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Laser Energy Necessary for Inertially
Confined Nuclear Fusion Plasma

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ABSTRACT:

The gain G of the thermonuclear fusion energy of very
dense spherical plasmas heated by lasers in times of
about 1 nsec is treated theoretically. Assuming as a
pessimistic estimate that fast ions leave the plasma
without contributing to fusion neutrons, we find that
for G = 1 laser energies of 3×10^7 joules are needed.
A kinetic theory for a one species gas starting with a
Gaussian density and velocity profile yields almost the
same result as found by a hydrodynamic model with adia-
batic expansion and without fast ion losses taken into
account. Therefore, the even-point of G = 1 is reached
at the well-known value for the laser energy of 4.6×10^6
joules for a D-T reaction.

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I. Introduction

Two different ways of attaining thermonuclear conditions are being studied at present: one is to heat a low-density plasma rather slowly and confine it sufficiently long by magnetic fields; the other is to heat a high-density plasma very rapidly, which then expands freely, this being called inertial confinement. The latter might be possible by using laser pulses, as is discussed here, or, by means of intense electron beams [2]. This paper continues the discussion of some theoretical aspects that was started some years ago [3, 4] to calculate the energies necessary to reach the even-point of energy production [+], i. e. where energy production equals energy input. In particular, the influence of fast ion losses is discussed.

Two models will be considered. In the first one the dynamics are described by a hydrodynamic theory [3, 4], while for fusion reactions certain assumptions about the loss of fast ions are made. These assumptions are probably too pessimistic, as can be seen from our second model, in which the dynamics are described by a one species kinetic theory including collisions. The time dependent problem with initial Gaussian density profile and a Gaussian velocity distribution is solved exactly. The fast ion losses are then automatically taken into account. It turns out that the result for the fusion gains is exactly the same as found from a hydrodynamic model with the same initial density profile and temperature, but without assumming any fast ion losses.

[+] For further references see, for example, the second paper of Ref. 4.

II. Hydrodynamic Model with Losses

An amount of energy E_o (Fig. 1) may be transferred within 10^{-9} sec into a pure deuterium or deuterium-tritium plasma of solid state density. The plasma will then expand adiabatically from an initial radius $R = R_o$ and an initial temperature T_o. This expansion is governed by the law

$$\dot{R} = \left(\frac{5kT_o}{m_o} \left(1 - (R_o/R)^2 \right) \right)^{1/2} \qquad (1)$$

where k is Boltzmann's constant, m_o is the averaged mass of the plasma particles, and $R = R(t)$ is the actual radius of the sphere at the time t. Using Eq. (1) together with the adiabatic law for the temperature T, we find for the fusion gain

$$G = \frac{\varepsilon_F}{E_o} \int_o^\infty dt \int_o^{R(t)} dx_1 dx_2 dx_3 \, \frac{n^2(R(t))}{A} \langle \sigma v \rangle \qquad (2)$$

where ε_F is the energy released by one fusion reaction, A is a factor equal to 2 for pure deuterium and equal to 4 for the D-T mixture. To determine the average value $\langle \sigma v \rangle$, where σ is the reacting cross section and v the relative velocity of two ions, we use for v a Maxwellian distribution modified in such a way as to take into account that the fast ions leave the plasma without reacting. This modification is found in the following way: The averaged distance $\langle \ell_F \rangle$ which an ion with a velocity v travels until it produces a fusion reaction is, because of numerous elastic collisions,

$$\langle \ell_F \rangle = \ell_c \sqrt{v_c/v_F} \qquad (3)$$

where v_F is the reaction frequency for fusion processes, v_c the ion collision frequency, and ℓ_c the mean free path for elastic collisions. If $\langle \ell_F \rangle$ is less than $\bar{R} = \frac{8}{9} R$, which represents an effective radius in the sense of averaging over position and flight direction of the particles, the probability that the particle will react is unity, and if $\bar{R} < \langle \ell_F \rangle$ the pro-

bability is $\bar{R}/\langle \ell_F \rangle$. For fusion gain resulting from Eqs. (1) and (2) we obtain by methods similar to those used before [4]

