# MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN


## The Effect of Plasma Flow on Toroidal Confinement


B.J. Green



IPP 6/101 January 1972



*Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.*

IPP 6/101 B.J. Green

The Effect of Plasma Flow on Toroidal Confinement.

January 1972 (in English)


ABSTRACT

An attempt is made to review recent developments in the study of plasma flow and its effect on toroidal plasma confinement. The topic is presently the object of active research, so that this summary will serve rather as a progress report of development. The different approaches to the problem are classified and discussed in such a way as to indicate the status of results which have been achieved and to highlight the outstanding problems.

## Contents

Introduction

A. Low Beta Case

(i) Local Approximation Treatments:

Resistivity

Resistivity and Perpendicular Viscosity

Resistivity and Parallel Viscosity

Resistivity and Parallel Thermal Conductivity

Resistivity, Viscosity and Thermal Conductivity

Weakly Collisional and Collisional Regimes

Numerical Calculations:

Resistivity

Resistivity and Bulk Viscosity

Resistivity and Parallel Viscosity;
Finite Larmor Radius Effects

Resistivity, Viscosity and Thermal Conductivity

Discussion

Summary of A (i)

(ii) Nonlinear Treatment

B. Finite Beta Case

(i) Local Approximation Treatments

(ii) Nonlinear Treatment

Conclusion

## Introduction

One of the central problems of the theoretical study of toroidal confinement is the calculation of the rate of plasma loss from a magnetic confinement region. Fundamental to this study was the investigation of static equilibrium (see for example the review [1] ). This established the existence of certain equilibria and allowed for the study of their detailed form with respect to stability. Although this study is by no means complete, people have attempted to make the confinement model more realistic by including plasma flows.

Non-ideal (resistive) plasmas were treated in terms of a quasi-static model [2]- [6] which neglects inertial effects, and an expression for plasma loss was obtained. This result was important as it could differ considerably from the classical resistive plasma loss in a magnetic field [7] . This difference is due to geometrical effects which require the existence of non-zero current parallel to the magnetic field. The associated ohmic dissipation is balanced by extra expansion energy.

(Such energy balance considerations can be extended to more complicated situations in the examination of plasma loss [8] ).
However, these quasi-static calculations showed that plasma flow could become large, so that neglect of inertia was suspect.

In the weakly collisional, and collisionless (here the trapping of particles must be treated explicitly) regimes, calculations of plasma transport [9] - [12] again showed large differences from the classical result. These are also due to geometrical effects. However, only small plasma flows were considered.

First attempts to include plasma inertia involved ordering procedures which raised questions concerning the definiteness of solution [13] - [16] . General consideration of ideal magnetohydrodynamic flow [17] - [20] has led to a deeper qualitative understanding of the flow problem, but quantitative results have been lacking.

The essential difficulties in the investigation of plasma flow in toroidal geometry are: (a) the multidimensional nature of the problem, (b) nonlinearity and increase in the order of the equations due to geometry, finite plasma beta, dissipation and flows themselves. Up until now, there exists no calculation which has successfully managed to include all these aspects of the problem. The problem has always been linearized in terms of one or more of the above features.

I wish here to summarise and review the recent work on plasma non-turbulent flow and its effect on toroidal confinement. The aim is to clarify the approximations which have been used in obtaining the various results and thereby obtain a clearer overall picture. The analytical calculations which I survey all fit into the schematic pattern of figure 1.

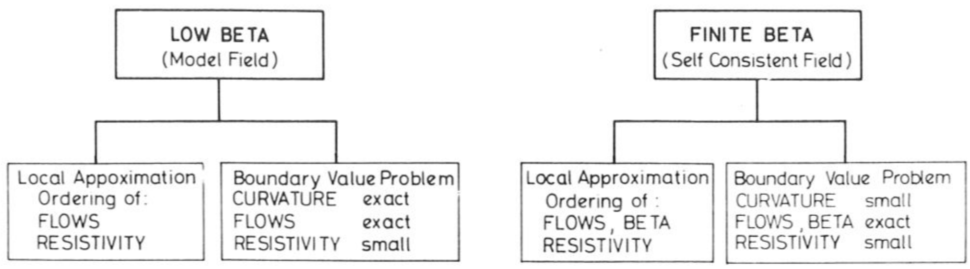


Fig. 1

Classification of analytical treatments of the problem of flow in toroidally confined plasmas.

The broadest classification is that of low or high plasma beta. The low beta calculations consider a given (model) magnetic field. This approximation assumes that the plasma reaction on the applied fields can be neglected. Most calculations fall into this category because of the considerable simplification involved.

Among the low beta calculations we can sub-classify according to whether or not the calculations use the largeness of the torus aspect-ratio to order other physical quantities. Such an ordering in small inverse aspect-ratio results in a local approximation, in the sense that the assumed ordering of physical quantities has in general only local validity. Thus the boundary-value problem along the minor radius can not be treated. Once again the simplification involved in such an ordering procedure is great, in fact so great that some important features may be lost. These features could be retained in a partially nonlinear treatment [21] , [22] which also formulated the appropriate boundary-value problem.

