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COLLISIONLESS DISSIPATION OF
A CROSS - FIELD ELECTRIC CURRENT

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Abstract

A detailed analysis is given of the development of the electrostatic turbulence excited by a cross-field current in a high density ($\omega_{pe} \gg \Omega_e$) collisionless plasma, as obtained from 1 D and 2 D computer simulations. The critical velocity for instability is found to be about half the value for the two-stream instability in contrast to the electron cyclotron drift instability. In 1 D the system remains selfsimilar in the turbulent phase, $\langle \tilde{E}^2 \rangle / 8\pi nT \approx \text{const}$, while the electron thermal velocity and the effective collision frequency increase linearly with time, $v_{eff} \propto \Omega_e t$, in agreement with the theory of coherent trapped-electron heating by Forslund et al. The switch-off drift velocity, however, is $v_{ds} \approx 2c_s$, and hence the thickness of a magnetic sheath would be $\Delta \sim c/\omega_{pi}$ instead of $\Delta \sim c/\Omega_e$. In 2 D the two projections are investigated, the $j - B$ plane (j = current) and the plane perpendicular to B , the latter being the more relevant. The heating is much less efficient than in 1 D, $v_{eff/\text{max}} \approx 5 \times 10^{-3} \omega_{pe}$. It is found that $v_{eff} \propto \langle \tilde{E}^2 \rangle / 8\pi nT$, indicating stochastic dissipation. The gross features of the 2 D behaviour correspond to the picture of ion-sound instability with electron runaway prevented by gyration.

1) INTRODUCTION

Experimental investigations of the microstructure of collisionless magnetic shock waves (especially in the so called resistive regime, $M_A \sim 2-3$) have revealed a high suprathermal level of electrostatic fluctuations, which seems to produce the anomalous resistivity in the shock front. The excitation mechanism of these fluctuations has been subject of numerous theoretical speculations. Since in general the electron-ion drift velocity v_d is smaller than the electron thermal velocity v_{the} , the ion-sound instability seems to be the most likely candidate. The high temperature ratio T_e/T_i required, is usually provided by the resistive heating itself; starting with $T_e \approx T_i$ in the upstream plasma, resistivity will heat the electrons preferentially and produce $T_e/T_i \gg 1$ within the shock.¹⁾²⁾ However this is not true in the high β experiments of Keilhacker and coworkers.³⁾⁴⁾ Here the initial ion temperature is substantially higher than the electron temperature, $T_i \approx 4 T_e$, and both become about equal at the rear of the shock. Nevertheless the value of the resistivity and even the shape of the turbulent spectrum appear to be quite similar to cases with $T_e/T_i \gg 1$ (compare Refs. 1 and 4). So the simple ion-sound picture seemed to be somewhat doubtful, and it was natural to look for an instability, which is not too sensitive to the temperature ratio. Taking into account the magnetic field explicitly in the dispersion relation, such instability exists and has been discussed by various authors⁵⁻⁸⁾. It consists of a coupling of electron Bernstein waves with ion waves, Doppler shifted by the electron-ion drift velocity, and is usually called electron cyclotron drift instability. This instability, however, has subsequently been shown⁹⁾ to stabilize at

very low amplitudes in the most interesting regime $\Omega_e \ll \omega_{pe}$, and thus do not seem to explain the experimentally observed micro-turbulence. On the other hand certain (one-dimensional) computer experiments do not show ultimate saturation at the predicted low amplitude but further growth of the turbulent energy and very efficient heating. This has been interpreted by Forslund et al.¹⁰⁾ to be a current driven nonlinear instability. A simple theoretical picture was presented in Ref. 10 explaining the electron heating and it was claimed that as in the case of the electron cyclotron drift instability the growth rate should only weakly depend on the temperature ratio. In Ref. 11 the dependence of the growth rate γ of this (nonlinear) cross-field instability on the parameters $\Omega_e, \omega_{pi}, v_d$ has been investigated in the limit $T_e \gg T_i$, using a simple numerical model. It was found that the scaling of γ is quite different from the electron cyclotron drift instability as well as the unmagnetized ion-sound instability.

However, the picture presented in Ref. 10 has not remained undisputed (see Ref. 9) and in addition, a number of problems, especially the physical relevance of the one-dimensional computations, are still open. In the present paper we give a detailed analysis of the different nonlinear phases of the instability in one and two dimensions. In section 2 we briefly discuss the essential points of the (linear) electron cyclotron drift instability and its irrelevance in shock experiments. The remaining sections describe the development of the instability at amplitudes greater than the saturation level of the electron cyclotron drift instability, as observed in one and two-dimensional

numerical simulations. Section 3 gives the numerical setup. In section 4 we present the results of 1 D computations. We investigate the conditions for instability, the effect of the turbulence produced and the process of ultimate stabilization. To analyse the effect of the dimensionality of the system, we performed 2 D computations, in the plane perpendicular to the magnetic field \underline{B} as well as in the plane including \underline{B} and the current \underline{j} . These cases are discussed in section 5. Section 6 contains a summary of the results and a comparison with experimental observations in magnetic shock waves.

2) LINEAR ELECTRON CYCLOTRON DRIFT INSTABILITY

We consider the electrostatic instability in a homogeneous plasma driven by an electric current perpendicular to a magnetic field. This corresponds for instance to the configuration in a magnetosonic shock wave. Since the wavelengths of the unstable modes are much smaller than the macroscopic scales of the shock wave, effects of density gradients and magnetic field gradients are neglected. The current is produced by an $E \times B$ drift of the electrons, while the ions are unmagnetized. In such a plasma there is an instability called the electron-cyclotron drift instability (ECDI), which results from coupling of electron Bernstein waves with Doppler-shifted ion waves. Since Bernstein modes are strongly Landau-damped for k outside a narrow fan around the plane perpendicular to B , the unstable modes are essentially confined to this plane, where they have a certain angular spread with respect to the current direction. Details of the instability calculations are found in Refs. 5 - 8. Applying a theory of Dum and Dupree¹², it was shown in Ref. 9 that the instability saturates at a rather low amplitude. This amplitude can be estimated by the following simple argument. The absence of damping of the Bernstein waves is due to the periodic gyro-motion of the electrons. If their orbits are perturbed by the presence of a turbulent electric field, damping occurs (this is similar to the damping introduced by a finite k_{\parallel} component of the wave). When the perturbation is so large that the shift of an electron's position after one gyroperiod is larger than about half a wavelength, then the particle runs out of phase with the wave.

In this case the cyclotron resonances in the dispersion relation are washed out. So the limiting level of the turbulence is given by the condition that the distance a particle diffuses in one gyroperiod is about $\frac{\pi}{k}$, the corresponding diffusion coefficient being

$$(1) \quad D \equiv \frac{(\Delta x)^2}{\Delta t} \approx \frac{\pi}{2} \frac{\Omega_e}{k^2}$$

Using a simple Fokker-Planck model of diffusion, D for thermal electrons is given approximately by

$$(2) \quad D = \sqrt{2\pi} \frac{\omega_{pe}^2}{\Omega_e^2} \frac{W}{nT} \frac{v_{the}}{\langle k \rangle}, \quad W = \frac{\langle E^2 \rangle}{8\pi}$$

By inserting (2) into (1) we obtain the fluctuation level above which a mode with wave number k will undergo electron Landau damping as in an unmagnetized plasma, i.e. the ECDI will be stabilized:

$$(3) \quad \frac{W_s}{nT} \approx \sqrt{\frac{\pi}{8}} \frac{\Omega_e^2}{\omega_{pe}^2} \frac{\Omega_e}{kv_{the}} \frac{\langle k \rangle}{k}$$

Since $kv_{the} \sim \omega_{pe}$, and in most cases of interest $.01 \leq \frac{\Omega_e}{\omega_{pe}} \leq .1$, W_s is very small. The term "strong turbulence" used in Ref.9 is only a technical term to distinguish the method (use of modified electron propagator) from the weak turbulence approach, with no relation to the physical importance of the turbulence level. In fact, in many laboratory experiments the thermal noise level is already sufficient to suppress the linear instability.

