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Hydromagnetic Equations for anisotropic Plasmas including Transport Effects

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Abstract:

A set of hydrodynamic equations is derived for an arbitrary anisotropic plasma allowing for COULOMB collisions among plasma particles. Using a generalized expansion in HERMITE polynomials for the distribution functions of plasma constituents (as described in Oraevskii et al.(1968) the collision integrals occuring in the hydrodynamic equations are calculated. Explicit results for the coefficients of ion-viscosity and heat conductivity of ions and electrons are derived.

1. INTRODUCTION

As is well known, plasma in a magnetic field often behaves as an anisotropic medium. The anisotropy may occur in two respects:

1) The velocity distribution of the plasma equilibrium is isotropic, f.i. Maxwellian, only small deviations from this equilibrium, due to transport effects are anisotropic (cf. f.i. BRAGINSKII, (1965) and HERDAN et al., (1960)).

2) Already the "equilibrium" distribution is anisotropic, f.i. bi-Maxwellian with different temperatures parallel and perpendicular to the magnetic field.

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It is the transport behaviour of an anisotropic plasma in this latter sense which will be investigated in the following.

Anisotropic plasmas of this kind may be found in space or may be obtained in laboratory by cyclotron heating, particle injection, or in the fast rising magnetic field of a magnetic shock wave (fast relative to the relaxation time of the anisotropy).

Hydrodynamic equations for a collisionless anisotropic plasma, neglecting transport phenomena, were first given by CHEW et al. (1956). Since then, a number of authors (KENNEL et al., 1966; MAC MAHON, 1965; FRIEMAN et al., 1966; BOWERS et al., 1968; ESPEDAL, 1969) have investigated the transport behaviour of an anisotropic plasma in the collisionless limit (assuming weak inhomogeneities on the length scale of the Larmor radius).

It is the purpose of this paper to derive a closed set of hydrodynamic equations for an anisotropic plasma taking care of transport properties and including the effect of collisions. In order to do this, we make use of an expansion of the distribution function which was described in ORAEVSKI et al. (1968).

Let be

ω_c^s	cyclotron frequency of species s
ν^s	collision frequency
r_g^s	gyration radius
λ^s	mean free path
t_o	macroscopic time scale of the plasma
L	macroscopic length scale

Then, besides the conditions of the collisionless case

$$\frac{1}{\omega_c^s t_o} \sim \frac{r_g^s}{L} \ll 1$$

we will also assume

$$\frac{1}{\nu^s t_o} \sim \frac{\lambda^s}{L} \ll 1 .$$

In the following section we rewrite the hydrodynamic equations of ORAEVSKII et al. (1968), including the collision terms and state the assumptions for calculating them. In Sect. 3 we give the results of calculating the collision integrals and finally in Sect. 4 in a simplified version for the case of only two species of colliding particles with very different masses (electrons and ions) we derive explicit expressions for ion-viscosity and electron- and ion-heat fluxes.

2. HYDRODYNAMIC EQUATIONS FOR AN ANISOTROPIC PLASMA ALLOWING FOR COLLISIONS

We assume a plasma composed of charged particles only that collide owing to their Coulomb interaction. Let be

m^s, q^s — the mass and the charge of a particle of the species s,

ρ^s, n^s — the mass density and particle density,

$\vec{V}^s$ — the hydrodynamic velocity,

$p_\perp^s$ — the thermal energy perpendicular to the magnetic field,

$\frac{1}{2} p_\parallel^s$ — the thermal energy parallel to the magnetic field

$p_{\alpha\beta}^s$ — the components of the pressure tensor:

$$p_{\alpha\beta}^s = p_\perp^s n_{\alpha\beta} + p_\parallel^s \tau_{\alpha\beta} + \pi_{\alpha\beta}^s = P_{\alpha\beta}^s + \pi_{\alpha\beta}^s$$

$\pi_{\alpha\beta}^s$ — the components of the viscosity tensor

$\vec{S}^{s\perp}$, $\vec{S}^{s\parallel}$ — the two heat flux vectors corresponding to the two thermal energies $p_\perp^s$ and $p_\parallel^s$

$\omega_c^s = \frac{q^s B}{m^s}$ — cyclotron frequency

I_α^{sr}, $J_{\alpha\beta}^{sr}$, $L_{\alpha\beta\gamma}^{sr}$ — the collision integrals due to collisions with particles of species r

h_α the components of unity vector in the direction of the magnetic field,

E_α the components of the electric field

$\frac{d^s}{dt}$ the operator $\frac{\partial}{\partial t} + V^s_\alpha \frac{\partial}{\partial x_\alpha}$.

Introducing the tensor components

$$\tau_{\alpha\beta} = h_\alpha h_\beta$$

$$n_{\alpha\beta} = \delta_{\alpha\beta} - \tau_{\alpha\beta}$$

$$\varepsilon_{\alpha\beta\gamma} = \begin{cases} 1 & \text{if } \alpha,\beta,\gamma \text{ is an even permutation of } 1,2,3 \\ -1 & \text{if } \alpha,\beta,\gamma \text{ is an odd permutation of } 1,2,3 \\ 0 & \text{if at least two indices are equal.} \end{cases}$$

the plasma may be described by the hydrodynamic equations

(2.1) $$\frac{\partial \rho^s}{\partial t} + \frac{\partial}{\partial x_\alpha} (\rho^s V^s_\alpha) = 0$$

(2.2) $$\rho^s \frac{d^s V^s_\alpha}{dt} + \frac{\partial}{\partial x_\beta} p^s_{\alpha\beta} - \rho^s \left[\frac{q^s}{m^s} E_\alpha + \omega^s_c \left[\vec{V}^s \times \vec{h}\right]_\alpha \right] = \sum_r I^{sr}_\alpha .$$

(2.3) $$\frac{d^s p^s_\perp}{dt} + \frac{\partial V^s_\alpha}{\partial x_\beta} \left[p^s_\perp \tau_{\alpha\beta} + 2p^s_\perp n_{\alpha\beta} + \pi^s_{\beta\gamma} n_{\alpha\gamma} \right] +$$

$$+ \frac{\partial}{\partial x_\alpha} S^{s\perp}_\alpha + S^{s\perp}_\alpha h_\alpha \frac{\partial}{\partial x_\beta} h_\beta + S^{s\parallel}_\alpha h_\beta \frac{\partial}{\partial x_\beta} h_\alpha + \frac{1}{2} \pi_{\alpha\beta} \frac{d^s}{dt} \tau_{\alpha\beta}$$

$$= \frac{1}{2} \sum_r J^{sr}_{\alpha\beta} n_{\alpha\beta}$$

(2.4)
$$\frac{d^s p_\parallel^s}{dt} + \frac{\partial V_\alpha^s}{\partial x_\beta}\left[p_\parallel^s n_{\alpha\beta} + 3p_\parallel^s \tau_{\alpha\beta} + 2\pi_{\beta\gamma}^s \tau_{\alpha\gamma}\right] +$$
$$+ \frac{\partial}{\partial x_\alpha} S_\alpha^{s\parallel} - 2S_\alpha^{s\parallel} h_\beta \frac{\partial}{\partial x_\beta} h_\alpha - 2S_\alpha^{s\perp} h_\alpha \frac{\partial}{\partial x_\beta} h_\beta - \pi_{\alpha\beta} \frac{d^s}{dt} \tau_{\alpha\beta}$$
$$= \sum_r J_{\alpha\beta}^{sr} \tau_{\alpha\beta} .$$

$$-\omega_c^s h_\delta \left(\varepsilon_{\alpha\gamma\delta}\pi_{\beta\gamma}^s + \varepsilon_{\beta\gamma\delta}\pi_{\alpha\gamma}^s\right) + p_\parallel \left(\frac{\partial V_\alpha^s}{\partial x_\delta}\tau_{\delta\beta} + \frac{\partial V_\beta^s}{\partial x_\delta}\tau_{\delta\alpha} - 2\frac{\partial V_\gamma^s}{\partial x_\delta}\tau_{\gamma\delta}\tau_{\alpha\beta}\right) +$$

(2.5)
$$+ p_\perp \left(\frac{\partial V_\alpha^s}{\partial x_\delta} n_{\delta\beta} + \frac{\partial V_\beta^s}{\partial x_\delta} n_{\delta\alpha} - \frac{\partial V_\gamma^s}{\partial x_\delta} n_{\gamma\delta} n_{\alpha\beta}\right) + \left(p_\parallel^s - p_\perp^s\right) \frac{d^s}{dt}\tau_{\alpha\beta}$$
$$= \sum_r \left[J_{\alpha\beta}^{sr} - \left(\frac{1}{2} n_{\alpha\beta} n_{\gamma\delta} + \tau_{\alpha\beta}\tau_{\gamma\delta}\right) J_{\gamma\delta}^{sr}\right] .$$

$$\left(p_\parallel^s \tau_{\beta\gamma} + p_\perp^s n_{\beta\gamma}\right) \frac{\partial}{\partial x_\beta}\left[\frac{p_\perp^s}{\rho^s}\left(\tau_{\alpha\gamma} + 2n_{\alpha\gamma}\right)\right] - \omega_c^s \left[\vec{S}^{s\perp} \times \vec{h}\right]_\alpha -$$

(2.6)
$$- \frac{1}{2}\left(P_{\alpha\beta}^s P_{\gamma\delta}^s + P_{\alpha\gamma}^s P_{\beta\delta}^s + P_{\beta\gamma}^s P_{\alpha\delta}^s\right) \frac{\partial}{\partial x_\delta} n_{\beta\gamma}$$
$$= \sum_r \left[\frac{1}{2} L_{\alpha\beta\gamma}^{sr} n_{\beta\gamma} - \frac{p_\perp^s}{\rho^s}\left(\tau_{\alpha\beta} + 2n_{\alpha\beta}\right) I_\beta^{sr}\right] .$$

$$\left(p_\parallel^s \tau_{\beta\gamma} + p_\perp^s n_{\beta\gamma}\right) \frac{\partial}{\partial x_\beta}\left[\frac{p_\parallel^s}{\rho^s}\left(3\tau_{\alpha\gamma} + n_{\alpha\gamma}\right)\right] - \omega_c^s \left[\vec{S}^{s\parallel} \times \vec{h}\right]_\alpha$$

(2.7)
$$- \left(P_{\alpha\beta}^s P_{\gamma\delta}^s + P_{\alpha\gamma}^s P_{\beta\delta}^s + P_{\beta\gamma}^s P_{\alpha\delta}^s\right) \frac{\partial}{\partial x_\delta} \tau_{\beta\gamma}$$
$$= \sum_r \left[L_{\alpha\beta\gamma}^{sr} \tau_{\beta\gamma} - \frac{p_\parallel^s}{\rho^s}\left(3\tau_{\alpha\beta} + n_{\alpha\beta}\right) I_\beta^{sr}\right] .$$

The upper (Roman) indices characterize the particle species, the lower (Greek) the components of a vector or tensor. The summation convention for equal component indices is applied, but not for species indices.