For finite beta the situation is very similar and most calculations are of the local approximation type. It is once again possible to carry through a partially nonlinear treatment where flow and beta effects are exactly retained and the appropriate boundary-value problem is formulated [23] .

Before a more detailed discussion of the various calculations, I give briefly the general physical picture of the effect of flow on toroidal confinement. This description, formulated by T.E.Stringer, is as follows: the toroidal magnetic field causes charge separation and resulting currents (at rest in the laboratory frame) in the confined dissipative plasma column. Dissipation is essential to the description for it prevents the complete neutralisation of space charge caused by the particle drifts in the toroidal field. If the column rotates, i.e., net solid-body rotation in a meridional plane, an observer in the plasma frame sees a rotating perturbation at the Doppler-shifted frequency.

The effect of this external driving force is to generate waves in the plasma, and the overall response will be governed by the plasma dielectric function. There is a resonance in this response when the rotation frequency coincides with that of a normal plasma mode. These critical frequencies (or rotation speeds) in the toroidally confined plasma can be derived from general kinematical arguments [20] . The magnitude of this resonance will be determined by the damping of the excited mode. A modified plasma loss is associated with this resonant wave generation.

In the weakly collisional case the situation is viewed as a resonant particle phenomenon,i.e.,those particles in phase with the wave can undergo large displacements from the magnetic surfaces and thus an increased plasma loss can be expected. This loss value clearly depends on the number of resonant particles, in other words on the number of particles in the distribution with velocities matching that of the generated plasma wave.

For the collisionless case (so-called banana regime) where the complexities of particle trapping must be considered, the effects of flow have not been analysed in detail. However the recent work of Rosenbluth et al., [24] is a step in this direction.

## A. Low Beta Case

The model field which is normally considered in the low beta approximation, is the axisymmetric one introduced in [3] , which in some sense simulates the field configurations of Stellarator, Levitron and Tokamak.

### A (i) Local Approximation Treatments

The main advantage of linearised treatments is that the physical model can incorporate many different physical effects and remain mathematically tractable. The following calculations include various different dissipative terms, i.e., they treat the effect of plasma flow in different plasma parameter regimes. To facilitate discussion, the calculations will be grouped essentially according to the type of plasma dissipation considered.

Resistivity

The initial calculation was carried out by Stringer[25] who treated a collision-dominated plasma with resistive dissipation only. Flow effects were investigated by introducing the poloidal rotation speed as a parameter determined by the radial electric field, which, in zeroth order in inverse aspect-ratio, is arbitrary. However, this procedure led to non-ambipolar loss which could be negative for some rotational speeds. Also the perturbation in inverse aspect-ratio became invalid on approaching the resonance condition, so that claims of very large plasma loss were premature.

Hazeltine et al., [26] treat the same model as Stringer but for small rotation speeds they follow the time evolution ( a time-scale ordering involving large aspect-ratio) of the rotation. They find that a zero rotation speed state is unstable with respect to rotation. That is, when the plasma rotates, the resistive plasma loss causes a local current to flow through a magnetic surface, so that the $\underline{j} \times \underline{B}$ force has a non-zero surface average which causes the plasma to rotate faster.

In the neighbourhood of the critical speed [20] (modified ion thermal speed $\Theta C_{th}$, where $C_{th}$ is the plasma thermal speed, and $\Theta$ the ratio of poloidal to toroidal magnetic fields) the treatment breaks down.

With a modified ordering of resistivity they arrive at a steady shock-like solution (suggested by Greene [27], and discussed elsewhere [28], [19]), and they calculate the loss rate in this state. The loss is enhanced over the Pfirsch-Schlüter value [3], by the factor $(R/r)^{1/2}$, where r, R are minor and major radii respectively. The small shock region (thickness $\sim\eta^{1/3}$ where $\eta$ is the resistivity) appears on the inside of the torus roughly in the equatorial plane.

However, a general treatment of ideal MHD flow [29] discusses the existence of such shock solutions and concludes that the periodicity (in the meridional plane) requirement for the entropy excludes them.

Resistivity and Perpendicular Viscosity

Rosenbluth and Taylor [30] introduced perpendicular viscosity (see for example [31]) to the resistive plasma, and considered the rotation time development. As for resistive plasma, the zero rotation speed is unstable with respect to rotation.

Upon rotation the plasma speeds up and approaches one of two states (depending on the sense of rotation) which were found to be stable with respect to rotation.

The time-scale for this rotational build-up to an ambipolar state is considerably shorter than the relatively slow resistive diffusion time-scale ( the time for significant alteration in the density). The steady ambipolar plasma loss in each of these two rotating states is calculated and compared with the quasi-static model result(Pfirsch-Schlüter value, [3] ). As the critical rotation speeds for the ambipolar states correspond to frequencies near normal modes, and the damping effect of perpendicular viscosity is weak, the enhancement of plasma loss could be considerable.