Typical plasma parameters in front of the shock wave are $n \approx 10^{14} \text{ cm}^{-3}$, $T \approx 2 \text{ eV}$, $\Omega_e \approx .02 \omega_{pe}$, such that the collision frequency is $\frac{\nu_e}{\omega_{pe}} \sim 10^{-3}$, and condition (1) is satisfied by a broad margin

$$(4) \quad D = \nu_e \rho_e^2 \sim 10^2 \frac{\Omega_e}{k^2}, \quad \text{with} \quad k\lambda_D \sim 1$$

(ρ_e = electron gyroradius).

Thus it seems clear that the anomalous resistivity observed in shock waves cannot be due to this instability. The question we want to discuss in this paper is what are the characteristic features of the cross-field current driven instability at amplitudes beyond the ECDI saturation, Eq. (3).

3) DESCRIPTION OF THE NUMERICAL MODEL

To describe the nonlinear phases of an instability, especially in a collisionless plasma, analytical methods are usually rather limited. Nowadays computer simulations seem to be the most promising way of obtaining an insight into the basic processes involved. We have followed the time development of the current instability in essentially collisionless plasmas of one and two spatial dimensions, using the standard PIC method ^{13),14)}. Since we are interested in times $T < \Omega_i^{-1}$, the magnetic force on the ions is neglected. A current density \underline{j} (x-direction) is imposed on the plasma by an initial drift v_d of the ions. The current, in particular its direction, is conserved by applying additional homogeneous electric fields E_{ox} , E_{oy} to the plasma, which exactly compensate the effect of the turbulent fields \tilde{E} . This constraint on the current simulates the conditions encountered in a plane magnetosonic shock wave, where no current is allowed to flow in the direction of shock propagation. Then the equations of motion of the particles are (\underline{B} is in Z-direction):

$$\begin{aligned}
 \dot{v}_{ex} &= -\frac{e}{m_e} (E_{ox} + \tilde{E}_x + \frac{v_{ey}}{c} B) \\
 (5) \quad \dot{v}_{ey} &= -\frac{e}{m_e} (E_{oy} + \tilde{E}_y - \frac{v_{ex}}{c} B) \\
 \dot{v}_{ez} &= -\frac{e}{m_e} \tilde{E}_z \\
 \dot{v}_{ix} &= \frac{e}{m_i} (E_{ox} + \tilde{E}_x) \\
 (6) \quad \dot{v}_{iy} &= \frac{e}{m_i} (E_{oy} + \tilde{E}_y) \\
 \dot{v}_{iz} &= \frac{e}{m_i} \tilde{E}_z
 \end{aligned}$$

An approximate expression for the driving field \underline{E}_0 in terms of the fluctuating quantities can easily be obtained. Since the change of the ion current Δj_i is smaller by a factor m_e/m_i than Δj_e , the ion dynamics can be neglected in this context. Multiplying (5) by n_e and averaging, and using the conditions imposed on the current $\langle j_x \rangle = n_0 e v_d = \text{const.}, \langle j_y \rangle = 0$, one obtains

$$(7) \quad \begin{aligned} n_0 E_{0x} &\approx - \langle \tilde{n}_e \tilde{E}_x \rangle \\ n_0 E_{0y} &\approx - \langle \tilde{n}_e \tilde{E}_y \rangle \end{aligned}$$

The size of the system L is chosen sufficiently large, so that $\lambda/L \ll 1$ (λ =typical wavelength of the turbulence) over the whole "observation time" in order to avoid finite box size effects influencing the plasma behaviour. In 1 D we take $L = 512 \lambda_{Do}$, $\lambda_{Do}^2 = T_{eo}/4\pi n_e$. In 2 D $L^2 = 128 \times 128 \lambda_{Do}^2$ is adequate, since heating effects which might bring the turbulent scales up to the system size, are much smaller in two-dimensional systems.

4) THE ONE-DIMENSIONAL CASE

One-dimensional simulations were reported in Ref. 10. These show very efficient electron heating, suggesting a coherent instead of a stochastic heating mechanism where the magnetic field plays an important role. The following concept was proposed and demonstrated by appropriate diagnostics applied to the computer experiments: The heating is due to trapped electrons being transported by an ion wave across the magnetic field. This process increases the perpendicular energy: $\dot{v}_y = \Omega_e v_x \approx \Omega_e v_{ph}$ for a plane wave propagating in the x-direction with phase velocity v_{ph} . After a time $t \gtrsim \Omega_e^{-1}$,

$t \gg \tau_b$ (τ_b = trapped electron bounce period) these electrons are released by the effect of the Lorentz force and randomize their directed energy in the y-direction by gyration.

This very intuitive picture, however, only describes the behaviour of single particles. To obtain net heating of the plasma, more electrons must be trapped at lower energy and untrapped at higher energy than vice versa. This question

has been investigated recently by the present authors¹¹⁾. In a wave of constant amplitude, we find that no net heating occurs because of the symmetry of the effective potential seen by an electron. In a self-consistent plasma, however, the wave amplitude is changed by the heating process. It increases when the electrons gain energy since in the reference frame of the electrons the wave energy has negative sign, and hence energy absorption by the electrons leads to an increase of the wave amplitude. Thus an electron trapped at a certain energy sees itself confined in a deeper and deeper potential well and will thus be transported up to energies several times the thermal energy until it becomes free because of the magnetic force. This process can easily be recognized when observing the electron motion in $x-v_y$ phase space (phase space pictures are shown in Ref. 10).

In the extensive one-dimensional computations which we report in this section, this picture of electron heating is confirmed and shown to have a number of interesting consequences. The numerical treatment of the one-dimensional case is attractive, since large systems can be followed over long times and the phenomena observed can readily be interpreted.

In addition it seems interesting to clarify the one-dimensional case in view of the discussions it has provoked. However, much care must be taken when applying the results of this special case to the interpretation of physical processes, as we shall see in section 5, where results of two-dimensional computer simulations will be given. We discuss three different phases in the development of the cross-field current instability: a) The onset of the instability, b) the strongly nonlinear phase, where the turbulence is fully developed and certain similarity laws are valid, and c) the saturation phase, where the instability is switched off.

a) Onset of the Instability

Since the ECDI, which is rather independent of the temperature ratio, has been shown to be too weak to explain the anomalous resistivity observed, the question of the conditions for instability, i.e. the critical drift velocity for a given T_e/T_i , is again open. The instability mechanism based on the heating of trapped electrons requires a certain amplitude to start, which in the absence of other kinds of fluctuations must be provided by the thermal noise. Loosely speaking, there must be a potential pulse of sufficient amplitude and lifetime, such that an electron can perform at least one trapping oscillation. It is, however, very difficult to formulate this condition more quantitatively. We have therefore investigated the point by numerical simulation. Imposing a certain drift velocity and temperature ratio we

check whether the plasma becomes unstable. Some results are indicated in the $v_d/v_{the} - T_e/T_i$ diagram in Fig. 1. The heavy line gives the critical velocity for the usual two-stream instability¹⁵⁾, the upper region being "two-stream" unstable. Open circles refer to systems which grow unstable.

If the initial conditions are too far below the critical curve, the system will not become unstable at once. However, the electrons will slowly be heated collisionally, so that the system advances in parameter space toward the critical curve until conditions for instability are favorable (dashed lines terminated by an open circle; note that v_d , not v_d/v_{the} is constant in our computations). These results indicate that there will be instability emerging from thermal noise only if the ion temperature is sufficiently low, in contrast to the ECDI. The initial conditions must be in the vicinity of the critical velocity for linear two-stream instability, since only then will the lifetime of a spontaneously generated ion wave pulse be sufficiently long. Quantitatively the drift velocity required for unstable growth is about a factor of 2 smaller than the critical drift for the two-stream instability (a similar result was obtained by Lampe et al.¹⁶⁾). No remarkable dependence on Ω_e was found.

