Tachyon Mechanics and Classical Tunnel Effect

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IPP 6/95 May 1971

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

Abstract

A model mechanics is established for classical, charged point tachyons, with omission of Cerenkov radiation. The principle of relativity provides that several observables, e.g. electric charge, momentum, and energy of tachyons, do not show the usual Lorentz covariance, but transform as "metascalars", "metavectors", etc.; Lorentz covariant quantities are associable with them, but the signs of some of these are not observable. A critical survey of tachyon kinematics and of "supraluminal reference frames" is given. In tachyon dynamics it follows from the principle of relativity that the sign of the observable kinetic energy is Lorentz invariant, while the sign of the observable electric charge is not. Any reinterpretation of negative energies is neither required nor possible. Classical tachyons may undergo pseudocreation and pseudoannihilation, which differ from ordinary creation and annihilation in that their world points depend on the frame of reference. Closed world lines may occur. Tachyons show a classical tunnel effect, which is discussed in detail. Classical tachyon mechanics may be useful for developing a Maxwell-Vlasov theory of tachyon plasmas.

I. Introduction

Tachyons are hypothetical physical objects (elementary particles) with imaginary rest mass (1-7). Experimental evidence of their existence has not been found so far (8-11), but three theoretical models have been proposed:

(1) Tachyons as classical point particles that move faster than light (12).

(2) Tachyons as wave groups of classical Klein-Gordon fields of imaginary rest mass (6, 18, 19).

(3) Tachyons represented by quantized Klein-Gordon fields of imaginary rest mass (4, 5, 20-24).

The correspondence between the particle model 1 and the wave models 2, 3 is not very good, however. In view of the dispersion relation ($\hbar=c=1$)

$$\text{(I.1)} \qquad \omega^2 = \underline{k}^2 - M^2 ,$$

where $M_0 = iM$ is the imaginary rest mass, the $\underline{k}$-spectrum is incomplete and contains only wave numbers larger than the inverse of the Compton wavelength if only real frequencies are taken into account. Hence the corresponding "particles" are not localizable (4-6, 25, 26). Furthermore, in models 2, 3 signals cannot propagate faster than with the velocity of light, which is the velocity of the wave fronts. The supraluminal group velocity

(I.2) $$\left|\frac{d\omega}{d\underline{k}}\right| = \left|\frac{k}{\omega}\right| = \left|\frac{k}{\sqrt{k^2 - M^2}}\right| > 1$$

has no significance for the propagation of signals or information ([5, 6, 19]). In the particle model 1, on the other hand, only special properties of the tachyon interaction could interfere with the use of point tachyons as carriers of supraluminal signals. The particle model shows the further difficulty that the electromagnetic and gravitational Cerenkov radiation leads to unphysical infinities ([16, 17, 27]). On the other hand, a consistent quantized field theory of tachyons with interaction has not been established up to now ([5]).

Under the circumstances it seems improbable that tachyons, if they could be found, would have any properties resembling those of point particles. Consideration of a classical point particle model of tachyons can nevertheless provide some useful information, in the heuristic sense. There has been some discussion about the Lorentz covariance properties of tachyon observables and about the sign of tachyon energy. The model mechanics below provides a unique solution to these questions. It also describes some qualitatively new effects not found with tardons ([28]). Pseudoprocesses occur that differ from normal processes in that the world point of their occurrence and the occurrence itself are not Lorentz invariant. Tachyon world lines may be closed, and tachyons may undergo a classical tunnel effect. For simplicity all these results are derived by means of an electromagnetic interaction of tachyons, but with omission of Cerenkov radiation in order to avoid infinities. The kinematical sections of this paper are not restricted to tachyons alone, but apply to all kinds of supraluminal objects, e.g. supraluminal wave groups propagating in dispersive media. Special chapters concern supraluminal frames of reference and length and time intervals associated with supraluminal objects.

II. Modes of Description, Covariance and Observability

If the equations of tachyon mechanics are to be written in a form such that only observable quantities appear, then all equations, including definitions must satisfy the principle of relativity, i.e. they must have the same form in all frames of reference. We shall call this type of description the "laboratory picture". Hence, as a real-valued orbit parameter of world lines we use, instead of the proper time τ, the orthochronous proper path $\tilde{s}$, viz.

(II.1) $$d\tilde{s}^2 = d\underline{r}^2 - dt^2 ,$$

(II.2) $$\tilde{Z} = \mathrm{sign}\,(dt/d\tilde{s}) = 1 .$$

The sign of a time-interval Δt between two neighbouring points of a tachyon world line is not invariant with respect to orthochronous Lorentz transformations (L.T.); see eq.(III.10) below. On the other hand, according to Section IV, tachyon world lines may bend backwards and forwards in time. It follows that the proper path $\tilde{s}$ is usually a nonmonotonic function of the 4-length of a tachyon world line. Also it is not a Lorentz invariant. Because the sign of $d\tilde{s}$ is defined as the sign of dt, dt and $|d\tilde{s}|$ being observable, the orthochronous proper path element $d\tilde{s}$ itself is an observable.

We shall denote all quantities of the laboratory picture by a tilde ($\sim$). Quantities defined in this picture will not, in general, be Lorentz covariant (in the usual sense) because $d\tilde{s}$ itself is not Lorentz invariant. This holds, in particular, for the observables charge, momentum, and energy (see Section VIII).

It is advantageous to use a second picture in which all quantities have the usual covariance properties, but are not all completely observable, i.e. their signs cannot be determined. We call this description the "world picture". Here a scalar proper path s is used, viz.

(II.3) $$ds^2 = d\underline{r}^2 - dt^2,$$

(II.4) $$z = \mathrm{sign}(dt/ds) = \pm 1.$$

The quantity Z is obviously not Lorentz invariant. Each world line allows for a choice of two different world pictures, with $ds_1 = -ds_2$. Evidently the signs of ds_1 or ds_2 are not observable since no measuring prescription exists or could be given. The same holds for the other "world quantities" except those that equal their observable counterparts. In particular, the signs of covariant "charge", "momentum", and "energy" are unobservable.

III. Tachyon Kinematics

In this section we present the kinematic equations for tachyons and a critical interpretation of them that makes use of the results of the previous section. Some of the formulas have, of course, appeared in the literature before ([29]), but are included for completeness. It is convenient to set up tachyon kinematics in the covariant world picture first and switch to the observable laboratory picture later. The definition of velocity

$$\text{(III.1)} \qquad v^{\kappa} = \frac{dx^{\kappa}}{dt}, \qquad \kappa = 1, 2, 3,$$

may be written by introducing the space-like 4-velocity as

$$\text{(III.2)} \qquad u^{\mu} = \frac{dx^{\mu}}{ds} = u^{0} v^{\mu}, \qquad \mu = 0, 1, 2, 3,$$

with $x^0 = t$, $v^0 = 1$. Hence

(III.3) $$\frac{dt}{ds} = u^0 = Z\gamma \gtreqless 0,$$

where

(III.4) $$Z = \text{sign}\, u^0,$$

(III.5) $$\gamma = \sqrt{\underline{u}^2 - 1} = 1/\sqrt{\underline{v}^2 - 1} \geqq 0,$$

with $\underline{u}^2 = |u_k u^k|$. In a space-favoring metric one has $u_\mu u^\mu = +1$ for tachyons. In contradistinction to the tardon case the variables $\underline{v}^2, \underline{u}^2, \gamma$ extend over the following ranges:

(III.6) $$\begin{cases} 1 < \underline{v}^2 \leqq \infty \\ \infty > \underline{u}^2 \geqq 1 \\ \infty > \gamma \geqq 0. \end{cases}$$

The 4 -acceleration of a tachyon may, of course, be introduced in an analogous fashion.

