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A Remark on Trapped Particle Pinch in a Tokomak

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IPP IV/20 March 1971

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Max-Planck-Institut für Plasmaphysik und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

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March 1971 (in English)

Abstract

Trapped particle pinch in a Tokomak is shown to be directly related to flux surface concentration. Study of flux surface motion allows an accurate estimate of particle drift in arbitrary axisymmetric geometries. The trapped particle pinch velocity is compared with the numerical computation of the particle trajectory in a Levitron.

It has been recently pointed out by Ware [1] that trapped particles in a Tokomak move towards the center of the discharge with an average radial velocity

$$v_r = -cE_\varphi / B_\theta , \tag{1}$$

(E_φ is the toroidal electric field and B_θ is the poloidal magnetic field) and that this phenomenon may have considerable bearing on Tokomak containment.

We wish to draw attention to the fact that this pinch effect may be viewed as the tendency of the trapped particles to follow a surface of constant flux. This behavior has long been recognized in the case of a linear-pinch. In toroidal geometry, however, since the axis of symmetry does not coincide with the axis of the current filament, the flux surfaces are to be defined with reference to the major-axis of the torus, rather than the local toroidal coordinates. Accordingly, we use the term "flux surface" or "surface of constant flux" instead of the frequently used expression "magnetic surface". In terms of this flux surface contraction, Ware's results may be generalized to arbitrary axisymmetric field geometries.

Writing down conservation of canonical angular momentum [2] of a particle in cylindrical coordinates (R, φ, z),

$$\frac{d}{dt}\left\{ R\left[mv_\varphi (R,\varphi,z) + \frac{e}{c} A_\varphi (R,z,t)\right]\right\} = 0, \tag{2a}$$

or,

$$\frac{d}{dt}\left\{ Rmv_\varphi (R,\varphi,z) + \frac{e}{2\pi c} \Phi (R,z,t)\right\} = 0, \tag{2b}$$

where, $$\Phi(R,z,t) = \int_0^R 2\pi R B_z(R,z,t)\,dR. \tag{3}$$

Eq.(2b) is equivalent to the statement:
"Points of the trajectory with equal angular momentum Rmv_φ adhere to the same surface Φ = constant."
Note that the flux Φ contains no contribution from the toriodal magnetic field B_φ .

For a Tokomak (or Levitron), the flux Φ may be divided into two components, $\Phi = \Phi_p + \Phi_d$, where Φ_p is the flux due to the primary winding of the ohmic heating transformer and Φ_d is the flux due to the current in the discharge. Fig.1 shows the flux Φ_o plotted as a function of R_o as well as its temporal evolution. The subscript "o" is used to denote quantities measured in the plane z=0. From Fig.1b we note that for Φ to remain constant, $\Delta R_o = (dR_o/d\,\Phi_o)\Delta\Phi_o$. Since, $d\Phi_o/dR_o = 2\pi R_o B_z(R_o,t)$ and $d\Phi_o/dt = 2\pi R_o E_\varphi\,(R_o,t)$, the velocity v_o of the surface of constant flux is given by,

$$v_o = dR_o/dt = -cE_\varphi\,(R_o,t)/B_z(R_o,t). \tag{4a}$$

It is readily seen that similar arguments hold for the motion along any path orthogonal to the surfaces of constant flux, and the velocity of a flux surface is given by

$$v_n = -cE_\varphi\,/B_t, \tag{4b}$$

B_t being the component of B which is tangential to the flux surface in the meridional plane.

If we now consider a trapped particle with reflection point (defined by $v_\varphi = 0$) lying on the flux surface corresponding to the point R_o in the plane z=0, the pinch velocity of the flux surface is once again given by Eq.(4a) for an axisymmetric system of arbitrary complexity. From Fig.1b it is evident that the particle can not stay trapped beyond the time t_2 as the surface on which the points of reflection must lie has by then definitely collapsed. Time t_2, therefore, gives an upper limit for the duration of the pinch.

We shall compare the value of v_o obtained by using Eq.(4) for a trapped particle pinch in a Levitron with the numerical computations of Ref.3. From the guiding center trajectory of an electron trapped in a Levitron field Gibson et al.[3], compute the radial velocity of the reflection points (A, B, C, D in Fig.1, Ref.3) to be 1.6×10^6 cm/sec. By approximate projection (<u>along the flux surface</u>) of the reflection points on the plane z=0, as well as by direct substitution[4] in Eq.(4a) of $E(R_o)$ and $B(R_o)$ from the data of Ref.3 we obtain $v_o \simeq 1.4 \times 10^6$ cm/sec. Thus, the computations of Gibson et al. are in agreement with the results obtained from the consideration of flux surface contraction.

In conclusion, we note[5] that if the θ component of ∇RA_φ is included, Eq.(6) of Ref.1 reads

$$\Delta r B_\theta - r \Delta \theta B_r + \Delta t c E_\varphi = 0.$$

This equation, written in toroidal coordinates, carries information identical to that contained in Eq.(4a) which is written in cylindrical coordinates in the plane z=0.