The conditions on drift velocity or temperature ratio are somewhat relaxed if the instability is triggered by some superthermal pulse. The reason becomes clear from the time dependence of the phase velocity of an unstable potential pulse, shown in Fig. 2. It is seen that soon after its appearance, at an amplitude ϕ about two times the thermal noise level, the pulse reaches a supersonic phase velocity $v_{ph} \approx 2c_s$, which strongly reduces the effect of ion Landau damping. Hence even at low amplitude such ion waves behave in a nonlinear way.

They resemble isolated solitary waves (solitons) rather than sinusoidal wave trains, and it is well known that solitons propagate at supersonic velocities ¹⁷⁾.

Anticipating the results of the behaviour in the strongly turbulent regime after the onset of ion trapping we find that the system may penetrate far into the "two-stream" stable region in the $v_d/v_{the}^{-T_e/T_i}$ plane. The solid curve in Fig. 1 shows the time history of a system which starts in the "two-stream" unstable region but in the later phases falls far below the critical velocity. Hence in the dynamic phase, which is the one usually observed in shock wave experiments, the conditions for current-driven turbulence to be present are less stringent than those necessary for starting the instability.

b) Nonlinear Phase

The period of exponential growth of the instability is terminated by strong ion trapping. This process is rather independent of the initial temperature ratio, since the waves are propagating with high phase velocity $v_{ph} \approx 2c_s$, clearly outside the bulk of the ion distribution. After the onset of ion trapping, i.e. irreversible ion heating, the ratio of field energy to thermal energy W/nT remains approximately constant. Figure 3 illustrates a run with $\frac{m_i}{m_e} = 1600$, $\frac{\Omega_e}{\omega_{pe}} = 0.02$ and the initial conditions $T_{eo}/T_{io} = 2$, $v_d/v_{the\ 0} = 1$.

Although the electron temperature no longer increases with an exponential law, it is in this regime where the main part of the heating takes place. We find that the thermal velocity increases linearly with time, $v_{the} \propto t$, which is clearly visible in Fig. 3 c. Table 1 summarizes the results of

six computer runs, which were performed to study the dependence of the electron heating rate and the final temperature on the parameters Ω_e/ω_{pe} and ω_{pi}/ω_{pe} . The ion mass was $m_i/m_e = 100, 400, 1600$ and the strength of the magnetic field was such that $\Omega_e/\omega_{pe} = 0.02$ and 0.04 . The other conditions $v_d/v_{theo} = 1$, $T_{eo}/T_{io} = 2$ were identical. All runs are qualitatively similar to the one illustrated in Fig. 3. The rate of change of v_{the} , $\dot{v}_{the} = \text{const}$, is given in Table 1 a. It shows that \dot{v}_{the} is nearly proportional to Ω_e , and that there is only a weak dependence on the mass ratio. These features can be explained by the theory of electron heating given in Ref. 10, as described above. If n_t is the fraction of electrons trapped at a certain moment, the increase of energy due to these particles is given by

$$(8) \quad \langle \dot{v}_y^2 \rangle \approx \langle n_t v_y \rangle \Omega_e v_{ph}$$

We may write approximately $\langle n_t v_y \rangle \approx n_t v_{the}$, since electrons are trapped at low values of v_y and released at $v_y > v_{the}$ and thus v_y has the same sign for most of the trapped electrons. Using $v_{ph} \approx v_d$ for $v_d \gg c_s$ (note that we are in the electron frame), we have

$$(9) \quad \dot{v}_{the} \approx \frac{1}{2} n_t \Omega_e v_d$$

Since the turbulent spectrum develops approximately in a self-similar way, i.e. $W/nT \approx \text{const}$, $\langle k \rangle \lambda_D \approx 0.5$, the fraction n_t is about constant, which allows integration of (9).

$$(10) \quad v_{the} \approx \frac{1}{2} n_t \Omega_e v_d t, \text{ for } v_{the} \gg v_{theo}$$

If $\frac{\Omega_e}{\omega_{pe}} \ll 1$ (to be more precise, $\Omega_e/\omega_{pe} \ll 1/2\pi$, see Ref. 10), n_t should not depend strongly on Ω_e and thus $\dot{v}_{the} \propto \Omega_e$ in agreement with the numbers in Table 1a. On the other hand, a smaller mass ratio m_i/m_e makes electron inertia effects more important. We expect that for decreasing $\frac{m_i}{m_e}$ electron trapping becomes less effective and hence n_t smaller, which would explain the $\frac{m_i}{m_e}$ - dependence of \dot{v}_{the} as seen in Table 1 a.

The effective collision frequency can, for instance, be obtained from the energy balance

$$(11) \quad n_o (\dot{T}_e + \dot{T}_i) = \eta j^2 \quad \eta = \frac{4\pi v_{eff}}{\omega_{pe}^2}$$

Since $\dot{T}_i < \dot{T}_e$, \dot{T}_i can be neglected approximately. Using Eq's (9) and (10) an expression can be given for v_{eff} ,

$$(12) \quad v_{eff} \approx \frac{1}{2} (n_t \Omega_e)^2 t$$

Hence the collision frequency is increasing linearly with time.

The maximum value is determined by the saturation value of v_{the} discussed in the following paragraph c).

The result Eq.(12) is in clear contradiction to a picture of stochastic electron heating, which forms the basis of the "quasilinear" approach.

If the electrons were heated because of stochastic scattering by ion fluctuations, a simple expression for the collision frequency in terms of the field energy would hold (see, for instance, Ref.18):

$v_{\text{eff}} \propto \omega_{pe} W/nT$. Since W/nT is constant (or even slightly decreasing), v_{eff} should be constant, too. The result (12) clearly implies that in one-dimensional systems electron heating is predominantly due to the coherent acceleration of trapped electrons by the effect of the magnetic field.

c) Switch-off Drift Velocity

It is seen in Fig.3c, that after a long period of linear increase, the thermal velocity saturates at some value v_{thes} . This implies that the instability is switched off, when the ratio v_d/v_{the} approaches a certain value v_d/v_{thes} . The switch-off drift velocity, which we call v_{ds} (v_{ds} is understood to be a certain function of v_{thes} ; note that the absolute value of the drift velocity, v_d , is constant in the computations), is an interesting quantity, particularly as regards the dependence on m_i and Ω_e . The linear theory of the ion acoustic instability gives $v_{ds} \approx c_s \approx \sqrt{T_e/m_i}$, while the HCDI is stabilized when v_d/v_{the} approaches Ω_e/ω_{pe} . It can be seen that if one assumes that the thickness Δ of the current sheath in a theta-pinch is determined by the condition that the current density is close to the marginal value for instability, the two cases $v_{ds} \sim c_s$ and $v_{ds} \sim v_{\text{the}} \Omega_e/\omega_{pe}$ lead to the following scalings, $\Delta \sim (c/\omega_{pe})(c/v_{\text{the}})(\Omega_e/\omega_{pi})$ and $\Delta \sim (c/\omega_{pe})(c/v_{\text{the}})$ respectively, i.e. $\Delta \sim c/\omega_{pi}$ and $\Delta \sim c/\Omega_e$ respectively for $\beta_e \sim 1$.

To decide between these alternatives, it is not convincing to argue that the ECDI is stabilized at low amplitude and thus the unmagnetic ion sound instability should determine the switch-off drift, since the magnetic field plays an important role in the heating process as seen in the previous section b).

To clarify this point, we have followed the six computer experiments summarized in Table 1 up to saturation of the electron temperature, Table 1b giving the values of v_d/v_{thes} . It can be seen that this quantity scales with the ion mass as $(m_e/m_i)^{1/2}$, i.e. $v_{ds} \sim c_s$. There is a weak dependence on Ω_e , too, but in the sense opposite to the ECDI result $v_{ds} \propto \Omega_e$, v_{ds} slightly decreasing with increasing Ω_e . Thus the prediction of the magnetic sheath $\Delta \sim (c/\omega_{pe})(c/v_{the})$ made in Ref.10 on account of the ECDI, is not correct.