Resistivity and Parallel Viscosity

However, Stringer pointed out that under typical experimental conditions the parallel component of viscosity would be more important than the perpendicular.

With the inclusion of this term for the collisional case [32] , he obtained the following qualitative picture: there are, in general, three ambipolar rotating states, of which the middle one (small rotation) is unstable, and the other two are stable. These rotation speeds correspond to the characteristic (electrostatic) wave speeds ( viz., electron drift and ion acoustic) in this axisymmetric situation. In certain regions of parameter space there is only one rotation state. The initial plasma potential distribution determines the sense of rotation (there is no symmetry with respect to the sense of rotation), and also the build-up time during which the ion and electron loss rates equalise, i.e., the particle loss becomes ambipolar.

Within the framework of this linearised treatment it is also possible to include, in first degree of smallness, the effects of helical fields (Stellarator windings) [32] . The magnetic particle drift in the helical field is small compared with the drift in the toroidal field, and in the collision-dominated case the effect on plasma loss is in general small.

Resistivity and Parallel Thermal Conductivity

Galeev [33] treated the collision-dominated case and included resistivity and parallel electron thermal conductivity [31] . He showed that in this case the ambipolar rotation states were far from resonance, so that the loss rate was essentially that of Pfirsch and Schlüter.

Resistivity, Viscosity and Thermal Conductivity

Pogutse [34] considered resistivity, thermal conductivity of both ions and electrons, and parallel viscosity,in the small Larmor radius limit and found only one ambipolar rotation state. This rotation state had the speed of the electron drift wave and had a potential distribution such that the axis was negative. Also in this case the plasma loss rate remained roughly at the Pfirsch-Schlüter level.

Not only was the plasma loss rate calculated but also the thermal loss rate (zero order temperature gradients and first order perturbations are present).

This was compared to the quasi-static calculation of Shafranov [35]. Of particular interest was the calculation of the transverse electric field which could become much larger than the Pfirsch-Schlüter value, and whose direction altered with variation of the collision frequency. However, at most, the particle and thermal fluxes were of the same order as their quasi-static values.

Maschke et al., [36] also carried through a calculation where resistivity, viscosity, parallel ion and electron thermal conductivities were included. A parameter search indicated that the ambipolar plasma loss rate remained of the same order as the Pfirsch-Schlüter value. The parameters used were $\lambda/L_c, a_i/r_n \Theta$: here $a_i$ is the ion Larmor radius, $r_n$ the radial scale-length for the density profile, $\lambda$ the collision mean free path, $L_c$ the connection length $(=2\pi r/\Theta)$. Parallel ion viscosity appeared to be the main damping mechanism.

This type of calculation which has been described above, is characterised by the ordering of all dissipative terms and time derivatives in $\epsilon$ ( $\epsilon = r/R$ the inverse aspect-ratio). In the lowest non-trivial order the particle (ion and electron fluxes through magnetic surfaces $\Gamma_i$, $\Gamma_e$ are expressed in terms of other quantities. In particular the plasma meridional rotation speed $V_o$ remains undetermined. In the next order in $\epsilon$ appears an equation of the form

$$\frac{\partial V_o}{\partial t} \propto (\Gamma_i - \Gamma_e).$$

For small $V_o$, the right hand side is proportional to $V_o$, i.e., the zero rotation state is unstable with respect to rotation. For $\frac{\partial V_o}{\partial t} = 0$ i.e. $\Gamma_i - \Gamma_e = 0$, the ambipolar rotation states are determined. Stability with respect to rotation is assessed by examining the sign of $\frac{\partial V_o}{\partial t}$ around the particular state.

Solution of the time variation equation gives the build-up time to the ambipolar state and allows the time-scale ordering to be checked. If temperature variations have been included it will also be possible to calculate thermal fluxes.

Weakly Collisional and Collisional Regimes

The initial calculation was carried out by Stringer [25], who treated the weakly collisional case where Landau damping seems to provide the dissipation (c.f. the collision-dominated case and resistive dissipation). The treatment closely paralleled the collision-dominated one and suffered from the same difficulty, viz., that the non-ambipolar loss could be negative for some rotational speeds.

However, consideration of the time evolution of the rotation using a time-scale ordering avoided this difficulty. This was explicitly shown for the case where parallel viscous terms were included [37]. The qualitative picture of the rotational time variation has been discussed above. However, on the inclusion of helical fields there are some differences to the collision-dominated case. Although the magnetic particle drift in the helical field is small compared with the drift in the toroidal field, in this weakly collisional case there may be many ions in phase with the helical field variation (the in-phase condition does not require large particle speeds). Thus the net particle loss due to flow in the helical field variation may be comparable or even exceed the loss in an axisymmetric field. Detailed ambipolar balance is reached when the electron loss rate in the axisymmetric field essentially balances the ion loss rate in the helical field variation.

