Max-Planck-Institut fuer Plasmaphysik,
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Force-Free Helical Equilibria

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IPP II/206

October 1972

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

FORCE-FREE HELICAL EQUILIBRIA

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Abstract - The equilibrium of a helically symmetric, current-carrying low- β plasma is investigated. The solution for a force-free magnetic field $\underline{B} = \underline{B}^* + \underline{B}^J$, generated by a current $\underline{J} = \nabla \times \underline{B}^J = s \underline{B}$ ($s = \text{const.}$) in a straight $\varrho=2$ Stellarator vacuum field $\underline{B}^*$, is calculated by solving the equation $\nabla \times \underline{B} = s \underline{B}$ self-consistently in helical symmetry. In order to impose the appropriate boundary conditions at the limiter, a self-consistent free-boundary problem is solved. The magnetic field configuration, the current distribution, the magnetic surfaces and the rotational transform are calculated.

1. INTRODUCTION

WE INVESTIGATE the equilibrium configuration of a straight $\varrho=2$ Stellarator containing a low- β plasma through which a strong ohmic-heating current is flowing. We take into account the presence of a limiter, which determines the boundary between the plasma and the vacuum. We shall use c.g.s. non-rationalized e.m. units (1.2).

The total magnetic field $\underline{B}$ is given by the sum of the vacuum Stellarator field $\underline{B}^*$ and the field $\underline{B}^J$ generated by the current $\underline{J}$ flowing in the plasma:

$$\underline{B} = \underline{B}^* + \underline{B}^J \quad (1.1)$$

$$\nabla \times \underline{B} = \nabla \times \underline{B}^* = \underline{J} \quad (1.2)$$

$$\nabla \cdot \underline{B} = 0 \quad (1.3)$$

$$\nabla \cdot \underline{J} = 0. \quad (1.4)$$

The total current $\underline{J}$ is given by the sum of the (externally induced) ohmic-heating current $\underline{J}^e$ and the equilibrium diamagnetic current $\underline{J}^d$:

$$\underline{J} = \underline{J}^e + \underline{J}^d. \quad (1.5)$$

The equilibrium is described by the equation

$$\underline{J} \times \underline{B} = \nabla p. \quad (1.6)$$

Since $\beta \ll 1$, we have $|\underline{J}||\underline{B}| \gg |\nabla p|$ and $|\underline{J}^e| \gg |\underline{J}^d|$. Hence we can split (1.6) into

$$\underline{J}^d \times \underline{B} = \nabla p \quad (1.7)$$

and

$$\underline{J}^e \times \underline{B} = 0, \text{ i.e. } \underline{J}^e = s\underline{B}. \quad (1.8)$$

To lowest order in β , (1.2) becomes

$$\nabla \times \underline{B} = \underline{J}^e \quad (1.9)$$

and the magnetic field is force-free:

$$\nabla \times \underline{B} = s\underline{B}. \quad (1.10)$$

Henceforth we shall neglect the small diamagnetic current $\underline{J}^d$, but we shall keep the high current $\underline{J}^e$ consistently.

Since $\underline{B}^*$ is helically symmetric, we expect the same of $\underline{B}$. It is easy to convince oneself of this the following way. If one wanted to solve (1.10) by iterations, one

would have $\nabla \times \underline{B}^{(1)} = s^{(0)} \underline{B}^{(0)}$, ... , $\nabla \times \underline{B}^{(n+1)} = s^{(n)} \underline{B}^{(n)}$. Since $\underline{B}^{(0)}$ is helically symmetric, so are $s^{(0)}$ ($\underline{B}^{(0)} \cdot \nabla s^{(0)} = 0$) and $\underline{B}^{(1)}$, ... , $\underline{B}^{(n)}$, $s^{(n)}$ and $\underline{B}^{(n+1)}$, and therefore $\underline{B}$.

