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Abstract

Numerical calculations of the inflection process of electrons into a compressor with an additional B_{ϕ} -field are presented. It is shown that, by choosing suitable values for the initial mean axial position and velocity, compact electron rings can be formed (i.e. the final betatron amplitudes can be made small).

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However, if the inflection takes place in a fast axial magnetic field, there are several methods to keep the axial collective oscillation amplitudes small:

- a) Relative damping of the laser currents in a compressor structure like ASPON.
- b) Application of an axial electric field pulse during the inflection interval.

c) Application of a time dependent radial magnetic field during the inflection to suppress axial forces⁶.

- d) Application of a magnetic gradient in the

I. Introduction

The inflection into an electron ring compressor without additional B_ϕ -field is normally performed by means of a fast axial magnetic field, which damps the radial betatron amplitudes and shifts the radial phase space to the desired area ^{1,2}.

In some compressor designs the application of an additional azimuthal magnetic field component, B_ϕ , seems to be favourable for shifting betatron tunes ³ and increasing Landau damping coefficients ^{4,5} of transverse collective instabilities. But in the presence of a B_ϕ -component the collective radial motion during the inflection process leads to axial collective oscillations and subsequent ring broadening.

However, if the inflection pulse is made of a fast axial magnetic field, there are several methods to keep the axial collective oscillation amplitudes small:

- a) Resistive damping of the image currents in a surrounding structure (like in ASTRON).
- b) Application of an axial electric field pulse during the inflection interval.
- c) Use of a time dependent radial magnetic field component during the inflection to suppress additional axial forces ⁶.
- d) Application of a suitable gradient in the inflector field.

- e) Choice of nonzero initial values for the phase space centers of the axial dimension and/or its velocity.

While the first four methods may probably give rise to disturbances of the main magnetic field shape, the last one can easily be applied and experimentally optimized.

To get a rough impression of the parameter range, let us first look at the linearized equations of motion to find inflection conditions for which the radial and the axial betatron amplitude vanish at the end of the inflection pulse (for the phase space center). After this the exact single particle calculations of the inflection in the presence of a B_{ϕ} -field will be presented. These calculations take into account all nonlinearities and also the relatively fast rise of the magnetic field of a proposed single-turn coil compressor⁵. The influences of the selffields of the beam and its images on the surrounding boundaries are neglected, which - especially at higher beam intensities - may imply different inflection behaviour.

II. Survey by linearization

To get an impression of the parameter range for initial axial position and velocity of the phase space center, let us look at the linearized equations of motion for the electron in the presence of a B_ϕ magnetic field component.

The inflection field may be described by

$$B_{\text{infl}} = -B_z \cdot \frac{\Delta X_0}{R} \cdot f(t) \cdot g(\varphi)$$

where B_z is the compressor axial magnetic field component, R is the corresponding radius and ΔX_0 is the radial separation between injection radius and R . $f(t)$ describes the time behaviour which may be a linear ramp:

$$f(t) = \begin{cases} \xi \left(1 - \frac{t}{T}\right) & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases}$$

The angular distribution consists of a zero and a first harmonic component:

$$g(\varphi) = \frac{1}{2} \left(1 - \cos \left[\frac{v_\phi}{R} (t - t_i) - \varphi_0 \right] \right),$$

where v_ϕ is the azimuthal electron velocity, t_i is the injection time, and φ_0 is the angle, where the inflector field is maximum ($\varphi_0 = 0$ opposite to snout).

If $x = r - R$, the equations of motion are

$$\frac{d^2 x}{dt^2} + \frac{v_\phi^2}{R^2} (1-n)x - \alpha \frac{v_\phi}{R} \frac{dz}{dt} = \frac{v_\phi^2}{R^2} \cdot \frac{\Delta X_0}{2} \xi \left(1 - \frac{t}{T}\right) \left(1 - \cos \left[\frac{v_\phi}{R} (t - t_i) - \varphi_0 \right] \right)$$

$$\frac{d^2 z}{dt^2} + \frac{v_\phi^2}{R^2} n z + \alpha \frac{v_\phi}{R} \frac{dx}{dt} = 0$$

for $t \leq T$, where $\alpha = \frac{B\phi}{B_z}$ and $n = -\frac{R}{B_z} \frac{\partial B_z}{\partial R}$.

