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A Simple Low-Loss Coupling Method
for RF Heating of Plasma Ions by
Electrostatic Ion-Cyclotron Waves

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ABSTRACT

It is proposed to excite TEM waves in a cylindrical coaxial waveguide consisting of a metallic outer conductor with the plasma acting as the inner conductor. It is found that for frequencies of the order of the ion-gyrofrequency, the penetration depth of the TEM wave fields into the plasma generally exceeds the wavelength of the electrostatic ion-cyclotron waves. The electrostatic ion-cyclotron waves thus excited propagate radially into the plasma and are collisionally absorbed if the time of energy propagation (at the wave's group velocity) over a distance of plasma diameter is of the order of or greater than the energy equipartition time due to ion-ion collisions. Numerical calculations for some representative examples indicate that this method could be used to heat plasma ions with a rather high efficiency.

I. Introduction

The principal rf heating schemes for obtaining thermonuclear temperatures (e.g., ion-cyclotron resonance and magnetic pumping) use coil structures within the plasma vessel. In addition to the inadvisability of using coils in the immediate vicinity of a neutron flux, the coils would dissipate an inordinately large amount of rf power.

A scheme for overcoming these limitations using slow-wave waveguide structures along the walls of the vacuum vessel and constructed out of the wall material itself has been proposed by Derfler [1]. According to Derfler, at the large magnetic fields and physical dimensions envisaged in a fusion reactor, the radial dimensions of the plasma vessel approach the free-space wavelength of the ion-cyclotron frequency and it should be possible to employ slow-wave structures at the cyclotron (or one of the harmonics) frequency. The electrostatic ion-cyclotron (EIC) waves [2] so launched have an electric field component in the magnetic field direction and are absorbed through collisionless Landau damping.

In this note we propose yet another coupling method that altogether eliminates structures in the vicinity of the plasma and is usable when the longitudinal dimensions of the apparatus become comparable to one half of the free-space wavelength of the rf heating frequency. It is proposed to excite TEM waves in a cylindrical coaxial waveguide consisting of a metallic outer conductor with the plasma acting as the inner conductor. It is found that for frequencies of the order of the ion-gyrofrequency, the penetration depth of the TEM wave fields into the plasma generally exceeds the wavelength of the electrostatic ion-cyclotron waves. The electrostatic ion-cyclotron waves thus excited propagate radially into the plasma and are collisionally absorbed if the time of energy propagation (at the wave's group velocity) over a distance of plasma diameter is of the order of or greater than the energy equipartition time due to ion-ion collisions. Numerical calculations for some representative examples indicate that this method could be used to heat plasma ions with a rather high efficiency.

A self-consistent analysis would involve the solution of the boundary value problem using the hot-plasma dispersion relation. Moreover, the effects due to the radial density and temperature profiles, the sheath structure, the interparticle collisions, as well as the finite conductivity of the outer wall, should be included.

In this paper, however, a considerably simplified model is employed. The plasma is assumed to be a homogeneous dielectric with a perfectly smooth boundary. The launching of the TEM and the EIC waves is treated as two independent problems, using the "cold-plasma" and the "hot-plasma" dispersion relations, respectively. The dissipation in the outer wall, as well as at the plasma boundary produced by the currents due to the TEM waves, are calculated using perturbation techniques. It is further assumed that the expressions for group velocity, energy density and energy flux are valid even though the dielectric constant varies rapidly as a function of frequency in the vicinity of an EIC resonance.

II. Launching the TEM Wave

As shown in Fig.1, the plasma acts as the inner conductor (diameter "a") of a cylindrical coaxial waveguide (outer wall diameter "b") and the rf energy is fed through a current loop. A uniform magnetic field B_o is present in the plasma (assumed to be cold) and is directed along the axis of the cylinder. Since the plasma readily supports a current in the magnetic field direction, TM-like coaxial waveguide modes are excited. These modes are not pure TM due to the finite plasma and wall conductivity. We shall restrict the following treatment to the lowest order, namely the TEM mode which has a radial electric and an azimuthal magnetic field.