We shall make the simplifying assumption that s is finite and constant in the plasma and vanishes outside. The assumption that s is constant is not too unrealistic. CHANDRASEKHAR and WOLTJER (1958) have shown that in resistive plasmas force-free configurations with constant s present minimum dissipation for a given magnetic energy. WOLTJER (1958) has proved that in perfectly conducting plasmas the state of minimum magnetic energy in a closed system corresponds to a force-free field with constant s . JETTE and SREENIVASAN (1969) have shown that in resistive plasmas s has to be either a function of space and time or a constant independent of both. JETTE (1970) has proved that in resistive plasmas a force-free field remains such in time, only if s is constant in space and time.

In the following we shall solve Eq. (1.10) in helical symmetry and determine the coefficients in the solution by imposing the appropriate boundary conditions.

Taking the divergence of (1.10),

$$\underline{B} \cdot \nabla s = 0. \quad (1.11)$$

This means that s is a surface function (KRUSKAL and KULSRUD, 1958). In particular, if s is finite in a certain

region and vanishes outside, the boundary between these two regions must coincide with a magnetic surface.

Since the pressure, to lowest order in β , vanishes both inside and outside the plasma, there are no surface currents and the magnetic field components have to match across the boundary. This must coincide with the magnetic surface S tangent to the limiter (see Fig. 1). This surface is in turn determined by the boundary conditions and the solution of the problem. We are therefore faced with a self-consistent free-boundary problem.

2. HELICAL FIELDS

We use a cylindrical coordinate system (r, φ, z) , where the z -axis coincides with the magnetic axis (see Fig. 2). All the relevant quantities depend only on the radius r and on

$$\theta = \varphi - az \quad (2.1)$$

where $a = 2\pi/L$, and L is the period length of the helical windings in the z -direction. The curves $r = \text{const.}$, $\theta = \text{const.}$ are helices with pitch equal to that of the helical windings. The whole configuration is invariant along these helices. For $\varphi = n$, L is n times the field period length.

The vector potential $\underline{A}$ is defined by: $\underline{B} = \nabla \times \underline{A}$.

The magnetic surfaces in the helically symmetric case are given by (MOROZOV and SOLOV'EV, 1966)

$$\psi = A_z + \frac{a r A}{\rho} = \text{const.} \quad (2.2)$$

In vacuum, $\nabla \times \underline{B} = 0$ and $\nabla \cdot \underline{B} = 0$. Hence $\underline{B} = \nabla \phi$ and $\nabla^2 \phi = 0$. The solution is

$$\underline{B} = B^0 \frac{e}{z} + \frac{b}{a} \frac{r}{n} \sum I'(n a r) \sin n \theta \quad (2.3)$$

$$\psi = B^0 \frac{r^2}{2} - \frac{b}{a} r \sum \frac{I'(n a r)}{n} \cos n \theta. \quad (2.4)$$

For $\varrho=2$, expanding around the axis, i.e. for $a r \ll 1$, the magnetic surfaces are given by

$$(r^0)^2 = r^2 \left(1 - \frac{b}{B^0} \cos 2\theta \right). \quad (2.5)$$

These are ellipses with "average radius" r^0 and eccentricity b/B^0 , where B^0 is the main toroidal field strength, and b measures the strength of the multipolar field. The relation of the latter to the current I^* flowing in the helical windings can be obtained by proceeding as in MOROZOV and SOLOV'EV (1966, p. 56)

$$b = a^2 A I^* \cos \gamma \frac{K'(2aA)}{2} \quad (2.6)$$

where A is the radius of the cylinder C on which the windings lie (Fig. 1), and $\tan \gamma = aA$.

