TOKAMAK WITH SUPERPOSED MULTIPOLE MAGNETIC FIELD

W. Feneberg

IPP III/2 Juni 1971

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

TOKAMAK WITH SUPERPOSED
MULTIPOLE MAGNETIC FIELD

W. Feneberg

IPP III/2 Juni 1971

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

IPP III/2 W. Feneberg Tokamak with Superposed Multipole Magnetic Field
June 1971 (in English)

Abstract

It is proposed that multipole magnetic fields be superposed on the Tokamak to produce an equilibrium such that the plasma is separated from the wall by a magnetic separatrix. The equilibrium is calculated for a special configuration with four conductors in which small deviations from circular symmetry are assumed.

The problems incurred in the Tokamak by the limiter can be investigated by superposing on the magnetic field of the discharge current a multipole magnetic field produced in external current carrying conductors. The resulting equilibrium configuration produces a separatrix which separates the interior region, where closed magnetic surfaces form around the magnetic axis, from the exterior region. On the outside the magnetic surfaces are closed around the current carrying conductors and intersect the vacuum wall if the separatrix lies wholly inside the vacuum wall. It may be assumed that the plasma pressure outside the separatrix is very small compared with that in the interior region because the plasma escapes to the wall along the field lines. The separatrix can thus be substituted for the limiter.

As an example of such an equilibrium we consider a configuration with four conductors. Inside a wall of radius a with infinitely good conductivity there are four conductors arranged as shown in Fig. 1. These each carry the same current I_Θ, and are a distance b from the centre of the discharge vessel. R, Θ, Z are a system of cylindrical coordinates. We require an approximate solution of the equation of the axisymmetric equilibrium [1]:

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)+\frac{\partial^2\psi}{\partial z^2}=Ri_\Theta=-II'-R^2p' \quad \text{-----} \quad (1)$$

As usual, the magnetic field is here derived from a flux function ψ. In a system of polar coordinates

$$Z = r\sin\vartheta$$

$$R = R_0 - r\cos\vartheta \quad \text{-------} \quad (2)$$

the poloidal magnetic field can be represented by

$$-B_r = \frac{1}{Rr}\frac{\partial\psi}{\partial\vartheta}, \qquad B_\vartheta = \frac{1}{R}\frac{\partial\psi}{\partial r} \quad \text{-------} \quad (3)$$

Equation (1) is treated for the special case

$$\alpha = -\frac{a^2}{2} I I' = \text{const}, \qquad \text{--------- (4)}$$

$$\gamma = -4\pi a^2 R_0^2 p' = \text{const},$$

and it is postulated that, on the boundary surface $\psi = \psi_g$ = const, which separates the magnetic surfaces in the plasma (ψ^p = const) from those in the vacuum (ψ^v = const), the plasma pressure p vanishes, and hence the magnetic field makes a continuous transition from the plasma to the vacuum region. For ψ_V we make the ansatz

$$\psi_V = \frac{R_0 I_\theta}{c} \ln\left(r^8 + b^8 - 2r^4 b^4 \cos 4\vartheta\right) + \tilde{\psi}_V, \qquad \text{--- (5)}$$

where c is the velocity of light and $\tilde{\psi}_V$ is a regular function in the region between the plasma and conducting wall.
The boundary conditions should be roughly such that the currents already flow in the four conductors at the beginning of the experiment, and that, in addition, a vertical magnetic field is applied. The total field is then assumed to be frozen into the conducting shell. Since expansion in the aspect ratio yields a flux function

$$\psi_A = \frac{R_0 I_\theta}{c} \ln\left(r^8 + b^8 - 2r^4 b^4 \cos 4\vartheta\right) + R_0 B_\perp r \cos\vartheta \qquad \text{--- (6)}$$

for such a field, the boundary condition is

$$\frac{\partial \psi_V (r=a)}{\partial \vartheta} = \frac{\partial \psi_A (r=a)}{\partial \vartheta} \qquad \text{--------- (7)}$$