The trapped electron heating mechanism driving the cross-field current instability requires $v_d > v_{ph}$, and since we observe $v_{ph} \approx 2c_s$ in the fully developed turbulence, we should expect $v_{ds} \approx 2c_s$. The results of the one-dimensional simulations, Table 1b, are remarkably close to this theoretical expectation. Using the corresponding saturation value of the thermal velocity, we give an approximate expression of the maximum value of the collision frequency reached

$$(13) \quad v_{eff/max} \approx 2 \frac{v_{thes}}{v_d} \frac{\dot{v}_{the}}{2} \propto \Omega_e \omega_{pe} / \omega_{pi}$$

The numbers obtained by using the values of Table 1 a and b are presented in Table 1 c. For large mass ratio, $m_i/m_e = 400, 1600$, we see that v_{eff}/\max in fact scales as indicated in eq. (13). This is in clear contrast to a theoretical prediction using unmagnetized ion-sound instability, which gives $v_{\text{eff}} \propto \omega_{pi}$. In particular v_{eff} may become much larger than ω_{pi} for large mass ratio m_i/m_e .

5) THE-TWO DIMENSIONAL CASE

The results of the previous section are valid for a one-dimensional system. This corresponds to the special case of a turbulent spectrum highly peaked in the current direction. In a real plasma, however, modes propagating within a finite angle with respect to the current are unstable and a broad angular spread of the spectrum can be expected as is indeed observed in shock wave experiments ¹⁾⁴⁾.

In a broad spectrum the efficiency of the coherent heating process observed in 1 D computations is strongly reduced. The time an electron remains trapped in a potential well, τ_{tr} , is determined by the coherence length $\sim k_{y,z}^{-1}$ of the spectrum perpendicular to the current. While in 1 D, where $k_{y,z} = 0$, τ_{tr} is only limited by the gyration effect, $\tau_{tr} \sim \Omega_e^{-1}$, it will be much shorter in higher dimensional systems, $\tau_{tr} \sim (k_{y,z} v_{the})^{-1}$, which for a broad spectrum $k_{y,z} \lambda_D \sim 1$, is only of the order of (or less than) the bounce period of a trapped particle. Consequently, the heating process will be more stochastic in this case.

Two-dimensional computation can be performed in two different projections, one in the plane perpendicular to \underline{B} , the other in the plane containing \underline{B} and the current. Because of the asymmetry introduced by the magnetic field, there may be differences between the two cases. The 2 D simulations have been restricted to the somewhat artificial case of high initial temperature ratio T_e/T_i . This is partly due to computer limitations and partly because of the possibility of comparison with the computer results of the nonmagnetic ion-sound instability. ¹⁸⁾

We first consider the plane perpendicular to \underline{B} . Figure 4 illustrates a run with $\frac{m_i}{m_e} = 1600$ and $T_e/T_i = 50$ initially. It is seen that v_{eff} reaches a maximum value at about the same time as the quantity W/nT , at an electron temperature such that $v_d \gg c_s$. This is in clear contrast to the one-dimensional case, where v_{eff} is growing until the switch-off condition $v_d \sim c_s$ is reached, and it indicates a stochastic heating process. In addition v_{eff} is much smaller numerically in this case. We find $v_{eff}/\omega_{pe} \approx 4.5 \times 10^{-3}$ nearly independent of the ion mass, as can be seen in Table 2, where the present run is compared with a similar one with $m_i/m_e = 400$. Comparing the decay of v_{eff} after passing through a maximum with the decrease of v_d/v_{the} in the present run Fig.4c and d, it is tempting to relate both by a simple formula $v_{eff} \propto v_d/v_{the}$. In a corresponding run with $m_i/m_e = 400$, however, v_{eff} decays faster than v_d/v_{the} . Thus the expression has to be corrected somewhat. Both runs would be consistent with the scaling

$$(14) \quad v_{eff} \propto \frac{v_d - \alpha c_s}{v_{the}}$$

with $\alpha \sim 2$. Eq.(14) is in agreement with the so called Sagdeev formula $v_{eff} \sim \omega_{pe} (v_d/v_{the}) (T_e/T_i)$. It would, however, be rather daring to claim that numerical experiments thus prove the Sagdeev formula. Since all 2 D runs have been made in the ion-sound regime, $T_{eo}/T_{io} \gg 1$, we have no evidence of the dependence of v_{eff} on the temperature ratio.

Apart from the fact that the electron distribution remains isotropic, the only direct evidence of the presence of the magnetic field we found, is a noticeable asymmetry of the spectrum with respect to the current direction. As seen in Fig. 4 e, the maximum of the spectrum is rotated by about 25° out of the current direction in the sense opposite to the electron gyration. We would like to emphasize that this asymmetry of the

spectrum was not an incidental phenomena but a systematic effect observed in several different runs. Apart from measurement of the spectrum itself, the driving field E_{oy} gives direct evidence of the observed asymmetry, since E_{oy} is consistently of negative sign, $|E_{oy}| \sim \frac{1}{3} |E_{ox}|$. Relation (7) implies $\langle \tilde{n}_e \tilde{E}_y \rangle \neq 0$, and assuming $\tilde{n}_e \propto \tilde{\phi}$, it follows that $\tilde{\phi}$ must be asymmetric $\phi(k_y) \neq \phi(-k_y)$. It would seem natural to relate this asymmetry to the properties of single modes. Since there is no analytical dispersion relation we have investigated this point by simulation of suitable 1 D systems, where the current was at a certain angle with respect to the computed system, i.e. the mode direction. However, no asymmetry was found in this way, the modes in current direction being the most unstable ones. Hence the asymmetry of the spectrum is due to the presence of many modes. We suppose that because of the rather stochastic nature of the electron scattering in this case, the asymmetry can be explained by the properties of the average distribution function under the combined influence of the magnetic field and a turbulent spectrum.

In the computations performed in the j, B -plane, the time behaviour of v_{eff} is qualitatively similar to the one shown in Fig. 4. However, the numerical value is larger by a factor 4, $v_{eff} \approx 1.6 \times 10^{-2} \omega_{pe}$. It is rather easy to understand this difference. In the j, B -plane a trapped electron can escape a certain potential well only by its motion parallel to B , τ_{tr} being

determined by the mean parallel component k_z of the spectrum, $\tau_{tr} \sim (k_z v_{the})^{-1}$.

Since the instability is particularly strong for modes nearly perpendicular to \underline{B} , i.e. along the current, the spectrum is expected to be peaked in the current direction. Since k_z is then small, τ_{tr} will be rather long, so that the one-dimensional heating mechanism should still be effective and hence v_{eff} be rather large. Indeed the spectrum found in this case in the numerical simulations is rather peaked. On the other hand, considering the plane perpendicular to \underline{B} , all modes within a certain angle θ with respect to the current compete with each other since the excitation mechanism is the same, leading to a broader spectrum, as is observed in the simulation (see Fig.4), and hence to a lower value of v_{eff} .

Computer limitations have precluded relevant three-dimensional computations. We suppose that in 3 D the maximum value of v_{eff} may be somewhat larger than the 2 D result in the plane perpendicular to \underline{B} , but it will be definitely smaller compared with the 2 D result in the j, B -plane, since the coherent heating process leading to the larger value of v_{eff} in this projection is inhibited by a broad spectrum perpendicular to \underline{B} .