For a force-free field (1.10), $\nabla \times \underline{B} = s \underline{B} = s \nabla \times \underline{A}$, and, for constant s , $\underline{B} = s \underline{A} + \nabla h$. Hence, for $s \neq 0$,

$$c + s\psi = c + s \left(\frac{A}{z} + ar\frac{A}{\phi} \right) = \frac{B}{z} + ar\frac{B}{\phi} \quad (2.7)$$

The components of a helical field can be expressed in terms of ψ and the plasma current (JOHNSON et al., 1958; SOLOV'EV, 1967). In the force-free case they can be written in terms of ψ only:

$$\begin{aligned} B_r &= -\frac{1}{r} \frac{\partial \psi}{\partial \theta} \\ B_\theta &= \frac{-\partial \psi / \partial r + ar(c + s\psi)}{1 + a^2 r^2} \\ B_z &= \frac{c + s\psi + ar \partial \psi / \partial r}{1 + a^2 r^2} \end{aligned} \quad (2.8)$$

Along a field line the following equalities hold:

$$\frac{dl}{B} = \frac{dr}{B_r} = \frac{r d\phi}{B_\theta} = \frac{dz}{B_z} = \frac{d\theta}{B \cdot \nabla \theta} \quad (2.9)$$

$$\text{where } B \cdot \nabla \theta = \frac{B}{r} \frac{\partial \theta}{\partial \phi} - aB_z$$

If the field components are given in terms of (r, θ) , and r (on the magnetic surfaces) can be expressed in terms of θ by an equation of the type of (2.5), from (2.9) one can calculate the magnetic lines in terms of the parameter θ :

$$\begin{aligned}
 r(\theta) &= \int d\theta \frac{B}{B \cdot \nabla \theta} \\
 \phi(\theta) &= \int d\theta \frac{B}{r B \cdot \nabla \theta} \\
 z(\theta) &= \int d\theta \frac{B}{B \cdot \nabla \theta}
 \end{aligned} \tag{2.10}$$

where the integrands are expressed in terms of θ . Here r is added for symmetry and ϕ, z are related by (2.1).

The magnetic lines lag behind the helices in their average rotation around the magnetic axis ($t < 1$). The rotational transform $2\pi t$ per length L is defined as the increment of ϕ along a field line as z is increased by L . Using (2.9), we have

$$\Delta\phi^0_{(\Delta z^0=L)} = 2\pi t = \int_0^L dz \frac{B}{r B} \frac{\phi}{z}$$

where the integrand is expressed in terms of z . The integration over a z -period makes the result independent of the end points. It is more natural, however, to use θ as a parameter, as follows. Differentiating (2.1), integrating in θ from 0 to 2π along a field line and using (2.10), we obtain

$$\Delta\theta = 2\pi = \Delta\phi - a\Delta z. \tag{2.11}$$

Here $\Delta z \neq L$, and $\Delta\phi \neq 2\pi t$. The increments $\Delta\phi$ and Δz are calculated over a θ -period and are independent of the end

points. We can add m adjacent field line segments (2.11) until we approximate $m\Delta z = nL$ and $m\Delta\theta = n\Delta\theta^0$ (with m, n large integers). Multiplying (2.11) by $m/n = L/\Delta z$, since $\Delta\theta m/n = \Delta\theta^0 = 2\pi t$, we finally obtain

$$t = 1 + L/\Delta z \quad (2.12)$$

$$\text{where } \Delta z = \int_0^{2\pi} d\theta \frac{B}{B \cdot \nabla \theta}.$$

It therefore seems that t is best calculated through the increment of z , rather than θ .

For the $\varrho=2$ vacuum field one obtains, for $ar \ll 1$,

$$t = 1 - [1 - (b/B^0)^2]^{1/2}. \quad (2.13)$$

Expanding for small b/B_0 ,

$$t \rightarrow 1/2 (b/B^0)^2. \quad (2.14)$$

3. FORCE-FREE HELICAL FIELDS

CHANDRASEKHAR and KENDALL (1957) give the general method of solution of Eq. (1.10) for $s = \text{const.}$ through the scalar equation

$$\nabla^2 f + s^2 f = 0 \quad (3.1)$$

$$\text{and } \underline{B} = \underline{\nabla} \times \underline{\nabla} \times \left(\frac{\underline{e}}{z} f \right) + s \underline{\nabla} \times \left(\frac{\underline{e}}{z} f \right).$$

