Neutron Flux Anisotropy of High Energy Deuterium Plasmas

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Abstract

The neutron flux anisotropy of high energy anisotropic deuterium plasmas is discussed for elliptic and monoenergetic velocity distributions of the deuterons.

Because the anisotropy coefficient of the differential cross section is not very well known at low bombarding energies, computations have been performed for two different laws corresponding to the results of BOOTH et al. /1/ and of THEUS et al./2/.

1. Introduction

The d-d reaction is quite strongly anisotropic. This fact offers the unique possibility of measuring directly the anisotropy of a sufficiently energetic deuterium plasma. It is just necessary to place scintillation counters side-on and end-on and thus to measure the corresponding neutron fluxes. Their ratio, if measured absolutely, may then be interpreted in terms of the anisotropy of the plasma. Absolute measurements may be difficult. But even relative measurements allow interesting conclusions if the ratio changes in time, i.e. it indicates increasing or decreasing anisotropy.

This method was proposed in an earlier paper /3/, and its experimental application has been discussed in references /4,5/.

A problem connected with this is that the differential cross sections are not very well known, especially at the low energies which are of main interest in plasma physics.

In our earlier computations we used the differential cross sections as given by BOOTH et al./1/, but we were well aware that this is perhaps not the last word. In the meantime J.NEUHAUSER has drawn our attention to the more recent measurements of THEUS /2/. His results do not really contradict the results of BOOTH et al./1/ if one takes into account the relatively large errors at low energies, but THEUS's results suggest another behaviour of the cross section as the energy approaches zero than BOOTH's results. So the situation is not really clear.

Figure 1 shows both the results of BOOTH et al. and of THEUS for the neutron branch of the d-d reaction. BOOTH et al. give the differential cross section in the following form

$$\sigma(g,\theta) = A(g)\left[1 + B(g)\cos^2\theta\right] \quad (1)$$

with $$B(g) = 0.31 + 0.0058\ E_d \quad (2)$$

where $E_d = \frac{m}{2} g^2$ is the bombarding energy in keV,

$\vec{g}$ is the relative velocity,

θ is the angle between $\vec{g}$ and the direction of neutron emission in the center-of-mass system,

and m is the deuteron mass.

In order to have a comfortable analytic representation of THEUS's results in the low energy region, we have chosen the arctg of figure 1. It should be noted, however, that this is a very bad fit for higher energies ($E_d > 300$ keV), but these are not important for our purposes. So we use

$$B(g) = 0.83\ \text{arctg}\ (0.02\ E_d), \quad E_d \text{ in keV} \quad (3)$$

Thus far we have discussed the neutron branch of the d-d reaction only. The situation is very similar for the proton branch. Both BOOTH et al. and THEUS also give its anisotropy. In the present paper we discuss only the neutron branch for two reasons: First, the neutron branch is more anisotropic and is thus better adapted to our plasma diagnostic purposes, i.e. the measurements based on the neutrons are more sensitive with respect to anisotropy. Second, the neutrons are easier to observe because they may freely leave the discharge region without being disturbed by the magnetic fields. This does not exclude similar measurements on the protons generated by fusion reactions. The general results of this report may be used for protons as well. The numerical results, however apply to the neutron branch only.

2. The flux anisotropy

Consider a deuterium plasma with a given velocity distribution of the deuterons. We fix our attention on a certain pair of particles with

center-of-mass-velocity $\vec{s}$

and relative velocity $\vec{g}$.

It is easy to show that the "differential reaction rate", i.e. the number of neutrons emitted per unit time and per unit plasma volume into the unit solid angle, is given by

$$\frac{dR}{d\Omega} = \frac{n^2}{2} \iint f(\vec{s} + \frac{\vec{g}}{2}) \cdot f(\vec{s} - \frac{\vec{g}}{2}) \cdot g \cdot \sigma(g,\theta)\ d^3s\ d^3g \qquad (4)$$

in the center-of-mass-system

f is the distribution of velocity in the plasma. This reaction rate should, in principle, be transformed into the laboratory system. If the plasma energy is much smaller than the reaction energy - in other words, if the center-of-mass velocities are much smaller than the velocities of the neutrons emitted - we can neglect the difference of the fluxes in the center-of-mass and laboratory systems. For this reason we use eq.(2) to compute the fluxes in the laboratory system also. We consider first an elliptic distribution of the deuterons, i.e. we assume

$$f = \frac{\beta_\perp \beta_\parallel^{1/2}}{\pi^{3/2}} \exp\left[- \beta_\perp u^2 - \beta_\parallel u^2 \right] \qquad (5)$$

$$\beta_\perp = \frac{m}{2kT_\perp} \qquad \beta_\parallel = \frac{m}{2kT_\parallel} \qquad (6)$$