I^{sr}_{α}, $J^{sr}_{\alpha\beta}$, and $L^{sr}_{\alpha\beta\gamma}$ are moments of the collision term of the Boltzmann equation. Putting M^{sr} for I^{sr}_{α}, $J^{sr}_{\alpha\beta}$ or $L^{sr}_{\alpha\beta\gamma}$ and denoting the collision term by $\left(\frac{\partial f}{\partial t}\right)^{sr}_{coll}$ these collisional moments are defined by

$$(2.8)\qquad M^{sr} = \int a^{s}(v) \left(\frac{\partial f}{\partial t}\right)^{sr}_{coll} d^3v$$

where $a^s(v)$ stands for v_γ, $(v_\alpha - V^s_\alpha)(v_\beta - V^s_\beta)$, and $(v_\alpha - V^s_\alpha)(v_\beta - V^s_\beta)(v_\gamma - V^s_\gamma)$ respectively.

The rate of change of the distribution function f due to COULOMB collisions of particles s with particles r is given in the LANDAU form (see f.i. TRUBNIKOV, 1965),

$$(2.9)\qquad \left(\frac{\partial f}{\partial t}\right)^{sr}_{coll} = -\frac{\partial}{\partial v_\alpha} j^{sr}_{\alpha}$$

where

$$(2.10)\qquad j^{sr}_{\alpha} = A^{sr} \int d^3v' \; u_{\alpha\beta} \left[\frac{f^s(\vec{v})}{m^r} \frac{\partial f^r(\vec{v}')}{\partial v'_\beta} - \frac{f^r(\vec{v}')}{m^s} \frac{\partial f^s(\vec{v})}{\partial v_\beta} \right]$$

$$(2.11)\qquad A^{sr} = 2\pi \ln \Lambda^{sr} \frac{q_s^2 \, q_r^2}{m^s}$$

$$(2.12)\qquad u_{\alpha\beta} = \frac{1}{g} \delta_{\alpha\beta} - \frac{1}{g^3} g_\alpha g_\beta$$

$\ln \Lambda^{sr}$ is the Coulomb logarithm,

$$\Lambda^{sr} = \frac{3\,D}{|q^s\, q^r|}\,\frac{m^s\, m^r}{m^s + m^r}\,\frac{1}{\beta^{sr}} \tag{2.13}$$

with β^{sr} as given by Eq. (2.15) and D is the Debye length:

$$D^{-2} = 4\pi e^2 n_e \left(\frac{Z}{T^i} + \frac{1}{T^e}\right) \qquad (q^i = Z\,e\,,\quad n_i = Z\,n_e). \tag{2.14}$$

We have assumed here that $\ln \Lambda^{sr}$ does not change very much if T is varied from $T_\perp$ to $T_\parallel$.

$\vec{g}$ is the relative velocity of two colliding particles, $g_\alpha = v_\alpha - v'_\alpha$, $g = \sqrt{g_\alpha\, g_\alpha}$.

Equations (2.5 - 2.7) are rewritten for the special case of straight magnetic field lines in Sect.4, Eqs. (4.3 - 4.12).

As was shown in ORAEVSKII et al. (1968), the set of Eqs. (2.1 - 2.7) corresponds to a generalization of GRAD's expansion of the distribution function f^s in a series of HERMITE polynomials and truncation after the third-order polynomial. In order to write down the distribution function, we introduce

$$\beta_\perp^s = m^s/T_\perp^s, \quad \beta_\parallel^s = m^s/T_\parallel^s, \quad w_\alpha^s = v_\alpha - V_\alpha^s\,. \tag{2.15}$$

Assuming at each space point a Cartesian coordinate system with x_3 parallel to the local magnetic field we have

$$f^s(\vec{w}^s) = f_o^s(\vec{w}^s)\Big\{1 + \frac{\beta_\perp^s}{2\rho^s}\Big[\beta_\perp^s \pi_{11}^s (w_1^{s2} - w_2^{s2}) + 2\beta_\perp^s \pi_{12}^s w_1^s w_2^s +$$

$$+ 2\beta_\parallel^s w_3^s (\pi_{13}^s w_1^s + \pi_{23}^s w_2^s)\Big] + \tag{2.16}$$

$$+ \frac{1}{\rho^s}\Big[(w_1^{s2} + w_2^{s2})\,\beta_\perp^{s2}\Big[\tfrac{1}{4}\beta_\perp^s (S_1^{s\perp} w_1^s + S_2^{s\perp} w_2^s) + \tfrac{1}{2}\beta_\parallel^s S_3^{s\perp} w_3^s\Big] +$$

$$+\frac{1}{2} w_3^{s2} \beta_\parallel^{s2} \left[\beta_\perp^s (S_1^{s\parallel} w_1^s + S_2^{s\parallel} w_2^s) + \frac{1}{3}\beta_\parallel^s S_3^{s\parallel} w_3^s \right] -$$

$$- \beta_\perp^{s2} (S_1^{s\perp} w_1^s + S_2^{s\perp} w_2^s) - \beta_\perp^s \beta_\parallel^s S_3^{s\perp} w_3^s -$$

$$\left. - \frac{1}{2} \beta_\perp^s \beta_\parallel^s (S_1^{s\parallel} w_1^s + S_2^{s\parallel} w_2^s) - \frac{1}{2} \beta_\parallel^{s2} S_3^{s\parallel} w_3^s \right] \Big\}$$

where $f_o^s(\vec{w}) = (2\pi)^{-3/2} n^s \beta_\perp^s \sqrt{\beta_\parallel^s} \exp\left[-\frac{1}{2} \beta_\perp^s (w_1^{s2} + w_2^{s2}) - \frac{1}{2}\beta_\parallel^s w_3^{s2} \right]$.

In calculating the collision integrals (2.8) we assume that the relative velocity

(2.17) $$d_\alpha = d_\alpha^{sr} = V_\alpha^s - V_\alpha^r$$

should be small relative to the reduced thermal velocity

(2.18) $$d_\perp \ll \sqrt{\frac{1}{\beta_\perp^s} + \frac{1}{\beta_\perp^r}} , \qquad d_\parallel \ll \sqrt{\frac{1}{\beta_\parallel^s} + \frac{1}{\beta_\parallel^r}}$$

and, in forming the product $f^r(\vec{v}') f^s(\vec{v})$, we will retain only terms which are linear in d, π or S.

3. CALCULATION OF THE MOMENTS OF THE COLLISION INTEGRAL

In this section we give the results of calculating the collisional moments I, J , L on the right-hand side of the hydrodynamic Eqs. (2.2 - 2.7).

Let us first introduce a collision frequency appropriate to an anisotropic plasma:

(3.1) $$\nu^{sr} = \sqrt{\frac{\pi}{2}} \frac{1}{\rho^s} \left(\frac{1}{m^r} + \frac{1}{m^s}\right) n^s n^r (q^s)^2 (q^r)^2 \ln\Lambda^{sr} \beta_\perp^{sr} \sqrt{\beta_\parallel^{sr}}$$

$\left(\frac{1}{m^r} + \frac{1}{m^s}\right)^{-1}$ is the reduced mass of the colliding particles,

(3.2) $$\beta_\perp^{sr} = \frac{\beta_\perp^s \beta_\perp^r}{\beta_\perp^s + \beta_\perp^r} , \qquad \beta_\parallel^{sr} = \frac{\beta_\parallel^s \beta_\parallel^r}{\beta_\parallel^s + \beta_\parallel^r} .$$

$1/\beta_\perp^{sr}$ and $1/\beta_\parallel^{sr}$ represent the sum of the squares of the thermal velocities of species s and r perpendicular and parallel to the magnetic field respectively.

The anisotropy of the distribution functions of the colliding particles is described by the ratio of these velocity squares:

(3.3) $$\alpha^{sr} = \beta_\parallel^{sr} / \beta_\perp^{sr} .$$

By definition, we have

(3.4) $$\nu^{rs} = \frac{\rho^s}{\rho^r} \nu^{sr}, \quad \beta^{sr} = \beta^{rs} , \quad \alpha^{sr} = \alpha^{rs} .$$

We further introduce for convenience the mass ratio

(3.5) $$\mu^s = \frac{m^s}{m^s + m^r}$$

and the ratio of squared thermal velocities

(3.6) $$b_\perp^s = \frac{\beta_\perp^s}{\beta_\perp^s + \beta_\perp^r} , \quad b_\parallel^s = \frac{\beta_\parallel^s}{\beta_\parallel^s + \beta_\parallel^r} .$$

Finally we define the integrals K_{LMN}^{sr},

(3.7) $$K_{LMN}^{sr} = \frac{1}{\pi} \int d^3u \; u^{-3} \, u_1^L \, u_2^M \, u_3^N \; \exp\left[- (u_1^2 + u_2^2 + \alpha^{sr} u_3^2)\right]$$

which are functions of α^{sr} only and are given in Table 1 for different values of K, L, M .

a) Momentum Transfer by Collisional Friction

For the collision moment of the first order

$$I_{\alpha}^{sr} = -m^s \int d^3v \; v_\alpha \frac{\partial}{\partial v_\beta} j_\beta^{sr}$$

we get

(3.8)
$$I_1^{sr} = -2\nu^{sr}\Big\{2K_{200}^{sr}\rho^s d_1 + 2(2K_{220}^{sr} - K_{200}^{sr})\, \beta_\perp^{sr}\big(S_1^{s\perp} - \frac{\rho^s}{\rho^r} S_1^{r\perp}\big) + (2\alpha^{sr} K_{202}^{sr} - K_{200}^{sr})\, \beta_\parallel^{sr}\big(S_1^{s\parallel} - \frac{\rho^s}{\rho^r} S_1^{r\parallel}\big)\Big\}.$$

and

(3.9)
$$I_3^{sr} = -2\nu^{sr}\Big\{2K_{002}^{sr}\alpha^{sr}\rho^s d_3 + 2(2K_{202}^{sr} - K_{002}^{sr})\, \beta_\parallel^{sr}\big(S_3^{s\perp} - \frac{\rho^s}{\rho^r} S_3^{r\perp}\big) + \alpha^{sr}\big(\tfrac{2}{3}\alpha^{sr}K_{004}^{sr} - K_{002}^{sr}\big)\, \beta_\parallel^{sr}\big(S_3^{s\parallel} - \frac{\rho^s}{\rho^r} S_3^{r\parallel}\big)\Big\}.$$

Using Eq. (3.4) we find

(3.10)
$$I_\alpha^{sr} = -I_\alpha^{rs}, \quad I_\alpha^{ss} = 0,$$

as demanded by momentum conservation.

