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Stellarator Fields with Twisted Coils

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ABSTRACT

The magnetic field of a set of circular and twisted coils is calculated. The number of coils is 16 or 12. It is shown that toroidal magnetic surfaces exist, which meet the requirements for stellarator experiments. In the l = 2 system the rotational transform can be increased to $\iota \approx 0.5$. The magnetic well depth of the configurations reaches values between 1 - 10 %. In the l = 3 system a rotational transform of $\iota \leqq 0.3$ on the separatrix can be obtained.

In change of the rotational transform with the variation of different parameters as coil radius, twist of the coil, superposition of a toroidal field and elliptic deformation of a coil is investigated.

I INTRODUCTION

The helical windings of a conventional stellarator are one of the major technical difficulties to be dealt with. Especially for the hypothetical stellarator reactor [1] the forces of the helical windings on the underlying blanket are very high [2]; in this case the support structure seems to become rather complicated.

Another problem is the construction phase of the helical windings: they have to be manufactured at the site of the reactor and after the installation of the reactor vessel and the blanket. Also maintenance and replacement of damaged parts of the reactor seem to be nearly impossible with the helical windings covering nearly the whole surface of the blanket.

These problems already affect large stellarator experiments like W VII [3] and in larger stellarator experiments the costs of the experiment will depend strongly on the technological implications of the helical windings. If a different way to produce the stellarator field is not found, the stellarator - as was pointed out by Bickerton et al. [1] - is going to lose the race with its major competitor as a fusion reactor: the tokamak.

A different method of magnetic field production has been chosen for the Tor - 2 - stellarator in the Lebedev-Institute, Moscow [4] . A system of planar elliptical coils creates a stellarator field, the rotational transform of which is changed by the superposition of a standard toroidal field. Electron beam measurements [4] and numerical calculations [5], [6] have shown the existence of magnetic surfaces. Unfortunately the rotational transform of the elliptical coil system is rather small - $\iota \cong 0.1$ - so that it is necessary to superimpose a large toroidal field in opposite direction in order to get higher values of iota.

In this paper we should like to discuss some theoretical aspects of this poloidal coil system and present some numerical calculations about the magnetic field of nonplanar poloidal coils, both circular and elliptical. These calculations have been carried through with modified Gourdon code.

II THEORY

The vacuum magnetic field of a stellarator is described by the scalar potential Φ and the toroidal flux function Ψ.

$$\underline{B} = \nabla\Phi = \nabla\Psi \times \nabla\chi \tag{1}$$

where
χ = const describes magnetic field lines.
The topology of a general stellarator field is shown in Fig. 1. The intersection Φ= const, χ = const is a curve closed in poloidal direction. The reason for this is the potential Φ which is singlevalued in the poloidal coordinate Θ.
Let us consider a toroidal surface $\dot{\Omega}$ which does not in general coincide with magnetic surface Ψ= const. If there are surface currents $\underline{j}$ on $\dot{\Omega}$ which produce the magnetic field $\underline{B}_i$ in the internal region Ω and $\underline{B}_e$ outside of $\dot{\Omega}$, these currents are given by

$$\underline{j} = \underline{n} \times (\underline{B}_i^* - \underline{B}_e^*) \tag{2}$$

where
$\underline{n}$ is the normal vector on $\dot{\Omega}$ and $\underline{B}_i^*$, $\underline{B}_e^*$ are the boundary values of the magnetic field on $\dot{\Omega}$. In the case of a straight stellarator with a cylindrical surface $\dot{\Omega}$ the solution of this problem can be found in [7].

Since $\underline{B}_i$ and $\underline{B}_e$ are derived from single valued potentials in Θ, the current lines of $\underline{j}$ on $\dot{\Omega}$ are poloidally closed curves. When these surface currents are replaced by coils of finite extent, the given magnetic field is approximated with an exactness increasing as the distance from the coils increases.

A special case is the coincidence of $\dot{\Omega}$ with a particular magnetic surface ψ = const. In this case the external field $\underline{B}_e^*$ is zero and there is no magnetic field in the whole external region. In reality due to the discrete spacing of the coils there exists a certain stray field outside the surface $\dot{\Omega}$, which will, however, be small. These optimal conditions might be of interest in order to avoid a large influence of magnetic materials like iron on the magnetic surfaces inside $\dot{\Omega}$.

In practice the currents $\underline{j}$ are not determined by the knowledge of the magnetic field but the magnetic fields are calculated from the given currents and a coincidence of one of the magnetic surfaces with the current surface is accidental.

Let us consider a straight stellarator with a cylindrical current surface.

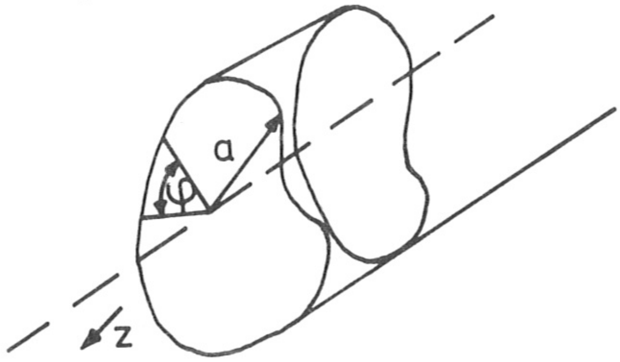

Fig. 2

The components of the surface currents are [7]

$$\underline{j}_z = \underline{j}_n \cos n\Theta$$
$$\underline{j}_\varphi = \underline{j}_n \tan\gamma \cdot \cos n\Theta + j_o \tag{3}$$

with $\Theta = \varphi - \alpha z$, $\alpha = 2\pi/L$, $n = 1, 2, 3 \ldots$, $\tan\gamma = \alpha \cdot a$, a = radius of the cylinder, and j_n, j_o = constant. By integrations we find the current lines

$$z(\Theta) = \frac{a}{n} \frac{j_n}{j_o} \sin n\Theta$$

$$\varphi(\Theta) = \Theta + \alpha \frac{a}{n} \frac{j_n}{j_o} \sin n\Theta \tag{4}$$

$z_0 = \frac{\alpha}{n} \frac{j_n}{j_o}$ is the maximum amplitude of the current line from the plane z = constant.