It is interesting to compare the 1 D and 2 D results of the cross-field current instability presented in this paper, Tables 1 and 2, with 1D, 2D and 3D computations of the unmagnetized ion-sound instability obtained earlier¹⁹⁾²⁰⁾. In Fig. 5 are given the maximum values v_{eff} seen in runs with $m_i/m_e = 400$ and the initial conditions $T_e/T_i = 50$ and $v_d/v_{the} = 1$, all performed with the $v_d = \text{const}$ constraint, for 1D, 2D and 3D. The largest difference is in 1 D, where the cross-field instability is particularly efficient, while for $B = 0$ the ion-sound instability is very

weak because of plateau formation in the electron distribution. In 2 D the difference is much smaller, especially when considering the cross-field instability in the plane perpendicular to \underline{B} . In 3 D, finally, the $B = 0$ case has a v_{eff} a factor of 2-3 larger than in 2 D and we expect the cross-field instability close to this value (it cannot be lower).

So we find that in higher dimensional system the cross-field instability is not much different from an ion-sound instability with stochastic electron heating, where runaway effects are suppressed by gyration.

6) CONCLUSIONS

We have presented a detailed analysis of the development of the instability driven by a current across the magnetic field. We have restricted ourselves to the experimentally most interesting regime of high plasma density,

$\Omega_e / \omega_{pe} \ll 1$, where the ECDI is stabilized at very low amplitude and hence is of no practical importance (in the simulations as well as in many experiments it is already suppressed by the thermal noise).

The main results of the 1 D simulations are: a) The critical velocity for instability as a function of the temperature ratio T_e/T_i is not very much different (factor 1/2) from the limiting drift for two-stream instability. However, in the turbulent phase this condition is strongly relaxed; the drift velocity may become much smaller than the two-stream limit without quenching the instability. b) In the turbulent phase of development the system remains approximately self-similar, in particular $W/nT \approx \text{const.}$ The electron thermal velocity increases linearly with time, $v_{the} \propto \Omega_e v_d t$, and so does the effective collision frequency. This behaviour is consistent with the theory of electron heating given in Ref. 10. c) There is a well defined value of the drift where the instability is switched off, $v_d \approx 2c_s$. Thus, loosely speaking, the instability behaves similarly to an ion-sound instability as regards its condition for being started and switched off. However, in 1 D the basic heating process driving the instability is coherent, not stochastic, and hence cannot be described by a quasilinear approach.

In the two-dimensional case, however, heating is much less efficient and is more stochastic. This is especially true when the computed system is the plane perpendicular to \underline{B} , where the one-dimensional coherent electron

acceleration is strongly perturbed by the presence of a broad cone of unstable modes. The only macroscopic effect produced by the magnetic field is an asymmetry of the spectrum with respect to the direction of the current. The collision frequency ν_{eff} behaves in a similar way to that found in the case of the unmagnetic ion-sound instability, in particular the maximum values of ν_{eff} being nearly equal. We thus conclude that the 1 D computations are of limited relevance to the interpretation of physical experiments; for instance, the strong dependence of ν_{eff} on Ω_e is an artefact of the 1 D system. Nevertheless the one-dimensional results are interesting since, in a sense, they give upper or lower bounds of certain quantities which cannot be studied in 2 D because of computer time limitations. Thus, for example, the fact that the switch-off drift velocity in 1 D, where magnetic effects are strongest, is not dependent on the magnetic field, $v_d \approx 2c_s$, suggests that v_d is independent of B in higher dimensions, too.

Finally, we briefly compare the results with measurements on "resistive" collisionless shock waves. The value $\nu_{\text{eff}} \approx 5 \times 10^{-3} \omega_{pe}$ found in the two-dimensional simulations (it may even be somewhat larger in 3D) is consistent with experimental observations. The asymmetry of the turbulent spectrum with respect to the current in the plane perpendicular to \underline{B} has not yet been measured experimentally, mainly because it was anticipated that the spectrum is symmetric. To discuss the interesting problem of understanding the results of Keilhacker and co-workers³⁾, we have to confine ourselves to the 1D computations. We have found that to start the instability for say $T_i = 4 T_e$, v_d must be larger than v_{the} initially.

In the later development the conditions for maintaining unstable growth are quite independent of T_e/T_i . However, it does not seem possible to have $T_e/T_i < 1$ in the turbulent phase; even starting with $T_e/T_i < 1$ the electrons are heated up to $T_e/T_i \gtrsim 1$ until strong ion heating sets in, which then tends to keep $T_e/T_i \approx 2-4$ over most of the heating period. Hence Keilhacker's experiments with $T_e/T_i < 1$ across the shock front cannot be fully explained. Since in this experiment β is rather high from the beginning, ∇B -drift may change the shape of the electron distribution appreciably. The effect of a magnetic field gradient is currently being investigated by means of numerical simulations. On the other hand, high β implies that there is no critical Mach number in the strict sense, but that, even at $M_A \sim 2-3$, a non-negligible fraction of ions is reflected, which can influence the processes occurring in the shock front.

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FIGURE AND TABLE CAPTIONS

- Figure 1. $(v_d/v_{the})-(T_e/T_i)$ conditions for instability. The heavy line is the critical velocity for two-stream instability. Open circles mean onset of instability. The dashed lines represent two systems being heated collisionally until they grow unstable. The solid lines represent the time history of two unstable systems.
- Figure 2. Time development of the phasevelocity of an unstable wave, as compared with the behaviour of the field energy.
- Figure 3. Plots illustrating a 1 D run with $m_i/m_e = 1600$, $\Omega_e/\omega_{pe} = 0.04$, $T_{eo}/T_{io} = 2$, $v_d/v_{theo} = 1$: a) logarithm of the field energy; b) logarithm of field energy over thermal energy; c) electron thermal velocity; d) logarithm of the ion and electron temperatures.
- Figure 4. Plots illustrating a 2 D run with $m_i/m_e = 1600$, $\Omega_e/\omega_{pe} = 0.04$, $T_{eo}/T_{io} = 50$, $v_d/v_{the} = 1$: a) field energy, b) field energy over thermal energy; c) effective collision frequency; d) drift-over thermal velocity; e) mode spectrum of the field energy, exhibiting an asymmetry with respect to the current (x-direction).

Figure 5.

Maximum values of $v_{\text{eff}}/\omega_{pe}$ in 1D, 2D, 3D for $B = 0$ (circles) and $j \perp B$, $\Omega_e/\omega_{pe} = 0.04$ (triangles). The initial conditions are identical: $m_i/m_e = 400$, $v_d/v_{\text{theo}} = 1$, $T_{eo}/T_{io} = 50$, $j = \text{const. constraint}$. The two different triangles in 2D represent the runs in the j - B plane (upper value) and the plane $\perp B$ (lower value). The 3D value in the $j \perp B$ case is only a supposition. No relevant computer run could be performed.

Table 1 Results of six 1 D computer runs with different m_i/m_e and Ω_e/ω_{pe} , but otherwise identical parameters $T_{eo}/T_{io}=2$, $v_d/v_{theo} = 1$. a) rate of increase of thermal velocity, \dot{v}_{the} , b) saturation thermal velocity v_{thes}/v_d corresponding to instability switch-off, c) maximum collision frequency v_{eff}/max .