We get I_2^{sr} from I_1^{sr} by replacing the index 1 by 2 in Eq.(3.8). Eqs. (3.8), (3.9) together with Eq. (2.2) show, as is well known from the transport theory of isotropic plasmas, that an electric field gives rise not only to a current d but also to a heat flux S, and so does the temperature gradient (Eqs. 3.20 - 3.31 and (2.6), (2.7)).

b) Collisional Heating and Temperature Relaxation

The collision moments of second order are defined by

$$J_{\alpha\beta}^{sr} = -m^s \int d^3v \; (v_\alpha - V_\alpha^s)(v_\beta - V_\beta^s) \frac{\partial}{\partial v_\gamma} j_\gamma^{sr}.$$

In particular, the change of $p_\perp^s$ and $p_\parallel^s$ due to collisions are given by

$$\left(\frac{\partial}{\partial t} p_\perp^s\right)_{coll} = \frac{1}{2} \sum_r J^{sr}_{\alpha\beta} n_{\alpha\beta} = \frac{1}{2}\sum_r (J^{sr}_{11} + J^{sr}_{22})$$

and

$$\left(\frac{\partial}{\partial t} p_\parallel^s\right)_{coll} = \sum_r J^{sr}_{\alpha\beta} \tau_{\alpha\beta} = \sum_r J^{sr}_{33}$$

where

(3.11)
$$\frac{1}{2}(J^{sr}_{11} + J^{sr}_{22}) = \frac{4\rho^s \nu^{sr}}{m^r + m^s} \left[2 K^{sr}_{200}(T^r_\perp - T^s_\perp) + \frac{m^r}{\beta^{rs}_\perp}(K^{sr}_{002} - K^{sr}_{200})\right] +$$
$$+ 4\nu^{sr} b^r_\perp \Big\{ K^{sr}_{200} \rho^s (d_1^2 + d_2^2) + (2 K^{sr}_{220} - K^{sr}_{200}) \beta^{sr}_\perp \Big[d_1 S_1^{s\perp} + d_2 S_2^{s\perp} - \frac{\rho^s}{\rho^r}(d_1 S_1^{r\perp} + d_2 S_2^{r\perp})\Big] + (\alpha^{sr} K^{sr}_{202} - \frac{1}{2} K^{sr}_{200}) \beta^{sr}_\parallel \Big[d_1 S_1^{s\parallel} + d_2 S_2^{s\parallel} - \frac{\rho^s}{\rho^r}(d_1 S_1^{r\parallel} + d_2 S_2^{r\parallel})\Big]\Big\}.$$

(3.12)
$$J^{sr}_{33} = \frac{8 \rho^s \nu^{sr}}{m^r + m^s} \alpha^{sr} \left[K^{sr}_{002} (T^r_\parallel - T^s_\parallel) + \frac{m^r}{\beta^{rs}_\parallel}(K^{sr}_{200} - K^{sr}_{002})\right] +$$
$$+ 8\nu^{sr} b^r_\parallel \Big[\alpha^{sr} K^{sr}_{002} \rho^s d_3^2 + (2 K^{sr}_{202} - K^{sr}_{002}) \beta^{sr}_\parallel (d_3 S_3^{s\perp} - \frac{\rho^s}{\rho^r} d_3 S_3^{r\perp}) + (\frac{1}{3}\alpha^{sr} K^{sr}_{004} - \frac{1}{2} K^{sr}_{002}) \alpha^{sr} \beta^{sr}_\parallel (d_3 S_3^{s\parallel} - \frac{\rho^s}{\rho^r} d_3 S_3^{r\parallel})\Big].$$

The first term of Eqs. (3.11) and (3.12) describes temperature relaxation between different species, the second term with $K_{002} - K_{200}$ describes the relaxation of anisotropy. These terms have already been derived by KOGAN (1961) and LEHNER (1967). The terms with d^2 and $d \cdot S$ describe Ohmic heating:

$$(J^{sr}_{33})_{ohm} = -2 b^r_\parallel I^{sr}_3 d_3$$

$$\frac{1}{2}(J^{sr}_{11} + J^{sr}_{22})_{ohm} = - b^r_\perp (I^{sr}_1 d_1 + I^{sr}_2 d_2)$$

The rate of change of thermal energy of species s by collisions with particles r is given by

$$\frac{\partial}{\partial t}\left(p_{\perp}^{s} + \frac{1}{2} p_{\parallel}^{s} \right) = \frac{1}{2}\left(J_{11}^{sr} + J_{22}^{sr} + J_{33}^{sr} \right). \qquad (3.13)$$

The rate of change of the total amount of thermal energy of all species is

$$\frac{1}{2} \sum_{s,r} J_{\alpha\alpha}^{sr} = - \frac{1}{2} \sum_{s,r} d_{\alpha}^{sr} I_{\alpha}^{sr} \qquad (3.14)$$

as demanded by energy conservation.

c) Momentum Transport by Viscosity

We now turn to Eq. (2.5) which describes the rate of change of the viscosity tensor π. For the collision terms on the right hand we get

$$J_{12}^{sr} = -8\nu^{sr} \left\{ \pi_{12}^{s} \left[(2b_{\perp}^{r}+\mu^{r}) K_{220}^{sr} + b_{\perp}^{s} K_{200}^{sr} \right] + \right.$$
$$\left. + \frac{\rho^{s}}{\rho^{r}} \pi_{12}^{r} \left[(2b_{\perp}^{r}+\mu^{r}) K_{220}^{sr} - b_{\perp}^{r} K_{200}^{sr} \right] \right\}. \qquad (3.15)$$

Replacing π_{12} by π_{11} in Eq. (3.15) we get an equation for $\frac{1}{2}(J_{11}^{sr} - J_{22}^{sr})$.

$$J^{sr}_{\alpha 3} = -4\nu^{sr}\Big\{ \pi^{s}_{\alpha 3} \Big[2\alpha^{sr}(b^{r}_{\perp}+b^{r}_{\parallel}+\mu^{r})K^{sr}_{202} + b^{s}_{\parallel}K^{sr}_{200} + \alpha^{sr}b^{s}_{\perp}K^{sr}_{002}\Big] +$$

(3.16)

$$+ \frac{\rho^{s}}{\rho^{r}}\pi^{r}_{\alpha 3} \Big[2\alpha^{sr}(b^{r}_{\perp}+b^{r}_{\parallel}+\mu^{r})K^{sr}_{202} - b^{r}_{\parallel}K^{sr}_{200} - \alpha^{sr}b^{r}_{\perp}K^{sr}_{002}\Big]\Big\}.$$

Eq. (3.16) holds for $\alpha = 1$ or 2.

d) Thermal Energy Transport

Finally, we have to calculate the collision integrals $L^{sr}_{\alpha\beta\gamma}$ for the rate of change of the two heat fluxes $\vec{S}^{s\perp}$ and $\vec{S}^{s\parallel}$ of Eqs. (2.6) and (2.7),

$$L^{sr}_{\alpha\beta\gamma} = -m^{s}\int d^{3}v \quad (v_{\alpha} - V^{s}_{\alpha})(v_{\beta} - V^{s}_{\beta})(v_{\gamma} - V^{s}_{\gamma})\frac{\partial}{\partial v_{\delta}} j^{sr}_{\delta}.$$

Without further assumptions, the formulas for $L^{sr}_{\alpha\beta\gamma}$ will be very long; we give them elsewhere (CHODURA and POHL, to be published). In this section we write them down for a two-component plasma only consisting of ions with charge number Z and electrons, indicated by indices i and e .

We further assume

(3.17)

$$\frac{T_e}{m_e} \gg \frac{T_i}{m_i} \qquad \beta^{e} \ll \beta^{i} .$$

Then we have $\alpha^{ee} = \alpha^{ie} = \alpha^{ei}$

and therefore $K^{ie}_{LMN} = K^{ei}_{LMN} = K^{ee}_{LMN}$.

For brevity we put

$$\alpha^{ii} = \alpha^{i}, \quad \alpha^{ee} = \alpha^{e}, \quad K^{ii}_{LMN} = K^{i}_{LMN}, \quad K^{ee}_{LMN} = K^{e}_{LMN}$$

$$V^{i}_{\alpha} - V^{e}_{\alpha} = d_{\alpha} .$$

$$\frac{1}{\nu^{sr}}(L^{sr}_{111}+L^{sr}_{221}) = \frac{2\rho^s}{\rho^{sr}_\perp} b^r_\perp d_1 \Big\{ (2\mu^r - 6b^r_\perp)K^{sr}_{220} + 2\mu^r K^{sr}_{202} + (3b^r_\perp - 1 - \mu^r)K^{sr}_{200} - \mu^r K^{sr}_{002} \Big\}$$

$$+ S^{s\perp}_1 \Big\{ 4b^r_\perp(\mu^r - 3b^r_\perp)K^{sr}_{420} + 8\mu^r b^r_\perp K^{sr}_{222} + 4\mu^r(1-2b^r_\perp)K^{sr}_{202} + 2\left[18(b^r_\perp)^2 - b^r_\perp(12+5\mu^r) + 3\mu^r\right]K^{sr}_{220} + (1-b^r_\perp)(9b^r_\perp - 3 - 2\mu^r)K^{sr}_{200} + 2\mu^r(b^r_\perp - 1)K^{sr}_{002} \Big\}$$

$$+ \frac{\rho^s}{\rho^r} S^{r\perp}_1 b^r_\perp \cdot \Big\{ 4(3b^r_\perp - \mu^r)K^{sr}_{420} + 8\mu^r \cdot (K^{sr}_{202} - K^{sr}_{222}) + 2(2 + 5\mu^r - 18b^r_\perp)K^{sr}_{220} + (9b^r_\perp - 2 - 2\mu^r)K^{sr}_{200} - 2\mu^r b^r_\perp K^{sr}_{002} \Big\} \tag{3.18}$$

$$+ \alpha^{sr} S^{s\parallel}_1 \Big\{ 4\alpha^{sr} b^r_\perp \left[(\mu^r - 3b^r_\perp)K^{sr}_{222} + \mu^r K^{sr}_{204}\right] + (1-b^r_\perp)\left[(3b^r_\perp - \mu^r)K^{sr}_{200} - \mu^r K^{sr}_{002} + 2\mu^r \alpha^{sr} K^{sr}_{004}\right] + 2b^r_\perp(3b^r_\perp - \mu^r)K^{sr}_{220} + \left[(1-b^r_\perp)\alpha^{sr}(2\mu^r - 6b^r_\perp) - 2\mu^r b^r_\perp\right]K^{sr}_{202} \Big\}$$