$$G = \frac{\varepsilon_F}{E_o} n_o V_o \sqrt{\frac{m_o}{5kT_o}} \int_0^\infty \frac{dR}{(1-(R_o/R))^{1/2}} \langle \sigma v \rangle \frac{n}{A} \qquad (4)$$

with

$$\langle \sigma v \rangle = \frac{\int_0^\infty Min\left(1; \frac{\bar{R}(R)}{\langle \ell_F(n,R,v) \rangle}\right) \sigma(v) v^3 exp\left(-\frac{\bar{m} v^2}{2kT}\right) dv}{(2kT(R)/\bar{m})^{3/2} \pi^{1/2}/8^{1/2}} \qquad (5)$$

where $\bar{m}$ is the reduced mass of the colliding nuclei.

The results of the calculation of G for varying laser energy E_o and different initial volumes V_o at the initial density $n_o = 6 \times 10^{22}$ cm^{-3} of the solid material are shown in Fig. 2. The maximum gain for different values of V_o increases with very good approximation as $E_o^{2/3}$, while the corresponding law for the case without losses is $E_o^{1/3}$. The even-point, G = 1, is reached with losses at $E_o = 3.5 \times 10^7$ joules, while without losses G was 1.6×10^6 joules [4]. From a set of plots of the kind in Fig. 2 we derive a formula for the maximum G valid for 10^3 joules $< E_o < 10^{10}$ joules, and 10^{20} cm$^{-3} < n_o < 10^{25}$ cm^{-3}, for a D-T plasma

$$G = \begin{cases} 4.42 \times 10^{-36} E_o^{2/3} n_o^{4/3}, & if\ E_o n_o^2 \lesssim 3.7 \times 10^{55} joules/cm^6 \\ 1.47 \times 10^{-17} E_o^{1/3} n_o^{2/3}, & if\ E_o n_o^2 \gtrsim 3.7 \times 10^{55} joules/cm^6 \end{cases} \qquad (6)$$

For a deuterium plasma we find for the same region of validity

$$G = 3 \times 10^{-39} E_o^{2/3} n_o^{4/3} \qquad (7)$$

At the limits of the range of validity the formulae may derivate from the numerical results by up to 10 %.

III. Kinetic Model

It is well known in hydrodynamics from numerical cal-
culations that an initially boxlike density profile becomes
Gaussianlike [5]. The same happens if the initial density pro-
file is triangular. As was shown analytically [6], an initially
Gaussian density profile of spatially constant temperature
conserves this profile self-similiarly. Because of these
facts and because it is not completely known, what the
real initial state of a plasma produced by laser is, the
approximation of the dynamics by a self-similarity expansion
of Gaussian density profiles appears to be justified. In
the following kinetic treatment, we therefore choose this
type of initial conditions.

We use a simplified model in which the dynamics of
only one particle species, namely deuterium ions, is taken
into account. From the results obtained in this way we
refer to what might occur in a D-T plasma. Neglecting the
electrons means neglecting space charges. The latter would
be true if the pressure of the electrons could be neglected.
This, of course, is not the case in real experiments, but
the order of magnitude should not be influenced by this.
Neglecting the dynamics of the electrons and ion-electron
collisions is justified by the small electron mass. We are
dealing with a distribution f ($\underline{x}$, $\underline{v}$, t) for the ions obeying
a kinetic equation

$$\frac{\partial}{\partial t} f + \underline{v} \frac{\partial}{\partial x} f = \left(\frac{\partial f}{\partial t} \right)_{coll} \tag{8}$$