The solution is

$$\begin{aligned}
 x = & A \sin \nu_1 t + B \cos \nu_1 t + C \sin \nu_2 t + D \cos \nu_2 t \\
 & + \frac{1}{1-n} \frac{\Delta k_0}{2} \xi \left(1 - \frac{t}{T}\right) + \frac{R}{v_\phi} \frac{\Delta k_0}{2} \frac{\xi}{T} \frac{2[\alpha^2 n + (1+n)^2]}{[\alpha^2 - n(1-n)]^2} \sin \left[\frac{v_\phi}{R} (t - t_i) - \varphi_0 \right] \\
 & - \frac{1-n}{\alpha^2 - n(1-n)} \frac{\Delta k_0}{2} \xi \left(1 - \frac{t}{T}\right) \cos \left[\frac{v_\phi}{R} (t - t_i) - \varphi_0 \right]
 \end{aligned}$$

$$\begin{aligned}
 z = & A' \sin \nu_1 t + B' \cos \nu_1 t + C' \sin \nu_2 t + D' \cos \nu_2 t \\
 & + \frac{1}{n(1-n)} \alpha \frac{R}{v_\phi} \frac{\Delta k_0}{2} \frac{\xi}{T} + \frac{\alpha}{\alpha^2 - n(1-n)} \frac{\Delta k_0}{2} \xi \left(1 - \frac{t}{T}\right) \sin \left[\frac{v_\phi}{R} (t - t_i) - \varphi_0 \right] \\
 & + \frac{R}{v_\phi} \frac{\alpha(2 - \alpha^2 - n(1-n))}{[\alpha^2 - n(1-n)]^2} \frac{\Delta k_0}{2} \frac{\xi}{T} \cos \left[\frac{v_\phi}{R} (t - t_i) - \varphi_0 \right]
 \end{aligned}$$

with

$$\nu_{1/2} = \frac{v_\phi^2}{R^2} \left[\frac{1 + \alpha^2}{2} \pm \sqrt{\left(\frac{1 + \alpha^2}{2}\right)^2 - n(1-n)} \right] \quad \text{and}$$

$$\frac{B}{A'} = \frac{A}{B'} = \sqrt{\frac{\nu_1^2 - n \frac{v_\phi^2}{R^2}}{\nu_1^2 - (1-n) \frac{v_\phi^2}{R^2}}} \quad \text{and} \quad \frac{D}{C'} = \frac{C}{D'} = \sqrt{\frac{\nu_2^2 - n \frac{v_\phi^2}{R^2}}{\nu_2^2 - (1-n) \frac{v_\phi^2}{R^2}}}$$

(see 7).

As an example, Fig. 1 gives the sum of the squares of the amplitudes in x and z for $R = 16$ cm, $n = 0.5$, $\alpha = 0.6$ and $T = 3 \frac{2\pi R}{v_p}$ after the end of the inflection pulse ($t \geq T$) as a function of initial axial position z_0 and axial velocity $\frac{v_{z0}}{C}$. This value is a relatively weak function of z_0 and ξ , but strongly dependent on v_{z0} . The optimum value for the initial axial velocity seems to be $v_{z0} = -0.053C$ at $z_0 \approx .1$ and $\xi = .4$ (i.e. $B_{infl}/\max = 12.9$ G). The angular distribution should have a maximum around 90° (measured from the injection snout).

These simplified calculations are only thought as a rough orientation for finding the center of the initial axial velocities and positions. The following exact calculations take into account the nonlinearities and the time dependence of the compressor field.

III. Single particle computer calculations

By solving the exact equation of motion

$$\frac{d}{dt}(m\vec{v}) = e(\vec{E} + \vec{v} \times \vec{B})$$

the nonlinearities and the time dependence of the magnetic field of a single turn coil compressor, as described in ⁵, are taken into account. A program description is given in ⁸.