$$+ \frac{\rho^s}{\rho^r} \alpha^{sr} S^{r\parallel}_1 b^r_\perp \cdot \Big\{ 4\alpha^{sr}\left[(3b^r_\perp - \mu^r)K^{sr}_{222} - \mu^r K^{sr}_{204} + \tfrac{1}{2}\mu^r K^{sr}_{004}\right] + 2(\mu^r - 3b^r_\perp)K^{sr}_{220} + (3b^r_\perp - 1 - \mu^r)K^{sr}_{200} - \mu^r K^{sr}_{002} + 2\left[\alpha^{sr}(1+\mu^r) + \mu^r - 3\alpha^{sr} b^r_\perp\right]K^{sr}_{202} \Big\}$$

$$
\begin{aligned}
\frac{1}{8\gamma^{sr}}\left(L^{sr}_{113}+L^{sr}_{223}\right) = \frac{\rho^s}{\beta_\perp^{sr}}\, d_3 \Big\{ & 2\alpha^{sr}\left[\mu^r(b^r_\parallel-2b^r_\perp) - b^r_\perp(b^r_\perp+2b^r_\parallel)\right] K^{sr}_{202} + 2\mu^r\alpha^{sr} b^r_\parallel K^{sr}_{004} \\
& + b^r_\parallel(2b^r_\perp-\mu^r)K^{sr}_{200} + \left[\alpha^{sr} b^r_\perp(b^r_\perp-1) - \mu^r b^r_\parallel\right] K^{sr}_{002} \Big\} \\
+ S_3^{s\perp}\Big\{ & 8\alpha^{sr}\left[-b^r_\perp(b^r_\perp+2b^r_\parallel) + \mu^r(-2b^r_\perp+b^r_\parallel)\right]K^{sr}_{222} + 4\mu^r\alpha^{sr} b^r_\parallel K^{sr}_{204} \\
& + (1-b^r_\parallel)\left[(4b^r_\perp-2-\mu^r)K^{sr}_{200} - \mu^r K^{sr}_{002} + (4\mu^r-8b^r_\perp)K^{sr}_{220}\right] \\
& + 2\Big[\alpha^{sr} b^r_\perp(4b^r_\perp+4b^r_\parallel-3) - 2\alpha^{sr} b^r_\parallel + \mu^r\big(4\alpha^{sr} b^r_\perp - 2\alpha^{sr} \\
& \qquad + 1 - (1+\alpha^{sr})b^r_\parallel\big)\Big] K^{sr}_{202} \\
& + \alpha^{sr}\left[(1-b^r_\perp)(2b^r_\perp-1)K^{sr}_{002} - 2\mu^r b^r_\parallel K^{sr}_{004}\right] \Big\} \\
+ \frac{\rho^s}{\rho^r} S_3^{r\perp}\Big\{ & 8\alpha^{sr}\left[b^r_\perp(b^r_\perp+2b^r_\parallel) + \mu^r(2b^r_\perp-b^r_\parallel)\right]K^{sr}_{222} - 4\mu^r\alpha^{sr} b^r_\parallel K^{sr}_{204} \\
& + b^r_\parallel\left[4(\mu^r-2b^r_\perp)K^{sr}_{220} + 2\mu^r\alpha^{sr}K^{sr}_{004} + (4b^r_\perp-\mu^r)K^{sr}_{200}\right] \\
& + 2\left[\alpha^{sr} b^r_\perp(1-4b^r_\perp-4b^r_\parallel-4\mu^r) + \mu^r(\alpha^{sr}+1)b^r_\parallel\right]K^{sr}_{202} \\
& + \left[\alpha^{sr} b^r_\perp(2b^r_\perp-1) - \mu^r b^r_\parallel\right]K^{sr}_{002} \Big\} \\
+ \alpha^{sr} S_3^{s\parallel}\Big\{ & \tfrac{2}{3}(\alpha^{sr})^2\Big[\left[\mu^r(b^r_\parallel-2b^r_\perp) - b^r_\perp(b^r_\perp+2b^r_\parallel)\right]K^{sr}_{204} + \mu^r b^r_\parallel K^{sr}_{006}\Big] \\
& + \alpha^{sr}\left[b^r_\perp(b^r_\perp+4b^r_\parallel-2) + \mu^r(2b^r_\perp-2b^r_\parallel+1)\right]K^{sr}_{202} \\
& + \alpha^{sr}\left[\tfrac{1}{3}\alpha^{sr} b^r_\perp(b^r_\perp-1) + \mu^r(1-2b^r_\parallel)\right]K^{sr}_{004} \\
& + (1-b^r_\parallel)(b^r_\perp-\tfrac{1}{2}\mu^r)K^{sr}_{200} + \tfrac{1}{2}\left[\alpha^{sr} b^r_\perp(1-b^r_\perp) - \mu^r(1-b^r_\parallel)\right]K^{sr}_{002} \Big\} \\
+ \frac{\rho^s}{\rho^r}\alpha^{sr} S_3^{r\parallel}\Big\{ & \tfrac{2}{3}(\alpha^{sr})^2\Big[\left[\mu^r(2b^r_\perp-b^r_\parallel) + b^r_\perp(b^r_\perp+2b^r_\parallel)\right]K^{sr}_{204} - \mu^r b^r_\parallel K^{sr}_{006}\Big] \\
& + \alpha^{sr}\left[2\mu^r(b^r_\parallel-b^r_\perp) - b^r_\perp(b^r_\perp+4b^r_\parallel)\right]K^{sr}_{202} + b^r_\parallel(b^r_\perp-\tfrac{1}{2}\mu^r)K_{200} \\
& + \alpha^{sr}\left[\tfrac{1}{3}\alpha^{sr} b^r_\perp(1-b^r_\perp) + 2\mu^r b^r_\parallel\right]K^{sr}_{004} + \tfrac{1}{2}\left[\alpha^{sr}(b^r_\perp-1)b^r_\perp - \mu^r b^r_\parallel\right]K^{sr}_{002} \Big\}
\end{aligned}
\tag{3.19}
$$

$$
\begin{aligned}
\frac{1}{\;+\nu^{sr}}\, L_{331} = \frac{\rho^{s}}{\beta_{\perp}^{sr}}\, d_1 \Big\{ & 2b_{\perp}^{r}\left(b_{\parallel}^{r} K_{002}^{sr} + 4\mu^{r} K_{220}^{sr}\right) - 2b_{\parallel}^{r}\left(b_{\parallel}^{r} + 2b_{\perp}^{r} + 2\mu\right) K_{202}^{sr} \\
& - \left[\frac{1}{\alpha^{sr}}\, b_{\parallel}^{r}(1-b_{\parallel}^{r}) + 2\mu^{r} b_{\perp}^{r}\right] K_{200}^{sr} \Big\} \\
+ S_1^{s\perp}\Big\{ & 8\mu^{r} b_{\perp}^{r} K_{420}^{sr} + 2\left[b_{\parallel}^{r}(b_{\parallel}^{r}-1)/\alpha^{sr} + 4\mu^{r}(1-2b_{\perp}^{r})\right] K_{220}^{sr} \\
& + 2b_{\parallel}^{r}\Big[(b_{\parallel}^{r}+4b_{\perp}^{r}-2+2\mu^{r}) K_{202}^{sr} - (2b_{\parallel}^{r}+4b_{\perp}^{r}+4\mu^{r}) K_{222}^{sr} \\
& \qquad + (1-b_{\perp}^{r}) K_{002}^{sr}\Big] \\
& + \left[\frac{1}{\alpha^{sr}}\, b_{\parallel}^{r}(1-b_{\parallel}^{r}) + 2\mu^{r}(b_{\perp}^{r}-1)\right] K_{200}^{sr} \Big\} \\
+ \frac{\rho^{s}}{\rho^{r}}\, S_1^{r\perp}\Big\{ & -8\mu^{r} b_{\perp}^{r} K_{420}^{sr} + 2\left[b_{\parallel}^{r}(1-b_{\parallel}^{r})/\alpha^{sr} + 8\mu^{r} b_{\perp}^{r}\right] K_{220}^{sr} \\
& + 2b_{\parallel}^{r}\left[2(b_{\parallel}^{r}+2b_{\perp}^{r}+2\mu^{r}) K_{222}^{sr} - (b_{\parallel}^{r}+4b_{\perp}^{r}+2\mu^{r}) K_{202}^{sr} + b_{\perp}^{r} K_{002}^{sr}\right] \\
& + \left[\frac{1}{\alpha^{sr}}\, b_{\parallel}^{r}(b_{\parallel}^{r}-1) - 2\mu^{r} b_{\perp}^{r}\right] K_{200}^{sr} \Big\} \\
+ \alpha^{sr} S_1^{s\parallel}\Big\{ & 2\alpha^{sr} b_{\parallel}^{r}\left[-(b_{\parallel}^{r}+2b_{\perp}^{r}+2\mu^{r}) K_{204}^{sr} + (b_{\perp}^{r}-1) K_{004}^{sr}\right] - 4\mu^{r} b_{\perp}^{r} K_{220}^{sr} \\
& + 8\alpha^{sr}\mu^{r} b_{\perp}^{r} K_{222}^{sr} + \left[\tfrac{1}{2}(1-b_{\parallel}^{r})(3b_{\parallel}^{r}-2)/\alpha^{sr} - \mu^{r}(1-b_{\perp}^{r})\right] K_{200}^{sr} \\
& + \left[b_{\parallel}^{r}(6b_{\parallel}^{r}+6b_{\perp}^{r}-5) - 4b_{\perp}^{r} + 2\mu^{r}\left(3b_{\parallel}^{r}+\alpha^{sr}(1-b_{\perp}^{r}) - 2\right)\right] K_{202}^{sr} \\
& + (1-b_{\perp}^{r})(3b_{\parallel}^{r}-2) K_{002}^{sr} \Big\} \\
+ \alpha^{sr}\frac{\rho^{s}}{\rho^{r}}\, S_1^{r\parallel}\Big\{ & 2\alpha^{sr} b_{\parallel}^{r}\left[(b_{\parallel}^{r}+2b_{\perp}^{r}+2\mu^{r}) K_{204}^{sr} - b_{\perp}^{r} K_{004}^{sr}\right] + 4\mu^{r} b_{\perp}^{r}\left(K_{220}^{sr} - 2\alpha^{sr} K_{222}^{sr}\right) \\
& + \left[b_{\parallel}^{r}(1-6b_{\parallel}^{r}-6b_{\perp}^{r}) + 2\mu^{r}(\alpha^{sr} b_{\perp}^{r} - 3b_{\parallel}^{r})\right] K_{202}^{sr} \\
& + \left[\frac{1}{2\alpha^{sr}}\, b_{\parallel}^{r}(3b_{\parallel}^{r}-1) - \mu^{r} b_{\perp}^{r}\right] K_{200}^{sr} + 3b_{\perp}^{r} b_{\parallel}^{r} K_{002}^{sr} \Big\}
\end{aligned}
\tag{3.20}
$$