The collision term is assumed to be a local one, des-
cribing only ion-ion collisions. It is easy to find a so-
lution which makes both sides of Eq. (8) equal to zero.
To make the left-hand side vanish, f must depend only on the
constants of motion $\underline{v}$ and $\underline{x}-t\underline{v}$. The right-hand side of Eq. (8)
postulates that f be an exponential function of the additive
collisional invariants $\underline{v}$ and v^2. Such a function is

$$f = \frac{n_o}{(2\pi k T_o/m_i)^{3/2}} \exp\left[-\frac{1}{R_o^2}(\underline{x} - \underline{v}t)^2 - \frac{1}{kT_o}\frac{m_i}{2}v^2\right] \qquad (9)$$

where T_o, R_o and n_o are constants and m_i is the ion mass. Equation (9) can be rewritten in the form

$$f = \frac{n_o}{(2\pi k \Theta/m_i)^{3/2}}\left(\frac{\Theta}{T}\right)^{3/2}\exp\left[-\frac{x^2}{R_o^2}\frac{\Theta}{T}\right]\exp\left[-\frac{m_i}{2k\Theta}\left(\underline{v} - \frac{2k\Theta}{m_i R_o^2}t\underline{x}\right)^2\right] \quad (10)$$

$$\text{with} \quad \Theta = T_o/\left(1 + \frac{2kT_o}{m_i R_o^2}t^2\right)$$

It can be shown that the dynamic expansion is like an adiabatic one with the ratio of the specific heats $\gamma = 5/3$. The local fusion rate is

$$\frac{1}{2}n^2\langle\sigma v\rangle = \frac{1}{2}\int d^3v_1\int d^3v_2\,\delta(|\underline{v}_1 - \underline{v}_2|)|\underline{v}_1 - \underline{v}_2|f(\underline{x},\underline{v}_1,t)f(\underline{x},\underline{v}_2,t) \qquad (11)$$

Introducing new coordinates

$$\underline{v} = \underline{v}_1 - \underline{v}_2 = \left(\underline{v}_1 - \frac{2k\Theta}{m_i R_o^2}t\underline{x}\right) - \left(\underline{v}_2 - \frac{2k\Theta}{m_i R_o^2}t\underline{x}\right)$$

$$\underline{V} = \frac{1}{2}\left(\underline{v}_1 - \frac{2k\Theta}{m_i R_o^2}t\underline{x} + \underline{v}_2 - \frac{2k\Theta}{m_i R_o^2}t\underline{x}\right)$$

we obtain

$$\frac{1}{2}n^2\langle\sigma v\rangle = \frac{n_o^2}{2}\int d^3V\int d^3v\,\,v\,\frac{\exp\left[-\frac{m_i/2}{2k\Theta}v^2\right]}{(2\pi k\Theta/(m_i/2))^{3/2}}\,\cdot$$

$$\cdot\,\frac{\exp\left[-\frac{2m_i}{2k\Theta}V^2\right]}{(2\pi k\Theta/(2m_i))^{3/2}}\exp\left[-\frac{2x^2}{R_o^2}\frac{\Theta}{T_o}\right]\left(\frac{\Theta}{T_o}\right)^3 \qquad (12\text{ a})$$

$$\frac{1}{2}n^2\langle\sigma v\rangle = \frac{m_i^2}{2}\left(\frac{\Theta}{T_o}\right)^3\exp\left[-\frac{2x^2\Theta}{R_o^2 T_o}\right]\int d^3v\,\delta(v)\frac{\exp\left[-(m/2)v^2/(2k\Theta)\right]}{(2\pi k\Theta/(m/2))^{3/2}} \quad (12\text{ b})$$

$$\frac{1}{2}n^2\langle\sigma v\rangle = \frac{n_o^2}{2}\left(\frac{\Theta}{T}\right)^3\exp\left[-\frac{2x^2}{R_o^2}\frac{\Theta}{T}\right]\langle\sigma v\rangle_o = \frac{n^2(x,t)}{2}\langle\sigma v\rangle_o \qquad (13)$$

the expression $\langle \delta v_o \rangle$ is identical with the one in Eq. (5) if no attention is paid to losses of fast ions, i. e. the function Min is set equal to unity.

Using a normalization of n_o such that the total number of ions with a Gaussian profile is the same as these with a box profile, and such that the initial central densities are the same, the ratio of the corresponding fusion gains is

$$\frac{G_{Box}}{G_{Gauss}} = \frac{\int n_o^2 \, d\tau}{\int n_{Gauss}^2 \, d\tau} = 2^{3/2} \qquad (14)$$

where G_{Box} is calculated without allowance for the fast ion losses. This fusion gain G_{Box} is therefore by definition the same as discussed earlier [4].