The fields of the TEM wave excite in the plasma the two cold-plasma waves. For $\epsilon_\perp = 1 - [\omega_{pi}{}^2/(\omega^2-\omega_{ci}{}^2)] \lesssim -1$, the extraordinary wave is very weakly excited and propagates into the plasma without attenuation, ω_{pi} and ω_{ci} being the ion plasma and cyclotron frequencies, respectively. The ordinary wave, on the other hand, has a much larger electric field but is evanescent with a penetration depth of $\delta_p = c/\omega_{pe}$ where c is the velocity of light in vacuum and ω_{pe} is the electron plasma frequency. Inclusion of electron-ion collisions introduces a weak attenuation in the extraordinary wave while adding a small real part in the propagation constant of the ordinary wave. The electric field of either plasma wave is predominantly along the radial direction. Due to the continuity of the normal component of the electric displacement across a boundary, the radial electric field E_o of the ordinary wave is given by $E_o \approx E_a/\epsilon_\perp$, E_a being the electric field just outside the plasma boundary. In order to couple a large electric field into the plasma, the rf frequency is chosen so that $\epsilon_\perp \sim O(1)$ resulting in $\omega \sim O(10\omega_{ci})$. Details of the solution outlined in this section will appear in a separate report (see also Section VI).

III. Excitation of the EIC Waves

The skin depth δ_p is typically of the order of or larger than the wavelength $\lambda_\perp$ of the EIC wave. The radial electric field of the TEM wave extending to the depth δ_p inside the plasma, therefore, launches the EIC wave propagating radially into the plasma column in the direction perpendicular to the static magnetic field B_o.

We shall assume that the EIC wave is uncoupled from the TEM waveguide mode and leaves it unaffected. This is largely justified because the energy carried by the EIC wave is a very small fraction of the energy in the TEM wave. Moreover, the electron and ion motions occur at right angles, the former carrying the longitudinal current of the TEM mode and the latter moving under the influence of radial electric field of the EIC wave. Almost the entire energy in the perpendicularly propagating EIC wave exists in the transverse motion of the ions. The wave, therefore, will be assumed to be completely absorbed if the energy propagation time over a plasma diameter equals or exceeds the ion-ion energy equipartition time.

Fredricks [3] has studied the full dispersion relation for the ion-cyclotron waves propagating perpendicular to the magnetic field and finds that for a given ω there exist in general two roots for the propagation constant $k_\perp$. He shows that if $\omega = N\omega_{ci} + \Delta\omega$ one of the roots $k_\perp \to \infty$ $(\lambda_\perp \to o)$, the wave phase velocity $v_p \ll c$ and the wave exhibits an electrostatic character with an electric field colinear with the propagation direction. For the other root $k_\perp \to o$ $(\lambda_\perp \to \infty)$ and the wave is electromagnetic. Thus it is primarily the electrostatic mode which would be excited by the radial electric field of the ordinary wave. Since for this mode the plasma diameter $a \gg \lambda_\perp$, we consider a planar geometry to calculate the amplitude of the EIC wave excited in the plasma. If the electric field of the ordinary wave is uniform with an amplitude E_o to a depth $\lambda_\perp/2$, the amplitude of the excited EIC wave would also be E_o. In practice, however, the electric field decays exponentially from the value E_o at the plasma boundary with a characteristic decay length δ_p. After Fourier transformation of this field in the wave number space and comparing the result with the case of a uniform E_o to a depth of δ_p it is readily shown that the amplitude of the excited EIC wave is given by

$$E = E_o / \left[1 + \left(\frac{2\pi\delta_p}{\lambda_\perp}\right)^{-2}\right]^{1/2} .$$

Since $E_o = E_a/\epsilon_\perp$ where E_a is the field of the TEM wave just outside the plasma boundary, one obtains

$$\begin{aligned} E &= RE_a, \\ R &= 1/\left[1 + (2\pi\delta_p/\lambda_\perp)^{-2}\right]^{1/2} \epsilon_\perp . \end{aligned} \tag{1}$$

IV. Propagation of the EIC Waves

In the electrostatic approximation, the dispersion relation for the EIC wavespropagating perpendicular to the magnetic field is given by [4]

$$K_\perp = \left(1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}}\right) - \frac{\omega_{pi}^2}{\omega_{ci}^2} \sum_{\substack{N=1 \\ ions}}^{\infty} \frac{e^{-\xi} I_N(\xi)}{\xi} \frac{2N^2\,\omega_{ci}^2}{\omega^2 - N^2\omega_{ci}^2} = 0 \tag{2}$$

where ω_{ce} is the electron cyclotron frequency, $\xi = k_\perp^2 r_{ci}^2$, r_{ci} being the ion-cyclotron radius and $I_N(\xi)$ is the Bessel function defined in the manner of Watson [5]. In Eq. (2) the contribution from the electron term has been determined assuming $\xi \ll (M/m)$. Also for the validity of the collisionless Boltzmann equation used in deriving the dielectric tensor, it is necessary thet $\lambda_\perp \gg \lambda_D$ where λ_D is the Debye length. We shall consider $\omega = N\omega_{ci} + \Delta\omega$ with $\Delta\omega \rightarrow o$ and shall ignore the terms other than the dominant term in the summation in Eq.(2). We now assume that $\xi \gg 1$ which together with $\xi \ll M/m$ implies