It is convenient to use this quantity instead of j_n and j_o.

The magnetic potential Φ_i inside the cylinder is given by

$$\Phi_i = B_o z + \frac{b}{\alpha} J_n(n\alpha r) \sin n\theta \tag{5}$$

where $B_o = \frac{4\pi}{c} j_o$, $b = B_o \alpha z n \alpha a K'(n\alpha a)$,

and $J_n(x)$, $K_n(x)$ denote Besselfunctions

The rotational transform per period is

$$\omega = (\alpha z_o)^2 (n\alpha a)^2 [K_n'(n\alpha a)]^2 \left[\frac{2}{n!} \left(\frac{n}{2}\right)^{n+1} \left\{ (1-\tfrac{1}{n}) (\alpha r)^{2n-4} + \frac{n}{2} (\alpha r)^{2n-2} + \ldots \right\} \right] \tag{6}$$

For $n\alpha a \ll 1$ the rotational transform is proportional to $\frac{z_o^2}{a^4}$

In this case the intersection between potential surface ϕ = const and the magnetic surface ψ = const can be found from the approximation

$$\phi_i \sim z + \frac{b}{2 B_o \alpha} (\alpha r)^2 \sin 2\theta$$

$$\psi \sim (\alpha r)^2 \left(1 - \frac{b}{B_o} \cos 2\theta\right) \quad ; \frac{b}{B_o} \ll 1 \tag{7}$$

Here we assume n = 2 (l = 2-stellarator)

The result is

$$z(\theta) = \frac{b}{2 B_o \alpha} \frac{(\alpha r_o)^2}{\left(1 - \frac{b}{B_o} \cos 2\theta\right)} \sin 2\theta \tag{8}$$

r_o is the mean radius of the magnetic surface.

The maximum amplitude is

$$z_{max} \simeq \frac{b}{2B\alpha}(\alpha r_o)^2 \quad \text{or} \quad z_{max} \simeq \frac{z_o}{2} 2\alpha a \; K_2'(2\alpha a)\,(\alpha r_o)^2 \tag{9}$$

If the mean radius r_o is equal to the radius a of the current cylinder we obtain:

$$z_{max} \leq \frac{z_o}{2}$$

(The maximum of $x^3 K'_2(x)$ is 4.)

From these results the following conclusion can be drawn:
A magnetic field with potential ϕ can be produced either by currents on the cylinder with radius a or by currents on the magnetic surface Ψ = const with mean radius a. In the second case the amplitude of the coils (z_{max}) is only half as big as in the first case (z_o).

Formula (6) has been evaluated for a $l = 2$-stellarator, Fig. 3 shows the relation between the rotational transform and the radius a and the amplitude z_o of the coils.

Formula (6) has been evaluated for a straight $l = 2$-stellarator. Fig.3 shows the curves with constant rotational transform per period. In order to simulate the toroidal case the number α is replaced by D/R_o, where R_o is the major torus radius and D the number of periods. The other quantities are described in the next chapter.

III NUMERICAL CALCULATIONS

1) The coil system

As pointed out in the previous chapter, the orthogonal trajectories to the field lines on a magnetic surface should be replaced by currents to create the desired magnetic field inside the torus. However this approach is not feasible because the coils must be as simple as possible for technical reasons.

For the numerical calculations reported in this paper the current filament which represents the coil of finite size was chosen according to the following formula:

$$x(\Theta) = r(\Theta)\cos(\Theta + \Theta_n)$$
$$y(\Theta) = r(\Theta)\sin(\Theta + \Theta_n)$$
$$z(\Theta) = F\sin(\Theta + \Theta_n) + C\sin 2(\Theta + \Theta_n) + E\sin 3(\Theta + \Theta_n)$$
$$r(\Theta) = B\cdot\left[1 + \gamma\cos(\Theta + \Theta_n) + \varepsilon\cos 2(\Theta + \Theta_n) + \mu\cos 3(\Theta + \Theta_n)\right]^{-\frac{1}{2}}$$

$B, \gamma, \varepsilon, \mu, F, C, E$ are constants, Θ_n is the phase angle of the n^{th} coil. The coordinate system with the z-axis along the axis of the coil is shown in Fig. 4 A set of N coils (16 or 12 in our calculation) was arranged on a circle of radius R_o. If each coil is turned by a constant circle relative to its neighbours, the azimutal components of the currents add up to the effective helical currents of the conventional stellarator. This is indicated in Fig. 5. In all calculations reported here the major radius was R_o = 333 cm, as this could be the size of a large stellarator experiment. Reactor conditions can be obtained by multiplying all dimensions by a factor of 3 or 4. A standard coil with the dimensions coil radius A = 150 cm, amplitude C= 40 cm γ, ε, μ = 0 is shown in Fig. 6 . A set of 16 of these coils arranged on the torus with radius R_o = 333 cm will be referred to as the "standard case", since alterations by changing parameters will be compared with this case. The rotation number D gives the number of coil periods around the torus, i.e.

$$D = N \frac{\Delta \Theta}{2\pi}$$

where $\Delta\Theta$ is the phase angle between two adjacent coils. The mumber D/R_0 is equivalent to the "wave" number α of the previous chapter. Calculations were done with coils of the circular l = 3-type. Elliptical coils are shown in Fig.7 and an l= 3-coil in Fig. 8.

2) The standard l = 2 case

The magnetic field lines were followed for 50 revolutions the long way around the torus thus giving a good picture of the magnetic surfaces. A plot of the intersection points of the field lines with the poloidal plane is shown in Fig. 9. The rotational transform of the standard case is shown in Fig. 10. The rotational transform exhibits negative shear. As has been pointed out by Popryadukhin [6] this negative shear is caused by the finite number of coils. This result was confirmed by our calculations in that a decrease of the number of coils to 12 increased negative shear. Furthermore we found a mean magnetic well in the standard case (Fig. 11 shows $V' = \lim \frac{1}{L} \int \frac{dl}{B}$ as function of R). The magnetic well depth is 13 % at maximum.