At first an example for the case $B_{\phi} = 0$ is given. From experimental point of view a rectangular azimuthal distribution of the inflection field with $\varphi_{\min} < \varphi < \varphi_{\max}$ is easy to obtain. Neglecting space charge effects, an optimum inflection will be achieved with the following parameters:

$$\varphi_{\min} = 100^{\circ}, \quad \varphi_{\max} = 240^{\circ} \text{ (measured from injection snout)}$$

$$/B_{\text{infl}}/\text{max} = 12.8 \text{ G and } T = 10. \text{ nsec.}$$

Three successive turns for these parameters are drawn in Fig. 2 at the end of the inflection pulse in radial phase space, where the circle indicates the initial phase space area. The areas contain only those particles, which are not lost at the snout. (As at the end of the inflection pulse the particles are at an azimuth different from that of the snout, even turn 3 doesn't strike the snout later on.) It turns out that the first turn, which can be inflected, has minimum mean betatron amplitudes, but only a fraction of the incoming current is inflected; the next two turns are well inflected but with higher mean oscillation amplitudes. Beam stacking is possible. The relatively high $\frac{dB_z}{dt}$ of the compressor, which causes fast inward motion of the closed orbit radius, is the reason for this relatively good inflection. The number of captured

electrons can be increased by increasing the radial velocity dimension (the compressor can be "flooded" with electrons).

For a compressor with $B_\phi \neq 0$ the initial center of the axial phase space (position and velocity) adds to the above mentioned parameters. An optimum of the inflection was found numerically at $\varphi_{\min} = 0^\circ$, $\varphi_{\max} = 170^\circ$, $T = 10. \text{ nsec}$, $/B_{\text{infl}/\text{max}} = 13.0 \text{ G}$, $Z_0 = .6 \text{ cm}$ and $U_{z0} = -.4.c$. This optimum is most sensitive to $U_{z0} \cdot \alpha = 0.6$. The time behaviour of the center of radial and axial phase space for this parameter set is given in Fig. 3, in which the small betatron amplitudes after the end of the inflection pulse (10 nsec) and the fast radial compression can be seen. The particle doesn't meet the snout, which is indicated (around 3.3 nsec) after one revolution.

The radial and axial phase space areas for the inflected electrons at the end of the inflection pulse (10.nsec) are given in Fig. 4 for two successive turns. They are 2-dim. projections of the four dimensional phase space volume and are drawn for variations in R- and z-space, respectively. The initial areas in R- and z-space are given by circles. As in the case without B_ϕ the first turn is partly lost but inflected with small betatron amplitudes, while the second turn is totally inflected (but with higher amplitudes). The inflected rings seem to be relatively compact. Also in this case we may profit of the fast radial compression.

While in all these calculations the initial kinetic energy of the electrons has been kept constant at $T_{\text{kin}} = 1.902 \text{ MeV}$, an example of inflection of electrons with an energy ramp in this coil geometry is given. This scheme of "spiral injection" into a compressor has already been proposed by C. Bovet⁹. Because of the high $\frac{dB_z}{dt}$ of our single turn

coil ⁵ this mechanism seems to be very favourable. Fig. 5 gives the time behaviour of R and z of the phase space center for an increasing energy ramp of approximately 2 % per revolution. All three turns are well inflected; the injection snout position is indicated.

Summary

The inflection of electrons into a compressor with an additional B_ϕ -field to form compact rings can be accomplished by the choice of suitable initial values for the axial position and velocity. The injection snout should make an angle of nearly 50 mrad with the compressor midplane. The inflector position in the presence of B_ϕ is quite different from the case with no azimuthal compressor magnetic field component. While the inflector center angle in the case with B_ϕ is somewhat smaller than $\pi/2$ (measured from the injection snout) for our compressor scheme (where one tune is $\nu_1 > 1$), the center should be located at somewhat less than π for the case $B_\phi = 0$. Although $\alpha = B_\phi / B_z$ may be changed initially in our experiment, the one tune ν_1 should be kept at $1 < \nu_1 < 1.2$, so that the above found parameter sets seem to be good working conditions. Since our compressor has a fast rising magnetic field, injection with an energy ramp seems to be favourable.