$$
\begin{aligned}
\frac{1}{12\, v^{sr}} L^{sr}_{333} = \frac{\rho^{s}}{\beta_{\perp}^{sr}}\, d_3 \Big\{ & b^{r}_{\parallel}\Big[(3b^{r}_{\parallel}-1)K^{sr}_{002} \; -2\alpha^{sr} b^{r}_{\parallel} K^{sr}_{004}\Big] \\
& + \mu^{r}\Big[4\alpha^{sr} b^{r}_{\parallel} K^{sr}_{202} \; -2b^{r}_{\parallel} K^{sr}_{200}\Big]\Big\} \\
+ S^{s\perp}_{3}\Big\{ & 2\alpha^{sr} b^{r}_{\parallel}\Big[8\mu^{r}K^{sr}_{222} \; -2b^{r}_{\parallel}K^{sr}_{204} \; +b^{r}_{\parallel}K^{sr}_{004}\Big] \\
& + (1-b^{r}_{\parallel})\Big[8\mu^{r}K^{sr}_{220} \; -2\mu^{r}K^{sr}_{200} \; +3b^{r}_{\parallel}K^{sr}_{002}\Big] \\
& + 2b^{r}_{\parallel}(3b^{r}_{\parallel} -3 -2\mu^{r}\alpha^{sr})K^{sr}_{202} \Big\} \\
+ \frac{\rho^{s}}{\rho^{r}} S^{r\perp}_{3}\, b^{r}_{\parallel}\Big\{ & 2\alpha^{sr}\Big[2b^{r}_{\parallel}K^{sr}_{204} \; -8\mu^{r}K^{sr}_{222} \; -b^{r}_{\parallel}K^{sr}_{004}\Big] \\
& + \mu^{r}(8K^{sr}_{220} \; -2K^{sr}_{200}) + (3b^{r}_{\parallel}-1)K^{sr}_{002} \\
& + 2(1 -3b^{r}_{\parallel} +2\mu^{r}\alpha^{sr})K^{sr}_{202} \Big\} \qquad (3.21) \\
+ \alpha^{sr}\, S^{s\parallel}_{3}\Big\{ & \tfrac{2}{3}(\alpha^{sr})^{2} b^{r}_{\parallel}\Big[2\mu^{r}K^{sr}_{204} \; -b^{r}_{\parallel}K^{sr}_{006}\Big] \\
& + \alpha^{sr}\Big[2\mu^{r}(1-2b^{r}_{\parallel})K^{sr}_{202} \; + \tfrac{1}{3}b^{r}_{\parallel}(10b^{r}_{\parallel}-7)K^{sr}_{004}\Big] \\
& + (1-b^{r}_{\parallel})\Big[(\tfrac{5}{2}b^{r}_{\parallel} -1)K^{sr}_{002} \; -\mu^{r}K^{sr}_{200}\Big]\Big\} \\
+ \alpha^{sr}\frac{\rho^{s}}{\rho^{r}} S^{r\parallel}_{3}\, b^{r}_{\parallel}\Big\{ & \tfrac{2}{3}(\alpha^{sr})^{2}\Big[b^{r}_{\parallel}K^{sr}_{006} \; -2\mu^{r}K^{sr}_{204}\Big] \\
& + \alpha^{sr}\Big[4\mu^{r}K^{sr}_{202} \; + \tfrac{1}{3}(1- 10b^{r}_{\parallel})K^{sr}_{004}\Big] \\
& (\tfrac{5}{2}b^{r}_{\parallel} - \tfrac{1}{2})K^{sr}_{002} \; -\mu^{r}K^{sr}_{200} \Big\}
\end{aligned}
$$

The equations for the $L^{sr}_{\alpha\beta\gamma}$ become much more simply in the case of equal particles (s=r). We introduce

$$(3.22) \qquad \Psi^s = (\alpha^s)^2 (K^s_{004} - K^s_{202}) + \frac{1}{2} \alpha^s (K^s_{200} - K^s_{002})$$

$$(3.23) \qquad \phi^s = 4K^s_{220} - 2K^s_{202} - K^s_{200} + K^s_{002} \,.$$

Then the collision integrals are of the following form:

$$(3.24) \qquad \frac{1}{4\,\nu^{ss}} L^{ss}_{331} = S^{s\perp}_1 \phi^s - S^{s\parallel}_1 \left[\psi^s + 6\alpha^s K^s_{202}\right]$$

$$(3.25) \qquad \frac{1}{12\,\nu^{ss}} L^{ss}_{333} = S^{s\perp}_3 \phi^s - S^{s\parallel}_3 \psi^s$$

$$(3.26) \qquad \frac{1}{8\,\nu^{ss}} (L^{ss}_{111} + L^{ss}_{221}) = S^{s\parallel}_1 \psi^s - S^{s\perp}_1 \left[\phi^s + 3K^s_{220}\right]$$

$$(3.27) \qquad \frac{1}{4\,\nu^{ss}} (L^{ss}_{113} + L^{ss}_{223}) = S^{s\parallel}_3 \psi^s - S^{s\perp}_3 \left[\phi^s + 12\alpha^s K^s_{202}\right].$$

4. TRANSPORT COEFFICIENTS FOR AN ELECTRON-ION PLASMA

We are now in a position to get explicit expressions for the transport coefficients of an anisotropic plasma, i.e. for viscosity, electrical and heat conductivity. For this purpose we have to solve Eqs. (2.6, 3.15, 3.16) for π^s and Eqs. (2.7, 2.8) and (3.20 - 3.31) for $S^{s\perp}$ and $S^{s\parallel}$ and insert S into the conductivity term I^s of Eq. (2.2). Even in the case of an electron-ion plasma this is a cumbersome task because equations for different species are coupled by collisions and equations for different components are coupled by the magnetic field.

Fortunately, in all practical cases electron viscosity will be small relative to ion viscosity (cf. BRAGINSKII, 1965).

Beyond this, in the ion-viscosity moments J^{ie}, terms with the electron viscosity π^e are weighted by a small factor $\frac{\nu^{ei}}{\nu^{ii}} \mu^e$ or $\frac{\nu^{ei}}{\nu^{ii}} b^e$ and thus can be neglected. We therefore give only expressions for the ion viscosity here.

Heat conduction parallel to the magnetic field in general will preferentially be carried out by electrons while in directions perpendicular to the field ions may be dominant. We therefore give expressions for the heat fluxes of them both. When calculating the ion-heat fluxes S^i contribution from S^e and d drop out. When calculating electron-heat fluxes S^e contributions by S^i to moments L^{ei} and I^{ei} are neglected being multiplied by a factor $\frac{\rho^e}{\rho^i}$.

Finally, by inserting the results for $S^{e\perp}$ and $S^{e\parallel}$ in the friction moment I^{ei} we get the electrical conductivity of the anisotropic plasma.

Let us first rewrite Eqs. (2.5) to (2.7) for a straight magnetic field with x_3-axis of the cartesian coordinate system parallel to the field lines.

In this case we have

(4.1) $$\underline{\underline{n}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \underline{\underline{\tau}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \underline{h} = (0,0,1)$$

(4.2) $$\underline{\underline{p}} = \begin{pmatrix} p_\perp + \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{12} & p_\perp - \pi_{11} & \pi_{23} \\ \pi_{13} & \pi_{23} & p_\parallel \end{pmatrix}$$

$$(4.3)\qquad -2\,\omega_c^s\,\pi_{12}^s + p_\perp^s\left[\frac{\partial V_1^s}{\partial x_1} - \frac{\partial V_2^s}{\partial x_2}\right] = \frac{1}{2}\sum_r (J_{11}^{sr} - J_{22}^{sr})$$

$$(4.4)\qquad 2\,\omega_c^s\,\pi_{11}^s + p_\perp^s\left[\frac{\partial V_1^s}{\partial x_2} + \frac{\partial V_2^s}{\partial x_1}\right] = \sum_r J_{12}^{sr}$$

$$(4.5)\qquad -\omega_c^s\,\pi_{23}^s + p_\parallel^s\frac{\partial V_1^s}{\partial x_3} + p_\perp^s\frac{\partial V_3^s}{\partial x_1} = \sum_r J_{13}^{sr}$$

$$(4.6)\qquad \omega_c^s\,\pi_{13}^s + p_\parallel^s\frac{\partial V_2^s}{\partial x_3} + p_\perp^s\frac{\partial V_3^s}{\partial x_2} = \sum_r J_{23}^{sr}$$

$$(4.7)\qquad p_\perp^s\frac{\partial}{\partial x_1}\frac{p_\parallel^s}{\rho^s} - \omega_c^s\,S_2^{s\parallel} + \frac{p_\parallel^s}{\rho^s}\sum_r I_1^{sr} = \sum_r L_{331}^{sr}$$

$$(4.8)\qquad p_\perp^s\frac{\partial}{\partial x_2}\frac{p_\parallel^s}{\rho^s} + \omega_c^s\,S_1^{s\parallel} + \frac{p_\parallel^s}{\rho^s}\sum_r I_2^{sr} = \sum_r L_{332}^{sr}$$

$$(4.9)\qquad 3p_\parallel^s\frac{\partial}{\partial x_3}\frac{p_\parallel^s}{\rho^s} + 3\,\frac{p_\parallel^s}{\rho^s}\sum_r I_3^{sr} = \sum_r L_{333}^{sr}$$

$$(4.10)\qquad 2p_\perp^s\frac{\partial}{\partial x_1}\frac{p_\perp^s}{\rho^s} - \omega_c^s\,S_2^{s\perp} + 2\,\frac{p_\perp^s}{\rho^s}\sum_r I_1^{sr} = \frac{1}{2}\sum_r (L_{111}^{sr} + L_{221}^{sr})$$

$$(4.11)\qquad 2p_\perp^s\frac{\partial}{\partial x_2}\frac{p_\perp^s}{\rho^s} + \omega_c^s\,S_1^{s\perp} + 2\,\frac{p_\perp^s}{\rho^s}\sum_r I_2^{sr} = \frac{1}{2}\sum_r (L_{112}^{sr} + L_{222}^{sr})$$

$$(4.12)\qquad p_\parallel^s\frac{\partial}{\partial x_3}\frac{p_\perp^s}{\rho^s} + \frac{p_\perp^s}{\rho^s}\sum_r I_3^{sr} = \frac{1}{2}\sum_r (L_{113} + L_{223})$$

In the following we assume that

$$\beta_\perp^e, \beta_\parallel^e \ll \beta_\perp^i, \beta_\parallel^i$$

Then

$$\alpha^{ie} = \alpha^{ei} = \alpha^{ee} = \alpha^e$$

and $K^{ie} = K^{ei} = K^{ee} = K^e$

We also put $K^{ii} = K^i$, $\vec{\underline{d}} = \vec{\underline{V}}^i - \vec{\underline{V}}^e$ in the following.