The symmetry of $\underline{B}$ suggests that we set $f = \sum_n f_n$, where

$$f_n(r, \theta) = g_n(r) \cos n\theta.$$

Equation (3.1) becomes

$$g_n'' + \frac{g_n'}{r} + \left(s^2 - n^2 a^2 - \frac{n^2}{r^2} \right) g_n = 0. \quad (3.2)$$

For $n = 0$, the solution is $g_0 = J_0(sr)$ and

$$B_0^{(1)} = c \left[\frac{e}{\phi} J_0(sr) + \frac{e}{z} J_0(sr) \right]. \quad (3.3)$$

For $n \neq 0$, since we can expect $|na| > |s|$, we have

$$g_n = I_n(r \sqrt{n^2 a^2 - s^2}) = I_n^* \quad (3.4)$$

and

$$B_n^{(1)} = c \left[\frac{e}{r} \left[\frac{\sqrt{n^2 a^2 - s^2}}{a} I_n^* + \right. \right. \quad (3.5)$$

$$\left. - \frac{s}{a^2 r} I_n^* \right] \sin n\theta +$$

$$+ \frac{e}{\phi} \left[\frac{n}{ar} I_n^* - \frac{s}{na^2} \sqrt{n^2 a^2 - s^2} I_n^* \right] \cos n\theta +$$

$$+ \frac{e}{z} \frac{s^2 - n^2 a^2}{na^2} I_n^* \cos n\theta \}. \quad (3.6)$$

The total field is

$$B^{(1)} = B_0^{(1)} + \sum_n B_n^{(1)}. \quad (3.6)$$

The singular solutions $g_0 = Y_0(sr)$ and $g_n = K_n^*$

correspond to a peak current on the axis. We do not

consider this case.

The equation for the magnetic surfaces is (2.7)

$$c + s\psi(1) = B(1) + arB(1). \quad (3.7)$$

Hence

$$c + s\psi(1) = c \left[J_0(sr) + arJ_1(sr) \right] + \quad (3.8)$$

$$+ \frac{c}{n} \frac{s}{na^2} \left[s I_n^* - ar \sqrt{na^2 - s^2} I_n^* \right] \cos n\theta.$$

This method of finding ψ for the force-free case is easier than directly solving the equation for ψ . For this equation, see JOHNSON et al. (1958), SOLOV'EV (1967), CORREA and LORTZ (1972).

As $s \rightarrow 0$, these formulae are reduced to the vacuum case (2.3) (2.4).

The coefficients c and c_1 must be determined from the boundary conditions.

4. THE BOUNDARY CONDITIONS

There are two boundary surfaces where field matching conditions are to be imposed: the magnetic surface S tangent to the limiter (of radius R), and the cylinder C (of radius A) on which the main and helical windings lie (see Fig. 1). These define three regions:

- 1) inside the magnetic surface S , $s \neq 0$, $\nabla \times \underline{B} = s\underline{B}$, and

the solution for $\underline{B}$ is as shown above (3.6);

2) between the surface S and the cylinder C , $s = 0$, and $\nabla \times \underline{B} = 0$. $\underline{B}$ is generated by the currents in the windings and in the plasma:

$$\begin{aligned} \underline{B}^{(2)} = & c \frac{e}{z} + c \frac{1}{r} \frac{e}{z} + \sum_n c \frac{\nabla I}{n^3} (nar) \sin n\theta + \\ & + \sum_n c \frac{\nabla K}{n^4} (nar) \sin n\theta; \end{aligned} \quad (4.1)$$

3) outside C the solution of $\nabla \times \underline{B} = 0$ which vanishes as $r \rightarrow \infty$ is

$$\underline{B}^{(3)} = c \frac{1}{s} \frac{e}{\varnothing} + \sum_n c \frac{\nabla K}{n^6} (nar) \sin n\theta. \quad (4.2)$$