We assume an expansion of ψ_V or ψ_P with respect to $\varepsilon = \frac{a}{R_0}$ and only take terms to first order in ε into account:

$$\psi = \psi_0 + \varepsilon \psi_1 \tag{8}$$

An approximate solution of eq.(1) is obtained by making the following ansatz for ψ_0 and ψ_1 in the plasma and in the vacuum region for $\frac{r}{b} \ll 1$:

$$\psi_{0,P} = \frac{1}{4}(\alpha+\gamma)\xi^2 + c_4 \xi^4 \cos 4\vartheta \tag{9}$$

$$\psi_{1,P} = -\frac{(\alpha+5\gamma)}{16}\xi^3 \cos\vartheta + a_1 \xi \cos\vartheta \tag{10}$$

$$\psi_{0,V} = a_0 \ln\xi + \left(b_4^{(1)}\xi^4 + b_4^{(2)}\xi^{-4}\right)\cos 4\vartheta \tag{11}$$

$$\psi_{1,V} = \frac{a_0}{2}\cos\vartheta\left(\frac{\xi}{2} - \xi\ln\xi\right) + \left(d_1\xi + d_2\xi^{-1}\right)\cos\vartheta \tag{12}$$

$$\left(\xi = \frac{r}{a}\right)$$

When these expressions are written down, it is also assumed that

$$\frac{c_4}{(\alpha+\gamma)}\xi^2 \ll 1 \tag{13}$$

i.e. the multipole fields are not too strong.

Owing to the symmetry of the configuration (Fig.1) the magnetic surfaces in the linear case can only contain the modes k = 4n (n being a positive whole number) after expansion of the flux function in a Fourier series, and so eqs.(9) and (11) constitute the simplest possible ansatz for calculating the deformation of the magnetic surfaces by the multipole field. The expressions (10) and (12) state that the shape of the magnetic surfaces is the same in toroidal geometry as in the linear case, but that the centres of the surfaces are shifted. This displacement is not governed by the multipole field. We therefore obtain here in first order in ε

the result of Shafranov [2], which is based on the assumption of circular magnetic surfaces in the linear case. As it is assumed in the following that for $\varepsilon = 0$ the magnetic surfaces should deviate very little from circular symmetry the influence of the multipole field can be neglected in first order in ε. The shape of the magnetic surfaces in the plasma region Ψ_P = const is expressed by the ansatz:

$$\xi = u + \delta_1 \cos 4\vartheta + \varepsilon \delta_2 \cos\vartheta \qquad --- (14)$$

If only terms linear in δ_1 and $\varepsilon \delta_2$ are used, this expression yields

$$\xi = u - \frac{2C_4}{(\alpha+\gamma)} u^3 \cos 4\vartheta + \varepsilon \left(\frac{(\alpha+5\gamma)}{8(\alpha+\gamma)} u^2 \cos\vartheta - \frac{2a_1}{(\alpha+\gamma)} \cos\vartheta \right) \qquad --- (15)$$

$0 \leq u \leq u_0$ (u_0 = plasma surface) describes the set of magnetic surfaces:

$$u = 2\sqrt{\frac{\Psi_P}{(\alpha+\gamma)}} \qquad ----- (16)$$

The boundary conditions on the plasma surface $u = u_0$

$$\frac{\partial \Psi_P}{\partial \xi} = \frac{\partial \Psi_V}{\partial \xi} \; , \qquad \frac{\partial \Psi_P}{\partial \vartheta} = \frac{\partial \Psi_V}{\partial \vartheta} \qquad ---- (17)$$

yield the relation between the constants of the vacuum and plasma solutions which can be obtained after a simple calculation:

$$b_4^{(1)} = \frac{3}{4} C_4 \; , \qquad b_4^{(2)} = \frac{1}{4} C_4 u_0^8 \qquad --- (18)$$

$$d_1 = \frac{\alpha}{4} u_0^2 \left(\ln u_0 - \frac{7}{4} \right) + \frac{\gamma}{4} u_0^2 \left(\ln u_0 - \frac{5}{4} \right) \qquad --- (19)$$

$$d_2 = -\frac{\alpha}{8} u_0^4 - \frac{\gamma}{8} u_0^4 + a_1 u_0^2 \qquad ---- (20)$$