Omitting all details of algebra we state that in this case we get

$$\frac{dR}{d\Omega} = \frac{\beta_\perp \beta_\parallel^{1/2} n^2}{\sqrt{2\pi}} \int_0^1 d\xi \int_0^\infty dg \exp\left[- \frac{\beta_\perp}{2} g^2 (1-\xi^2) - \frac{\beta_\parallel}{2} g^2 \xi^2 \right] \tag{7}$$

$$\cdot\, g^3 A(g) \left\{ 1 + \frac{B(g)}{2} \left[(1 - \xi^2) \sin^2\alpha + 2\xi^2 \cos^2\alpha \right] \right\}$$

α is the angle between the direction of observation and the parallel axis.

We are not interested in the absolute value of $dR/d\Omega$.

We consider just the ratio

$$V = \frac{\left(\frac{dR}{d\Omega}\right)_{\alpha = 90^\circ}}{\left(\frac{dR}{d\Omega}\right)_{\alpha = 0^\circ}} \tag{8}$$

i.e. the ratio of fluxes side-on and end-on.

$$V = \frac{\int_0^1\int_0^\infty \exp\left[- \frac{1}{2}\beta_\perp g^2 (1-\xi^2) - \frac{\beta_\parallel}{2} g^2 \xi^2 \right] g^3 A(g) \left[1 + \frac{1}{2} B(g) (1-\xi^2) \right] dg\, d\xi}{\int_0^1\int_0^\infty \exp\left[- \frac{1}{2}\beta_\perp g^2 (1-\xi^2) - \frac{\beta_\parallel}{2} g^2 \xi^2 \right] g^3 A(g) \left[1 + B(g) \xi^2 \right] dg\, d\xi} \tag{9}$$

Introducing mean values with respect to both g and ξ, which are defined as follows:

$$\langle\langle F(g,\xi) \rangle\rangle = \frac{\iint \exp\left[- \frac{1}{2}\beta_\perp g^2 (1-\xi^2) - \frac{\beta_\parallel}{2} g^2 \xi^2 \right] g^3 A(g)\, F(g,\xi)\, dg\, d\xi}{\iint \exp\left[- \frac{1}{2}\beta_\perp g^2 (1-\xi^2) - \frac{\beta_\parallel}{2} g^2 \xi^2 \right] g^3 A(g)\, dg\, d\xi} \tag{10}$$

V may be written as

$$V = \frac{1 + \frac{1}{2}\left[\langle\langle B(g) \rangle\rangle - \langle\langle B(g)\, \xi^2 \rangle\rangle \right]}{1 + \langle\langle B(g)\, \xi^2 \rangle\rangle} \tag{11}$$

Let us now discuss more special cases. First we consider a two-dimensional Maxwellian for which $\beta_\parallel \Rightarrow \infty$.

Either by taking this limit of equation (9) or by introducing a two-dimensional Maxwellian directly into eq. (4) we obtain

$$V = 1 + \frac{1}{2} \frac{\int_0^\infty \exp(-\frac{1}{2}\beta_\perp g^2)\, g^2\, A(g)\, B(g)\, dg}{\int_0^\infty \exp(-\frac{1}{2}\beta_\perp g^2)\, g^2\, A(g)\, dg} \tag{12}$$

or

$$V = 1 + \frac{1}{2} \langle B(g) \rangle_2 \tag{13}$$

where $\langle B(g) \rangle_2$ is the mean value appearing in eq. (12). Similarly we obtain for a one-dimensional Maxwellian ($\beta_\perp \Rightarrow \infty$)

$$\frac{1}{V} = 1 + \frac{\int_0^\infty \exp(-\frac{1}{2}\beta_\parallel g^2)\, g\, A(g)\, B(g)\, dg}{\int_0^\infty \exp(-\frac{1}{2}\beta_\parallel g^2)\, g\, A(g)\, dg} \tag{14}$$

or

$$V = \frac{1}{1 + \langle B(g) \rangle_1} \tag{15}$$

Another example is a two-dimensional monoenergetic distribution, i.e.