In Fig. 3 we report the results of $G = G_{Gauss}$ for a D-T plasma with an initial solid-state density. In Fig. 4 $G = G_{Gauss}$ is shown for pure deuterium. Interpolation formulae for the maximum fusion gains for 1 joule $< E_o < 10^{15}$ joule and 10^{16} cm$^{-3} < n_o < 10^{25}$ cm^{-3} are

$$G_{Gauss} = 3.93 \times 10^{-18} \, n_o^{2/3} \, E_o^{1/3} \qquad \text{(D-T)}$$

$$G_{Gauss} = 1.79 \times 10^{-19} \, n_o^{2/3} \, E_o^{1/3} \qquad \text{(D-T)}$$

Therefore, a D-T plasma will reach the even-point at $G_{Gauss} = 4.6$ MJ and for pure deuterium $G_{Gauss} = 4.2 \times 10^4$ MJ.

IV. Conclusions

We have shown that according to a rather pessimistic estimate laser energies of about 3×10^7 joules are necessary to reach the even-point, or according to a probably more realistic estimate only 4.6×10^6 joules. These figures are perhaps not beyond future capabilities, especially as regards chemical lasers.

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References

1   F. Floux, D. Cognard, L. G. Denoeud, G. Pair, D. Parisot,
    J. L. Bobin, F. Delobeau, C. Fauquignon, Phys. Rev. A1,
    821 (1970); N. G. Basov, O. N. Krokhin and P. G. Kriukov,
    Paper 2.1, Sixth Int. Quantum Electronics Conf. Kyoto,
    Sept. 1970.

2   F. Winterberg, Phys. Rev. 174 , 212 (1968)

3   N. G. Basov and O. N. Krokhin, Proceed. 3rd Int.
    Quantum Electronics Conference, Paris 1963;
    N. Bloembergen and P. Grivet eds., (Dunod Paris, 1964)
    Vol. II, p. 1373;
    J. M. Dawson, Phys. Fluids 7, 981 (1964);
    A. G. Engelhardt, Bull. Amer. Phys. Soc. 9, 3o5 (1964).

4   H. Hora, Inst. f. Plasmaphysik, Garching, Report 6/23
    (1964) USAEC Rept. NRC-TT 1193 (1965); Application of
    Laser Produced Plasmas for Controlled Thermonuclear
    Fusion in Laser Interaction and Related Plasma Phenomena,
    H. Schwarz and H. Hora, eds. (Plenum Press, New York,
    1970) , p. 427

5   W. Fader, Phys. Fluids, 11, 22oo (1968)

6   H. Hora, Some Results of the Self-similarity Model,
    in Laser Interaction and Related Plasma Phenomena,
    H. Schwarz and H. Hora, eds. (Plenum Press, New York,
    1970) , p. 365

Fig. 1   Laser energy E_o is transferred to a D-T plasma
          producing a box-like density profile at given
          initial values for a following free expansion.

Fig. 2   Fusion gain G with ion losses at varying initial
          values of the laser energy E_o and volume V_o for
          an initial density of solid deuterium
          $n_o = 6 \times 10^{22}$ cm^{-3}.

Fig. 3   Fusion gains $G = G_{Gauss}$ for a D-T plasma, for
          varying laser energy E_o and volume V_o and for
          an initial density of $n_o = 6 \times 10^{22}$ cm^{-3}.