3) Superposition of a toroidal field

The rotational transform of the standard case can be increased by the superposition of a toroidal field opposite to the field created by the twisted poloidal coils. Fig. 12 shows the rotational transform as a function of the superimposed toroidal field. With increasing iota the separatrix moves inward (Fig. 13 shows the shape of the magnetic surface at $\iota \simeq 0.25$).

4) Variation of coil amplitude

A change of the amplitude C changes the effective helical currents, i.e. a change of C varies the rotational transform. Fig. 14 indicates the dependence of $\iota\!\!\!-$ on the coil amplitude C.

5) Variation of the coil radius

A modification of the coil radius changes the aspect ratio of the system. A smaller radius of the coils makes the resulting helical fields more effective. The increase of iota with decreasing coil radius is shown in Fig. 15. Also an example of the magnetic surface at a higher value of iota is given (Fig. 16).

6) Elliptical deformations of the coils

A deformation of the coils into an elliptic shape (Fig. 7) also increases the effect of helical currents, so that the rotational transform increases (Fig. 17). Fig. 18 shows a set of 12 elliptic coils which create the magnetic surfaces shown in Fig. 19 . We also observe the formation of islands at $\iota\!\!\!- = 0.4$. The advantage of elliptical coils is that they can be adjusted more easily to the shape of the magnetic surfaces than circular coils. This fact might be of importance for a stellarator reactor as was pointed out in [5].

7) l = 3-coil system

Calculations with an l = 3-coil system (Fig. 20) also show the existence of magnetic surfaces (Fig. 21). The rotational transform on the separatrix is $\iota\!\!\!- = 0.3$.
It turned out to be difficult to reach higher values of iota, since the amplitude of the coils becomes very large in that case.

8) Comparison with a straight stellarator

The numerical results with the l = 2 system were compared with the analytical results of a straight stellarator. The numerical results are shown in Fig.3. The rotational transform in a torus is smaller than the corresponding value of the straight stellarator, probably resulting from the torus curvature and the discreteness of the coils. However, the difference is sufficiently small so that the analytical results may be used to approximate the toroidal case.

IV DISCUSSION AND CONCLUSIONS

The calculations have shown that stellarator fields with rotational transform $\iota \leqq 0.5$ can be achieved by circular but twisted coils. The necessary number of coils (16 in most calculations) is small enough, so that the coils in projection on the equatorical plane do no overlap. This fact is important for a stellarator reactor, since in this case it can built together of modules.

In all configurations mentioned in this paper there exists a magnetic well depth ($V''(\psi) < 0$) but since we did not trace the sparatrix in all cases we cannot give the exact values of the magnetic well depth. In the socalled "standard case" the well dpth is 13 % but it decreases with increasing rotational transform. For a large experiment or a prototype reactor the above mentioned values of iota and the magnetic well depth seem to be sufficient. Whether they also are sufficient for an economic stellarator reactor depends on the maximum ß which can finally be obtained in these configurations. The formula $\beta_c = \frac{\delta U}{U} \iota^2$ which was given by Shafranov et al. [8] yields to small values of ß in our cases. But this formula does not take into account the effect of shear and also holds for all modenumber of the hydromagnetic instability. Therefore the formula of Shafranov gives a lower estimate of the critical ß in a stellarator, more theoretical and experimental work on this point has to be done, before the problem of maximum ß in a stellarator can be solved.

For large experiments or a stellarator reactor the forces on the coils are an important problem. Since the effective helical current of the coil system does not flow parallel to the main toroidal field there occur bending forces on the coils. All forces on the coil are radially outward but inhomogeneous, therefore a bending results. The magnitude of these bending forces is comparable with the forces of a helix on the vacuum tube of a conventional stellarator but here the act upon the main coils. Because of the inhomogeneity of the main toroidal field these coils have to be reinforced by structural material in any case. Therefore the problem of bending forces on the twisted coils does not seem to be more serious than the same problem for equivalent circular coils. Besides these bending forces there also occur forces which want to tilt the coils. But these forces compensate each other around the torus so tilting of the coils can be prevented by supporting the coils mutually. The radial forces, which are due for the main magnetic field, are of the same magnitude as in the case of equivalent circular coils.
The results given above are rough estimates on the problem of forces, detailed numerical calculations are under preparation.

If the configurations described above are appropriate for a stellarator reactor the system of circular or elliptical twisted coils has several advantages compared with a conventional stelllarator.

1) There is more and easier access to the blanket and the core of the reactor than in a device with helical windings.

2) The reactor can be built of modules which is of importance for maintainance and repair problems [9].

3) The magnetic field is created by one set of coils which are all identical, at least the superconducting core of the coils. This fact diminishes the construction costs.

4) The coils can be manufactured separately and at the same time as the other components of the device. In a conventional stellarator the helical windings can only be wound after completion of the reactor core and the blanket. Therefore the concept of twisted coils also reduces the construction time of the reactor.

5) As shown by Popryadukhin [6] there exists the possibility of a poloidal divertor From the technical point of view the concept of twisted coils is more advantageous than the conventional stellarators, arguments against it may come from plasma physics: The critical ß might be lower than in the equivalent stellarator with helical windings and the discreteness of the coils could introduce a large field ripple, which gives rise to extra losses. These problems will be studied in more detail in the near future.

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FIGURE CAPTIONS

Fig.1 Schematic shape of a magnetic surface in a stellarator

Fig.3 Lines of constant rotational transform per period in a straight stellarator. The points are results of numerical calculations for a toroidal stellarator

Fig.4 Coordinate system for a twisted coil

Fig.5 A set of 16 twisted circular coils arranged on a torus (R_o = 333 cm, A = 150 cm, A = coil radius

Fig.6 A circular l = 2-coil

Fig. 7 An elliptical and twisted l = 2-coil

Fig.8 Circular l = 3-coil

Fig.9 Magnetic surfaces of the standard case (R_o = 333 cm, A = 150 cm, C = 40 cm D = 1.0)

Fig.10 Rotational transform of the standard case

Fig.11 Magnetic well of the standard case.

Fig.12 Variation of the rotational transform on the axis as a function of a superimposed toroidal field.

Fig.13 Magnetic surfaces at 10 % of toroidal field superimposed in opposite direction.

Fig.14 Variation of the rotational transform on the axis with varying amplitude of the coil. All other parameters are kept fixed.