a) Ion Viscosity

Solving Eqs. (4.3 - 4.6, 3.15, 3.16) for π^i we get for the ion visity

$$\begin{pmatrix} \pi^i_{11} \\ \pi^i_{12} \end{pmatrix} = \frac{1}{a^2 + 4(\omega^i_c)^2} \left\{ \begin{pmatrix} -a \\ 2\omega^i_c \end{pmatrix} p^i_\perp \left[\frac{\partial V^i_1}{\partial x_1} - \frac{\partial V^i_2}{\partial x_2} \right] - \begin{pmatrix} 2\omega^i_c \\ a \end{pmatrix} p^i_\perp \left[\frac{\partial V^i_1}{\partial x_2} + \frac{\partial V^i_2}{\partial x_1} \right] \right\} \tag{4.13}$$

$$\begin{pmatrix} \pi^i_{13} \\ \pi^i_{23} \end{pmatrix} = \frac{1}{b^2 + (\omega^i_c)^2} \left\{ \begin{pmatrix} -b \\ \omega^i_c \end{pmatrix} \left[p^i_\parallel \frac{\partial V^i_1}{\partial x_3} + p^i_\perp \frac{\partial V^i_3}{\partial x_1} \right] - \begin{pmatrix} \omega^i_c \\ b \end{pmatrix} \left[p^i_\parallel \frac{\partial V_2}{\partial x_3} + p^i_\perp \frac{\partial V_3}{\partial x_2} \right] \right\} \tag{4.14}$$

with $a = 24\, \nu^{ii}\, K^i_{220} + 8\, \nu^{ie}\, K^e_{200}$

$$b = 24\, \nu^{ii}\, K^i_{202}\, \alpha^i + 4\, \nu^{ie} (K^e_{200} + \alpha^e\, K^e_{002})$$

For $a = b = 0$ we come back to the collision-less limit of part 1.

b) Electron- and Ion Heat Conductivity

Solution of Eqs.(4.7-4.12) and (3.20-3.31) for $S^{\perp}$ and $S^{\parallel}$ may be put in the following general form:

$$\underline{S}^{s\perp} = \left[\left(\omega_c^2 + a_{\parallel} b_{\perp} - a_{\perp} b_{\parallel}\right)^2 + \omega_c^2 (a_{\perp} + b_{\parallel})^2\right]^{-1} \frac{p_{\perp}^s}{m^s}$$

$$\Big\{\left[a_{\perp} (\omega_c^2 + b_{\parallel}^2) - a_{\parallel} b_{\perp} b_{\parallel}\right] 2 (\nabla_{\perp} T^{s\perp} - \underline{f}^{s\perp}) +$$

$$+ a_{\parallel} (\omega_c^2 + a_{\parallel} b_{\perp} - a_{\perp} b_{\parallel}) (\nabla_{\perp} T^{s\parallel} - \underline{f}^{s\parallel}) -$$

(4.15)

$$- \omega_c (\omega_c^2 + a_{\parallel} b_{\perp} + b_{\parallel}^2) \left[2 (\nabla T_{\perp}^s - \underline{f}^{s\perp}) \times \underline{h}\right] +$$

$$+ \omega_c a_{\parallel} (a_{\perp} + b_{\parallel}) \left[(\nabla T^{s\parallel} - \underline{f}^{s\parallel}) \times \underline{h}\right]\Big\} +$$

$$+ (c_{\perp} e_{\parallel} - c_{\parallel} e_{\perp})^{-1} \frac{p^{s\parallel}}{m^s} \left\{e_{\parallel} (\nabla_{\parallel} T^{s\perp} - \underline{k}^{s\perp}) - 3c_{\parallel} (\nabla_{\parallel} T^{s\parallel} - \underline{k}^{s\parallel})\right\}$$

$$\underline{S}^{s\parallel} = \left[(\omega_c^2 + a_{\parallel} b_{\perp} - a_{\perp} b_{\parallel})^2 + \omega_c^2 (a_{\perp} + b_{\parallel})^2\right]^{-1} \frac{p_{\perp}^s}{m^s}$$

$$\Big\{b_{\perp} (\omega_c^2 + a_{\parallel} b_{\perp} - a_{\perp} b_{\parallel}) 2 (\nabla_{\perp} T^{s\perp} - \underline{f}^{s\perp}) +$$

$$+ \left[b_{\parallel} (\omega_c^2 + a_{\perp}^2) - a_{\perp} a_{\parallel} b_{\perp}\right] (\nabla_{\perp} T^{s\parallel} - \underline{f}^{s\parallel}) +$$

(4.16)

$$+ \omega_c b_{\perp} (a_{\perp} + b_{\parallel}) \left[2 (\nabla T^{s\perp} - \vec{\underline{f}}^{s\perp}) \times \underline{h}\right] -$$

$$- \omega_c (\omega_c^2 + a_{\perp}^2 + a_{\parallel} b_{\perp}) \left[(\nabla T^{s\parallel} - \vec{\underline{f}}^{s\parallel}) \times \underline{h}\right]\Big\} +$$

$$+ (c_{\parallel} e_{\perp} - c_{\perp} e_{\parallel})^{-1} \frac{p^{s\parallel}}{m^s} \left\{e_{\perp} (\nabla_{\parallel} T^{s\perp} - \vec{\underline{k}}^{s\perp}) - c_{\perp} 3 (\nabla_{\parallel} T^{s\parallel} - \underline{k}^{s\parallel})\right\},$$

where $\nabla_{\parallel} = \underline{h}(\underline{h}\cdot\nabla)$ and $\nabla_{\perp} = \nabla - \nabla_{\parallel}$. s stands for e or i. The index s was omitted in the abbreviations a to e and in ω_c. These abbreviations mean the following:

Electrons: s = e ; α stands for α^e, K_{LMN} for K^e_{LMN}

$$\omega_c = - \frac{e\,B}{m_e}$$

$$a_{\perp} = -4\,\nu^{ee}\,(3K_{220} + \phi^e) + 16\nu^{ei}\,(-2K_{420} + 2K_{222} + 3K_{220} - K_{202} - \tfrac{1}{2}K_{200})$$

$$a_{\parallel} = 4\,\nu^{ee}\,\psi^e + 4\,\alpha\,\nu^{ei}\left[-8\alpha\,K_{222} + 4\,\alpha\,K_{204} + 4K_{220} + 2(\alpha-1)K_{202} - K_{200}\right]$$

$$b_{\perp} = 4\,\nu^{ee}\,\phi^e + 4\,\nu^{ei}\left[2(4K_{420} - 10K_{222} - 4K_{220} + 5K_{202}) + \frac{1}{\alpha}(2K_{220} - K_{200})\right]$$

$$b_{\parallel} = -4\,\nu^{ee}\,(6\alpha\,K_{202} + \psi^e) + 4\nu^{ei}\,(8\alpha^2\,K_{222} - 10\,\alpha^2\,K_{204} - 4\,\alpha\,K_{220} + 6\,\alpha\,K_{202} - \tfrac{1}{2}\,K_{200})$$

$$c_{\perp} = -4\,\nu^{ee}\,(6\,\alpha\,K_{202} + \frac{\phi^e}{2}) + 4\,\alpha\,\nu^{ei}(-32\,K_{222} + 4\,K_{204} + 10\,K_{202} - 2\,K_{004} - K_{002})$$

$$c_{\parallel} = 2\,\nu^{ee}\,\psi^e + 4\,\alpha^2\,\nu^{ei}\,(-\tfrac{8}{3}\alpha\,K_{204} + \tfrac{2}{3}\,\alpha\,K_{006} + 4\,K_{202} - (1 - \tfrac{\alpha}{3})K_{004} - \tfrac{1}{2}K_{002})$$

$$e_{\perp} = 12\,\nu^{ee}\,\phi^e + 12\,\nu^{ei}\,(16\alpha\,K_{222} - 4\,\alpha\,K_{204} - 2(2\,\alpha - 1)K_{202} + 2\alpha K_{004} - K_{002})$$

$$e_{\parallel} = -12\,\nu^{ee}\,\psi^e + 4\,\nu^{ei}\,\alpha\,(4\,\alpha^2\,K_{204} - 2\,\alpha^2\,K_{006} - 6\,\alpha\,K_{202} + 4\,\alpha\,K_{004} - \tfrac{3}{2}\,K_{002})$$

$$\underline{f}^{\perp} = 4\,\nu^{ei}\,m^{e}\,\underline{d}_{\perp}(\phi^{e} - K_{200})$$

$$\underline{f}^{\parallel} = 4\,\nu^{ei}\,m^{e}\,\underline{d}_{\perp}(6\,K_{202} - 2\,\phi^{e} - \frac{1}{\alpha}\,K_{200})$$

$$\underline{k}^{\perp} = -4\,\nu^{ei}\,m^{e}\,\underline{d}_{\parallel}\left[2\,\psi^{e} - \alpha^{2}\,(6\,K_{202} - K_{002})\right]$$

$$\underline{k}^{\parallel} = -4\,\nu^{ei}\,m^{e}\,\underline{d}_{\parallel}\left[-4\,\psi^{e} + \alpha\,(2\,\alpha\,K_{004} + K_{002})\right].$$

Ions:

$$\omega_c = \frac{Z\,e\,B}{m_i}$$

$$a_{\perp} = -4\,\nu^{ii}\,(3K^{i}_{220} + \phi^{i}) - 12\,\nu^{ie}\,K^{e}_{200}$$

$$a_{\parallel} = 4\,\psi^{i}\,\nu^{ii}$$

$$b_{\perp} = 4\,\phi^{i}\,\nu^{ii}$$

$$b_{\parallel} = -4\,\nu^{ii}\,(6\,\alpha^{i}\,K^{i}_{202} + \psi^{i}) - 4\,\nu^{ie}\,(K^{e}_{200} + 2\,\alpha^{e}\,K^{e}_{002})$$

$$c_{\perp} = -4\,\nu^{ii}\,(6\,\alpha^{i}\,K^{i}_{202} + \frac{1}{2}\,\phi^{i}) - 4\,\nu^{ie}\,(2\,K^{e}_{200} + \alpha^{e}\,K^{e}_{002})$$

$$c_{\parallel} = 2\,\psi^{i}\,\nu^{ii}$$

$$e_{\perp} = 12\,\phi^{i}\,\nu^{ii}$$

$$e_{\parallel} = -12\,\psi^{i}\,\nu^{ii} - 12\,\nu^{ie}\,\alpha^{e}\,K^{e}_{002}$$

$$\underline{f}^{i\perp} = \underline{f}^{i\parallel} = \underline{k}^{i\perp} = \underline{k}^{i\parallel} = 0$$

$\emptyset$ and ψ are defined in Eqs. (3.18) and (3.19).