$$f = \frac{1}{\pi} \delta(u_\perp^2 - u_0^2)\, \delta(u_\parallel) \tag{16}$$

which yields

$$\frac{dR}{d\Omega} = \frac{n^2}{2\pi} \int_0^{2u_0} \frac{g\, A(g) \left[1 + B(g) \frac{\sin^2\alpha}{2} \right]}{\left(u_0^2 - \frac{1}{4} g^2\right)^{1/2}} dg$$

so that

$$V = 1 + \frac{1}{2} \frac{\int_0^{2u_o} (4u_o^2 - g^2)^{-1/2} gA(g)\, B(g)\, dg}{\int_0^{2u_o} (4u_o^2 - g^2)^{-1/2} gA(g)\, dg} \quad (17)$$

or

$$V = 1 + \frac{1}{2} \langle B(g) \rangle_{mono2} \quad (18)$$

3. Approximations

The results for two-dimensional Maxwellians, one-dimensional Maxwellians and two-dimensional monoenergetic distributions, i.e. eq.(12), (14), (17) may be approximated for both branches of the d-d reaction in the following manner.

For the Maxwellian cases it is known that most reactions occurring correspond to a relative energy E_d, which is approximately given by

$$E_d \approx 13 \cdot (kT)^{2/3} \quad (19)$$

(E_d and kT in keV, see reference /3/ for details), and thus the mean values $\langle B(g) \rangle_2$ and $\langle B(g) \rangle_1$ may both be approximately obtained just by choosing g corresponding to eq. (19):

$$\langle B(g) \rangle_2 \approx B(g) \quad \text{for} \quad \frac{m}{2}g^2 = 13\ (kT_\perp)^{2/3} \quad (20)$$

$$\langle B(g) \rangle_1 \approx B(g) \quad \text{for} \quad \frac{m}{2}g^2 = 13\ (kT_\parallel)^{2/3} \quad (21)$$

For the two-dimensional monoenergetic distribution the main contribution to the averaged value of B(g) comes from g-values close to the upper limit of the integral in eq.(17), first because the cross section A(g) increases with g, and second because of the large weight given to this g - values by the root in the denominator. So we may approximately write

$$\langle B(g) \rangle_{mono2} \approx B(g=2u_o) \quad (22)$$

As mentioned already, these approximations may be used for both the neutrons and the protons produced. In the next section we give numerical results for the neutron branch only, and comparison shows that approximations (20) to (22) are rather good.

4. Numerical results

Because the behaviour of the anisotropy is not really clear we have computed the flux ratios V defined in section 2 for both the linear dependence for B(g) of BOOTH (eq. (2)) and for the arctg dependence (eq.(3)), the latter of which fits the results of THEUS at low energies.

Figures 2 and 3 give V for elliptic distributions with perpendicular and parallel temperatures from 0 to 10 keV.

Figures 4 gives the corresponding results for two-dimensional monoenergetic distributions, also in the 0 to 10 keV range. In the case

$$B(g) = 0.31 + 0.0058\ E_d$$

V is not really defined for zero temperature. For instance,
$V = 1.155$ for a two-dimensional Maxwellian with $kT_\perp \Rightarrow 0$ (see eq.13),
$V = 0.76$ " " one " " " $kT_\parallel \Rightarrow 0$ (see eq.15).

Actually any value between 1.155 and 0.76 can be obtained by fixing the ratio $T_\parallel / T_\perp$ and then going to the limit of zero temperatures.

Thanks are due to J.NEUHAUSER, who drew our attention to the measurements of THEUS.

Note added in proof.