Fig.15 Variation of the rotational transform with the radius of the coil

Fig.16 Magnetic surfaces and rotational transform at a coil radius of $A = 130$ cm.

Fig.17 Increase of the rotational transform on the axis with increasing excentricity of the coils. The mean radius of the coils $\frac{A+B}{2}$ is kept fixed.

Fig.18 Schematic picture of 12 elliptical coils on a torus

Fig.19 Magnetic surfaces and rotational transform of the system with elliptical coils (Fig.18)

Fig.20 A set of 12 circular $l = 3$ coils

Fig.21 Magnetic surfaces and rotational transform of the $l = 3$ system

In all plots of magnetic surfaces the scale length-indicated by bars at the coordinate axis - is one tenth of the major torus radius (unit = 33,3 cm).

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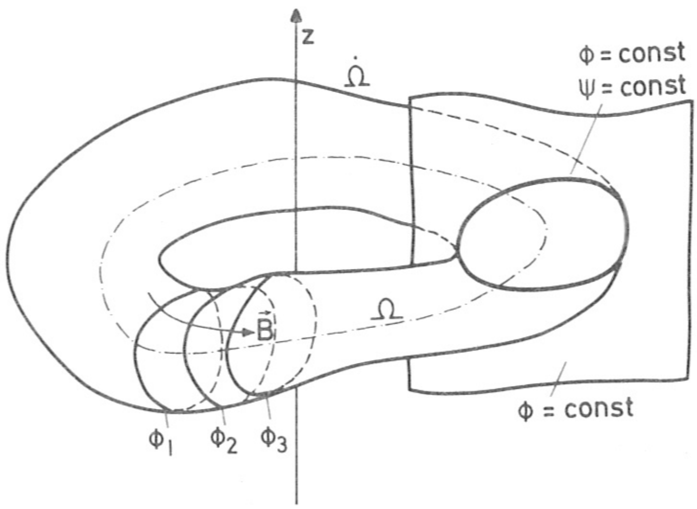