In the case of a strong magnetic field, i.e. for $\omega_c \gg \nu$ the results of Eqs. (4.15) and (4.16) are obviously simplified.

If a species r happens to be isotropic, i.e. $T_{\perp}^{r} = T_{\parallel}^{r}$, only one heat flux S^r is attributed to this species, given by

$$\vec{S}^{r} = 2\vec{S}^{r\perp} + \vec{S}^{r\parallel}$$

(4.18)
$$\frac{2}{5} S_1^r = 2S_1^{r\parallel} = S_1^{r\perp}$$

$$\frac{1}{5} S_3^r = \frac{1}{3} S_3^{r\parallel} = S_3^{r\perp} .$$

This definition for S^r is equivalent with

$$\vec{S}^{r} = \int m^r \, (w^r)^2 \, \vec{w}^r \, f^r \, d^3v .$$

5. SPECIAL CASE: ANISOTROPIC IONS AND ISOTROPIC ELECTRONS

There is a special interest in a situation where in a two-component plasma only the ions are anisotropic whereas the electrons, due to their higher collision rate, are nearly isotropic.

In this case $K^{ee}_{LMN} = K^{ei}_{LMN} = K_{LMN}$ $(\alpha = 1)$ of table 1.

Thus the friction force becomes

(5.1) $$I^{ei}_{\alpha} = 2 \nu^{ei} \left(\frac{4}{3} \rho^{e} d_{\alpha} + \frac{2}{5} \beta^{e} S^{e}_{\alpha} \right), \quad d_{\alpha} = v^{i}_{\alpha} - v^{e}_{\alpha}$$

Deviating from page 3 we define the viscosity tensor of the electrons $\underline{\underline{\pi}}^{e}$ in this case by

(5.2) $$p^{e}_{\alpha\beta} = p^{e} \delta_{\alpha\beta} + \pi^{e}_{\alpha\beta}, \quad \pi^{e}_{\alpha\alpha} = 0$$

(while in the anisotropic case $\pi_{\alpha\beta} n_{\alpha\beta} = 0$ and $\pi_{\alpha\beta} \tau_{\alpha\beta} = 0$ separately).

The change of the isotropic part of the pressure tensor p^{e} is now given (for straight field lines) by

(5.3) $$3 \frac{\partial p^{e}}{\partial t} + 5 p^{e} \frac{\partial v^{e}_{\alpha}}{\partial x_{\alpha}} + 2 \pi^{e}_{\alpha\beta} \frac{\partial v^{e}_{\alpha}}{\partial x_{\beta}} + \frac{\partial s^{e}_{\alpha}}{\partial x_{\alpha}} = J^{ei}_{\alpha\alpha}$$

with $J^{ei}_{\alpha\alpha} = -\frac{16}{3} \nu^{ei} \frac{\rho^{e}}{m^{i}} \left(3 T^{e} - 2 T^{i}_{\perp} - T^{i}_{\parallel} \right)$

$\pi^{e}_{\alpha\beta}$ for $\alpha \neq \beta$ is given by (4.13) and (4.14), replacing ω^{i}_{c} by $-\omega^{e}_{c}$ and putting $a = b = \frac{16}{5} \nu^{ei}$.

π^{e}_{11}, π^{e}_{22} and π^{e}_{33} are given by:

(5.4) $$\pm 2\omega^{e}_{c} \pi^{e}_{12} + 2 p^{e} \begin{pmatrix} \frac{\partial v_1}{\partial x_1} \\ \frac{\partial v_2}{\partial x_2} \end{pmatrix} - \frac{2}{3} p^{e} \frac{\partial v^{e}_{\alpha}}{\partial x_{\alpha}} = -\frac{16}{5} \nu^{ei} \begin{pmatrix} \pi^{e}_{11} \\ \pi^{e}_{22} \end{pmatrix}$$

(5.5) $$2 p^{e} \frac{\partial v^{e}_{3}}{\partial x_3} - \frac{2}{3} p^{e} \frac{\partial v^{e}_{\alpha}}{\partial x_{\alpha}} = -\frac{16}{5} \nu^{ei} \pi^{e}_{33}$$

Ion viscosity is given by (4.13) and (4.14) with $K^{e}_{200} = K^{e}_{002} = -\frac{2}{3}$

Finally the heat-flux for the electrons may be derived:

(5.6) $$\underline{S}^e = 5\frac{p^e}{m_e}\left(\frac{b\,\nabla_\perp T^e + \omega_c^e\,\nabla_\perp T^e \times \underline{h}}{b^2 + (\omega_c^e)^2} + \frac{1}{b}\nabla_\parallel T^e\right)$$

$$- a\,p^e\left(\frac{b\,d_\perp + \omega_c^e\,\underline{d} \times \underline{h}}{b^2 + (\omega_c^e)^2} + \frac{1}{b}\,d_\parallel\right)$$

$$a = -\,8\,\nu^{ei}\ ,\quad b = -\,\frac{32}{15}\,\nu^{ee} - \frac{52}{15}\,\nu^{ei}$$

The ion-heat flux is given by (4.15) and (4.16) with

$$K^e_{200} = K^e_{002} = \frac{2}{3}$$

In the absence of a temperature gradient there still exists a heat flux $\underline{S}^e$ due to the current $\underline{d}$. This heat-flux tends to reduce the friction force I^{ei}_3 parallel to the magnetic field, as can be seen from (5.1). On the other hand, for strong magnetic fields ($\omega_c^e \gg \nu^{ee}, \nu^{ei}$) $S^e_\perp = 0$ and no reduction of $I^{ei}_\perp$ occurs. Thus, under this conditions, the resistivity

$$\frac{\eta_\parallel}{\eta_\perp} = \frac{I^{ei}_3}{I^{ei}_\perp} = \frac{\frac{4}{3} - \frac{2}{5}\frac{4}{b}}{\frac{4}{3}} = 1 - \frac{36\,Z^2 n_i}{16\sqrt{2}\,n_e + 52\,Z^2 n_i}$$

6. SOME DETAILS ABOUT CALCULATING THE INTEGRALS

Inserting eq.(2.10) into (2.8) we have by partial integration

$$(6.1) \qquad M^{sr} = A^{sr} \int d^3v \; d^3v' \; f^s(\vec{v}) \; f^r(\vec{v}') \; U$$

with

$$(6.2) \qquad U = \frac{1}{m^s} \sum_{\alpha,\beta} u_{\alpha\beta} \frac{\partial^2 a(\vec{v})}{\partial v_\alpha \partial v_\beta} - \frac{2}{g^3}\left(\frac{1}{m^s} + \frac{1}{m^r}\right) \sum_\alpha g_\alpha \frac{\partial a(\vec{v})}{\partial v_\alpha}$$

Because g occurs in the integrand, we introduce $\vec{g}$ and $\vec{h}$ instead of $\vec{v}$ and $\vec{v}'$ as integration variables. Defining

$$(6.3) \qquad \begin{aligned} v_\alpha - V_\alpha^s &= h_\alpha + \sum_\beta b_{\alpha\beta}^{rs}(g_\beta - d_\beta^{sr}) \\ v'_\alpha - V_\alpha^r &= h_\alpha - \sum_\beta b_{\alpha\beta}^{sr}(g_\beta - d_\beta^{sr}) \end{aligned}$$

with

$$d_\beta^{sr} = V_\beta^s - V_\beta^r$$

$$b_{\alpha\beta}^{sr} = b_\perp^s \, n_{\alpha\beta} + b_\parallel^s \, \tau_{\alpha\beta}$$

we have $\dfrac{\partial(\vec{v},\vec{v}')}{\partial(\vec{g},\vec{h})} = 1$

In the coordinate system used in eq.(2.16) we have

$$(6.4) \qquad \begin{aligned} f_o^s(\vec{v}) \; f_o^r(\vec{v}') &= (2\pi)^{-3} \, n^s n^r \; \beta_\perp^s \beta_\perp^r \sqrt{\beta_\parallel^s \beta_\parallel^r} \\ &\times \exp\left[-\tfrac{1}{2}\beta_\perp^{sr}(g_1^2+g_2^2) - \tfrac{1}{2}\beta_\parallel^{sr} g_3^2 - \tfrac{1}{2}\gamma_\perp^{sr}(h_1^2+h_2^2) - \tfrac{1}{2}\gamma_\parallel^{sr} h_3^2\right] \\ &\times \exp\left[\beta_\perp^{sr}(d_1 g_1 + d_2 g_2 - \tfrac{1}{2}d_1^2 - \tfrac{1}{2}d_2^2) + \beta_\parallel^{sr}(d_3 g_3 - \tfrac{1}{2}d_3^2)\right] \end{aligned}$$

Assuming (2.18) we expand the last term of eq.(6.3) as follows:

$$(6.5) \qquad \exp\left[\beta_\parallel^{sr}(d_3 g_3 - \tfrac{1}{2}d_3^2)\right] = 1 + \beta_\parallel^{sr} d_3 g_3$$

Note, that (6.5) strongly fails only in regions, where the integrand of (6.1) is practically zero.

We insert (6.3), (6.4), (6.5) and similar expressions for the other components into (2.16). Using (6.3) to replace $\vec{v},\vec{v}'$ by $\vec{g},\vec{h}$ we neglect $\vec{d}$ in (6.3) and get

$$f^s(\vec{v})\ f^r(\vec{v}') = (2\pi)^{-3}\ n^s n^r\ \beta_\perp^s \beta_\perp^r \sqrt{\beta_\parallel^s \beta_\parallel^r}$$

$$\times \exp\left[-\tfrac{1}{2}\beta_\perp^{sr}(g_1^2+g_2^2) - \tfrac{1}{2}\beta_\parallel^{sr} g_3^2 - \tfrac{1}{2}\gamma_\perp^{sr}(h_1^2+h_2^2) - \tfrac{1}{2}\gamma^{sr} h_3^2\right]$$

$$\times \Big\{ 1 + \beta_\perp^{sr} d_1 g_1 + \beta_\perp^{sr} d_2 g_2 + \beta_\parallel^{sr} d_3 g_3$$

$$+ \frac{\pi_{11}^s}{2\rho^s}(\beta_\perp^s)^2\left[(h_1 + b_\perp^r g_1)^2 - (h_2 + b_\perp^r g_2)^2\right] + ..$$

$$(6.6)\qquad + \frac{\pi_{11}^r}{2\rho^r}(\beta_\perp^r)^2\left[(h_1 - b_\perp^s g_1)^2 - (h_2 - b_\perp^s g_2)^2\right] + .. \Big\}.$$

The points stand for the terms containing the other π-components and the heat flux components. Second-order-terms, like $\pi^2, d\cdot S, \pi\cdot S, ..$ are neglected; (6.6) represnts an expansion up to terms which are small in first order.