Although a simple example was used for the purpose of illustration, we believe that the flux surface motion can be used for an accurate estimate of particle drifts in more complex geometries like plasma current multipoles [6]. Also, flux surface motion allows an easy visualization of particle drifts.

References

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4 The expression used to determine the magnetic field $B(R_o)$ due to a circular current filament is taken from J.A. Stratton, "Electromagnetic Theory", McGraw Hill, New York (1941), p. 263

5 In Ref. 3 the origin of this discrepancy is attributed to Ware's assumption of circular cross section for the magnetic surfaces.

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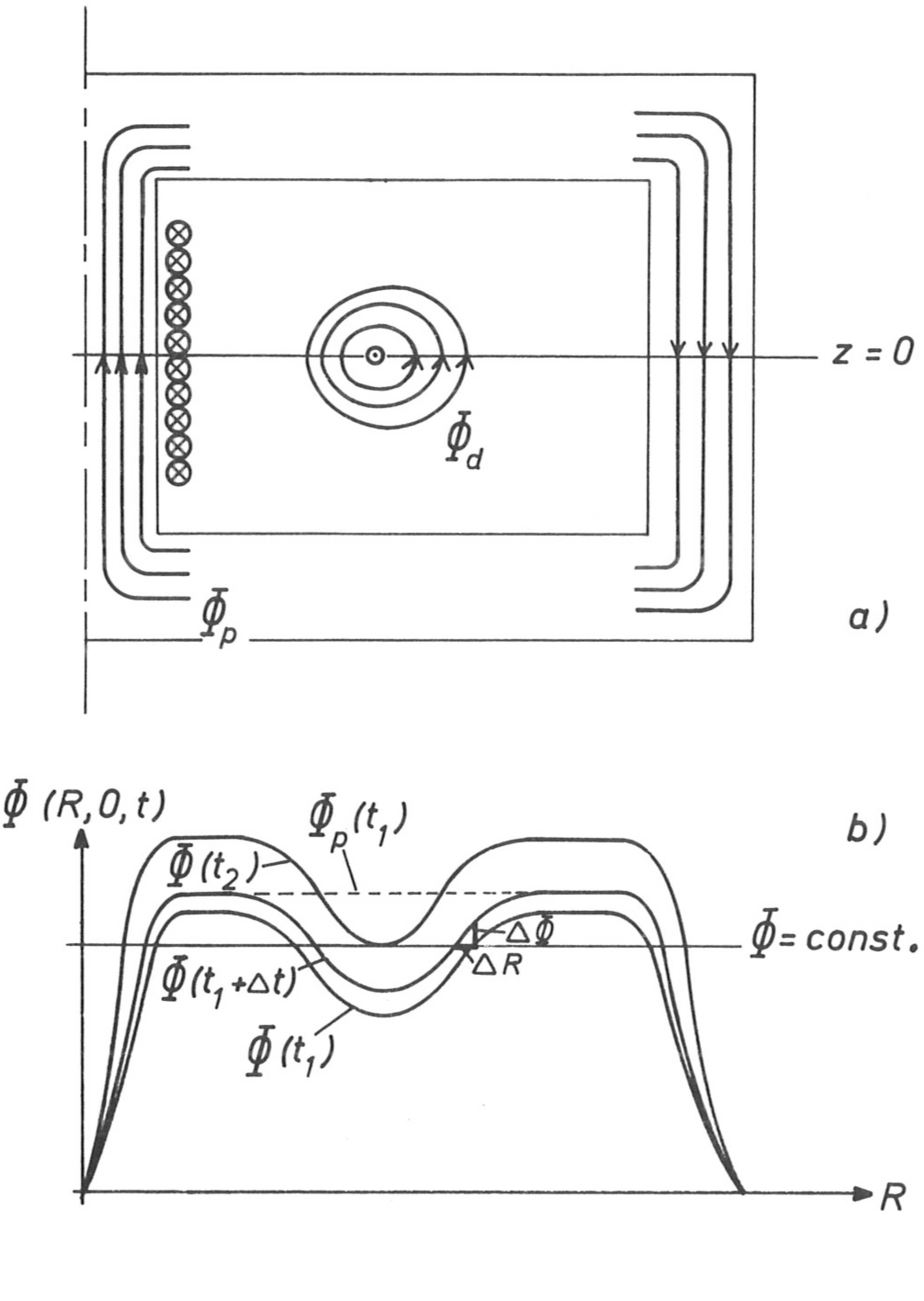

Fig.1 Radial distribution and temporal evolution of flux $\Phi = \Phi_p + \Phi_d$, where Φ_p is the flux due to the primary winding of the ohmic heating transformer and Φ_d is the flux due to the current in the discharge.