Fig. 1

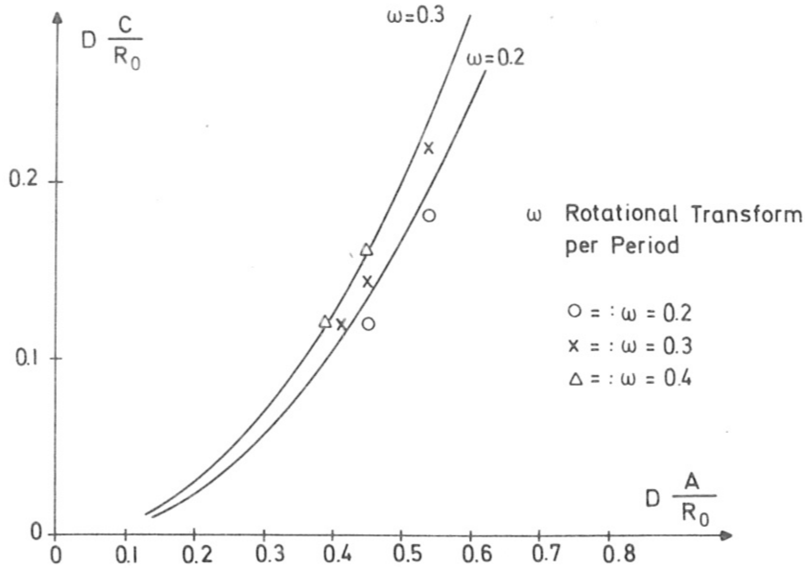


Fig.3

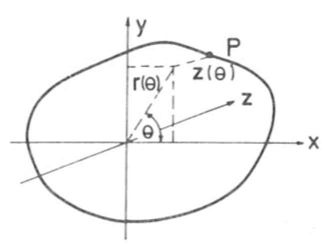


Fig.4

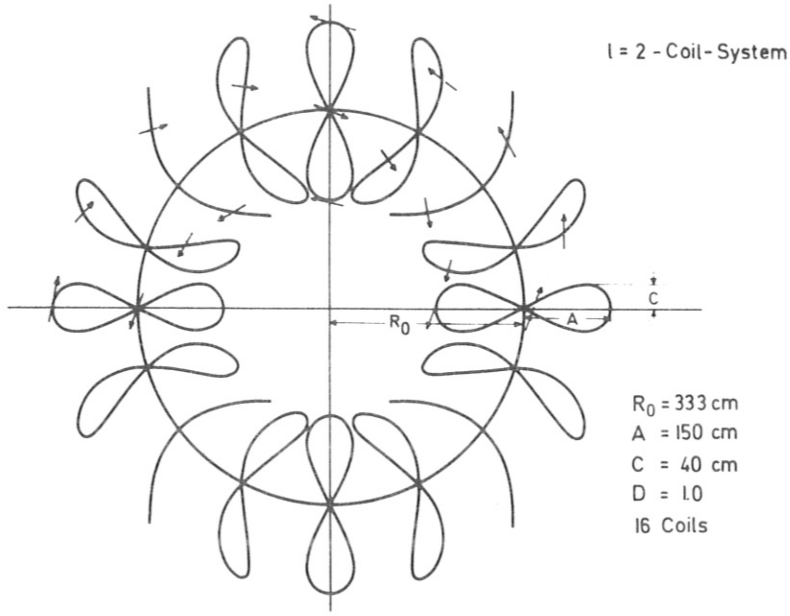


Fig. 5

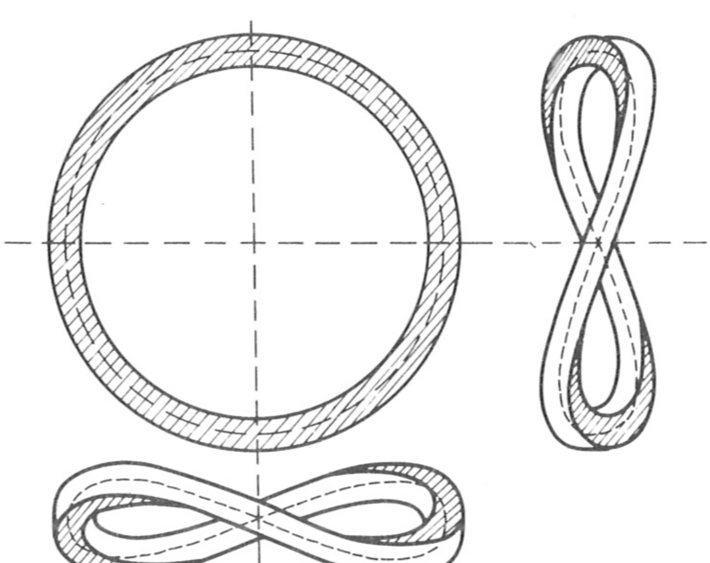


Fig. 6

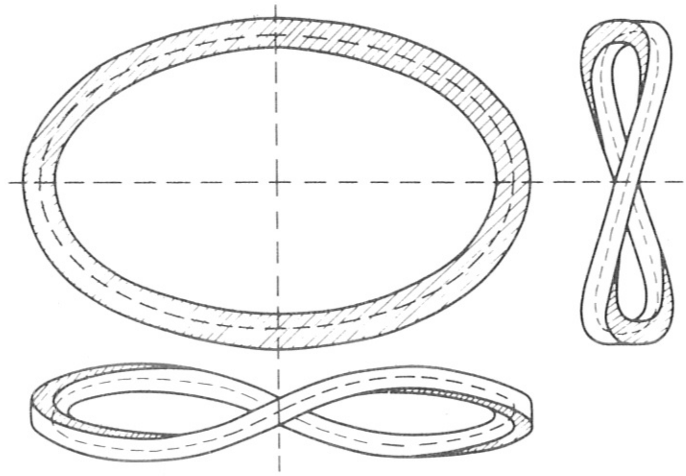


Fig.7

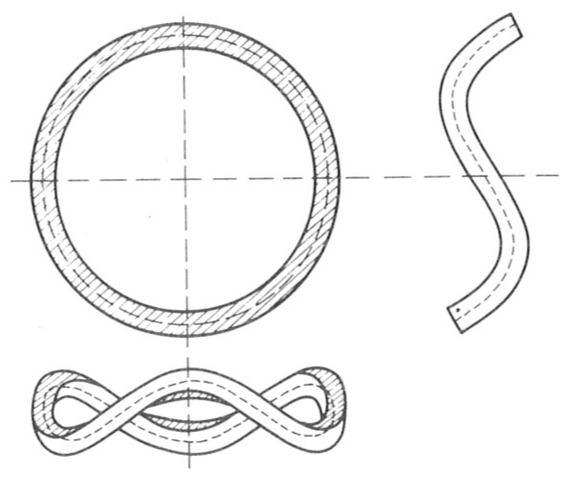


Fig.8

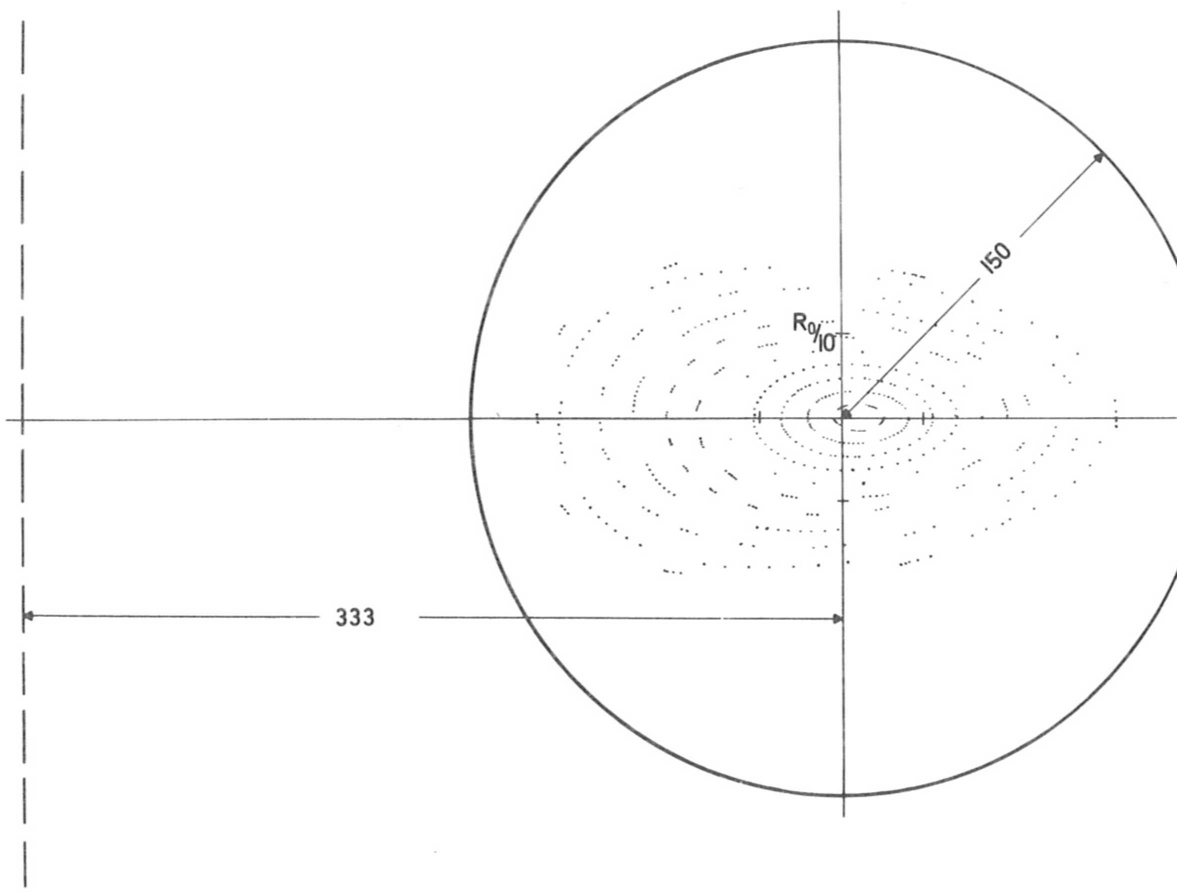
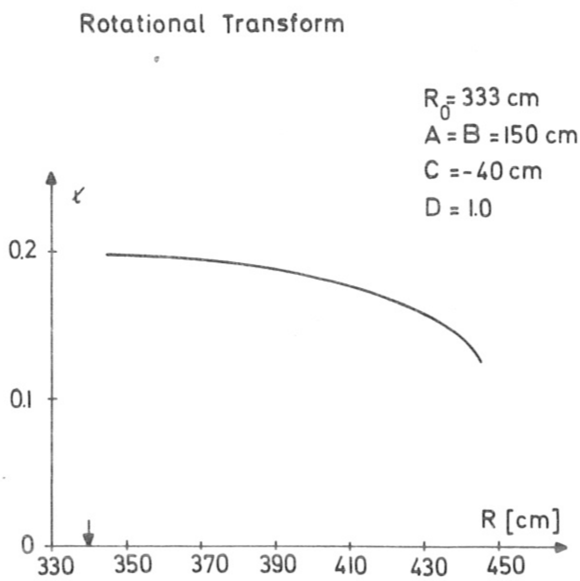
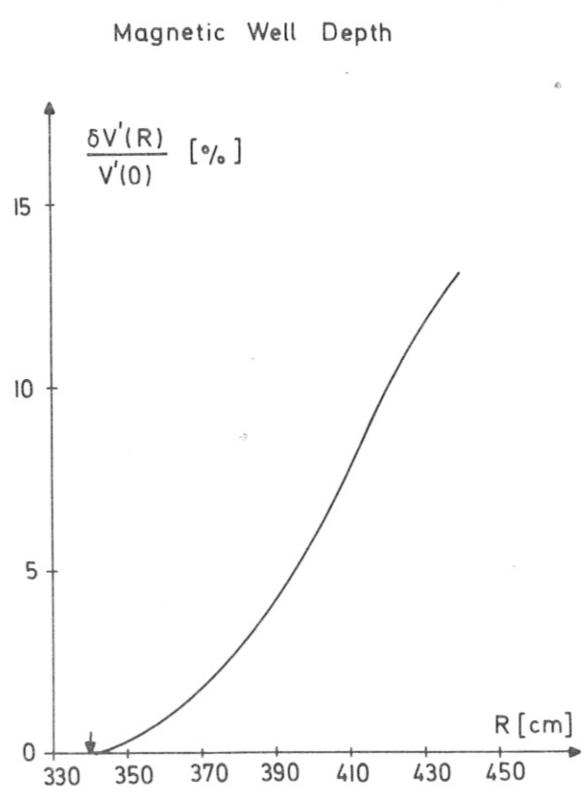