The currents in the windings make the problem non-homogeneous and determine the coefficients. On the cylinder C , proceeding as in MOROZOV and SOLOV'EV (1966, p. 56), we have the surface current components

$$\begin{aligned} i_z &= i^* \cos \gamma \cos 2\theta \\ i_\varnothing &= i^* \sin \gamma \cos 2\theta + i^0 \end{aligned}$$

where $\tan \gamma = aA$. Here i^0 is the surface current generating the main field, and $i^* = I^*/A$ is the $\varnothing=2$ helical-winding surface current.

The boundary conditions on C are:

$$\begin{aligned} B_{r(in)} &= B_{r(out)} \end{aligned}$$

$$B_{\varnothing(in)} = B_{\varnothing(out)} - i z$$

$$B_{z(in)} = B_{z(out)} + i \varnothing$$

Equating the Fourier components we find

$$c_0 = i^0 = B^0$$

$$c_2 = c_5$$

$$c_{32} = a A^2 i^* \cos \gamma K'_2(2aA) = \frac{b}{a}$$

$$c_{3n} = 0 \quad (n \neq 2)$$

$$c_{62} - c_{42} = \frac{b}{a} \frac{I'_2(2aA)}{K'_2(2aA)}$$

$$c_{6n} - c_{4n} = 0 \quad (n \neq 2)$$

Here B^0 , b are the coefficients of the vacuum field B^* .

The fields $B^{(1)}$ and $B^{(2)}$ must coincide on the surface S . This must be a magnetic surface for both fields. We do not know S , since it is determined by the conditions on S itself. There are no standard methods for solving free-boundary problems. We shall assume that the surface section is not too different from a circle.

The magnetic surfaces in region 1 are given by (3.8), and in region 2 by

$$\psi^{(2)} = B^0 \frac{ar^2}{2} - c_2 \lg(r/R) - br \frac{I^0(2ar)}{2} \cos 2\theta + \quad (4.3)$$

$$- ar \sum_n c_n \frac{K'_n(nar)}{n} \cos n\theta.$$

If the surface sections are approximately circles, the terms containing the trigonometric factors are small, say of order $\epsilon \ll 1$, and we can expand both $\psi^{(1)}$ and $\psi^{(2)}$ for $r(\theta) = r^{(0)} + \epsilon r^{(1)}(\theta)$. We then equate the two expressions obtained for $r^{(1)}$, the derivatives of $\psi^{(1)}$ and $\psi^{(2)}$, i.e. the components of $\underline{B}^{(1)}$ and $\underline{B}^{(2)}$ (2.8), expand these in ϵ , use the expressions for $r^{(1)}$, and equate the Fourier components. After some algebra, expanding also for small $ar^{(0)}$, we obtain to the lowest significant order

$$c = B^0$$

$$c_{12} = \frac{b}{(1-s^2/4a^2)(1-s/4a)}$$

$$c_{1n} = 0 \quad (n \neq 2)$$

$$c_2 = \frac{s}{2} R^2 B^0$$

$$c_{42} = \frac{b a^2 s R^4}{4(1-s/4a)}$$

$$c_{4n} = 0 \quad (n \neq 2)$$

$$S = I^0 / n R^2 B^0$$

where I^0 is the total ohmic-heating current.

5. CONCLUSIONS

The configuration in the plasma (region 1) is determined by c and c_{12} .

Expanding (3.8) for small ar , the surfaces are given by

$$r^2 \left[1 - \frac{b}{B^0(1-s/4a)} \cos 2\theta \right] = \text{const.} \quad (5.1)$$

The surface sections are again ellipses as in the vacuum case, but their eccentricity depends on s , i.e. on the ohmic-heating current I^e . If $s > 0$, i.e. if $\underline{J}^e$ has the same direction as the magnetic field, the surface sections tend to be more eccentric, i.e. flatter. If $s < 0$, i.e. if $\underline{J}^e$ is antiparallel to the magnetic field, the surface sections are less eccentric, i.e. closer to circles.