After the completion of this report the problem of the d-d reaction and its anisotropy at zero energy has again been discussed theoretically by Boersma /6/. His conclusion is that an anisotropy should exist even in the limit of zero energy as pointed out by Beiduk et al./7,8/ in 1950 already. The theoretical results confirm the experimental results of Eliot et al./9/ and of Booth et al./2/, while the corresponding curves of Theus et al./1/ seem to need modification.

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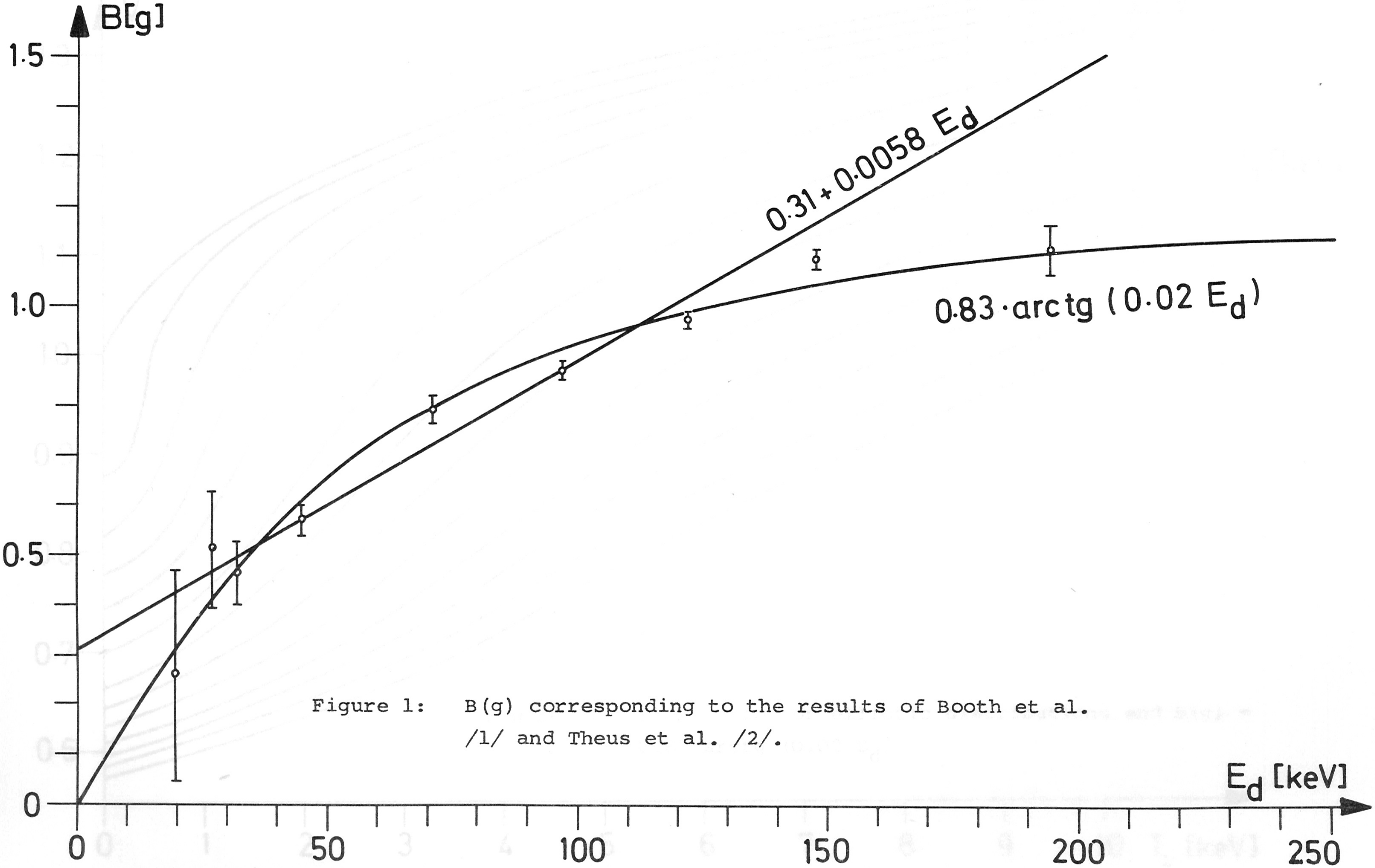

Figure 1: B(g) corresponding to the results of Booth et al. /1/ and Theus et al. /2/.