It follows from Section II that u^μ and all quantities representable only by coordinate differentials and odd powers of ds are not observable. We obtain the observable quantities of the laboratory picture by using the orthochronous proper path $\tilde{s}$ and postulating that the kinematic equations holding for the observables are derived from eqs.(III.1) to (III.5) by substituting the pertinent observables into them. It follows that

$$\text{(III.7)} \quad \tilde{x}^{\mu} = x^{\mu}\,; \; \tilde{u}^{\mu} = Z u^{\mu}\,; \; \tilde{v}^{\mu} = v^{\mu}\,; \; \tilde{\gamma} = \gamma\,; \; \tilde{Z} = 1.$$

Obviously, $\tilde{u}^{\mu}$ is not a 4-vector. We may agree upon the following definition. Let $\underline{\alpha}$ be a 4-tensor of rank r, and let us call $\tilde{\underline{\alpha}} = Z\underline{\alpha}$ a "metatensor" of rank r (30). With this definition $d\tilde{s}$ is a metascalar, and $\tilde{u}^{\mu}$ is a metavector. Next, we discuss the transformation formulas for the 3-velocity $\underline{v}$. Upon a L.T. of the normal form

$$\text{(III.8)} \quad x' = \Gamma(x + Vt)\,; \; y' = y\,; \; z' = z\,; \; t' = \Gamma(Vx + t),$$

with $\Gamma = (1 - V^2)^{-1/2}$, V = relative velocity in the x-direction, between the two reference frames, the transformed velocity components are:

$$\text{(III.9)} \quad v_1' = \frac{v_1 + V}{1 + v_1 V}\,; \; v_2' = \frac{v_2}{\Gamma(1 + v_1 V)}\,; \; v_3' = \frac{v_3}{\Gamma(1 + v_1 V)}.$$

The formulas are the same for tachyons as for tardons. They do not give information, however, on "relative velocities between tachyons" because there is no real-valued L.T. that would transform a tachyon into a state of rest; see Section VI. Hence in eqs.(III.9) V is always subluminal, i.e. $|V| < 1$. It is seen that infinite velocities are not invariant. An observer who slowly moves in the direction of a tachyon, paradoxically, measures a greater tachyon speed than his colleague "at rest". When he accelerates his motion he finally measures an infinite, and then a reversed tachyon velocity. In view of the time reversal occurring at $\vartheta = \infty$ no kinematic contradictions follow as might appear at first sight. It is seen that upon L.T. the components ϑ_2, ϑ_3 change their signs if and only if the tachyon flight time changes sign, i.e. for

$$\text{(III.10)} \qquad \frac{dt'}{dt} = \frac{u'^0}{u^0} = \frac{Z'\gamma'}{Z\gamma} = \Gamma(1 + \vartheta_1 V) < 0.$$

For completeness we give the transformation of the flight time Δt of a free tachyon, i.e. the time interval between two points on a world line. It is

$$\text{(III.11)} \qquad \Delta t = u^0 \Delta t_0 = \frac{Z\,\Delta t_0}{\sqrt{\underline{\vartheta}^2 - 1}} ,$$

where $\Delta t = \Delta t_0$ for $\vartheta = \sqrt{2}$, or $u^0 = 1$. The pertinent flight distance $\underline{\Delta x}$ transforms as

$$\text{(III.12)} \qquad \underline{\Delta x} = \underline{\vartheta}\, u^0 \Delta t_0 = \frac{\underline{\vartheta}\, Z\, \Delta t_0}{\sqrt{\underline{\vartheta}^2 - 1}} .$$

Equation (III.11) may be transformed directly into eq.(V.8) to give the velocity dependence of the frequency of a supraluminal oscillator. On the other hand, eq.(III.12) is, of course, different from eq.(V.3), which gives the velocity dependence of the length of a supraluminal rod. In eqs.(III.11) and (III.12) the quantity Z is well-defined if the two endpoints of the interval are numbered.

IV. Pseudoprocesses and Conservation of Particles

In this section the qualitative aspects of tachyon kinematics are discussed by considering various cases of accelerated motion. It is advisable to treat the uniform motion of tachyons as a limiting case of the accelerated one, because otherwise there are frames of reference in which a tachyon has infinite velocity and cannot be localized in 3-space. New phenomena with tachyons, not present with tardons, are the occurrence of infinite velocities and the reversal of direction in time of a world line, when a suitable acceleration is effective (Fig. 1). In the world picture one may speak of a world line that bends backwards in time. In the laboratory picture two tachyons move forward in time and, as if by appointment, meet and vanish together in a world point A. A second observer, who is at rest in another frame of reference, observes a different annihilation point A', or no annihilation at all. Hence this phenomenon will be called "pseudoannihilation". World lines can also occur that are observed in the laboratory picture as a creation of two tachyons out of nothing (Fig. 2). We call this "pseudocreation" for the same reason. Pseudocreation or pseudoannihilation may be obtained by applying a L.T. to a piece of world line that does not exhibit such pseudoprocesses (Figs. 1, 2). Hence, these phenomena are purely kinematic. World lines exist which show the phenomenon of pseudoannihilation in one frame of reference S and the phenomenon of pseudecreation in another frame of reference S' (Fig. 3). In the example in Fig.3 the quantities v_x and v_x' must both go through zero. In a world with only one spatial dimension this would be impossible, because $|v| > 1$ for tachyons. The same holds for the occurrence of closed world lines (Fig. 4). These are observed in any frame of reference as the temporal sequence of pseudocreation and pseudoannihilation, with the possible exception of frames such that t = const on the world line. Closed world lines are exceptional in so far as the proper path s is a many-valued function of the world points on this line. This does not have the consequence that the world line is covered by infinitely many tachyons. A closed world line should be understood to represent one tachyon in the world picture, and two (or more) tachyons in the laboratory picture. An arbitrarily small perturbation of a closed world line may lead to a spiraling, open world line that represents an enormous number of tachyons in the laboratory picture. This discontinuity is not a physical one, however, since the "perturbation" is a mathematical device, not an experimental procedure. An example of a closed world line will be given in Section IX ([31]).

The number of tachyons is not a Lorentz invariant ([4]), as can easily be seen from the examples considered. Furthermore, classical tachyons do not obey conservation in time of the observable number of particles. However, there exists an analogous theorem for the unobservable quantity Z. It is true that in the world picture the observable number $\tilde{Z}$ is constant along world lines, i.e.

$$\text{(IV.1)} \quad d\tilde{Z}/ds = 0 \quad ; \quad \tilde{Z} \equiv 1.$$

In the laboratory picture, however, one has only

$$\text{(IV.2)} \quad \sum Z = \text{constant in time,}$$

where the sum $\sum$ extends over the isochronous points of the world line. The situation is different in the case of conservation of charge; see Section VIII.

V. Transformation of Length and Oscillation Frequency of Supraluminal Objects

Supraluminal "objects" may be of a mathematical (kinematical) or of a physical nature, as in the theory of waves or in the theory of spatially distributed tachyons ([13]). We shall regard "tachyon rods" and "tachyon clocks" primarily as fictitious or mathematical objects that may be defined with reference to subluminal observers. We do not mean to introduce any "supraluminal observers" having measuring rods and clocks at their disposal because the notion of "supraluminal frames of reference" appears to introduce more problems than advantages; see Section VI.

A "tachyon rod" is defined by the position of its two endpoints, viz.

$$(V.1) \qquad \underline{x}_i = \underline{v}t + \underline{L}_i , \quad i = 1, 2,$$

with $\underline{v}$ and $\underline{L}_i$ constant in time. The rod length is given by the vector

$$(V.2) \qquad \underline{L} = \underline{L}_2 - \underline{L}_1 .$$

A simple calculation shows that upon L.T. of the normal form [eq.(III.8)] the rod length transforms thus:

$$(V.3) \quad \begin{cases} L'_x = \Gamma (1 - v'_x V) L_x \\ L'_\perp = L_\perp - \Gamma V v'_\perp L_x , \end{cases}$$

where $\perp$ stands for y and z. If $\underline{v}$ and $\underline{L}$ are parallel, then

$$(V.4) \qquad L' = \Gamma (1 - v'V) L = L / [\Gamma (1 + vV)] .$$

These equations hold for tardons, luxons, and tachyons. By eliminating V from eq.(V.4) one obtains for tachyons:

$$(V.5) \qquad L = L_0 / u^0 = L_0 \sqrt{\underline{v}^2 - 1} \; \mathrm{sign}(dt/ds) .$$

The reference length L_0 equals the rod length for $|v| = \sqrt{2}$. The length L becomes infinite for $v = \infty$, it may become negative (the two ends of the rod change their sequence in space), and for $|v| \to 1$ the length goes to zero. Hence, one has cases of Lorentz contraction and Lorentz dilatation as compared to L_0.