$$(M/m) \gg (V_{thi}/V_p)^2 \gg 1,$$

V_{thi} and V_p being the ion-thermal speed and the wave phase velocity, respectively. Using the approximation $\exp(\xi)\ I_N(\xi) \rightarrow (2\pi\xi)^{-1}$, Eq.(2) becomes

$$K_\perp = (1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}}) - \frac{\omega_{pi}^2}{\omega_{ci}^2}\ \frac{1}{K_\perp^3\, r_{ci}^3}\ \frac{1}{(2\pi)^{1/2}}\ \frac{N\omega_{ci}}{\omega - N\omega_{ci}} = 0 \tag{3}$$

From Eq.(3) we can immediately deduce the wavelength, phase and group velocity, energy density W and the non-electromagnetic energy flux [6] T due to coherent motion of the charges,

$$\lambda_\perp = r_{ci}\,(2\pi)^{7/6}\, N^{-1/3}\, (1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}})^{1/3}\, (\frac{\omega_{ci}}{\omega_{pi}})^{2/3}\, (\frac{\Delta\omega}{\omega_{ci}})^{1/3} \tag{4}$$

$$V_p = V_{thi}\ (2\pi)^{1/6}\ N^{2/3}\, (1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}})^{1/3}\, (\frac{\omega_{ci}}{\omega_{pi}})^{2/3}\, (\frac{\Delta\omega}{\omega_{ci}})^{1/3} \tag{5}$$

$$V_g = V_{thi}\ 3\,(2\pi)^{1/6}\, N^{-1/3}\, (1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}})^{1/3}\, (\frac{\omega_{ci}}{\omega_{pi}})^{2/3}\, (\frac{\Delta\omega}{\omega_{ci}})^{4/3} \tag{6}$$

$$W = \frac{1}{4} R^2\, E_a^2\, \frac{\partial}{\partial\omega}(\omega K_\perp) = \frac{1}{4} R^2 N\, E_a^2\, (1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}})(\frac{\omega_{ci}}{\Delta\omega}) \tag{7}$$

$$T = -\frac{\omega}{4} R^2\, E_a^2\, \frac{\partial K_\perp}{\partial k_\perp}$$

$$= -\frac{3}{4} V_{thi}\ R^2\, E_a^2\, N^{2/3}\, (1 + \frac{\omega_{pi}^2}{\omega_{ci}\,\omega_{ce}})\, (2\pi)^{1/6}\, (\frac{\omega_{ci}}{\omega_{pi}})^{2/3}\, (\frac{\Delta\omega}{\omega_{ci}})^{1/3} \tag{8}$$

The power absorbed in the plasma per unit length of the coaxial waveguide is given by

$$P_{abs}^{p} = \pi a T \tag{9}$$

There is also power absorbed by the ohmic dissipation of the waveguide current I on the plasma surface. Since this power does not reach the plasma bulk, it would be considered lost. For the case $\sigma_p \ll \omega \epsilon_p$ the power dissipated per unit waveguide length is given by

$$P_{diss}^{p} = \frac{1}{2}\left(I \frac{\sigma_p}{\omega \epsilon_p}\right)^2 \left(\frac{1}{\pi a \delta_p \sigma_p}\right)$$

$$\simeq \frac{\pi a c}{2} \quad \frac{\nu_{ei}}{\omega_{pe}} \; E_a^{\,2} \tag{1o}$$

Similarly, the power dissipated on the outer waveguide walls is given by

$$P_{diss}^{w} = \frac{1}{4} \; \frac{\pi a^2 \delta_w}{b} \; \omega E_a^2 \tag{11}$$

where δ_w is the skin depth for the guide wall. Still another mechanism of power absorption by the plasme electrons through the attenuation of the extraordinary wave shall be ignored in the present treatment but shall be included in a future report. From Eqs. (9) - (11), the heating efficiency (disregarding all losses in the launching system external to the waveguide) is given by

$$\eta = \frac{P_{abs}^{p}}{P_{abs}^{p} + P_{diss}^{p} + P_{diss}^{w}} = \frac{P_{abs}^{p}}{P}$$

The stored energy per unit length and the quality factor are given respectively as

$$S = \frac{\pi}{8} a^2 \; \ln \frac{b}{a} \; E_a^2$$

$$Q = \frac{\omega S}{P} \; .$$

V. Some Representative Results

Table I summarizes the calculations for three different deuterium plasmas. The harmonic number N is selected so as to make $\epsilon_\perp \sim O(1)$. In order to assure a uniform heating of the plasma volume, $\Delta\omega/\omega_{ci}$ was selected so as to make $V_g = a/\tau_{ii}$ where τ_{ii}, the ion-ion energy equipartition time is calculated according to Spitzer [7]. All other calculations follow in a straight-forward manner.