Table 2 Maximum collision frequency v_{eff}/max for 2 D run in the two projections, j-B plane and plane $\perp B$, for different mass ratio (no $m_i/m_e = 1600$ run was perturbed in the j-B plane). $T_{eo}/T_{io} = 50$, $v_d/v_{theo} = 1$.

$$\dot{v}_{the}/v_d \omega_{pe}$$

| Ω_e/ω_{pe} \ m_i/m_e | 100 | 400 | 1600 |
|------------------------------------|----------------------|----------------------|----------------------|
| 0.02 | 1.3×10^{-3} | 2.0×10^{-3} | 2.5×10^{-3} |
| 0.04 | 1.5×10^{-3} | 3.7×10^{-3} | 4.5×10^{-3} |

a)

$$v_{thes}/v_d$$

| Ω_e/ω_{pe} \ m_i/m_e | 100 | 400 | 1600 |
|------------------------------------|-----|------|------|
| 0.02 | 5.7 | 9.0 | 16.5 |
| 0.04 | 5.3 | 11.5 | 19.0 |

b)

$$v_{eff}/\max/\omega_{pe}$$

| Ω_e/ω_{pe} \ m_i/m_e | 100 | 400 | 1600 |
|------------------------------------|----------------------|----------------------|----------------------|
| 0.02 | 1.4×10^{-2} | 4.0×10^{-2} | 8.0×10^{-2} |
| 0.04 | 1.6×10^{-2} | 7.7×10^{-2} | 1.6×10^{-1} |

c)

Table 1

$$v_{\text{eff}/\text{max}}/\omega_{\text{pe}}$$

| | | |
|------------------|----------------------|----------------------|
| m_i/m_e | 400 | 1600 |
| plane $\perp B$ | 4.3×10^{-3} | 4.5×10^{-3} |
| \perp, B plane | 1.6×10^{-2} | |

Table 2

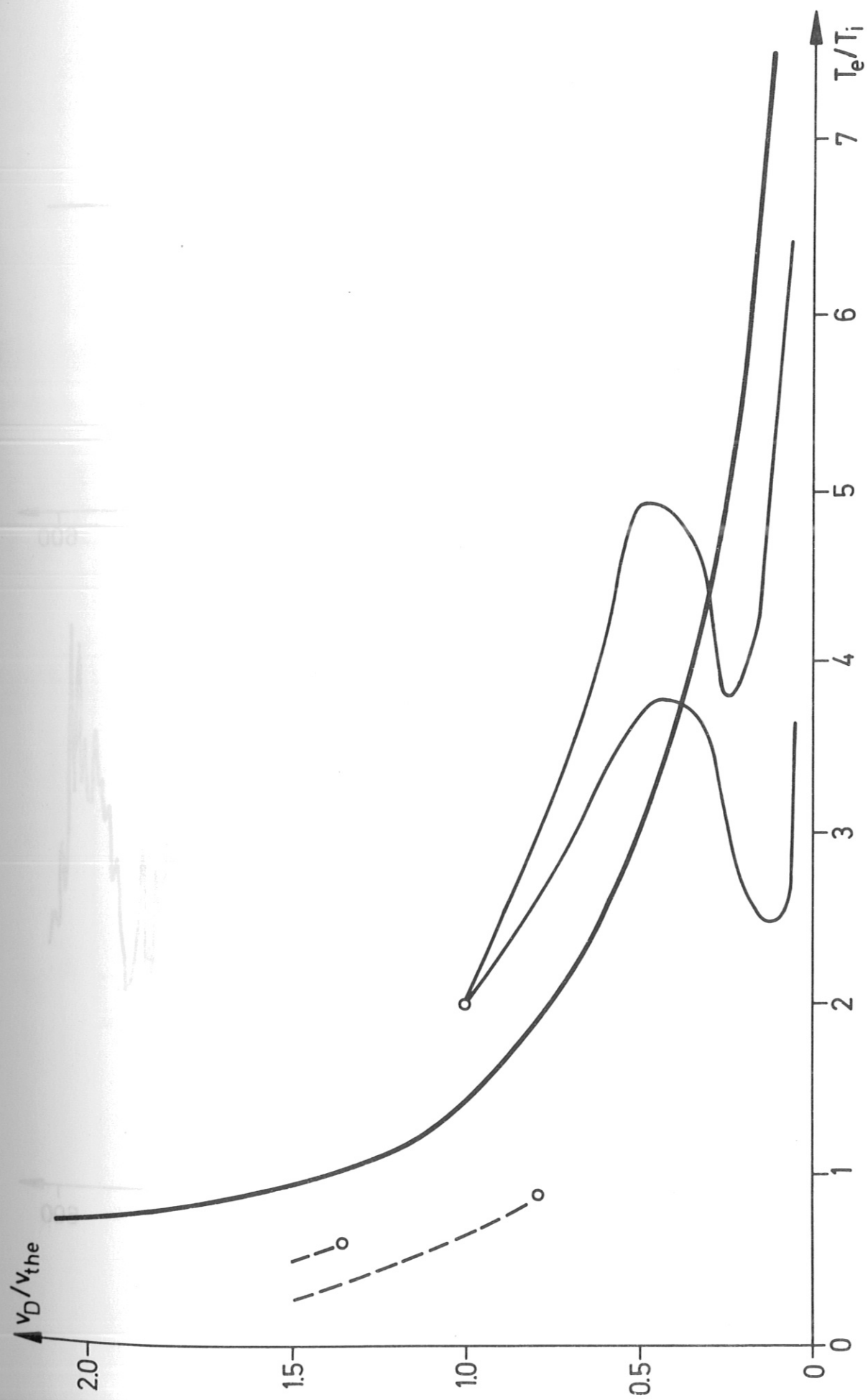


Fig.1

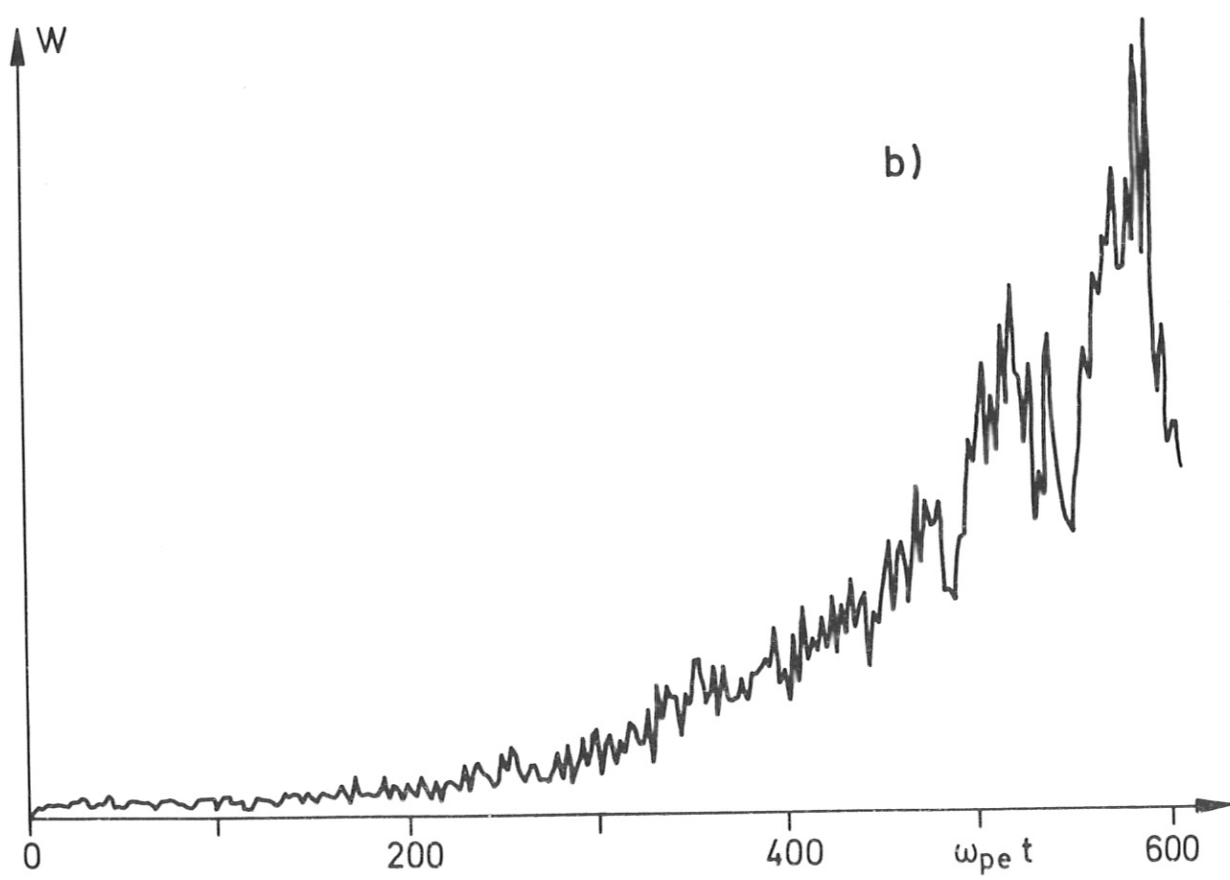
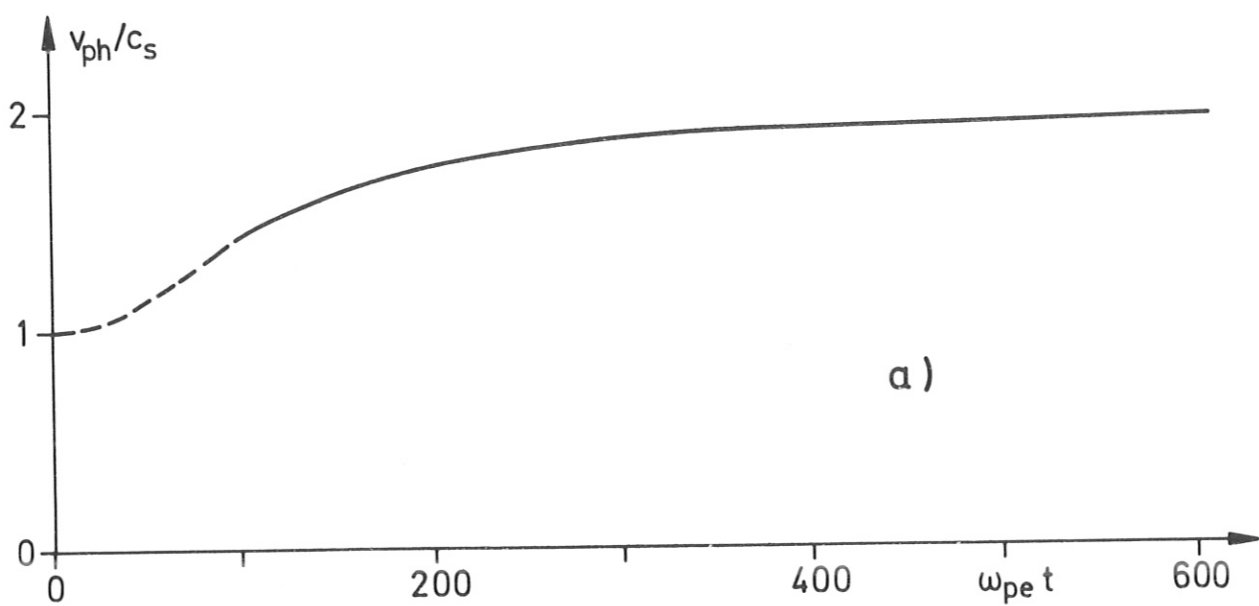