Fig. 4   Fusion gains $G = G_{Gauss}$ for pure deuterium for
          varying laser energy E_o and volume V_o and for
          an initial density of $n_o = 6 \times 10^{22}$ cm^{-3}.
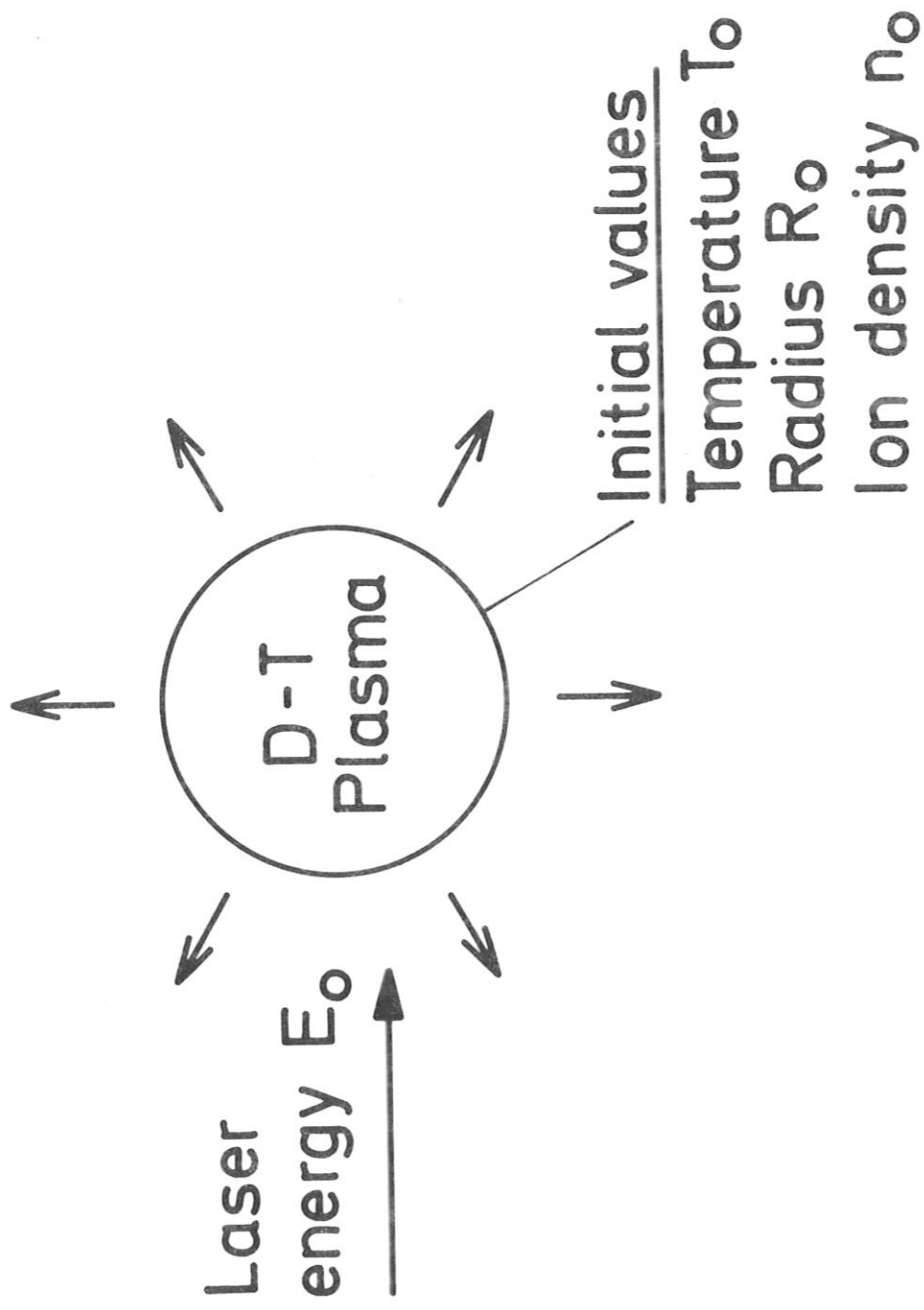