L.J. Laslett, Lawrence Berkeley Laboratory, Berkeley, California, Report ERAN - 159

A.U. Luccio, MPI für Plasmaphysik Report in preparation

C. Bovet, Lawrence Berkeley Laboratory Report ERAN - 87 (June 1970)

Figure captions

Fig. 1 Sum of the squares of oscillation amplitudes for different parameters.

Fig. 2 Radial phase space of inflected electrons for three successive turns (at the inflection pulse end); $B_{\phi} = 0$.

Fig. 3 Time behaviour of R and z for the phase space center.

Fig. 4 Radial and axial phase space areas at the end of the inflection pulse; $B_{\phi} \neq 0$.

Fig. 5 R- and z-time dependence for injection with an energy ramp.

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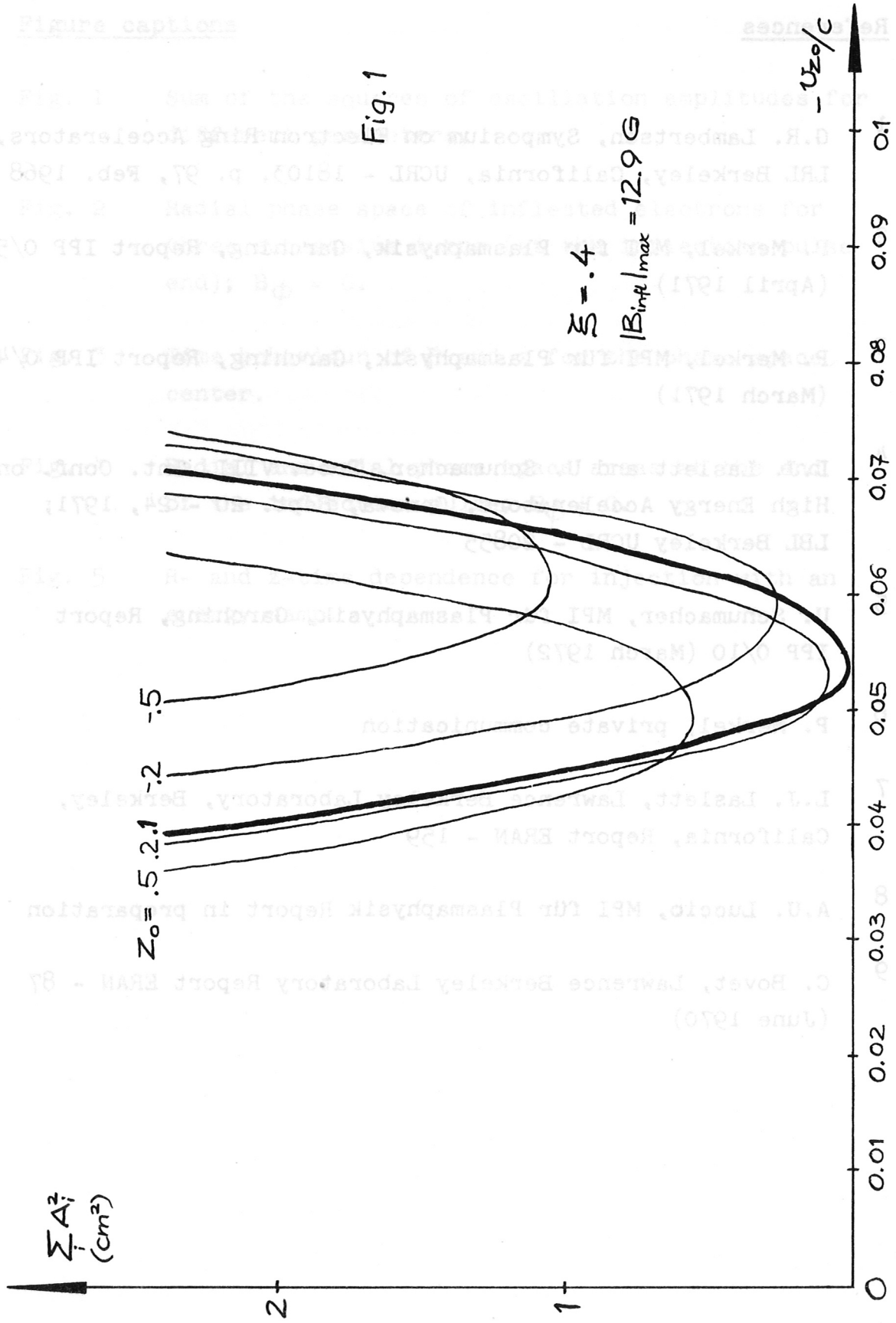
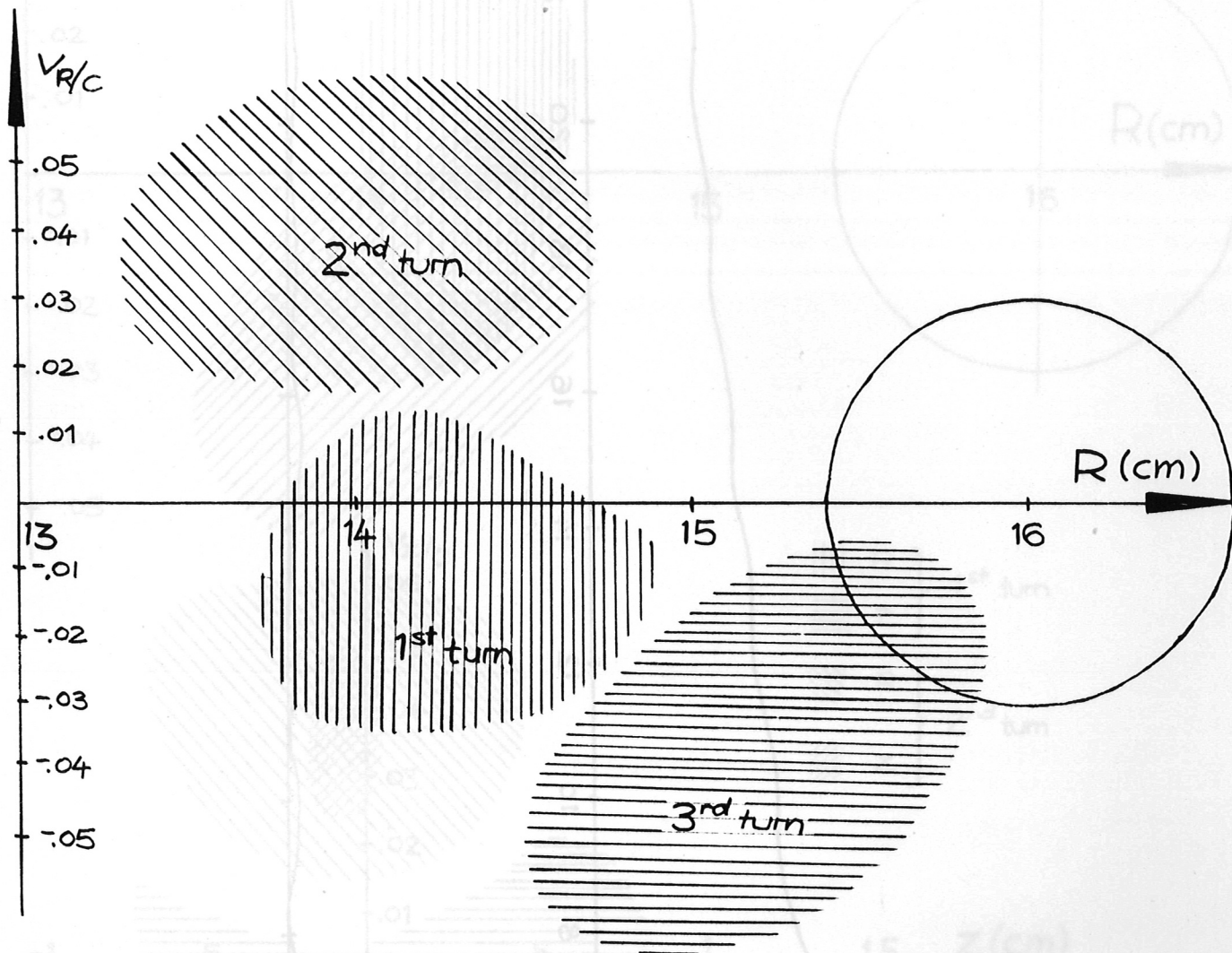