In forming the expression for M^{sr} by introducing (6.3) into (6.2) and inserting in (6.1) only terms linear in $\vec{d}, \pi$ or S are retained. An exception was made for the calculation of $J_{11}+J_{22}$ and J_{33} where we had to include all terms in U in order to retain JOULE-heating and to fulfill energy conservation. Also the collision integrals $J_{11}-J_{22}$ and $J_{\alpha\beta}$, $\alpha\neq\beta$, would contain second-order-terms with $d\cdot S$ and d^2; but these terms would enter the energy equation (2.3) and (2.4) only in the term $\pi\frac{\partial V}{\partial x}$ and these corrections are small by a factor $\frac{V}{L\nu}$ (L=macroscopic length scale, see introduction) as compared to the JOULE heating terms.

We may write M^{sr} as a sum of integrals of the type

$$\hat{I} = \int d^3h\, d^3g\, h_1^I\, h_2^J\, h_3^K\, g_1^L\, g_2^M\, g_3^N\, g^{-3}$$

$$\times \exp\left[-\tfrac{1}{2}\gamma_\perp^{sr}(h_1^2+h_2^2) - \tfrac{1}{2}\gamma_\parallel^{sr}h_3^2 - \tfrac{1}{2}\beta_\perp^{sr}(g_1^2+g_2^2) - \tfrac{1}{2}\beta_\parallel^{sr}g_3^2\right]$$

I,J,K,L,M,N being integers ≥ 0

$\hat{I}$ vanishes, if at least one of the numbers I,J,K,L,M,N is odd.

$\hat{I}$ diverges if I=J=K=L=M=N=O; but this case cannot occur due to eq. (6.2)

If I,J,K,L,M,N are even, we have

$$\hat{I} = \sqrt{2}\,(2\pi)^{-5/2}\, \beta_\perp^{sr}\sqrt{\beta_\parallel^{sr}}\,(I-1)!!\,(J-1)!!\,(K-1)!!\,K_{LMN}$$

$$\times \left(\gamma_\perp^{sr}\right)^{-1-\frac{I+J}{2}} \left(\gamma_\parallel^{sr}\right)^{-\frac{1+K}{2}} \left(\tfrac{1}{2}\beta_\perp^{sr}\right)^{-1-\frac{L+M}{2}} \left(\tfrac{1}{2}\beta_\parallel^{sr}\right)^{-\frac{1+N}{2}}$$

with $(K-1)!! = 1*3*5*7*...*(K-1)$

$(-1)!! = 1$

We calculate the K_{LMN} introducing spherical coordinates in the $u_1u_2u_3$ space.

Using the relations

$$K_{LMN} = K_{MLN}$$

$$K_{4ON} = 3K_{22N}$$

$$K_{6ON} = 5K_{42N}$$

we may reduce all K_{LMN} with $L + M + N \leq 6$ to the K_{LMN} given in table I.

Tab. 1 $K_{LMN}(\alpha)$ as defined in Eq. (3.7) as function of

$$X = \alpha - 1$$

L M N	K_{LMN}	K_{LMN} for $X \ll 1$
2 0 0	$X^{-1}\left[-1 + (1+X)\,\varphi(X)\right]$	$\frac{2}{3} - \frac{2}{15}X$
0 0 2	$X^{-1}\,2\left[1 - \varphi(X)\right]$	$\frac{2}{3} - \frac{2}{5}X$
2 2 0	$X^{-2}\,\frac{1}{8}\left[3 + X + (1+X)(X-3)\,\varphi(X)\right]$	$\frac{2}{15} - \frac{4}{105}X$
2 0 2	$X^{-2}\,\frac{1}{2}\left[-3 + (3+X)\,\varphi(X)\right]$	$\frac{2}{15} - \frac{4}{35}X$
0 0 4	$X^{-2}\left[2 + \frac{1}{1+X} - 3\,\varphi(X)\right]$	$\frac{2}{5} - \frac{4}{7}X$
4 2 0	$X^{-3}\,\frac{1}{32}\left[-15 - 4X + 3X^2 + (15+9X-3X^2+3X^3)\varphi(X)\right]$	$\frac{4}{105}(3-X)$
2 2 2	$X^{-3}\,\frac{1}{16}\left[15 + X + (-15 - 6X + X^2)\,\varphi(X)\right]$	$\frac{4}{105}(1-X)$
2 0 4	$X^{-3}\,\frac{1}{4}\left[-13 - \frac{2}{1+X} + (15 + 3X)\,\varphi(X)\right]$	$\frac{4}{105}(3-5X)$
0 0 6	$X^{-3}\,\frac{1}{2}\left[8 + \frac{9}{1+X} - \frac{2}{(1+X)^2} - 15\,\varphi(X)\right]$	$\frac{4}{7} - \frac{4}{3}X$

with $\varphi(X) = \frac{1}{\sqrt{X}}\,\mathrm{atan}\,(\sqrt{X})$.

Tab. 2 Moments of Distribution Function

Q	$\int Q\, f^s(v)\, d^3v$	
1	n^s	particle density
$\frac{1}{n^s}\, v_\beta$	V^s_β	hydrodynamic velocity β-component
$\frac{1}{n^s}\, \frac{m^s}{2}\, w^2$	$\frac{3}{2}\, T^s$	averaged kinetic energy of a particle; $w_\beta = v_\beta - V^s_\beta$
$\frac{m^s}{2}\, (w_1^2 + w_2^2)$	$p^s_\perp$	components of pressure tensor taken in a coordinate sy- system with x_3 parallel to the magnetic field
$m^s\, w_3^2$	$p^s_\parallel$	
$m^s\, w_i w_k$	π^s_{ik} $(i \neq k)$	
$\frac{m^s}{2}\, (w_1^2 - w_2^2)$	π^s_{11}	
$m^s\, w^2\, w_\beta$	S^s_β	heat flux vector, β-component
$\frac{m^s}{2}\, w_\perp^2\, w_\beta$	$S^{s\perp}_\beta$	If x_3 is parallel to the magnetic field, we have $w_\perp^2 = w_1^2 + w_2^2$ and $w_\parallel^2 = w_3^2$
$m^s\, w_\parallel^2\, w_\beta$	$S^{s\parallel}_\beta$	

Table 3 Symbols Used in this Paper

We give here for each symbol used in this paper the number of the equation defining the symbol. If the defining equation does not have a number, we write, for example,

a after (4.14)

which means, that the equation defining a is situated one or a few lines after eq.(4.14).

A^{sr}	(2.11)	h_α	before (2.1)
$a^s(v)$	after (2.8)	I_α^{sr}	before (3.8)
a	after (4.14)	$\hat{I}$	after (6.4)
$a_\perp, a_\parallel$	after (4.16)	$J_{\alpha\beta}^{sr}$	before (3.11)
$b_\perp, b_\parallel$	after (4.16)	K_{LMN}^{sr}	(3.7)
$b_\perp^s, b_\parallel^s, b_\perp^r$	(3.6)	$\vec{\underline{k}}^{e\perp}, \vec{\underline{k}}^{e\parallel}$	after (4.16)
b	after (4.14)	$L_{\alpha\beta\gamma}^{sr}$	(3.17)
B	magnetic field	m^s, n^s	before (2.1)
$c_\perp, c_\parallel$	after (4.16)	M^{sr}	(2.8)
D	(2.14)	$n_{\alpha\beta}$	before (2.1)
d_α, d_α^{sr}	(2.17)	$p_\perp^s, p_\parallel^s$	before (2.1)
$d_\perp, d_\parallel$	(2.18)	q^s	before (2.1)
$\vec{\underline{d}}_\parallel = \{0,\ 0,\ v_3^i - v_3^e\}$		r, s	species index
$\vec{\underline{d}}_\perp = \{v_1^i - v_1^e;\ v_2^i - v_2^e;\ 0\}$		$S_\alpha^{s\perp}, S_\alpha^{s\parallel}$	heat flux, before (2.1)
$e_\perp, e_\parallel$	after (4.16)	$T_\perp^s, T_\parallel^s$	temperature
E_α	electric field (2.2)	$u_{\alpha\beta}$	(2.12)
$\vec{\underline{f}}^{e\perp}, \vec{\underline{f}}^{e\parallel}$	after (4.16)	v_α	particle velocity
$f^s(w)$	(2.16)	V^s	before (2.1)
$f_o^s(w)$	after (2.16)	w^s	(2.15)
g	after (2.14)	Z	ion charge

Table 3 (continuation) Greek symbols

α, β, γ	component index	$\Pi^s_{\alpha\beta}$	viscosity, before (2.1)
α^e	after (4.12)	ρ^s	mass density, before (2.1)
α^{sr}	(3.3)	σ	electric conduct. before (5.8)
$\beta^s_\perp, \beta^s_\parallel, \beta^r_\perp$	(2.15)	$\tau_{\alpha\beta}$	before (2.1)
$\beta^{sr}_\perp, \beta^{sr}_\parallel$	(3.2)	Φ^s	(3.23)
$\delta_{\alpha\beta}$	1 for $\alpha=\beta$ 0 " $\alpha\neq\beta$	Ψ^s	(3.22)
$\varepsilon_{\alpha\beta\gamma}$	before (2.1)	ω^s_c	before (2.1)
Λ^{sr}	(2.13)		
μ^s, μ^r	(3.5)		
ν^{sr}	(3.1)		

REFERENCES

BOWERS, E. and HAINES, M.G. (1968) Phys. Fluids 11, 2695

BRAGINSKII, S.I. (1965) Review of Plasma Physics, M.A. Leontovich Ed., Consultants Bureau, New York, Vol. 1

CHEW, G.F., GOLDBERGER, M.L., and LOW, F.E. (1956) Proc.Roy.Soc. A 236, 112

CHODURA R. and POHL, F. (to be published) Report of the Institut für Plasmaphysik, Garching / Munich

ESPEDAL, M.S. (1969) Proc. 9th Int. Conf. on Phenomena in Ionized Gases, Bucharest

FRIEMAN, E. DAVIDSON, R. , and LANGDON, B. (1966) Phys. Fluids 9, 1475

HERDAN, R. and LILEY, B.S. (1960) Rev.Mod.Phys. 32, 731

KENNEL, C.F. and GREEN, J.M. (1966) Ann. Phys. (N.Y.) 38, 63

KOGAN, V.J. (1961) Plasma Physics and the Problem of Controlled Thermonuclear Reactions, M.A. Leontovich Ed., Pergamon Press, New York, Vol. 1

LEHNER, G. (1967) Z. Physik, 206, 284

MAC MAHON, A. (1965) Phys. Fluids 8, 1840

ORAEVSKII, V., CHODURA, R., and FENEBERG, W. (1968) Plasma Phys. 10, 819

TRUBNIKOV, B.A. (1965) Review of Plasma Physics, M.A. Leontovich Ed., Consultants Bureau, New York, Vol. 1