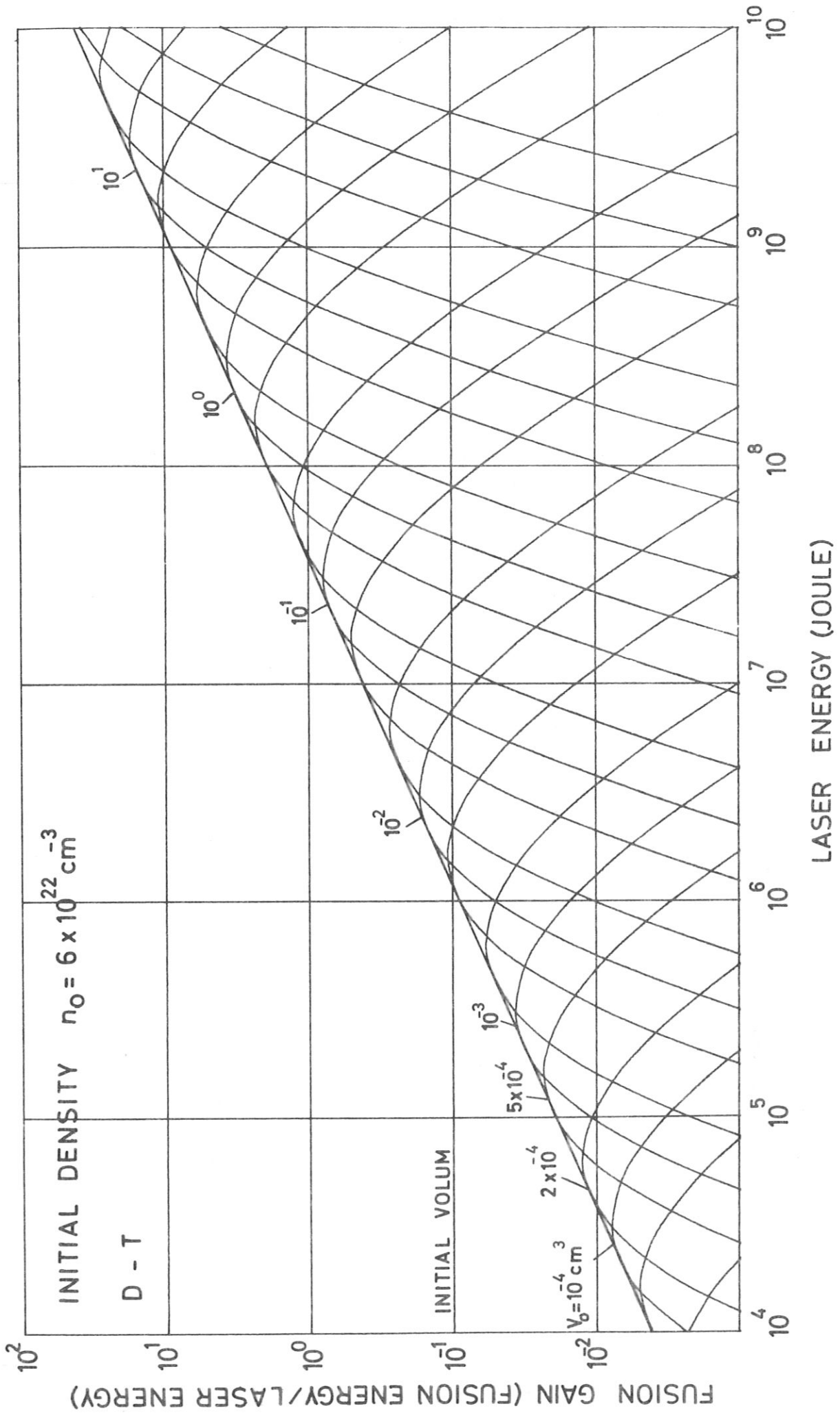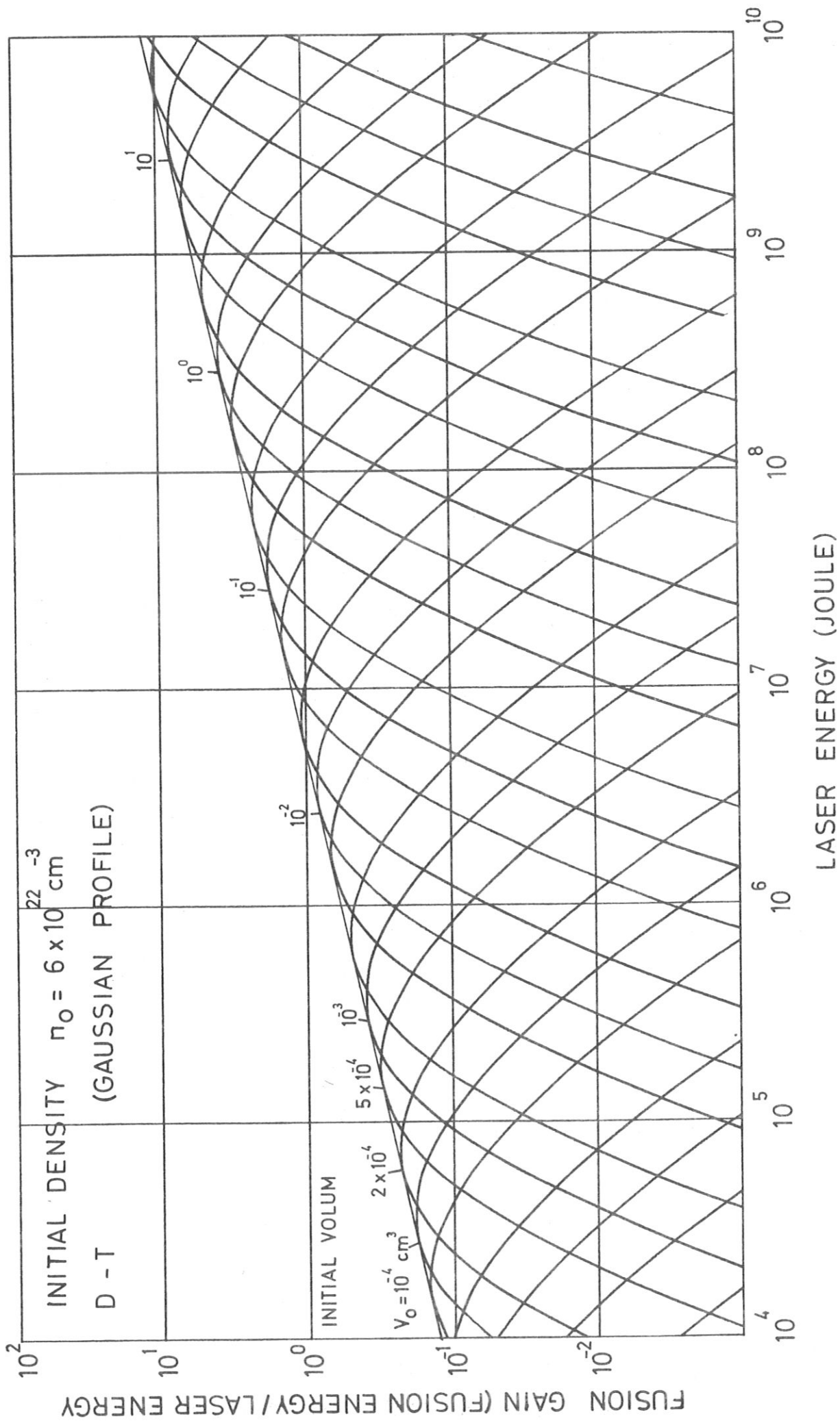
Laser energy E_o