Fig. 1



$$B_{\phi} = 0$$

Fig. 2

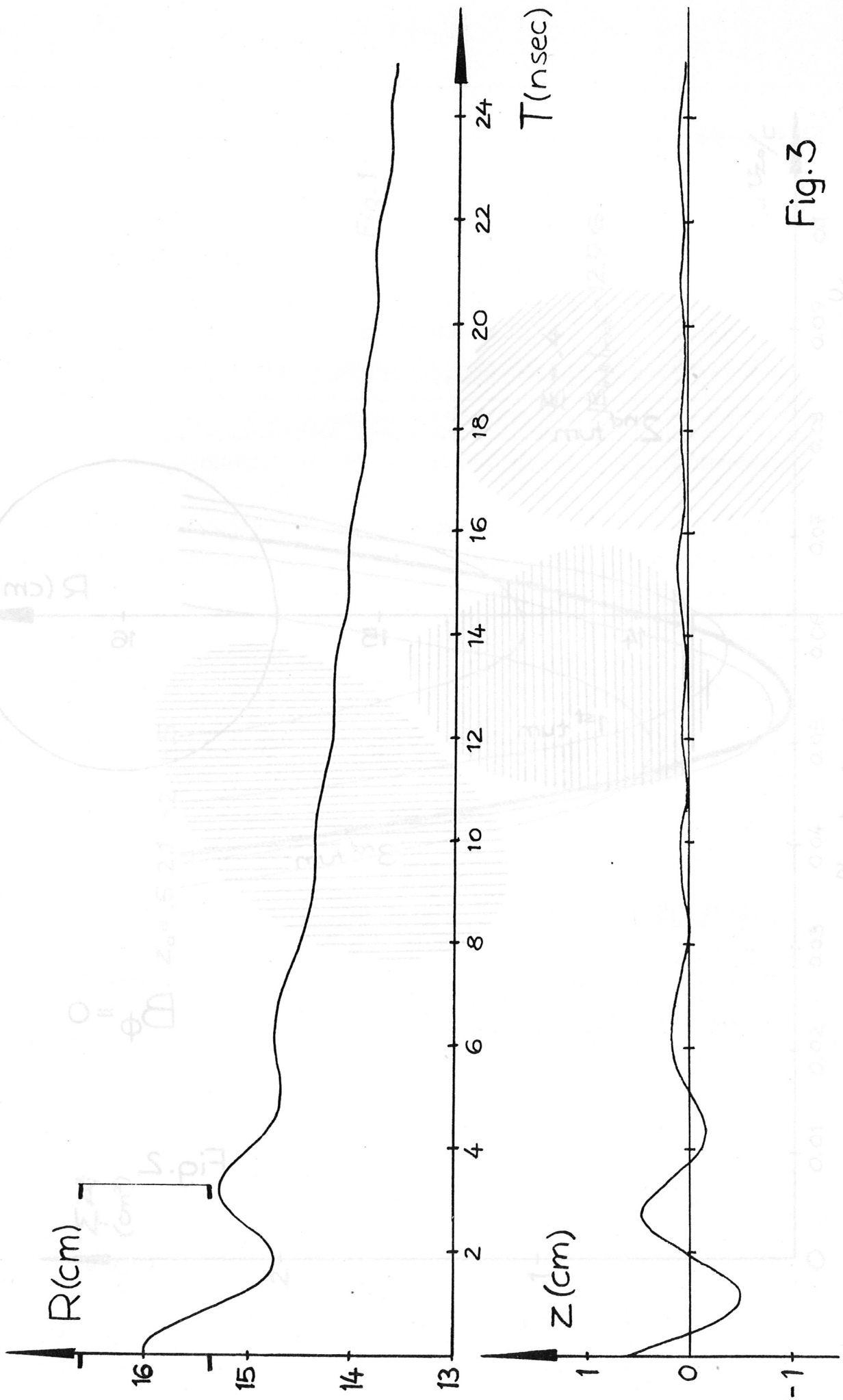