VI. Discussion

One notes that the heating efficiency is rather high and a glance at the table shows that the method lends itself to a wide range of plasma parameters. The coupling method has a disarming simplicity. Since the TEM wave's magnetic field does not penetrate into the plasma bulk, no serious distortion of the magnetic surfaces is produced. It is still a matter of conjecture whether the large electromagnetic fields at the plasma surface could actually contribute to improved confinement.

The method has, however, its share of pitfalls, the most serious objections arising from the possible importance of density profile and sheath structure at the plasma boundary. This problem assumes special importance at high temperatures and densities when δ_p becomes of the order of and even less than the ion-cyclotron radius. While we intend to treat the effects of density gradients in a reasonably comprehensive manner in a forthcoming report, the sheath problem remains an open question. Considerations of efficient coupling compel a resort to very high harmonics of the cyclotron frequency. Unfortunately, practically no experimental work exists at present to warrant the reliability of the theoretical work on the EIC waves at such high harmonic numbers.

On the technical side, the extremely large Q of the system would cause difficult matching problems (presumably using electronically controlled double stub tuning) between the generator and the plasma. Since $\Delta\omega/\omega_{ci}$ is rather small, the magnetic field must be held constant and uniform to close tolerances. The problem could be alleviated by controlling ω so as to keep $\Delta\omega$ constant, although this procedure may merely accentuate the matching problem mentioned already.

In an effort to focus attention on the essentials, we have deliberately avoided consideration of a toroidal geometry. Since the magnetic field is no longer uniform, the EIC waves are not excited uniformly on the plasma surface. Also a resonance would exist at some value of the torus´major radius where $\lambda_{\perp} \rightarrow \lambda_D$ and the wave energy would be locally absorbed.

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Outer wall material	Cu	Cu	O_S
Plasma density, n/ccm	5×10^{11}	5×10^{12}	5×10^{14}
Plasma temperature, eV	10	100	5000
Magnetic field, kG	10	10	100
Plasma diameter, cm	10	10	100
Outer wall diameter, cm	20	20	200
Ion cyclotron frequency, MHz	7.6	7.6	76
EIC wave harmonic number, N	8	20	20
RF frequency, MHz	61	152	1500
Ion-cyclotron radius, mm	0.73	2.3	1.6
Debye length, mm	0.03	0.03	0.02
Electric field penetration depth into the plasma, mm	7.5	2.4	0.24
Wavelength of EIC waves, mm	0.31	0.29	0.16
$\omega/(\omega-N\omega_{ci})$	5.6	13	26
Ion-ion energy equiparti-time, msec	0.08	0.2	0.7
Group velocity, cm/sec	1.2×10^{5}	4.9×10^{4}	1.4×10^{5}
Phase velocity, cm/sec	1.9×10^{6}	4.4×10^{6}	2.5×10^{7}
Heating efficiency, %	67	81	75
Quality factor Q	592	1896	31000

Table I. Representative calculations for three different set of parameters. The gas used is deuterium in all the three cases.

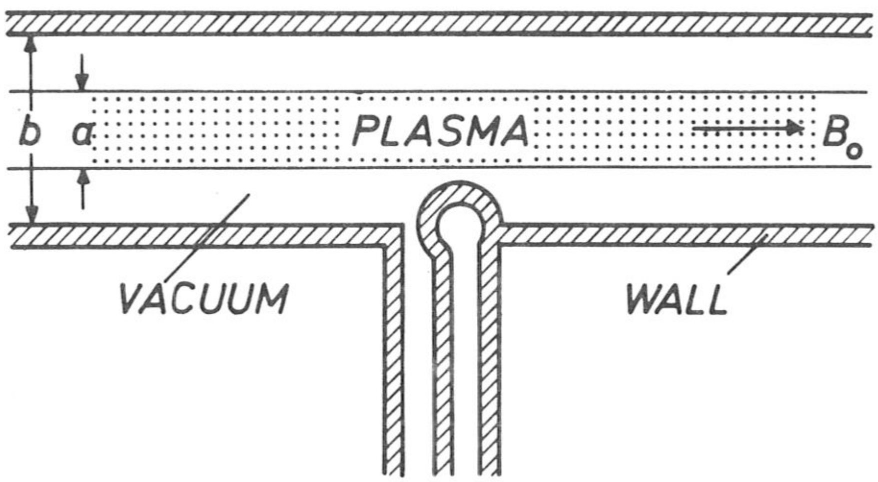

Fig. 1

The coupling geometry