Fig. 9



Rotational Transform
R_0= 333 cm
A = B = 150 cm
C = -40 cm
D = 1.0
ι
0.2
0.1
0
R [cm]
330
350
370
390
410
430
450

Fig. 10



Magnetic Well Depth
$\frac{\delta V'(R)}{V'(0)}$ [%]
15
10
5
0
R [cm]
330
350
370
390
410
430
450

Fig. 11

Superposition of a Toroidal Field

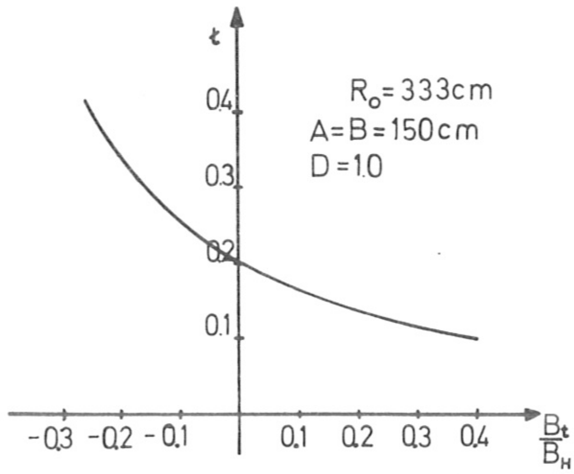


Fig.12

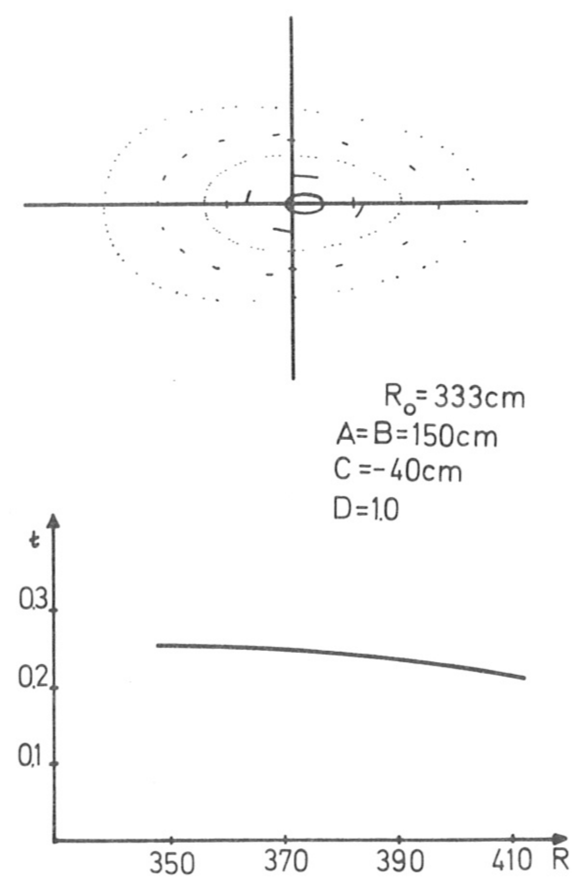
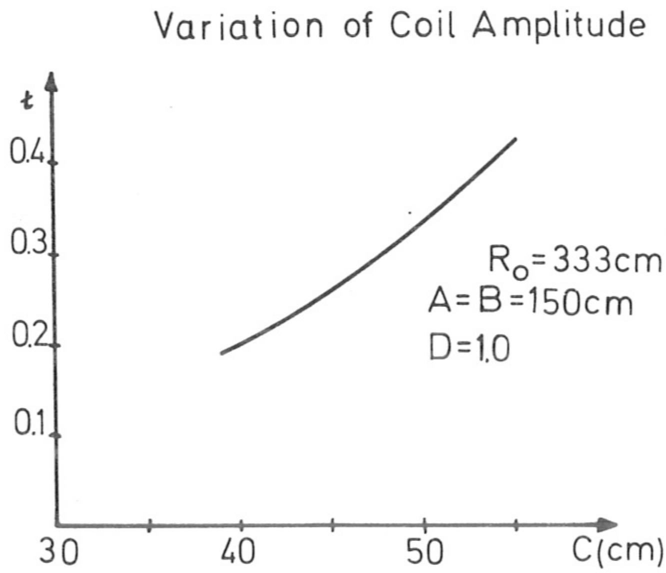