In this linearised approach Stringer and Connor [38] found it possible (using guiding-centre equations with a model collision term) to derive equations valid over a wide range of collision frequencies, thus unifying the treatments of the collision-dominated and weakly collisional regimes. They evaluate the resulting ambipolar loss rates in certain limiting regions of parameter space [39], i.e., for certain ranges of the dimensionless parameters

$$\frac{a_i}{r_{n\Theta}}, \quad \frac{2\pi\lambda}{L_c}, \quad \frac{T_e}{T_i} .$$

(here $T_e$ and $T_i$ are the electron and ion temperatures respectively).

The losses are then compared with the results of Pfirsch and Schlüter [3] (for the collision-dominated case), and of Galeev and Sagdeev [9] (for the weakly collisional case), and situations are found where considerable loss enhancement is possible. The physical picture of this resonance behaviour has been discussed above.

Most recently Stringer [40] reconsidered the build-up of rotation, and found that for consistency a re-ordering of time derivatives was necessary ($\frac{\partial V_o}{\partial t}$ now provides a correction to the lowest order equations). The resulting equations were then solved iteratively. This re-ordering modified the rotational build-up times. However, the identification of the modes responsible for this rotational instability could not be completely carried out, as in some cases the mode involved went unstable on too short a time scale for the iterative scheme to be adequate.

Indeed for $a_i/r_{n\Theta} < 2$, a growing resistive mode causes growth, for $a_i/r_{n\Theta} > 2$ an acoustic mode is responsible. This latter result was first obtained in [41] and is discussed below.

The modified situation is as follows: when three ambipolar states exist the central one ($V_o \backsim 0$) is always unstable and the unstable modes can be identified.

For the zero Larmor radius case see [42], for the finite Larmor radius case [41].) Further, each of the remaining rotating states (previously thought stable) can be unstable in certain regions of parameter space and the modes responsible can be identified [41].

Stringer [40] further investigated the possible build up of parallel ion and electron flow and found that a differential build-up (i.e., current build-up) was possible, leading to a steady current in the final ambipolar rotating state. The calculation was done for the weakly and highly collisional regimes, but is complementary to the bootstrap-current calculations [43], [44] in the banana (collisionless) regime (here the current depends on the existence of trapped particles).

In the collision-dominated case the current arises as a result of pressure anisotropy and temperature variation (i.e., presence of ion viscosity and finite ion and electron thermal conductivities). For scalar pressure or for equations of state where $p_{\perp,\parallel} = p\ (n)$

($p_{\perp,\parallel}$ is the perpendicular or parallel pressure, n the particle number density), there is no current build-up. In the weakly collisional regime the current can be interpreted in terms of resonant particle effects. Although estimated to be measurable, this current has not been observed in the Proto-Cleo Stellarator experiments [45].

## Numerical Calculations

Although the low beta numerical calculations do not really fit the local approximation category A (i), the checking of such codes has, up to now, been done with the results of linearized analyses. These analyses which have not been referred to above will now be discussed together with the corresponding numerical calculations.

The numerical approach to the problem of plasma flow in toroidal devices is extremely complex and still in an experimental stage, but in principle allows the simultaneous consideration of many different physical processes without any specific ordering. For example the one-fluid codes of the Princeton group have, up to now, included: resistivity, inertia, Hall term, viscosity and gyro-viscous terms. These codes use the model field [3] and the numerical problem is a three dimensional ( 2 space, 1 time ) mixed initial, boundary-value problem. As above we order the discussion according to the dissipative effects considered.

Resistivity

The development of the numerical programme proceeded from a treatment of the resistive flow problem in a cylinder, to the toroidal case where in the initial time phase (before inertial effects become important) a reasonable agreement with the Pfirsch-Schlüter loss rate was obtained [46] . On this time-scale (acoustic time-scale) the numerical calculations indicated the existence of a modified acoustic mode (the so-called geodesic mode [47] ). At later times the calculations showed a build-up of density variations on magnetic surfaces. These,of course,are due to the finite plasma inertia.

The numerical studies for the resistive case [48] show qualitative agreement with the results of analysis [42] . The initial plasma state is so chosen that not too much energy is put into wave motion i.e., the plasma profiles are chosen reasonably "close" to their steady-state (on the acoustic time-scale) values.

During an initial noisy period characterised by acoustic and geodesic wave motion, the plasma meridional rotation on each magnetic surface builds up and saturates at a value close to the modified acoustic speed ($\Theta c_{th}$, [20] ). The initial profiles determine the sense of rotation. Plasma loss rates in this case were typically of the order of the Pfirsch-Schlüter rate.

Analyses of the different wave modes [42] showed that for initially small flow ($V_o \backsim 0$) the geodesic mode is damped, the acoustic mode may or may not be, but a resistive mode is a growing one. For the case of large initial flow (large and small flow here mean plasma rotation relative to the modified acoustic speed $\Theta c_{th}$), the resistive and geodesic modes are damped but the acoustic mode grows.

In the neighbourhood of the rotation saturation, an appropriate re-ordering of terms [49] (expansion about this state in terms of $\epsilon^{1/2}$) gives rather good agreement with the numerical results. However, in these resistive calculations numerical viscosity was encountered and it was decided to add physical viscosity to swamp such effects.