Fig.2

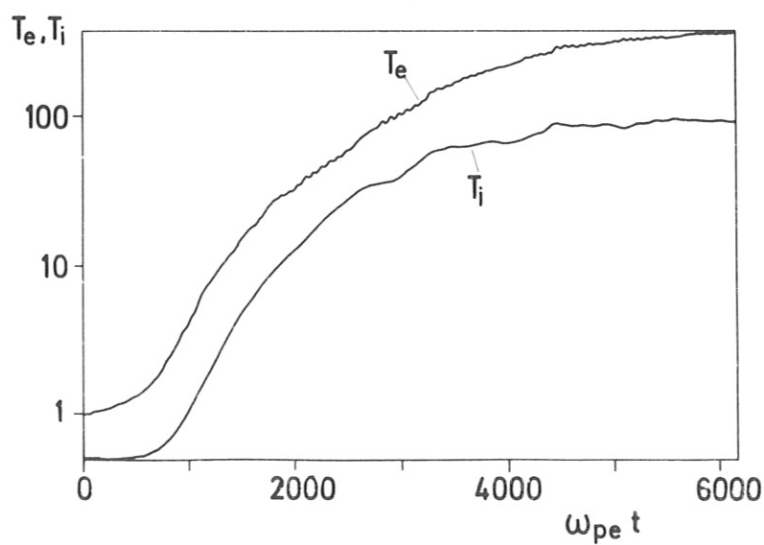
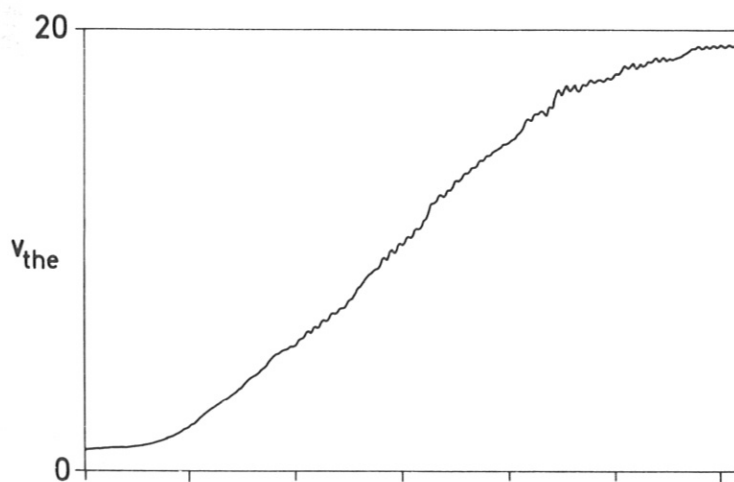
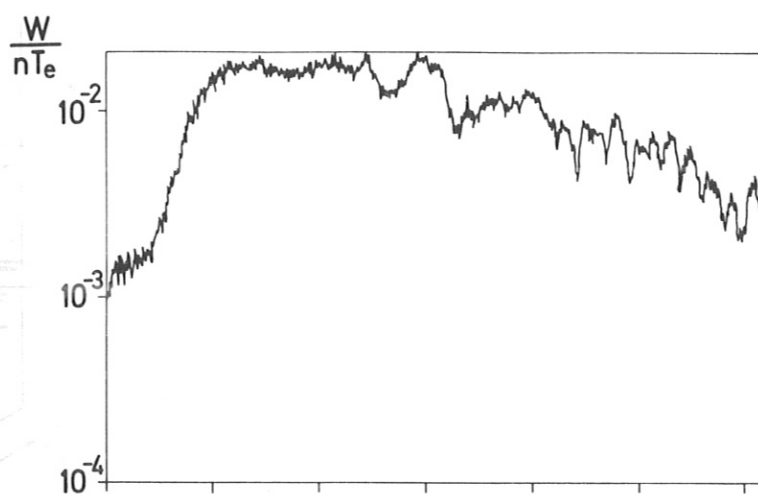
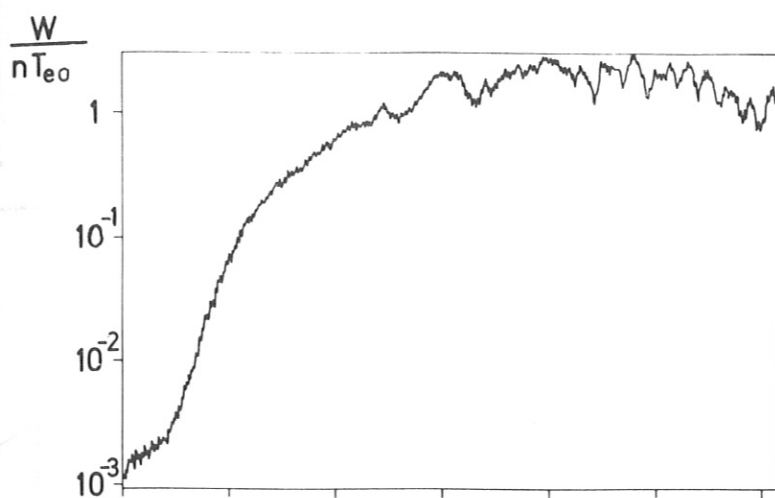