Fig.13

Fig. 14


Variation of Coil Amplitude
t
0.4
0.3
0.2
0.1
30
40
50
C(cm)
R₀=333cm
A=B=150cm
D=1.0

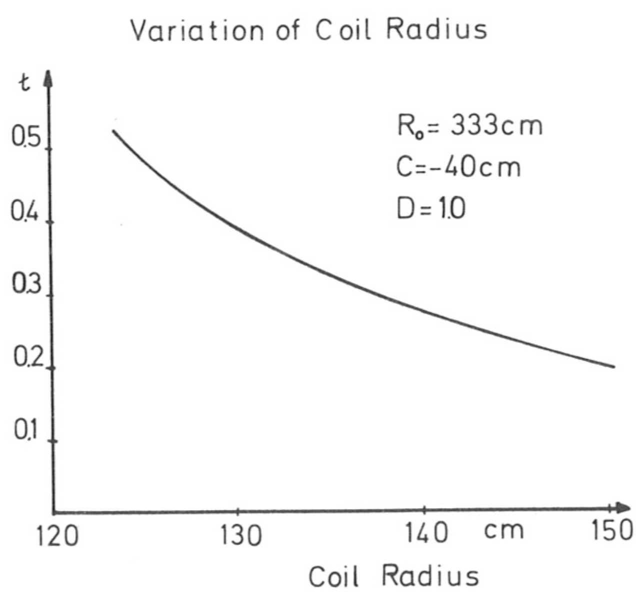
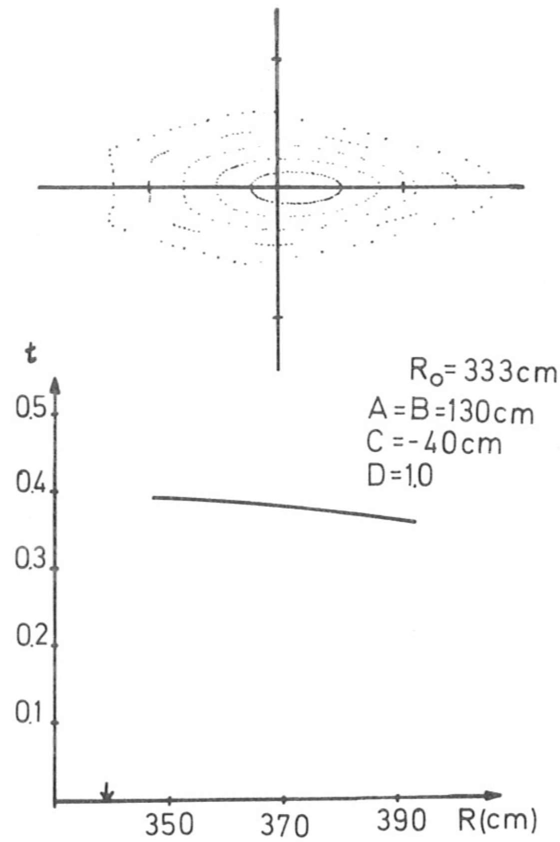
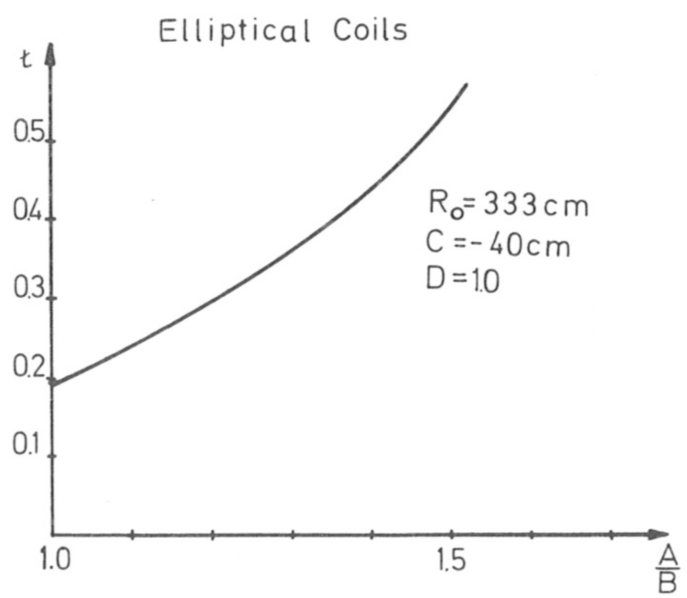

Variation of Coil Radius
t
0.5
0.4
0.3
0.2
0.1
120
130
140 cm
150
Coil Radius
R₀= 333cm
C=-40cm
D=1.0

Fig. 15



t
0.5
0.4
0.3
0.2
0.1
350
370
390
R(cm)
R_o= 333cm
A = B = 130cm
C = -40cm
D = 1.0