Resistivity and Bulk Viscosity

Introduction of a phenomenological bulk viscosity for ions and electrons [50] eliminates degeneracy with respect to the sense of plasma rotation, so that the ambipolar state is such that the axis is left at positive potential. Once again plasma losses were calculated to be of the order of the Pfirsch-Schlüter value.

Resistivity and Parallel Viscosity; Finite Larmor Radius Effects

A further development [41] was the introduction to the resistive model of parallel viscosity and finite Larmor radius effects (Hall term and gyro-viscous corrections to the stress tensor). Here, with a particular ordering, the rotational build-up was analytically studied for collision mean free paths (a) much greater than (b) much less than, the connection length $L_c$.

Resistivity, Viscosity and Thermal Conductivity

One of the most recent developments in this category is the work of Grimm and Johnson [51]. These authors treat plasma inertia in the presence of resistivity, collisional viscosity (with a phenomenological bulk contribution) and thermal conductivity. The model is a single fluid one where $\Omega\ \tau \gg 1$ ($\Omega_i$ ion gyrofrequency, $\tau$ ion-ion collision time), but $C_{th}\tau/R < 1$ i.e., there are no trapped particle effects. Thus finite Larmor radius effects are not included and the ion-electron temperature relaxation is treated in a highly phenomenological manner. The linearized analysis determines the possible steady rotational states (steady on the acoustic time-scale, slowly varying on the longer resistive time-scale), and investigates the stability.

One of the main results is that there is a restriction on the class of stable steady-states. Some of these states are calculated numerically.

This new restrictive element is traced to the parallel viscosity terms associated with magnetic field curvature. These terms had previously been neglected by other authors. The investigation shows that it is possible to have steady rotating states without net toroidal flow, in contrast to the resistive calculation [42] where large flows on some surface were necessary.

The characteristic modes of the system are discussed (thermal conductivity introduces entropy modes additional to the modes previously discussed in the resistive case [42]), but because of the complexity of the dispersion relation, numerical methods had to be used. In one case (where there is no net parallel flow) the dispersion relation can be treated analytically, the modes identified and their stablility discussed. Calculations for plasma loss were not presented.

Discussion

The analytical results of the linearised analyses can be summarised (see also [52] ) as follows: (a) $\lambda \gg L_c$.

Here trapped-particle effects were not included and resistivity was set identically zero, but parallel viscosity and gyro-viscosity were retained. The analysis shows only one stable equilibrium at a speed less than the ion drift and with a negative potential on the axis. Numerical experiment appears to support this result of linearised theory. Plasma loss was of the same order as that of Pfirsch-Schlüter, as a numerical run with Princeton ST Tokamak parameters showed. (b) $\lambda/L_c < 1$. Behaviour of the plasma here depends on whether $\lambda/L_c$ is greater or less than a critical value ($\propto 1/\Theta^6$ ). For $\lambda/L_c$ greater than this value a stable rotation state was found (at a speed less than the electron drift) with positive potential on the axis. The modes causing this build-up of flow were analysed and the drift modifications to the known modes (c.f. the resistive case [42] ) discussed. Under certain conditions the acoustic mode goes unstable causing rotational build-up. Under other conditions this mode is damped and the resistive mode goes unstable (just as in the pure resistive case [42] for subsonic flow). This corresponds to Stringer's results [40] coming from a more consistent time-scale ordering.

Once again numerical experiment appears to give qualitative agreement with this analysis, indeed the final rotation state and density variations show semi-quantitative agreement. Plasma loss is of the order of that of Pfirsch-Schlüter, as a calculation with appropriate C Stellarator parameters shows.

However, for $\lambda/L_c$ less than this critical value (which roughly is a measure of the comparative effects of viscosity and resistivity), flow builds up until a shock-like state develops. Here the density variations are large on the inside of the torus. By a re-ordering procedure (c.f. [49] ) the position of the shock and its width can be estimated. A calculation of particle loss shows typically an order of magnitude increase over the Pfirsch-Schlüter value. Numerical experiment showed qualitative agreement with this analysis.

Summary of A (i)

To briefly summarise: because of the relative "simplicity" involved, most of the recent research has taken place in the area defined above. The relative strengths or weaknesses of this approach can be formulated as follcws:

(a) The most important success for this approach, whose advantage is that it can include many real plasma effects, is the qualitative agreement with experiment with respect to the sense of rotation. (e.g., C Stellarator, ST Tokamak [51] )).

(b) The weaknesses of such an approach are:

(1) Quantitative comparsion with experiment is difficult if not impossible because of the local nature of the approximations used. The orderings are valid only in certain regions of parameter space, and an experimental plasma generally has a radial variation which spans more than one such region. Thus the whole radial region should be considered i.e., the appropriate boundary-value problem formulated.

(2) As can be seen from the detailed discussion, results tend to depend critically on rather complicated orderings. The inherent difficulties of such procedures are inconsistency and breakdown of validity.