The rotational transform per length L is given by (2.12)

$$t = 1 - (1 - s/2a) \left(1 - \frac{b}{B^0(1-s/4a)} \right)^{1/2}. \quad (5.2)$$

Expanding for a small multipolar field ($b/B^0 \ll 1$) and for a small current ($s/a \ll 1$), we see that the total transform is equal to the sum of the transform t^* due to the vacuum field and of that due to the current (t^J):

$$t \rightarrow 1/2 (b/B^0)^2 + s/2a = t^* + t^J. \quad (5.3)$$

For $s > 0$ the transform is larger and for $s < 0$ smaller than in the vacuum case. We see the same behavior as in the vacuum case, where a higher transform corresponds to flatter

surfaces.

The motivation for the present work was the experiments on the Wendelstein II b (WIIb) Stellarator (GRIEGER et al., 1971, 1972). The analysis of the field structure was necessary to help interpret some of the results (BARBERIO-CORSETTI, 1972). The experimental results show an instability when the sum of the rotational transforms due to the vacuum field and to the current is rational. Eq. (5.3) shows that this sum is in fact approximately equal to the total transform.

The current distribution is derived from (1.8) and (3.6). Taking a $z = \text{const.}$ section, averaging over ϕ (which makes the contribution from (3.5) vanish) and expanding (3.3) for $sr \ll 1$, we obtain

$$\langle j_z \rangle = \langle s_B \rangle \rightarrow s_B^0 \left[\frac{e}{\phi} sr/2 + \frac{e}{z} (1-s^2 r^2/4) \right]. \quad (5.4)$$

A large current would presumably distort the surface shape very strongly. The problem can be solved analytically only for $s/a \ll 1$, i.e. for a small current and for almost circular surface sections. For finite s/a the problem is non-linear and must be solved numerically. The solution is always of the form shown in Sec. 3, where the coefficients have to be determined as functions of s by solving an initial-value problem. The solution is known for $s = 0$. In general S is given by

$$\psi(r, \theta, s) = \psi(R, 0, s) \quad (5.5)$$

(assuming that S touches the limiter at $\theta = 0$) and by

$$r = r(\psi, \theta, s). \quad (5.6)$$

By varying s , the variation of r on S is found through

$$\frac{\partial \psi}{\partial r} \frac{dr}{ds} + \frac{\partial \psi}{\partial s} = \frac{d\psi}{ds} \quad (5.7)$$

where we have differentiated the left and right-hand sides of (5.5) with respect to s .

Acknowledgements - This work was done during the author's stay at the MPIPP Stellarator Division as a research guest. Thanks are due to Dr. G. GRIEGER for his hospitality, and for suggesting the problem. Helpful conversations with Drs. H. WOBIG and J. HUGILL are gratefully acknowledged. The MPIPP Documentation Service and the computerized Literature Search Facility were very helpful in finding the relevant literature. Manuscript prepared by ROFF.

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C: CYLINDER
L: LIMITER
S: SURFACE

List of symbols used.

β	beta
γ	gamma
∂	partial derivative
Δ	delta
∇	del (grad; $\nabla \cdot$ div; $\nabla \times$ curl; ∇^2 div $\cdot$ grad)
ϵ	epsilon
θ	theta
$\bar{i}$	iota bar
π	pi
Σ	sum (capital sigma)
ϕ	phi
Φ	phi (capital)
ψ	psi
$\varnothing$	el

$\int$ integral

$\square$ square brackets

()
< > curly brackets
()

$\sqrt{\quad}$ square root

∞ infinity

Superscripts.

$\cdot$	D
e	e
H	H
J	J
n	n

C: CYLINDER
L: LIMITER
S: SURFACE

FIG. 1

FIG. 2