A "supraluminal clock", or "supraluminal oscillator" will be defined by its position in ordinary space and by its Lorentz invariant phase, viz.

$$\text{(V.6)} \quad \underline{x} = \underline{v} t \; ; \; \varphi = \omega t ,$$

with $\underline{v}$ and ω constant in time. Again a simple calculation gives the transformed frequency

$$\text{(V.7)} \quad \omega' = \Gamma (1 - v_x' V) \omega = \frac{\omega}{\Gamma (1 + v_x V)} .$$

Hence, for tachyons:

$$\text{(V.8)} \quad \omega = \omega_0 / u^0 = \omega_0 \sqrt{\underline{v}^2 - 1} \; \operatorname{sign}(dt/ds) ,$$

in agreement with eq.(III.11). Infinite frequencies occur for $v = \infty$, and "time reversal" (negative ω) may occur as well as red shift (time dilatation) and blue shift (time contraction) relative to ω_0. In eqs. (V.5) and (V.8) the scalar proper path element ds can be made well-defined if the rod ends are numbered and if positive and negative times can be distinguished on the "clock".

VI. Supraluminal Frames of Reference

Let us consider the question in what sense "supraluminal frames of reference" could be defined (32-34) even though no real-valued L.T. exists that transforms a tachyon into a state of rest.

Some authors (33, 34) have endeavored to establish new theories by supplementing the mathematical definition of supraluminal reference frames by additional postulates of a physical nature. Since all of physics, including tachyons, can be formulated completely using subluminal frames of reference, it appears, however, that there is no freedom to introduce additional postulates. Consider, for instance, the proposal (33, 34) of generalizing the principle of relativity in such a way that a "supraluminal observer" could not decide whether he is moving more slowly or faster than light. Of course, the same would have to hold for a subluminal observer as well. A reciprocity relation between mutually supraluminal reference frames would have to hold. But this contradicts the fact that two tardons which move side by side on straight, parallel paths can exchange light signals, while two tachyons moving in the same way cannot. This fundamental asymmetry cannot be "postulated away". Parker (34) himself asserts that his proposal could be realized only for a space of one dimension.

Another possibility seems to be introducing supraluminal frames of reference as a mathematical device only. In this way one gains another mathematical representation of the same tardon or tachyon physics. For instance, one may employ the following pseudo-Lorentz transformation [33, 34]

$$\text{(VI.1)} \quad x' = \Gamma^*(x + Vt)\,;\; y' = y\,;\; z' = z\,;\; t' = \Gamma^*(Vx + t),$$

with $|V| > 1$, $\Gamma^* = (V^2 - 1)^{-1/2}$. However, because of

$$\text{(VI.2)} \quad ds^2 = -(dx')^2 + (dy')^2 + (dz')^2 + (dt')^2$$

it follows that in the new "frame of reference" the 3-space has a metric that is not only anisotropic, but indefinite. The analogy with the Schwarzschild metric inside the Schwarzschild sphere is evident. The new representation is thus more complicated than the old one, new physical results cannot be derived in this way, and in order to make theoretical calculations, one will rather prefer spatially isotropic frames of reference.

The following question seems to lie behind the proposals for supraluminal frames of reference: "Provided that supraluminal observers having their own measuring procedures for length and time intervals existed, could information on the natural physical definition of length and time valid for such observers be obtained from general principles only?" In the case of subluminal frames of reference the answer to the corresponding question is yes; in essence, it is sufficient here to employ the principle of relativity and the constancy of the velocity of light, so that detailed theories of measuring rods and clocks are not needed. The same would not be true of "supraluminal observers" because the principle of relativity does not apply in the sense of a reciprocity between subluminal and supraluminal observers. Only detailed theories of supraluminal measuring procedures and apparatus could be of help, if they could be meaningfully formulated at all. In particular, this is true if one wishes to define "relative velocities between tachyons" in an operational sense and not just as a mathematical expression devoid of physical content.

As a result "supraluminal frames of reference" appear to offer neither mathematical simplification nor new physical information.

VII. Tachyon Dynamics

If the momentum and energy of a free tachyon form a space-like 4-vector - as is true in the world picture - then the sign of the energy changes upon suitable L.T. ([4]), cf.eq. (III.10). We shall show below that the observable energy does not change its sign, however, and that the occurrence of negative observable kinetic energies, can easily be excluded without any ad hoc reinterpretation.

In the following, we consider the classical Hamilton dynamics of a charged tachyon moving in an electromagnetic field. The Cerenkov radiation and the Cerenkov self-force (13, 16, 17) are omitted because both lead to divergence difficulties in the frame of classical theory. Hence, this mechanics can be coupled in a consistent way to Maxwell's equations with tachyonic sources only if the Vlasov approximation (35-37) is employed. Apart from the omission of Cerenkov radiation, the electromagnetic field introduced in this manner is a perfectly normal e.m. field that acts on tardons and tachyons alike. As mentioned earlier, this model mechanics is useful in that it allows discussing the definition of the observables, charge, momentum, and energy, and it demonstrates novel effects such as the classical tunnel effect, or the occurrence of closed world lines.

The Hamiltonian mechanics of a charged tachyon in the world picture is obtained by formal analogy with tardon mechanics, i.e. by substituting

(VII.1) $$m_0 = \pm i M \; ; \quad d\tau = \pm i ds \; ; \quad H_0 = \pm i H$$

in the equations of motion of a charged tardon. Here H_0, m_0, τ are the Hamiltonian, the rest mass, and the proper time of a tardon; H and M are the Hamiltonian and the "proper mass" of a tachyon, M being a real-valued scalar. Hence, the Hamiltonian of a charged tachyon, in the world picture, is

(VII.2) $$H = \frac{\sigma}{2M}\left(p_\lambda - qA_\lambda\right)\left(p^\lambda - qA^\lambda\right),$$

and the Hamilton equations read

(VII.3) $$\frac{dx^\lambda}{ds} = \frac{\partial H}{\partial p_\lambda}\,; \quad \frac{dp^\lambda}{ds} = -\frac{\partial H}{\partial x_\lambda}.$$

Here $q \gtrless 0$ is the invariant charge, p^λ is the covariant, canonical 4-momentum, A^λ is the 4-potential, and $\sigma = \pm 1$ is a constant scalar yet to be fully determined. As mentioned, all quantities are Lorentz-covariant, but not necessarily fully observable. From eq. (VII.3) it follows in the usual way that

(VII.4) $$p^\lambda = \sigma M u^\lambda + qA^\lambda,$$

(VII.5) $$\frac{dp^\lambda}{ds} = q u_\mu \frac{\partial A^\mu}{\partial x_\lambda}.$$

The kinetic momentum and energy are defined by the 4-vector

(VII.6) $$W^\lambda = \sigma M u^\lambda = p^\lambda - qA^\lambda.$$

Hence, the equations of motion are

(VII.7) $$\frac{dW^\lambda}{ds} = q u_\mu F^{\lambda\mu}\,; \quad F^{\lambda\mu} = A^{\mu,\lambda} - A^{\lambda,\mu},$$

or in vectorial form:

$$\text{(VII.8)} \quad \begin{cases} \dfrac{d\underline{W}}{dt} = q(\underline{E} + \underline{v} \times \underline{B}) \\ \dfrac{dW^0}{dt} = \underline{v} \cdot \dfrac{d\underline{W}}{dt} = q\underline{v} \cdot \underline{E}, \end{cases}$$

with

$$\text{(VII.9)} \quad W^0 = \sigma Z M_\gamma; \quad \underline{W} = \sigma Z M_\gamma \underline{v} = W^0 \underline{v}.$$

As is seen, the equations of tachyon dynamics are almost literally the same as those for tardons.

To obtain the observable quantities and the equations of the laboratory picture, one must again introduce the orthochronous proper path element $d\tilde{s} = Z\,ds$ and employ the implicit definition that the dynamical equations in the laboratory picture are obtained by replacing all covariant quantities by the pertinent observables (designated by $\sim$). The quantities $\tilde{M}$ and $\tilde{\sigma}$ must be defined as scalars, too. It is convenient to choose $\tilde{M} = M, \tilde{\sigma} = \sigma$. It follows that the observables are given by

$$\text{(VII.10)} \quad \tilde{H} = H; \quad \tilde{q} = Zq; \quad \tilde{p}^\lambda = Zp^\lambda; \quad \tilde{W}^\lambda = ZW^\lambda.$$

Here $\tilde{q}$ is the observable charge, $\tilde{p}^\lambda$ is the observable canonical 4-momentum, and $\tilde{W}^\lambda$ is the observable kinetic 4-momentum. One notices the possible difference in sign of charge, momentum, and energy in the two pictures. The constant σ will be determined below.