An example of inconsistency in the resistive model is the simultaneous ordering of poloidal flow speed $= \Theta C_{th}$, and toroidal flow speed = 0. In actual fact the only possible ordering (in the steady state) for the toroidal flow speed is $= C_{th}$. (This will be discussed more fully in A(ii)). In the time development investigation, an initial ordering may cease to be valid. Because of the many inequalities to be satisfied, a check on consistency is often difficult.

A(ii) Nonlinear Treatment

Because of mathematical intractability a nonlinear treatment can in general be carried through only with some simplification of the model itself i.e., the number of physical effects considered at any one time has to be restricted. A first step in the direction of an analytical nonlinear treatment is the resistive low beta plasma calculation [21], [22]. Here there is no expansion in inverse aspect-ratio (i.e., the whole radial boundary-value problem is treated), and the plasma inertia term is fully retained (i.e., no ordering of plasma flow components).

The investigation, which considers the class of possible steady-states and not the approach to steady-state, requires only one small parameter $\delta$, a normalised resistivity.

$$\delta = \frac{\eta ne}{B_e \epsilon \iota^2} \backsim \frac{V_{PS}}{\epsilon U_{dr}}$$

Here $\iota$ is the rotational transform ($\simeq \frac{R}{r}\Theta$), $B_o$ the toroidal magnetic field on axis. $V_{PS}$ is the surface-averaged Pfirsch-Schlüter loss speed ($= \frac{\eta p}{B_o^2 \iota^2 r_n}$), and $U_{dr}$ is a plasma drift speed ($= C_{th}^2 / r_n \Omega_i$). A well-defined perturbation expansions in $\delta$ was carried out. Other orderings (e.g. [42]) can be regained as special cases of this.

The main results of this calculation are:

(1) steady-state continuous solutions (where physical variables have a continuous spatial

variation) are not possible for all values of the poloidal rotation speed ($V_o$). There exist three regimes: $V_o \leq (\Theta C_{th})_-$ (subsonic), $V_o \geq (\Theta C_{th})_+$ (supersonic) and a finite transonic regime $(\Theta C_{th})_- \leq V_o \leq (\Theta C_{th})_+$ where continuous solutions do not exist. Various authors [26], [27], [28] took this as an indication for the existence of shock-like solutions. Thus the system evolves through either sub-(rotational build-up) or super-sonic (rotational slow-down) states to the transonic region where the flow takes on a shock nature. Other dissipative effects modify this shock formation process. These analyses have been discussed in A (i) above.

(2) In the subsonic regime the possible steady states had associated plasma loss of the same order as the Pfirsch-Schlüter value [3]. The result for plasma loss in the shock flow case [28] also indicates losses of this order. (c.f. [51] which gives an order of magnitude enhancement).

Linearised treatments of this problem claimed plasma losses at the critical flow speed (sonic speed) larger than this [25], but in actual fact the perturbation expansion used broke down in this region.

An important feature of this nonlinear treatment, where plasma inertia is treated exactly, is that for a poloidal flow speed $\sim \Theta\, C_{th}$, there is no freedom in the choice of the corresponding (steady-state) toroidal flow speed, which must be $\sim C_{th}$.

## B. Finite Beta Case

The calculations described above have all employed the model field [3]. Although this field satisfies div $\underline{B}=\underline{0}$, it requires for its generation a toroidal current $\underline{j}$ = curl $\underline{B}$, which is radially distributed. For real plasma traps this is an inconsistency which can be removed and the validity of the model field checked, only by the self-consistent (finite beta) approach. Here the full information contained in $\underline{j}$ = curl $\underline{B}$ replaces the subset of information contained in div $\underline{j}$ = 0 (the low beta approach). Although finite beta effects have been considered in the quasi-static case (see for example review [53] pp.141-148), inertial effects were not included.

Because of the added complexity of this situation, it has not received as much consideration as the low beta case (section A). The introduction of self-consistent field results in the appearance of a new time-scale, the Alfven wave period. In usual situations (beta small) this is a very fast time-scale. (i.e.$\gg$ acoustic time-scale). The low beta description of the effects of plasma flow can be extended to this case with the added possibility of rotation approaching the frequency of the normal <u>electromagnetic</u> modes of the system.

## B (i) Local Approximation Treatments

Hazeltine et al., [54] extended their low beta treatment [26] of the resistive isothermal model, and showed that although the skin penetration time for the poloidal magnetic field (due to the induced toroidal electric field, i.e., Tokamak configuration) is comparable to the build-up time for plasma rotation, the finite beta effects are typically small. They examine the rotational instability behaviour for small rotations and find that the growth rate is essentially that of the low beta case.

The result of rotational build-up was investigated in [55]. As discussed in terms of general kinematical arguments [20] there exists a critical speed (corresponding to a normal electromagnetic mode) in this toroidal finite beta plasma i.e., $\Theta C_A$, where $C_A$ is the Alfven speed in the toroidal field. For a low beta plasma this speed is much greater than the critical speed ( $\Theta C_{th}$) for "zero" beta situations (section A). Dobrowolny et al., [55] showed that for an ideal plasma, rotation near the critical speed gives rise to distortion of the magnetic surfaces.

