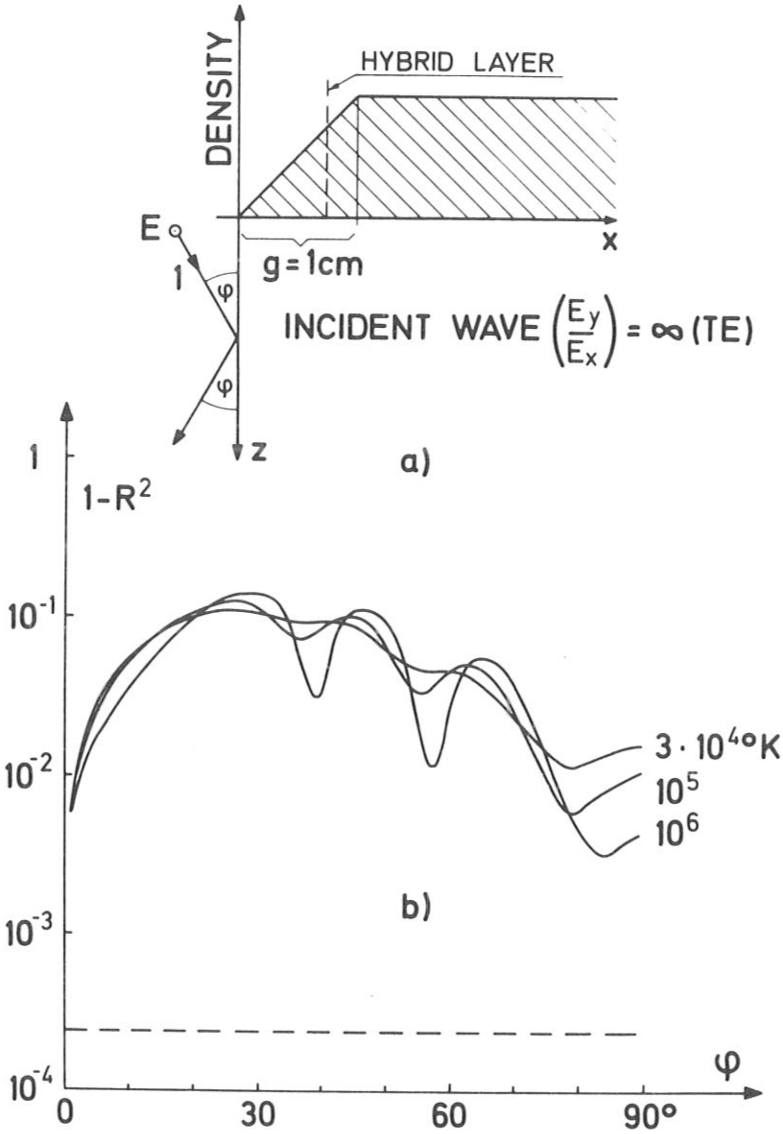

Fig. 3 Transmitted energy as a function of and the profile width "g" for an incident TM wave.

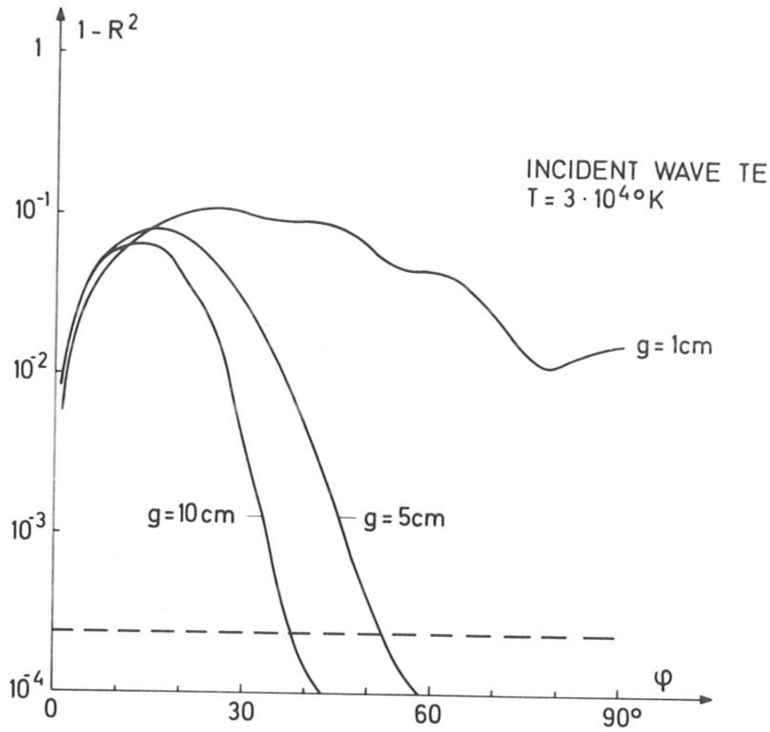

Fig. 4 Transmitted energy as a function of and "g" for an incident TE wave.