VIII. Charge and Energy of Tachyons

The properties of the electric charge of tachyons are discussed first. Inspection of the equations in the preceding section reveals the following: (see also Fig. 5)

1. The magnitudes of q and $\tilde{q}$ are equal, but their signs may differ.
2. While q is a scalar, the observable charge $\tilde{q}$ is a metascalar, i.e. the sign of $\tilde{q}$ may change upon L.T.([38]).
3. While q is constant along a world line when s is defined as a monotonic function of the 4-length, $\tilde{q}$ changes its sign along a world line at all points with $\vartheta = \infty$.
4. The sign of q has no physical meaning since it depends on the direction of increasing s, which can be chosen arbitrarily. One may, for instance, agree upon a definition of s such that $q \geqq 0$ for all tachyon world lines.

It follows that the theorem of charge conservation assumes the following two forms for tachyons. In the world picture one has

$$\text{(VIII.1)} \qquad \frac{dq}{ds} = 0,$$

while in the laboratory picture one has instead

(VIII.2) $\sum \tilde{q}$ = constant in time,

where the sum extends over the isochronous points of the world line. Contrary to the conservation of the number of particles (Section IV), the laboratory form of the conservation theorem holds for the observable quantity $\tilde{q}$.

Next, tachyon energy is discussed. The relations $\tilde{W}^0 = Z W^0$ and $\tilde{p}^0 = Z p^0$ are reminiscent of Bilaniuk, Deshpande, and Sudarshan's reinterpretation of negative energies [1,4]. However, no reinterpretation is required or even possible in our theory since W^0 and p^0 are not observables. The observables $\tilde{W}^0$ and $\tilde{p}^0$ have been well defined to begin with. From the preceding section one derives the following results relating to energies (see also Fig. 5):

1. Observable energies $\tilde{W}^0, \tilde{p}^0$ and covariant energies W^0, p^0 may differ by their signs, their magnitudes being equal.
2. While covariant energies are, of course, zero components of 4-vectors, observable energies are zero components of metavectors. Hence, the sign of an observable kinetic energy is Lorentz invariant, while the sign of a covariant kinetic energy is not [38].
3. The sign of the observable kinetic energy $\tilde{W}^0$ is essentially constant along a world line, while the sign of W^0 changes at all points with $v = \infty$.

4. The sign of W^0 is unobservable because it depends on the definition of s.

5. The sign of the observable kinetic energy $\tilde{W}^0$ is zero or equal to σ, i.e. one could introduce two classes of tachyons, with $\sigma = +1$ and $\sigma = -1$ respectively. However, negative-energy tachyons are undesirable because they lead to thermodynamic non-equilibrium if they interact with tardon matter ([39]). Hence $\sigma = +1$ is to be chosen in Section VII in order to obtain tachyons with non-negative definite kinetic energies:

$$\tilde{W}^0 = M_\gamma = M/\sqrt{\underline{v}^2 - 1} \geqq 0 . \tag{VIII.3}$$

Finally, the conservation of total energy (= canonical energy) is discussed. For time-independent fields the conservation law in the world picture reads:

$$\frac{dW^0}{ds} + q\frac{d\phi}{ds} = 0, \tag{VIII.4}$$

or $W^0 + q\phi$ = const along a world line. In the laboratory picture one has instead:

$$\sum\left(\tilde{W}^0 + \tilde{q}\phi\right) = \text{constant in time}, \tag{VIII.5}$$

where, again, the sum extends over the isochronous points of the world line, and $\nabla\phi = -\underline{E}$. Analogous formulas hold if momentum is conserved.

Upon a L.T. of the normal form, see eq. (III.8), observable charge $\tilde{q}$ and observable kinetic energy-momentum transform in the following way:

(VIII.6) $$\tilde{q}' = \alpha \tilde{q}$$

and

(VIII.7) $$\begin{cases} \tilde{W}'^{0} = \alpha \Gamma (\tilde{W}^{0} + V \tilde{W}^{x}) \\ \tilde{W}'^{x} = \alpha \Gamma (\tilde{W}^{x} + V \tilde{W}^{0}) \\ \tilde{W}'^{y} = \alpha \tilde{W}^{y} \; ; \; \tilde{W}'^{z} = \alpha \tilde{W}^{z} \end{cases} .$$

with

(VIII.8) $$\alpha = Z Z' = \mathrm{sign}(1 + v_x V).$$

Of course, the transformation of the observable, canonical energy-momentum agrees with eq. (VIII.7).

IX. Closed World Lines and Tachyon Gyration in a Magnetic Field

As an application of the equations of motion we now consider the gyration of a charged tachyon in a homogeneous $\underline{B}$-field, first with $\underline{E} \equiv 0$. For a $\underline{B}$-field in the z direction the tachyon orbit is simply

$$
\text{(IX.1)} \quad \begin{cases} x - x_0 = R\cos(\Omega t) = R\cos(\Omega_0 s) \\ y - y_0 = -R\sin(\Omega t) = -R\sin(\Omega_0 s) \\ z - z_0 = v_z t \end{cases}
$$

with

$$
\text{(IX.2)} \quad \begin{cases} \Omega_0 = qB/M \; ; \; \Omega = \Omega_0/u^0 \; ; \\ R = |v/\Omega| = |Mu/qB| \; ; \\ t = u^0 s \; ; \; u^0 = \text{const} \, . \end{cases}
$$

The solution is formally identical with that holding for a charged tardon. However, the case $v = \infty$ is qualitatively new. In this case the world line is closed (Fig. 6a), and $t \equiv 0$, $\Omega = \infty$, $R = |M/qB|$. The laboratory picture becomes singular, similar to the case of a free tachyon with $v = \infty$. The world picture remains regular. By a L.T. in the x-direction one obtains crossed $\underline{E}$ and $\underline{B}$ fields (with $E'_y \neq 0$), and the closed world line now extends over a non-vanishing, finite time interval, $t'_1 \leq t' \leq t'_2$ (see Fig. 6b). This is a dynamical example for the closed world lines introduced in Section IV. Even though in the example the electric and magnetic fields extend through the whole space, this is not necessary; clearly it suffices for the fields to extend over a finite volume that encloses the tachyon orbit.

X. Classical Tunnel Effect of Tachyons

A classical, charged tardon can reach the other side of a potential hill only if $p^0 \geqq (q\phi)_{max}$ holds. In quantum mechanics passage through the hill is also possible if $p^0 < (q\phi)_{max}$; this is the well-known quantum tunnel effect. Surprisingly, the same possibility also exists in classical tachyon mechanics, although for a different reason. We call this phenomenon the "classical tunnel effect". It is associated with a real charge reversal, in the laboratory picture.

We shall (theoretically) demonstrate this effect and give the conditions for its occurrence. For simplicity we limit ourselves to the case of a static electric field $\underline{E}(x)$ without magnetic field. The equations of motion assume the following form in the world picture:

$$
\text{(X.1)} \quad \begin{cases} \dfrac{d\underline{u}}{ds} = \dfrac{q}{M} u^0 \underline{E} \ ; & \dfrac{d\underline{x}}{ds} = \underline{u} \ ; \\[2ex] \dfrac{du^0}{ds} = \dfrac{q}{M} \underline{u} \cdot \underline{E} \ ; & \dfrac{dt}{ds} = u^0 . \end{cases}
$$

Since p^0, u^y and u^z are constants of the motion, one has for $\underline{E}(x)$ in the x-direction (plane geometry):

$$(X.2)\quad \begin{cases} u^0 = \dfrac{p^0 - q\phi}{M} \\ u^x = \sigma^x \left(\dfrac{(p^0 - q\phi)^2}{M^2} - u_\perp^2 + 1 \right)^{1/2} \\ u^y = \text{const} \; ; \quad u^z = \text{const} , \end{cases}$$

with

$$(X.3)\quad u_\perp^2 = (u^y)^2 + (u^z)^2 \; ; \quad \sigma^x = \text{sign}\, u^x .$$

These equations demonstrate that a tachyon that hits a potential hill shows a qualitatively different behavior, depending on whether upon increasing $(q\phi)$ it is u^x or u^0 that goes to zero first. The first case, ordinary reflection of the tachyon, occurs for $|v_x| < 1$, i.e. $u_\perp > 1$. The second case, the classical tunnel effect, occurs for $|v_x| > 1$, i.e. $u_\perp < 1$. Since in the case of the tunnel effect the sign of u^0 changes twice, the sign of the observable charge $\tilde{q}$ also changes twice, once in front of the potential hill, and once at the rear. In the case $|v_x| \equiv 1$, i.e. $u_\perp = 1$, a singular form of motion occurs with an orbit that approaches, in a finite time interval but asymptotically in space, the plane $p^0 - q\phi(x) = 0$, with $v_\perp$ becoming infinite.