Since $\tilde{\psi}_v$ (eq.5) represents a function that is regular in the entire vacuum region, it can be approximated well in the whole region by

$$\tilde{\psi}_v = \frac{1}{2}(\alpha+\gamma)u_0^2 \ln \xi + \frac{3}{4}\xi^4 \cos 4\vartheta + \frac{1}{4} C_4 u_0^8 \xi^{-4} \cos 4\vartheta + \tag{17}$$
$$+ \frac{2R_0}{c} I_\theta \xi^4 \left(\frac{a}{b}\right)^4 \cos 4\vartheta + \varepsilon \Delta^S \cos\vartheta$$

where $-\varepsilon\Delta^S$ denotes the well-known Shafranov shift. If the boundary condition (eq.7) is satisfied, one gets

$$C_4 = \frac{8 R_0}{c(3+u_0^8)} I_\theta \left(\frac{a}{b}\right)^4 \quad - - - - - \tag{18}$$

The quantities α and γ in eq.(1) can be expressed by the net plasma current I_p and by $\beta_p = \frac{2\pi P_0 c^2 a^2 u_0^2}{I_p^2}$ (P_0 being the pressure on the magnetic axis).

$$\alpha = \frac{4R_0 I_p}{c u_0^2}\left(1-\frac{1}{2}\beta_p\right) + O(\varepsilon^2) \quad - - - \tag{19}$$

$$\gamma = \frac{2R_0 I_p}{c u_0^2}\beta_p + O(\varepsilon^2)$$

This yields the following result for the set of magnetic surfaces in the plasma:

$$\xi = u + \frac{4 I_\theta a^4 u^3 u_0^2}{3 b^4 I_p}\cos 4\vartheta + \tag{20}$$
$$+\frac{\varepsilon}{2}\left\{\frac{u^2}{4} + \frac{u^2\beta_p}{2} + \ln u_0 - \frac{u_0^2}{2} + \frac{1}{4} - \frac{1}{2}\beta_p - \frac{B_\perp R_0 c}{I_p}\right\}$$

and for the flux function in the vacuum:

$$\tilde{\psi}_v = \frac{2R_0}{c} I_p \ln\xi + \frac{2R_0}{3c} I_\theta u_0^8 \left(\frac{a}{b}\right)^4 \cos 4\vartheta\left(\xi^4 - \xi^{-4}\right) + \varepsilon\Delta^S\cos\vartheta \tag{21}$$

The position of the separatrix is given by two stagnation points (ξ_s, ϑ_s) in which the poloidal magnetic field vanishes. For $\frac{I_p}{I_\theta} > 0$ these points are located at $\vartheta_s = 0$ and $\vartheta_s = \pi$,

$$1 - 4\left(\frac{I_\theta}{I_p}\right)\xi_s^4 \frac{a^4}{b^4} + \frac{4}{3}\frac{I_\theta}{I_p} u_0^8 \frac{a^4}{b^4}\left(\xi_s^4 + \xi_s^{-4}\right) \pm \varepsilon A = 0$$

$$A = \xi_s \ln u_0 - \xi_s \ln \xi_s - \frac{3}{4}\xi_s + \xi_s^{-1}\ln u_0 + \tag{22}$$
$$+ \frac{1}{4}\xi_s^{-1} - \frac{c B_\perp R_0}{I_p}\xi_s^{-1} - \frac{1}{2}\beta_p\left(\xi_s + \xi_s^{-1}\right)$$

where the plus sign applies to $\vartheta_s = 0$ and the minus sign to $\vartheta_s = \pi$.

It can be seen that a separatrix will end up inside conducting walls if realistic dimensions are taken as a basis. There then exists a sufficiently large region where the magnetic surfaces deviate little from the circular shape, so that almost unperturbed Tokamak conditions are to be expected there.
It should also be possible to solve the divertor problems in the Tokamak using these proposed multipole conductors.