Fig.3

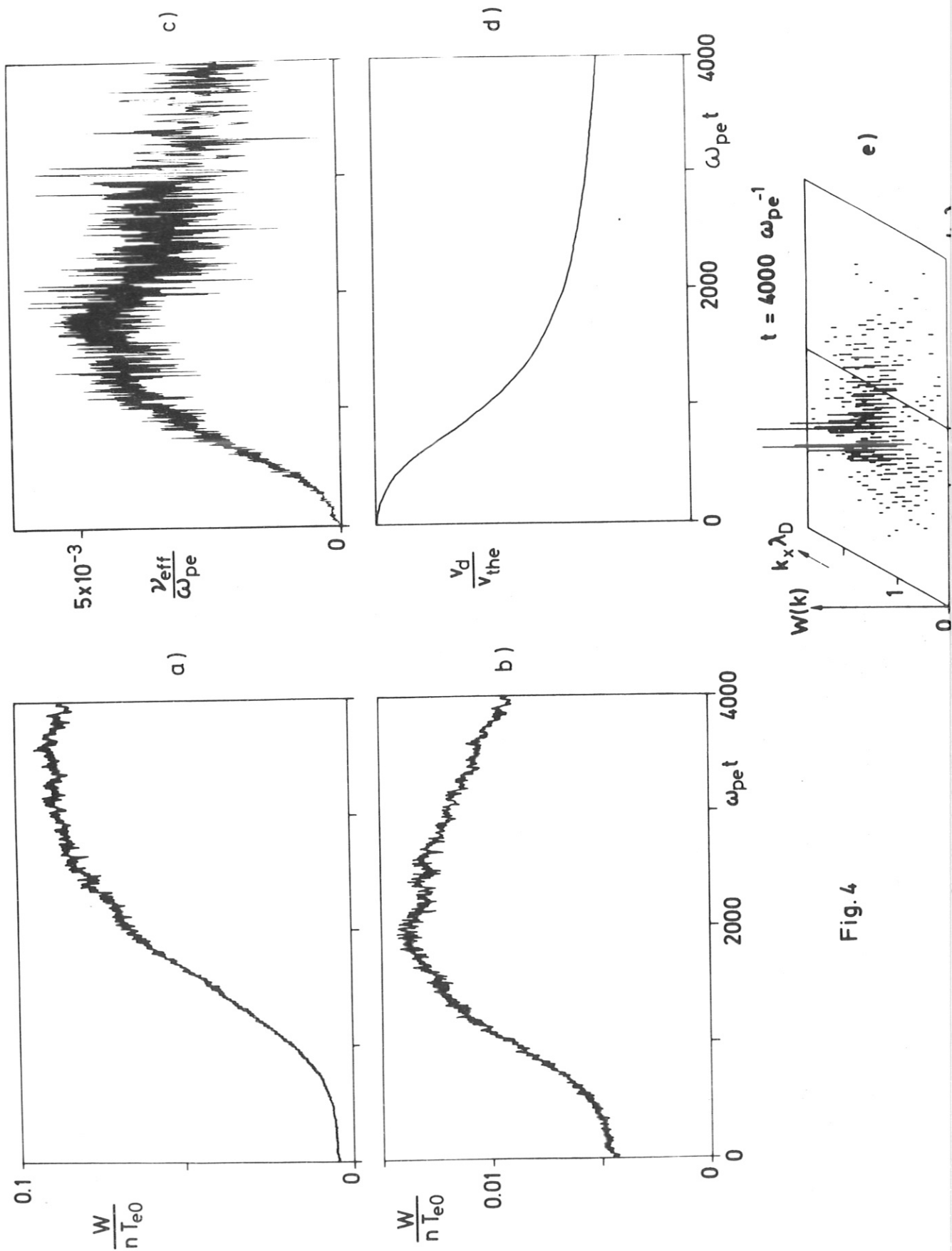


Fig. 4

$\frac{V_{eff}/\max}{\omega_{pe}}$

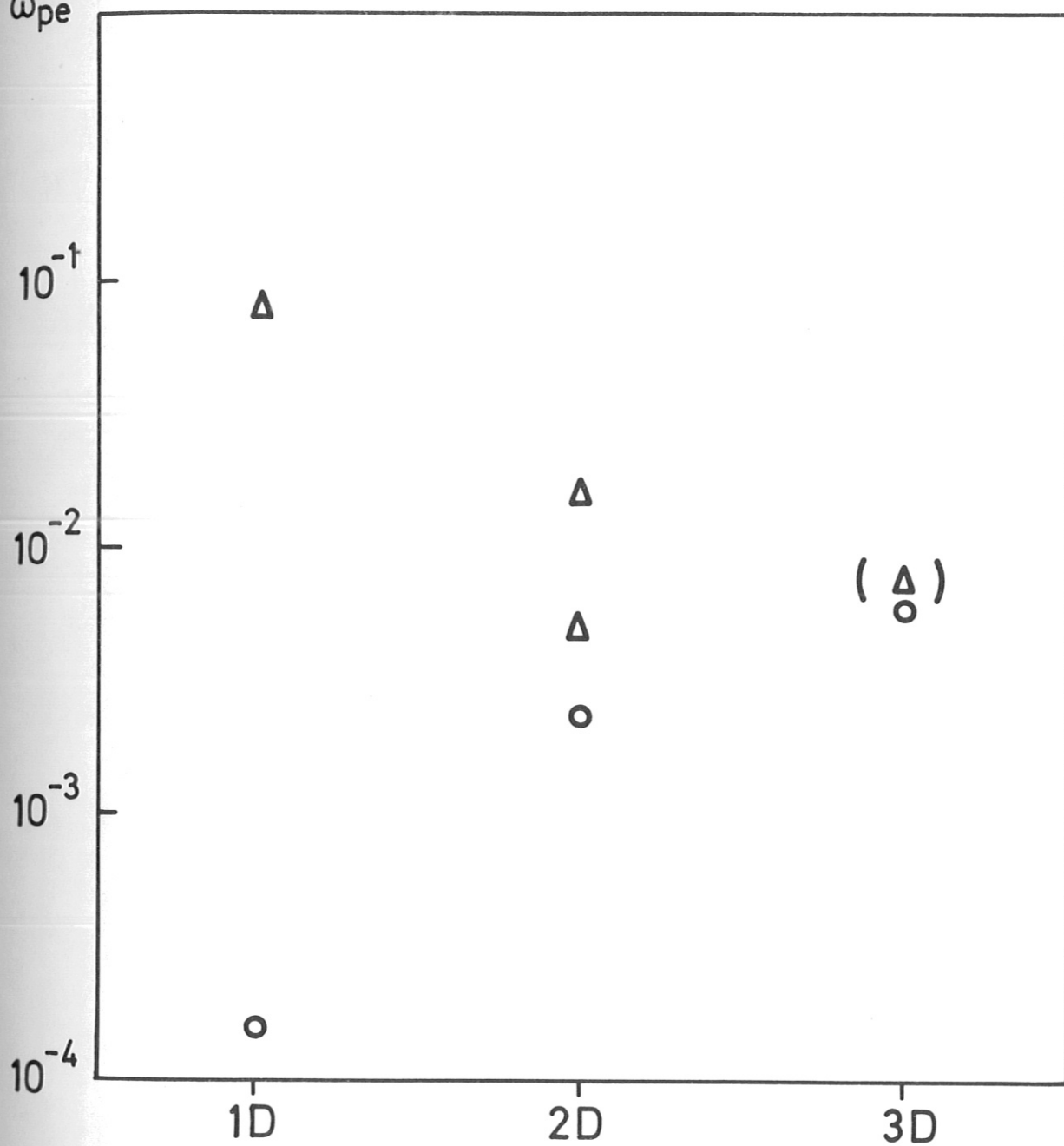


Fig.5

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