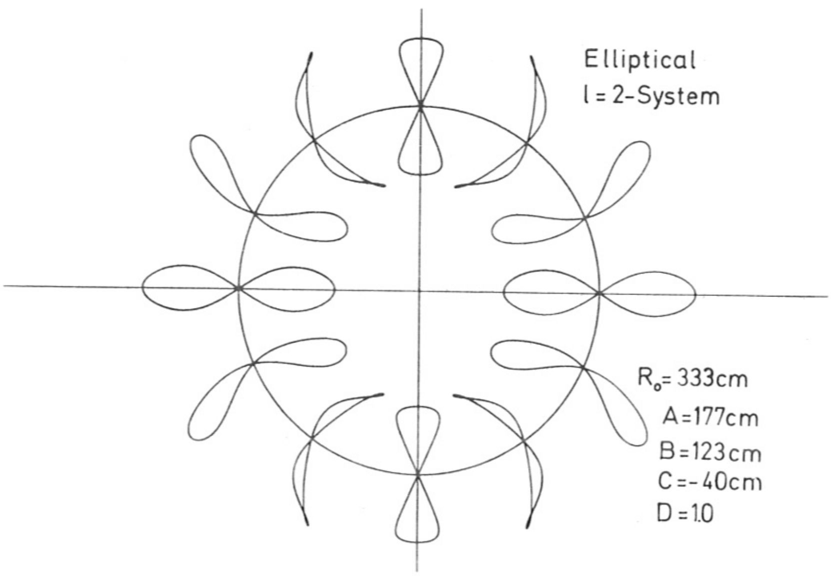
Fig.16



Elliptical Coils
t
0.5
0.4
0.3
0.2
0.1
1.0
1.5
A/B
R_o= 333 cm
C = -40cm
D = 1.0

Fig.17

Fig.18


Elliptical
l = 2-System
R_o= 333cm
A=177cm
B=123cm
C=-40cm
D=1.0

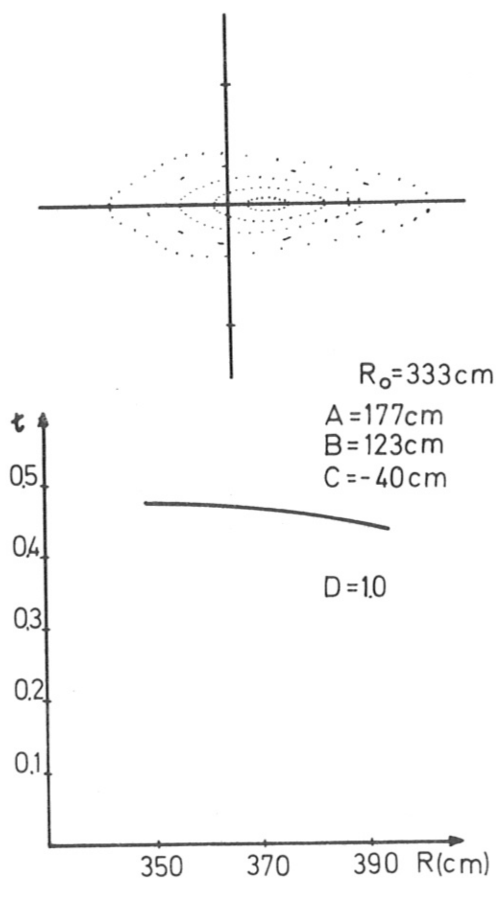
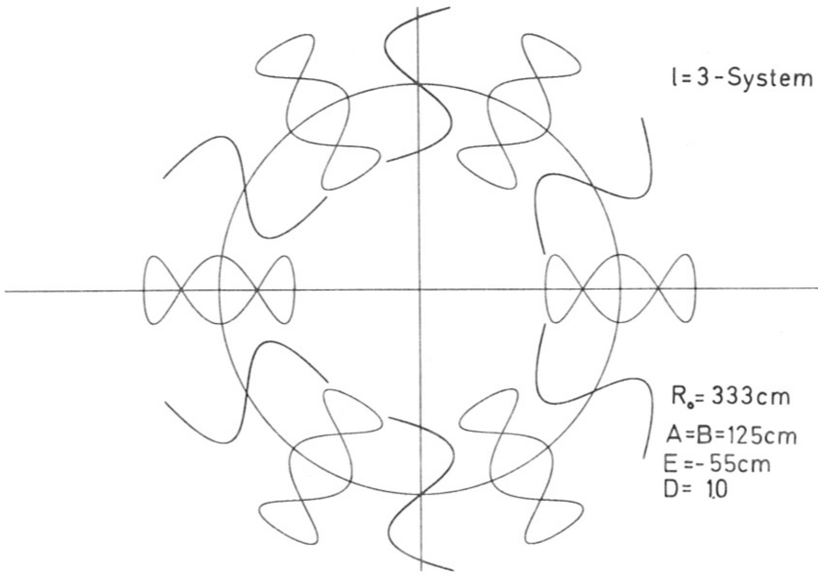

R_o=333cm
A=177cm
B=123cm
C=-40cm
D=1.0
t
0.5
0.4
0.3
0.2
0.1
350
370
390
R(cm)

Fig.19

Fig. 20


l = 3 - System
R₀= 333cm
A=B=125cm
E =- 55cm
D= 1,0

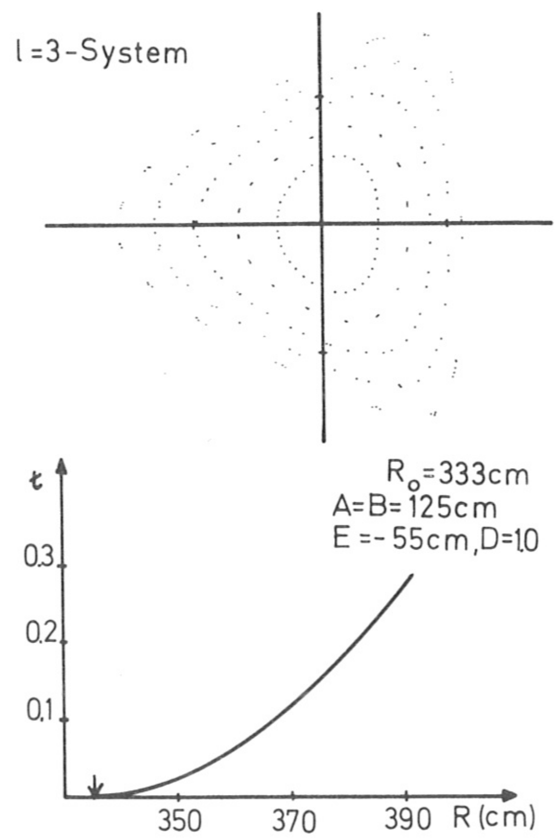

l =3 - System
t
0,3
0,2
0,1
350
370
390
R (cm)
R₀=333cm
A=B=125cm
E =- 55cm, D=1,0

Fig. 21