1 Lüst, R.,A.Schlüter, Zeitschr.f.Naturforschg.129,850(1957)
2 Shafranov,V.D., Reviews of Plasma Physics, M.A.Leontovich, Ed., Consultants Bureau, New York, 1966, Vol. 2, p. 103

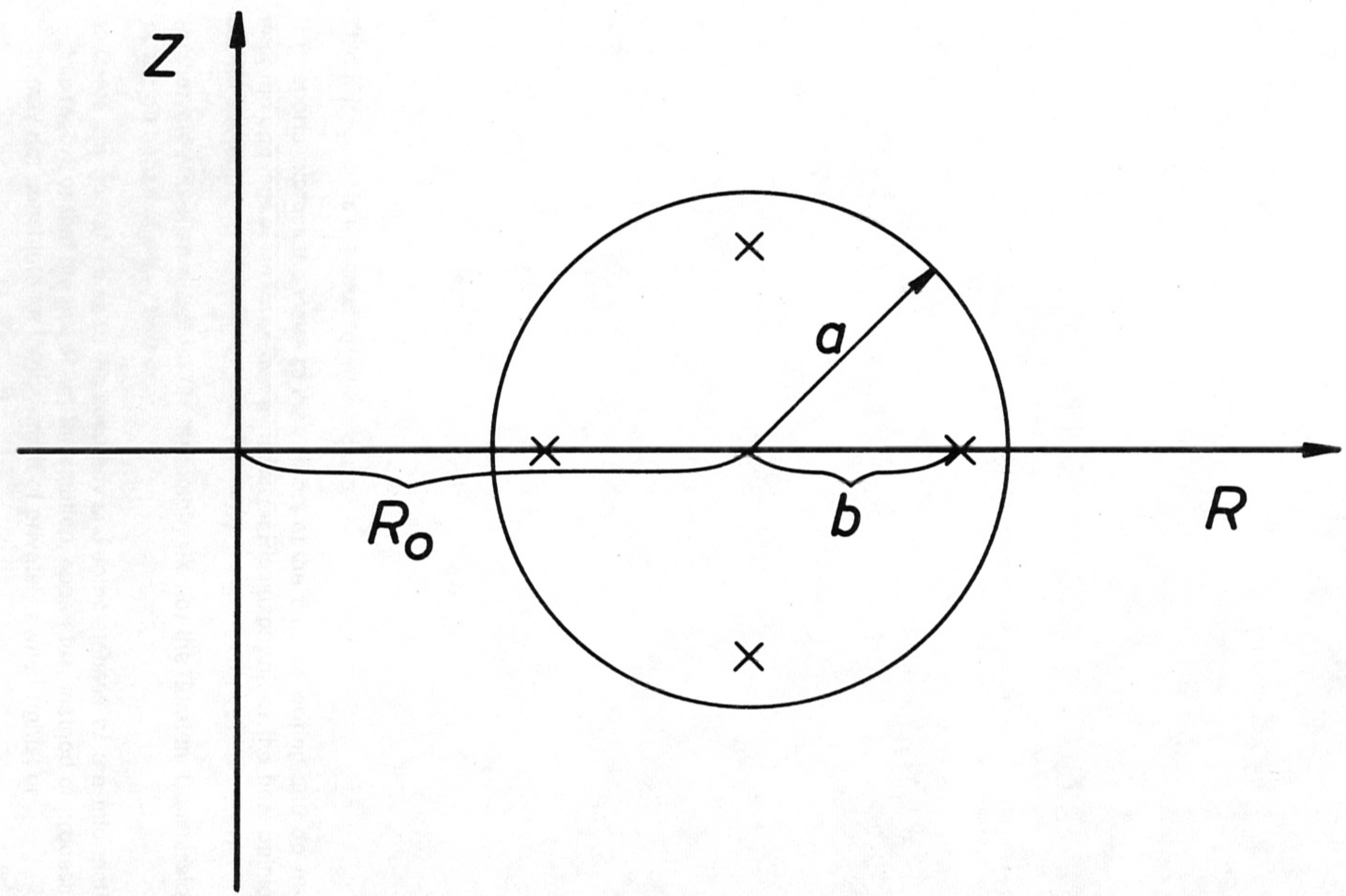

Fig. 1 Multipole-Tokamak configuration with 4 conductors