Fig.3

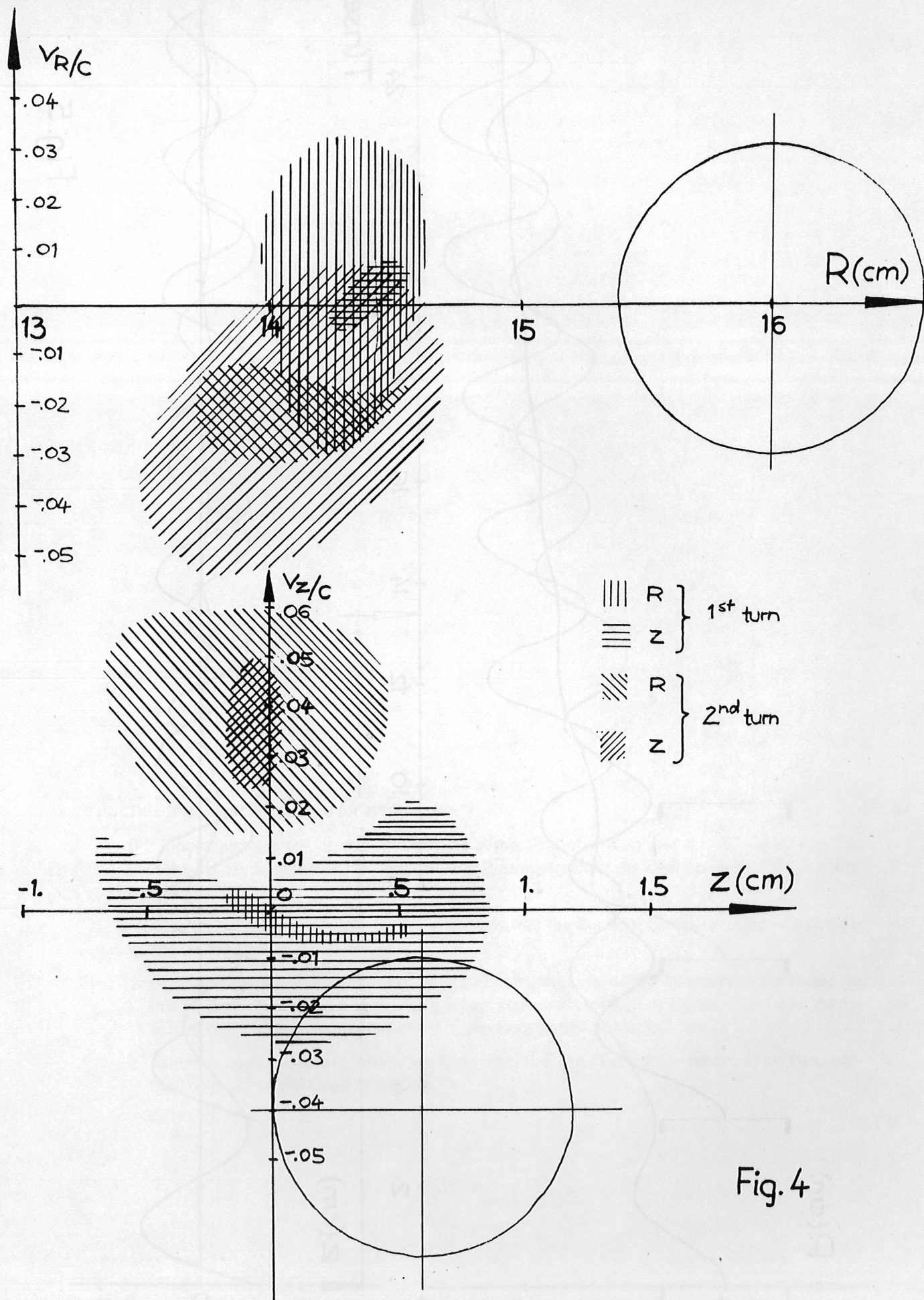