We first consider the special case $u_\perp = 0$ of the classical tunnel effect. Figure 7a shows the potential hill $q\phi(x) \geqq 0$ and the orbit of the tachyon. Within the hill the direction of the velocity and the sign of the observable charge $\tilde{q}$ are reversed. Hence, the tachyon leaves the inside of the potential hill earlier on the right-hand side than it enters the inside on the left-hand side. Figure 7b shows the temporal progress of the tunneling motion. Because of the charge reversal conservation of energy holds throughout the whole process, in contradistinction to the quantum tunnel effect.

The general case of tunneling, $u_\perp < 1$, is visualized most easily in the $\tilde{u}$-space. Figure 8 shows the function $\tilde{u}^0(\tilde{u}^x)$ at the point of entry, $x = x_1$, and at the point of exit, $x = x_2$. The arrows indicate the positive direction of time. It follows that safe confinement of charged tachyons by electric fields is not possible.

XI. Conclusion

We have shown that a consistent classical point mechanics of charged tachyons without Cerenkov self-force can be established. Problems of causality have not been considered [40-46]. As compared to tardons the following new phenomena may occur with classical tachyons:

1. Closed world lines.

2. Real charge inversion by L.T.

3. Pseudoprocesses, with violation of conservation of the particle number.

4. Classical tunnel effect.

5. Length reversal and Lorentz dilatation for "tachyon rods"; time reversal and time contraction for "tachyon clocks".

6. Some observables transform differently from the corresponding tardon observables, viz. as "metascalars", "metavectors", etc.

It is the very principle of relativity that provides for the tachyon observables not to be Lorentz covariant in general. Negative kinetic energies are not observable in the mechanics considered; hence, they do not form a problem that would have to be "postulated away".

As mentioned in the introduction, it is questionable whether classical point particle theory could describe any tachyon phenomena, provided tachyons existed. The main reasons for this scepticism are the apparent disagreement with field theory, the divergence of Cerenkov radiation, and, perhaps, reasons of macrocausality. On the other hand, it is thought that the results concerning covariance properties are of heuristic value - in fact they have their analogues in field theoretical results obtained by Ecker ([5]). Furthermore, the classical tachyon mechanics considered yields results, such as the classical tunnel effect, that seem interesting in themselves ([47]).

Acknowledgments

The author thanks Dr. G. Küppers, Dr. D. Lortz, Professor D. Pfirsch, and Dr. H. Tasso for valuable discussions.

This work was performed under the terms of the agreement on association between Max-Planck-Institut für Plasmaphysik and Euratom.

References

(1) O.M.P. Bilaniuk, V.K. Deshpande, E.C. G. Sudarshan, Am. J. Phys. 30, 718 (1962).

(2) O.M.P. Bilaniuk, E.C.G. Sudarshan, Physics Today 22 (5), 43 (1969).

(3) O.M.P. Bilaniuk et al., Physics Today 22 (12), 47 (1969).

(4) G. Feinberg, Phys. Rev. 159, 1089 (1967).

(5) G. Ecker, Annals of Phys. 58, 303 (1970). Further references are given there.

(6) R. Fox. C.G. Kuper, S.G. Lipson, Proc. Roy. Soc. Lond. A 316, 515 (1970).

(7) J. Strnad, Fortschritte der Physik 18, 237 (1970). Further references are given there.

(8) T. Alväger, M.N. Kreisler, Phys. Rev. 171, 1357 (1968).

(9) C. Baltay, G. Feinberg, N. Yeh, Phys. Rev. D 1, 759 (1970).

(10) M.B. Davis, M.N. Kreisler, T. Alväger, Phys. Rev. 183, 1132 (1969).

(11) G. Feinberg, Sci. Am. 222 (2), 68 (1970).

(12) Some of the earlier papers, e.g. Refs. (1-3), do not clearly define the model used. At any rate equations of motion for classical point particles are not set up there. Sommerfeld ([13]) considers classical tachyons with a continuous distribution of charge within a finite volume. Glück ([14]) investigates the classical motion of a point

tachyon in a Coulomb field and introduces Bohr-Sommerfeld quantization rules, but does not discuss the principles of tachyon mechanics. Leiter ([15]) introduces classical charged point tachyons including Cerenkov radiation. However, his theory leads to infinite tachyon self-acceleration ([13, 16, 17]). Neither Glück nor Leiter discusses the observability of the mechanical tachyon variables or the general consequences of the theory.

(13) A. Sommerfeld, Gesammelte Schriften, Vol. II (Vieweg & Sohn, Braunschweig, 1968).

(14) M. Glück, Nuovo Cim. 67A, 658 (1970).

(15) D. Leiter, Lett. Nuovo Cim. 1, 395 (1971).

(16) R.G. Cawley, Phys. Rev. D2, 276 (1970).

(17) H.K. Wimmel, Lett. Nuovo Cim. 1, 645 (1971).

(18) H. Schmidt, Z. Physik 151, 365 and 408 (1958).

(19) R. Fox, C.G. Kuper, S.G. Lipson, Nature 223, 597 (1969).

(20) M.E. Arons, E.C.G. Sudarshan, Phys. Rev. 173, 1622 (1968).

(21) J. Dhar, E.C.G. Sudarshan, Phys. Rev. 174, 1808 (1968).

(22) D. Korff, Z. Fried, Nuovo Cim. 52A, 173(1967).

(23) E. Marx, Int. J. Theor. Phys. 3, 299 (1970).

(24) S. Tanaka, Progr. Theor. Phys. 24, 171 (1960).

(25) D. Pfirsch, private communication.

(26) A. Peres, Lett. Nuovo Cim. 1, 837 (1969).

(27) A. Peres, Phys. Letters 31A, 361 (1970).

(28) Normal, subluminal particles, i.e. with $|v| < c$, are called "tardons", "tardyons", "bradyons" or "ittyons", depending on the author(s) involved.

(29) Compare, for example, Ref. ([4]).

(30) The prefix "meta" is to recall "metarelativity", a term coined by Bilaniuk et al. ([1]) in connection with tachyons.

(31) Closed tachyon world lines were proposed by H. Tasso, private communication.

(32) K.H. Mariwalla, Am. J. Phys. 37, 1281 (1969).

(33) V.S. Olkhovsky, E. Recami, Lett. Nuovo Cim. 1 (4), 165 (1971).

(34) L. Parker, Phys. Rev. 188, 2287 (1969).

(35) D.C. Montgomery, D.A. Tidman, Plasma Kinetic Theory (McGraw Hill Book Co. 1964).

(36) Ta-You Wu, Kinetic Equations of Gases and Plasmas (Adison-Wesley Pub. Co. 1966).

(37) E.S. Weibel, Plasma Physics 9, 665 (1967).

(38) An analogous result is obtained by Ecker from quantum field theory of free tachyons; see Ref. ([5]).

(39) Ya. P. Terletskii, Paradoxes in the Theory of Relativity (Plenum Press, New York, 1968).

(40) Y. Aharonov, A. Komar, L. Susskind, Phys. Rev. 182, 1400 (1969).

(41) G.A. Benford, D.L. Book, W.A. Newcomb, Phys. Rev. D 2, 263 (1970).

(42) O.M. Bilaniuk, E.C.G. Sudarshan, Nature 223, 386 (1969).

(43) R.G. Newton, Science 167, 1569 (1970).

(44) F.A.E.Pirani, Phys. Rev. D 1, 3224 (1970).