D - T Plasma

Initial values
Temperature T_o
Radius R_o
Ion density n_o

Fig. 1

Fig. 2

INITIAL DENSITY $n_o = 6 \times 10^{22} \text{ cm}^{-3}$

D – T

(GAUSSIAN PROFILE)

10^1

10^0

10^{-1}

10^{-2}

10^{-3}

5×10^{-4}

2×10^{-4}

INITIAL VOLUM

$V_o = 10^{-4} \text{ cm}^3$

10^2 10^1 10^0 10^{-1} 10^{-2}

FUSION GAIN (FUSION ENERGY/LASER ENERGY)

10^4 10^5 10^6 10^7 10^8 10^9 10^{10}

LASER ENERGY (JOULE)

Fig. 3

INITIAL DENSITY $n_0 = 6 \times 10^{22}$ cm^{-3}

D – D   (GAUSSIAN PROFILE)

INITIAL VOLUM

$V_0 = 5 \times 10^{-5}$ cm^3

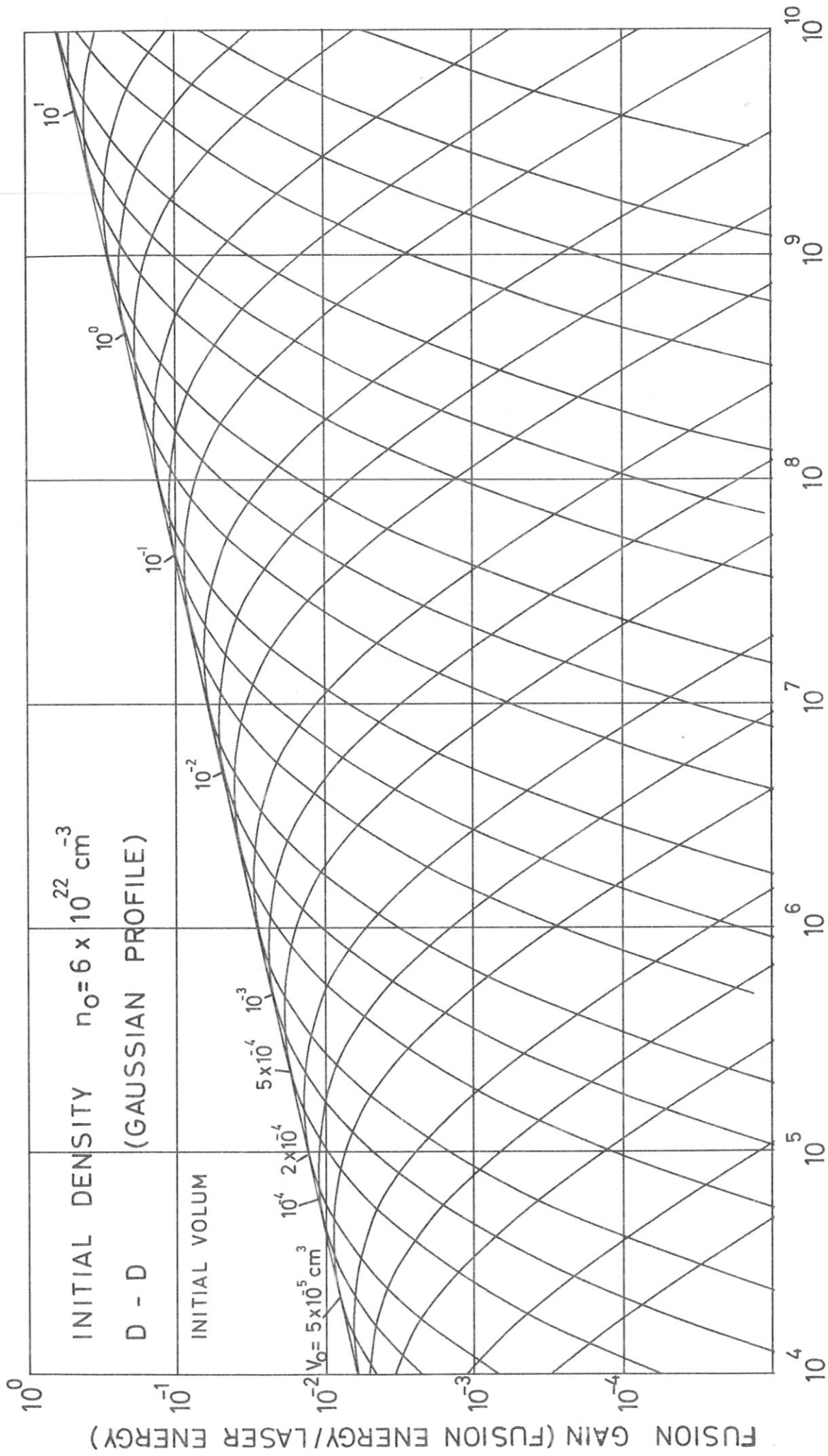10^1 10^0 10^{-1} 10^{-2} 10^{-3} 5×10^{-4} 2×10^{-4} 10^{-4}

FUSION GAIN (FUSION ENERGY/LASER ENERGY)

LASER ENERGY (JOULE)

Fig. 4