Fig. 4

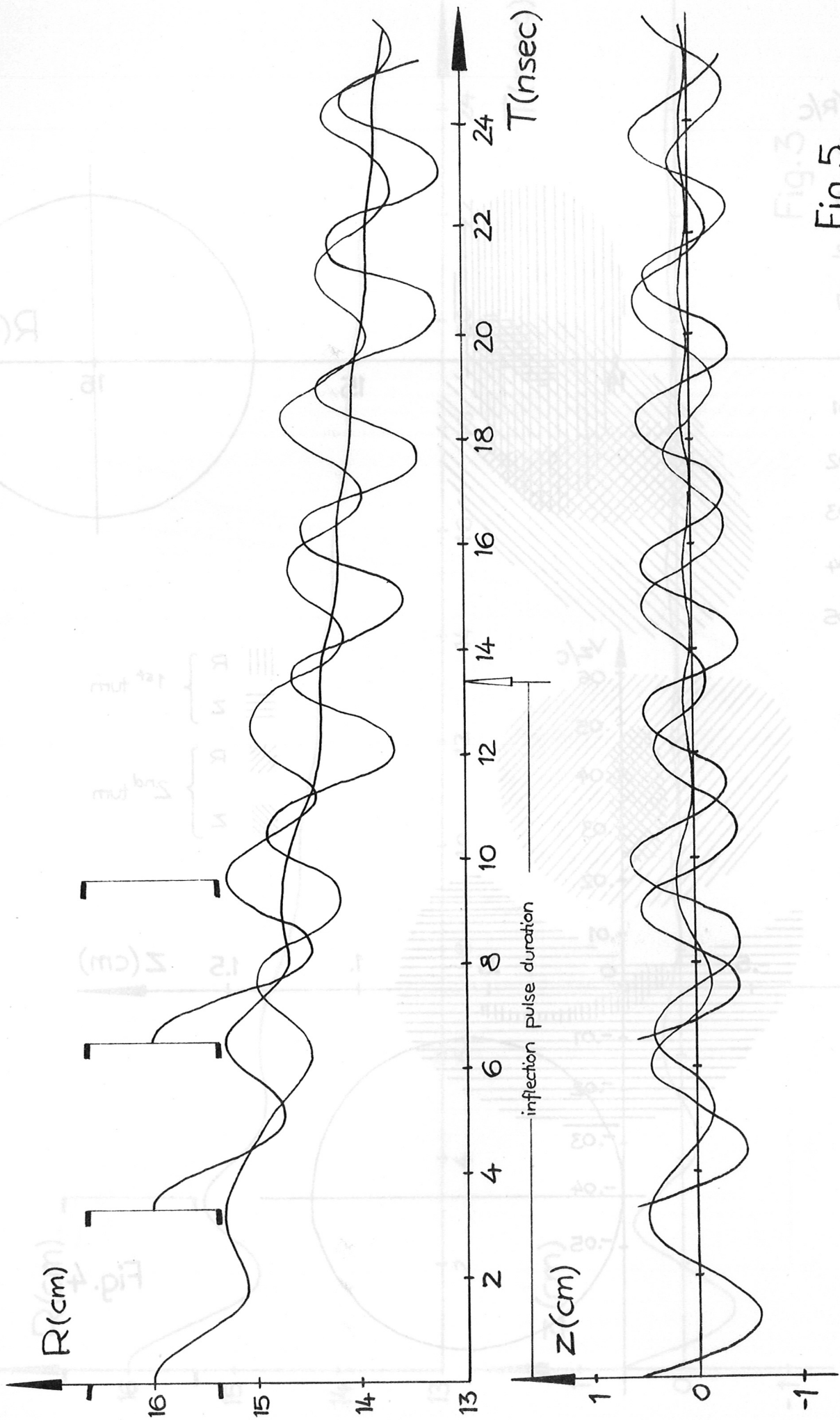


Fig. 5