(45) W.B. Rolnick, Phys. Rev. 183, 1105 (1969).

(46) R.G. Root, J.S. Trefil, Lett. Nuovo Cim. 3, 412 (1970).

(47) After completing this paper an article by M. Baldo, G. Fonte, E. Recami, Lett. Nuovo Cim. 4, 241 (1970), came to the attention of the author. There a discussion of the principle of relativity and of the definition of charge is given also, though under the aspect of quantum theory.

Figure Captions

Fig. 1 Tachyon world lines with pseudoannihilation.

Fig. 2 Tachyon world lines with pseudocreation.

Fig. 3 Tachyon world line (projection in x-t plane) with pseudoannihilation at $\underline{A}$ in the frame S, and pseudocreation at $\underline{C}'$ in the frame S'.

Fig. 4 Closed tachyon world line with $z =$ const, extending in the time interval $t_1 \leqq t \leqq t_2$.

Fig. 5 The signs of electric charge and kinetic energy along a tachyon world line (projection in x-t plane).

Fig. 6 a: Tachyon in a magnetic field gyrating with infinite gyrofrequency.
b: Application of L.T. to a.

Fig. 7 a: Potential energy versus x, and tunneling path of tachyon.
b: Tunneling motion in the x-t diagram.

Fig. 8 Tachyon tunneling in the $\tilde{u}^x - \tilde{u}^0$ diagram, for the entry point $x = x_1$ and the exit point $x = x_2$. Thearrows indicate the direction of increasing time.