However, the linearised treatment used breaks down at precisely this point, so the singular nature of this situation has still to be investigated. Taniuti [20] has pointed out that because of this high speed compared to the critical acoustic speed, and the possibility of shock formation, it is unlikely that this rotation state will be reached experimentally.

Haines [56] considered a fluid model including parallel viscosity, Hall effect, finite Larmor radius terms and induced electric field. The ion and electron temperatures were taken as uniform. The equations are expanded in terms of inverse aspect-ratio ($\epsilon$) so that the lowest order represents equilibrium in cylindrical geometry. In higher order in $\epsilon$, appear first-order nonlinear couped ordinary differential equations. These determine the time development of the system, i.e., it is an initial-value problem. These equations have been numerically integrated and show a rotational build-up tending to an apparently steady state. An expression for plasma loss is derived but no estimates have been given.

Unfortunately, the choice of co-ordinate system (the magnetic surfaces are not co-ordinate surfaces) makes it difficult to compare the resulting expression with other calculations of plasma loss across magnetic fields. It is shown in this axisymmetric case that although angular momentum about the major axis is conserved, that about the minor axis is not.
Finally, Asano and Taniuti [29] consider steady ideal MHD flow. They use the characteristic theory of hyperbolic

partial differential equations and re-derive the critical speeds (see [20] ) for this type of flow. As mentioned previously they discuss shock transitions and conclude that the periodicity of entropy with respect to poloidal angle excludes them (c.f. [26] ). They employ a Tokamak ordering, using $\epsilon^{1/2}$ as the small parameter and obtain low beta solutions. These results are well approximated by the zero-beta continuous solutions of Zehrfeld and Green [21] , [22] (upon appropriate expansion of the latter in $\epsilon^{1/2}$ ).

B (ii) Nonlinear Treatment

The magnetohydrostatic problem [1] has been the object of research for a long time. Flow effects have received much less attention (see however [53] p.141 ). The inclusion of plasma inertia for finite beta in a nonlinearised form has recently been accomplished [23] . The discussion of flow and finite beta effects is at present important with respect to the assessment of plasma confinement in Tokamak devices.

Zehrfeld and Green [23] treat the steady-state problem where the resistivity is small, and obtain and discuss the effects of flow and finite beta on:

(1) the Bennet-pinch relation (i.e.,average balance of forces),

(2) the restrictions to the class of ideal steady flow solutions ( such restrictions had been found in the low beta case [21] , [22] ),

(3) the toroidal equilibrium plasma displacement (c.f. the magnetohydrostatic and quasi-static results [57] , [58] respectively).

It is possible without expanding in a small quantity, to obtain some exact integrals for ideal flow. In next order in resistivity the plasma loss can be calculated and the effect of flow assessed. Detailed results can be obtained for the case of large aspect-ratio, but it must be stressed that no ordering of flows has been assumed, and finite beta effects and the boundary-value nature of the problem have been retained.

## Conclusion

It is clear from the above that although the low beta problem has been actively discussed, the finite beta problem is still largely unexplored (both analytically and numerically). Obviously this latter area requires more research. This comment is even more pertinent when applied to the situation involving linear and nonlinear treatments.

The containment problem is essentially a boundary-value problem. Too little attention has been paid to this point. On the other hand unified treatments [38],[39] (although linear) which span large regions of parameter space represent progress, in that they reduce the jig-saw puzzle nature of previous ordering approaches.

The area which holds great promise is that of numerical simulation. However, this is still in a largely experimental stage, for instance a three-dimensional (2 space, 1 time) finite beta code for toroidal situations has yet to be developed. Furthermore, the treatment of boundary conditions in the existing low beta codes does not appear adequate to investigate the long time confinement behaviour. This and other problems (e.g.,numerical viscosity and accuracy) detract somewhat from the attractiveness of this approach.

To generate confidence, these codes must be checked extensively against analytical results. This clearly indicates that only a <u>combined</u> numerical-analytical approach will succeed in solving the highly complex problem of toroidal plasma containment.

References

1. J.M.Greene, J.L.Johnson, Advances in Theoretical Physics 1 (1965) 195.
2. M.D. Kruskal, R.M.Kulsrud, Phys.Fluids 1 (1958) 265
3. D.Pfirsch, A.Schlüter, MPI/PA/7/62, Max-Planck-Institut für Physik und Astrophysik, Munich (1962)
4. B.B. Kadomtsev, V.D. Shafranov, Sov.Phys.Doklady 11 (1967) 794.
5. E.K.Maschke, Plasma Physics 13 (1971) 905; EUR-CEA-FC-429 (1967)
6. J.L.Johnson, S.von Goeler, Phys.Fluids 12 (1969) 255.
7. L.Spitzer Jr., "Physics of Fully Ionized Gases", Interscience (1962) p. 43
8. H.K. Wimmel, Nuclear Fusion 10 (1970) 117.
9. A.A. Galeev, R.Z. Sagdeev, Sov.Phys.JETP 26 (1968) 233.
10. P.H. Rutherford, Phys. Fluids 13 (1970) 482
11. L.M. Kovrizhnikh, Sov.Phys.JETP 29 (1969) 475
12. E.A. Frieman, Phys.Fluids 13 (1970) 490
13. G. Knorr, Phys.Fluids 8 (1965) 1334
14. M.Bineau, Phys.Fluids 10 (1967) 1540
15. E.T. Karlson, Arkiv f.Fysik 35 (1967) 539
16. K.S. Viswanathan, Phys.Fluids 11 (1968) 1104
17. L.Woltjer, Astrophys. J.130 (1959) 405