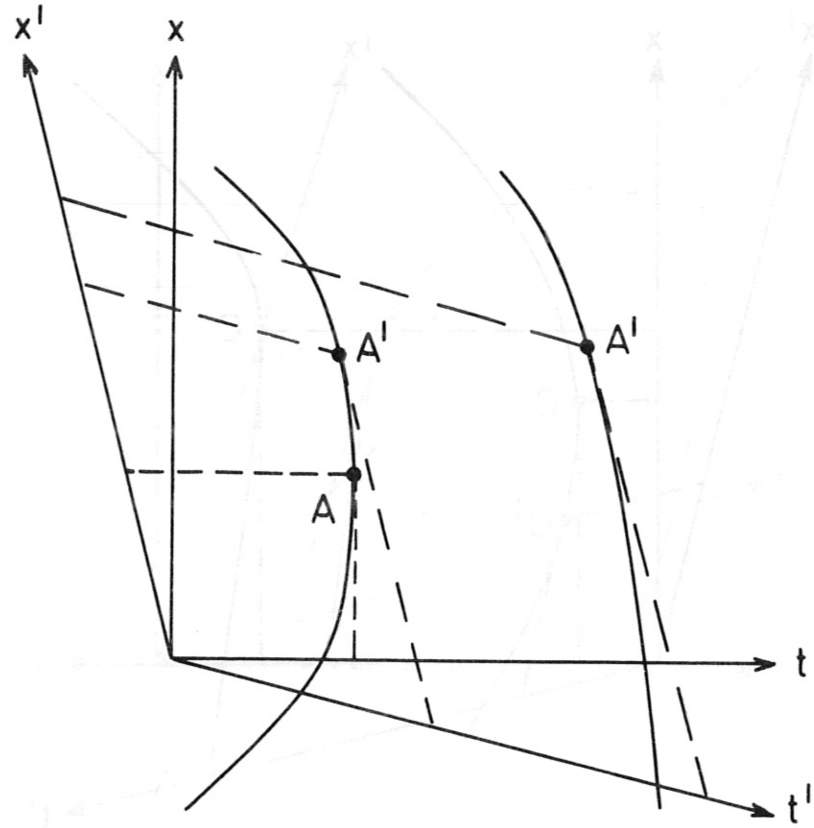

Fig. 1

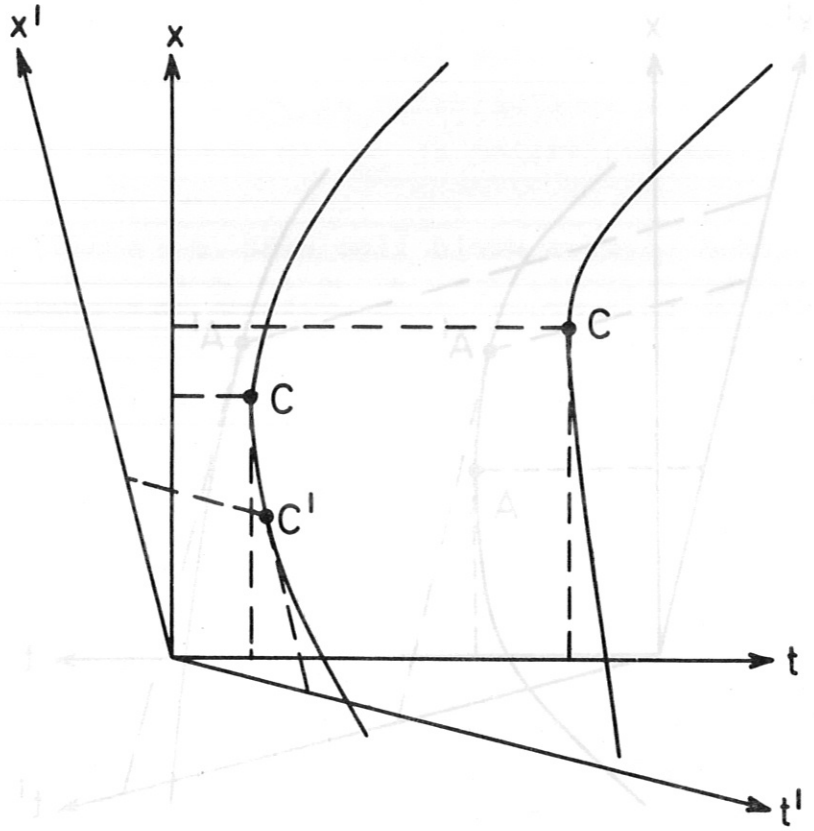

Fig. 2

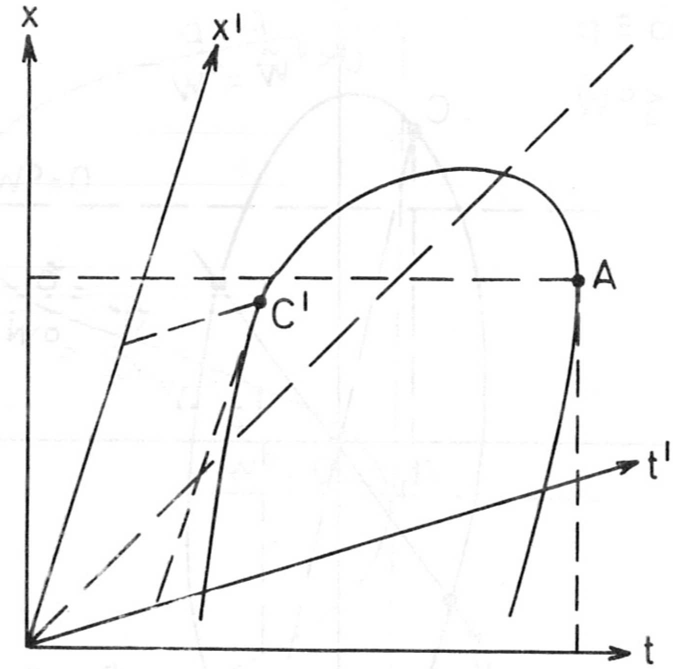

Fig. 3

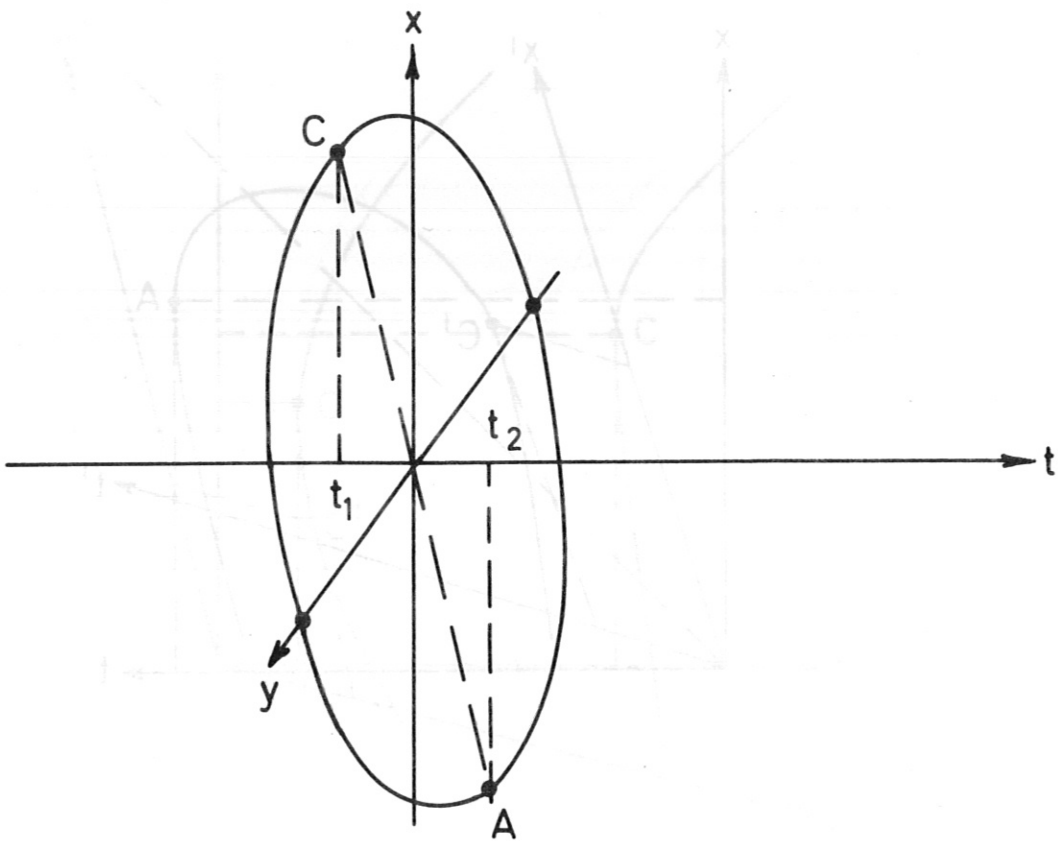

Fig. 4

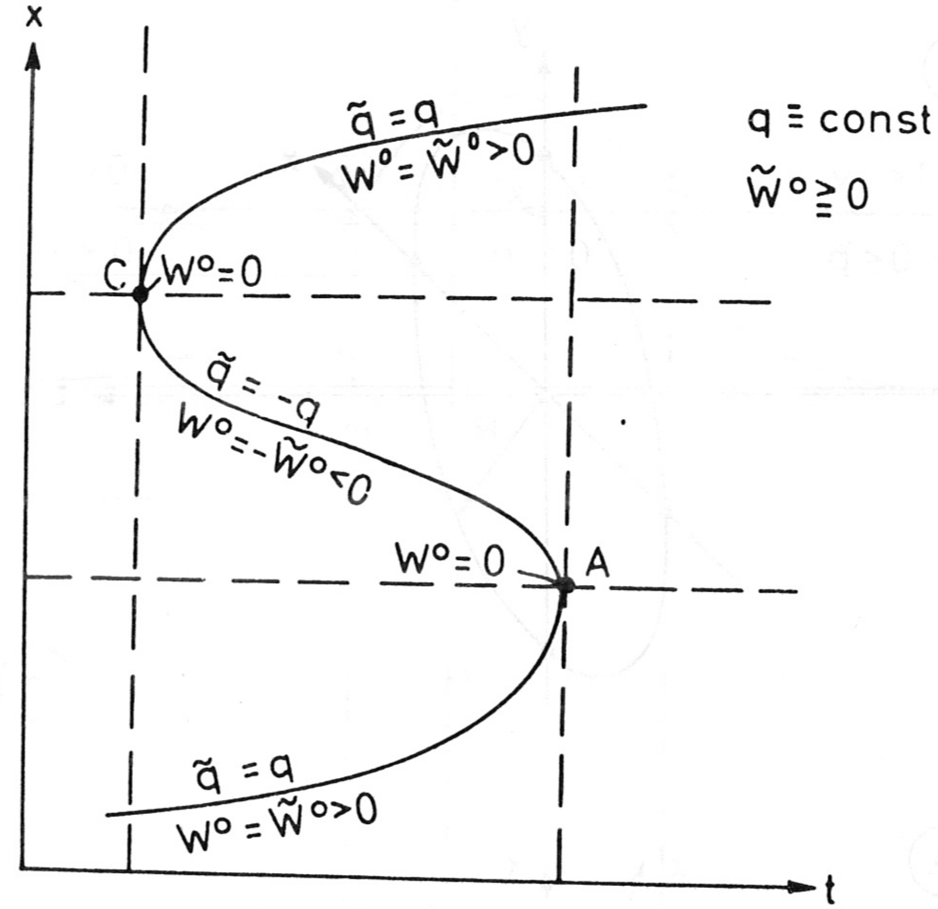
x
$\tilde{q} = q$
$W^o = \tilde{W}^o > 0$
$q \equiv \text{const}$
$\tilde{W}^o \geqq 0$
C
$W^o = 0$
$\tilde{q} = -q$
$W^o = -\tilde{W}^o < 0$
$W^o = 0$
A
$\tilde{q} = q$
$W^o = \tilde{W}^o > 0$
t

Fig. 5

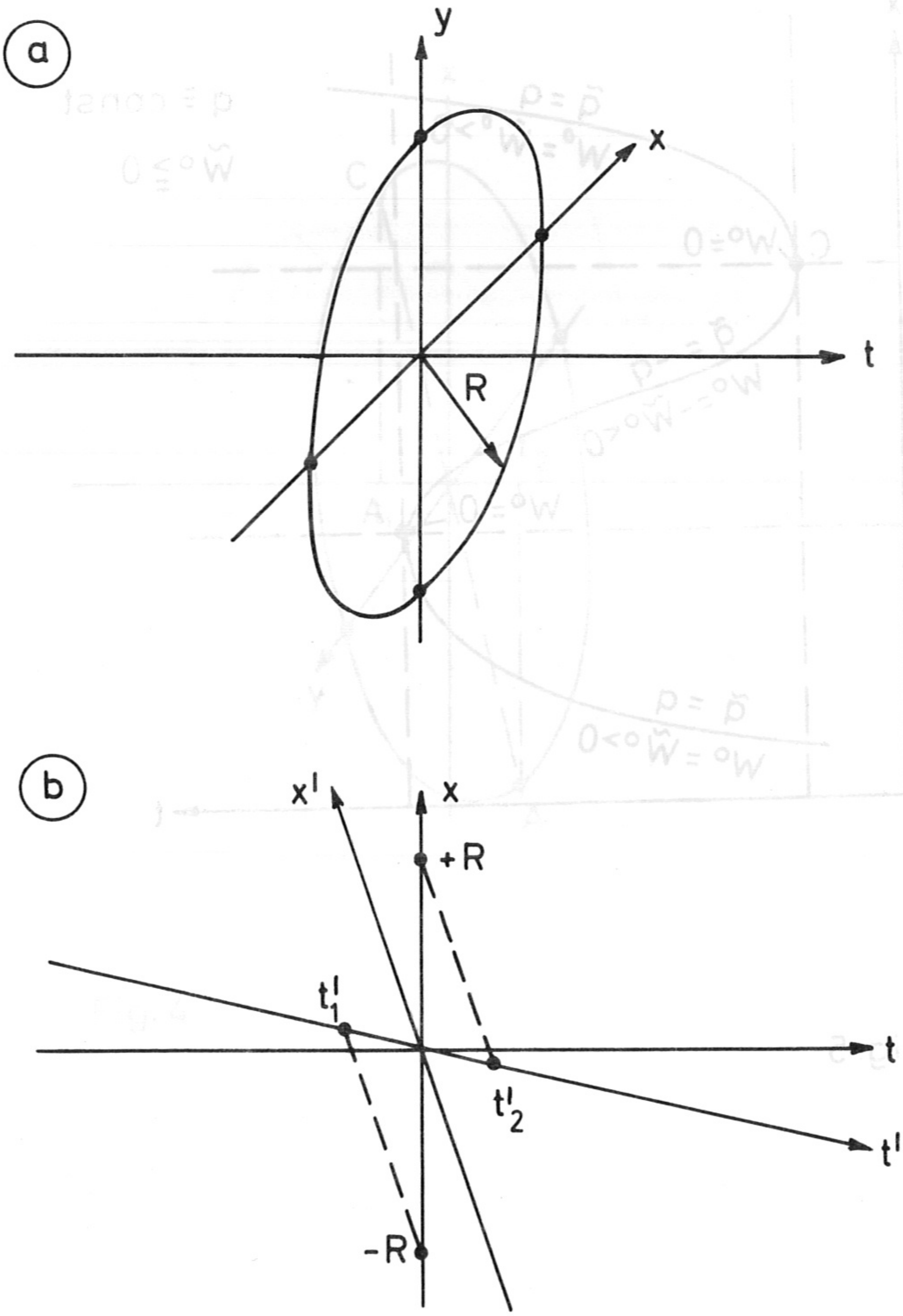

Fig. 6

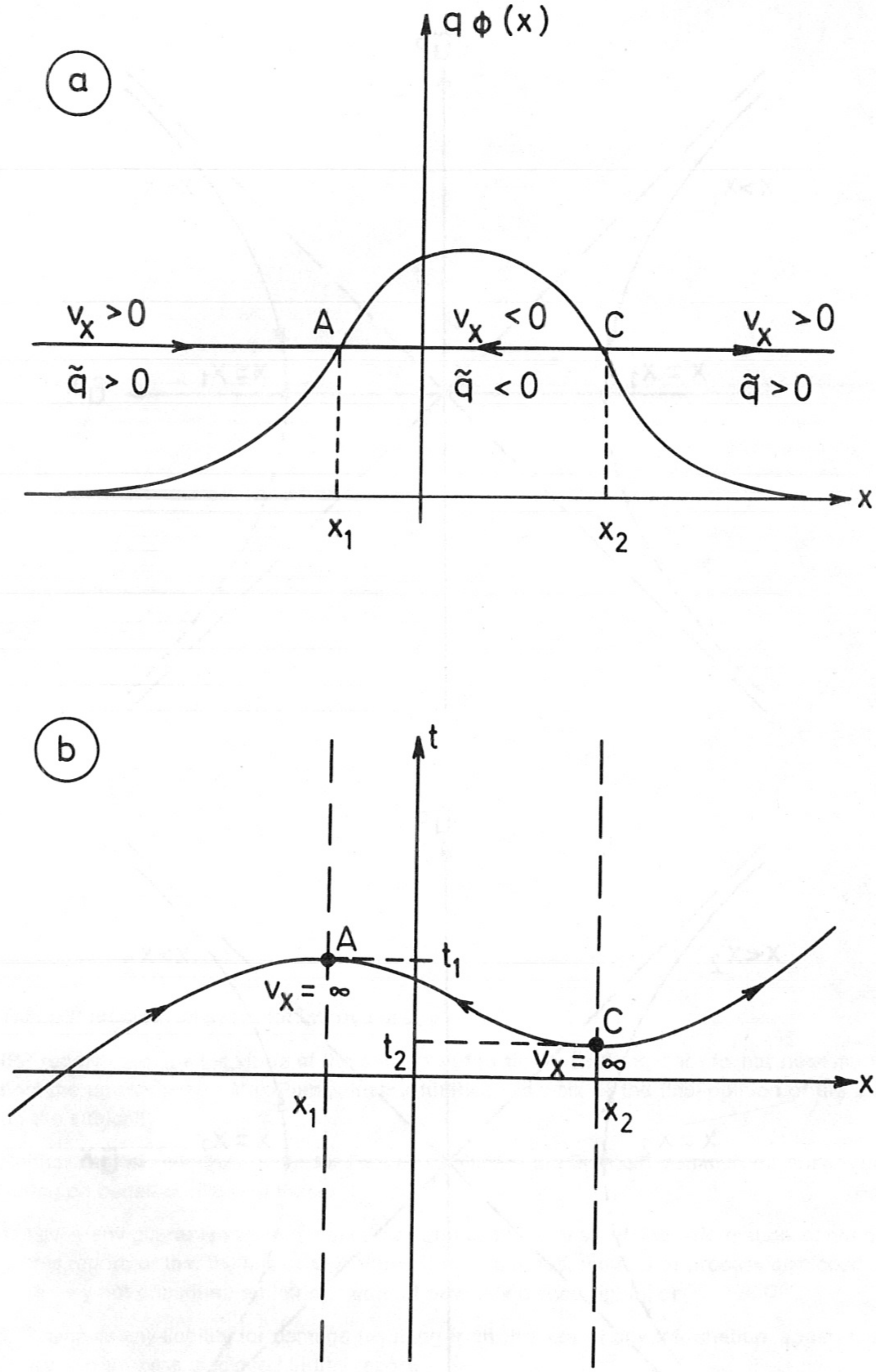

Fig. 7

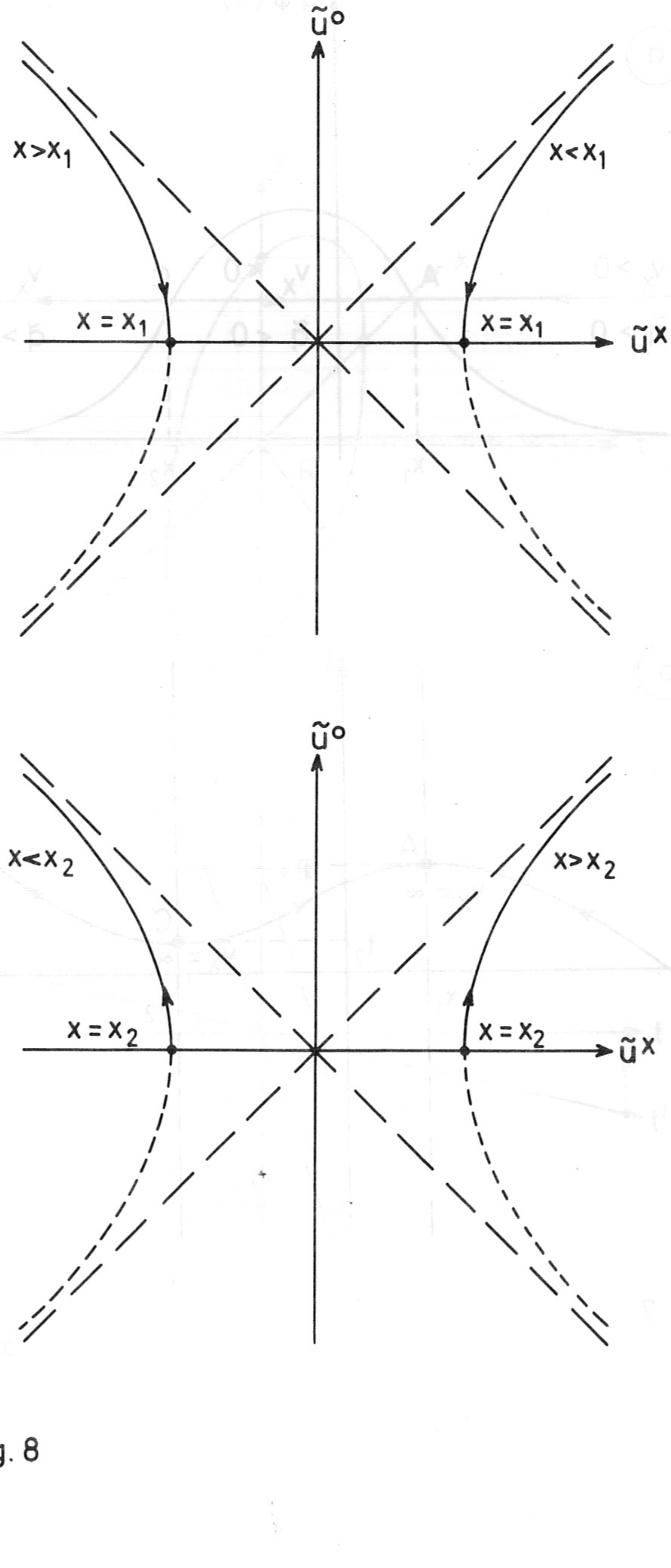

Fig. 8