18. J.M. Greene,E.T.Karlson, Phys.Fluids 12 (1969) 561

19. D.Dobrott, J.M.Greene, Phys.Fluids 13 (1970) 2391

20. T.Taniuti, Phys.Rev.Letters 25 (1970) 1478

21. H.P.Zehrfeld, B.J.Green, Phys.Rev.Letters 23 (1969)961

22. H.P.Zehrfeld, B.J.Green, Nuclear Fusion 10 (1970) 251

23. H.P.Zehrfeld, B.J.Green, (to be published)

24. M.N.Rosenbluth, P.H.Rutherford, J.B.Taylor, E.A.Frieman, L.M.Kovrizhnikh, Paper CN-28/C-12, IAEA Conference, Madison (1971)

25. T.E.Stringer, Phys.Rev.Letters 22 (1969) 770

26. R.D.Hazeltine, E.P.Lee, M.N.Rosenbluth, Phys. Fluids 14 (1971) 361

27. J.M.Greene, Dubna Conference (1969)

28. M.N.Rosenbluth, Dubna Conference (1969)

29. N.Asano, T.Taniuti, Dept. of Physics, Nagoya University DPNU-13, May (1971)

30. M.N.Rosenbluth, J.B.Taylor, Phys.Rev.Letters 23 (1969) 367

31. S.I.Braginskii ,"Reviews of Plasma Physics", Consultants Bureau (1966), vol.1 p.229

32. T.E.Stringer, Phys.Fluids 13 (1970) 1586

33. A.A.Galeev,JETP Letters 10 (1970) 225

34. O.P.Pogutse,Nuclear Fusion 10 (1970) 399

35. V.D.Shafranov, Atomic Energy 19 (1965) 120

36. E.K.Maschke, R.Pellat, M.N.Rosenbluth, Bull.Am.Phys. Soc. 15 (1970) 1423

37. T.E.Stringer, Phys. Fluids 13 (1970) 810

38. T.E.Stringer, J.W.Connor, Phys.Fluids 14 (1971) 2177

39. J.W. Connor, T.E.Stringer, Phys.Fluids 14 (1971) 2184

40. T.E. Stringer, Paper CN-28/F-3, IAEA Conference, Madison (1971)

41. E.Bowers, N.K.Winsor, Phys. Fluids 14 (1971) 2203.

42. J.M.Greene, J.L.Johnson, K.E.Weimer, N.K.Winsor Phys.Fluids 14 (1971) 1258

43. A.A.Galeev, R.Z.Sagdeev, JETP Letters 13 (1971) 113

44. R.J.Bickerton, J.W.Connor, J.B.Taylor, Nature 229 (1971) 110

45. R.A.E.Bolton, J.Hugill, D.J.Lees, W.Millar, P.Reynolds, Paper CN-28/H-6, IAEA Conference, Madison (1971)

46. J.L.Johnson, N.K.Winsor, J.M.Dawson, Bull Am. Phys. Soc. 13 (1968) 1797
Los Alamos Conference on Numerical Simulation of Plasma (1968) Paper D-4

47. N.K.Winsor, J.L.Johnson, J.M.Dawson, Phys.Fluids 11 (1968) 2448

48. N.K. Winsor, J.L.Johnson, J.M.Dawson, J.Comp.Phys. 6 (1970) 430

49. J.M. Greene, N.K.Winsor, private communication

50. J.M. Dawson, N.K.Winsor, E.C.Bowers, J.L.Johnson, IVth. European Conference on Controlled Fusion and Plasma Physics, Rome (1970), Proceedings p.9

51. R.C.Grimm, J.L.Johnson, (to be published)

52. N.K.Winsor, E.C.Bowers, J.M.Dawson, Paper CN-28/F-4, IAEA Conference, Madison (1971)

53. V.D.Shafranov, "Reviews of Plasma Physics", Consultants Bureau (1966), Vol. 2

54. R.D.Hazeltine, E.P.Lee, M.N.Rosenbluth, Phys.Rev. Letters 25 (1970) 427

55. M.Dobrowolny, L.Kovrizhnykh, R.Pellat, Phys.Rev. Letters 25 (1970) 639

56. M.G.Haines, Phys.Rev. Letters 25 (1970) 1480

57. V.D.Shafranov, Nuclear Fusion 3 (1963) 183

58. H.P.Zehrfeld, B.J.Green, IPP III/1 Garching Report (1970)