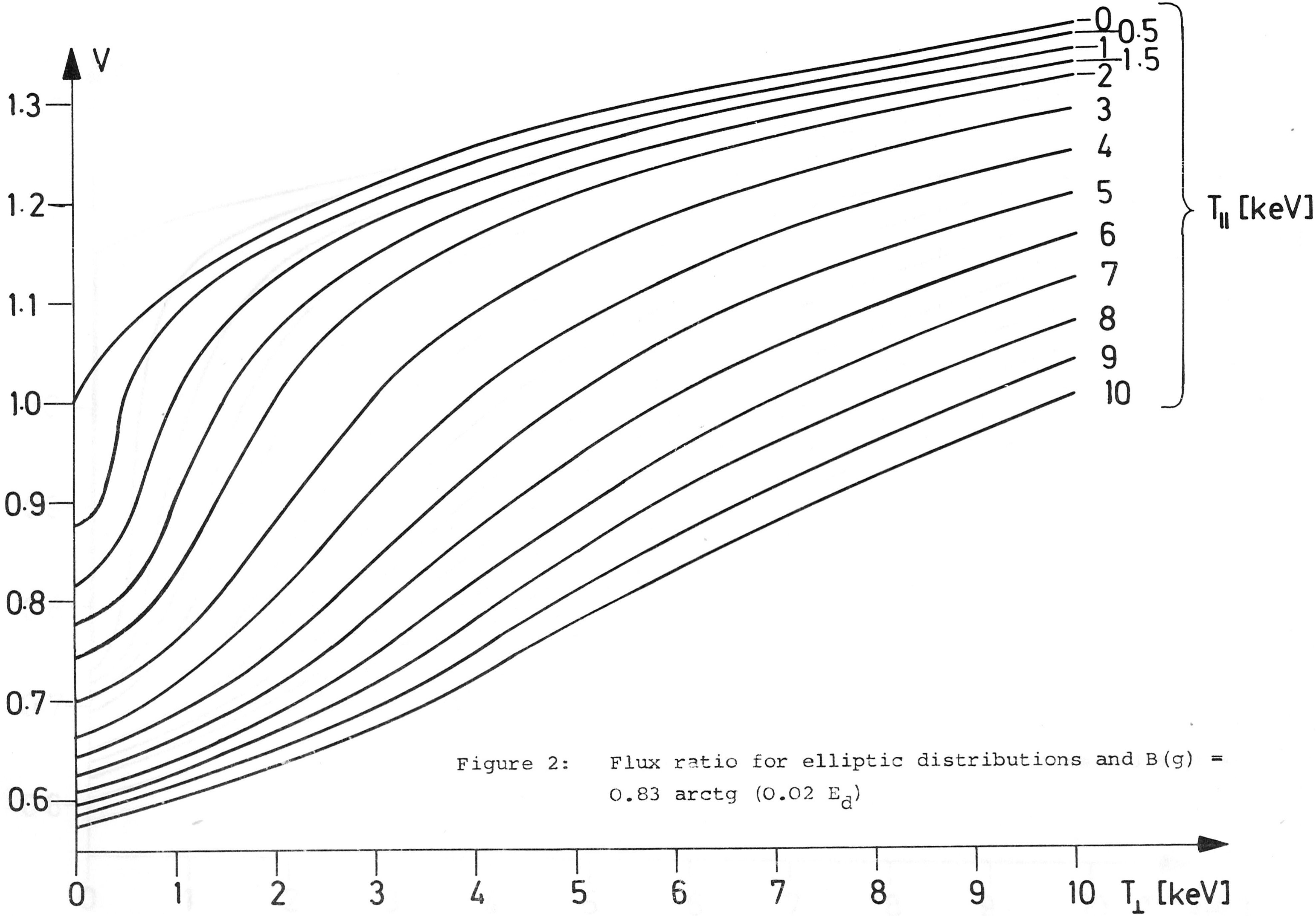

Figure 2: Flux ratio for elliptic distributions and B(g) = 0.83 arctg (0.02 E_d)

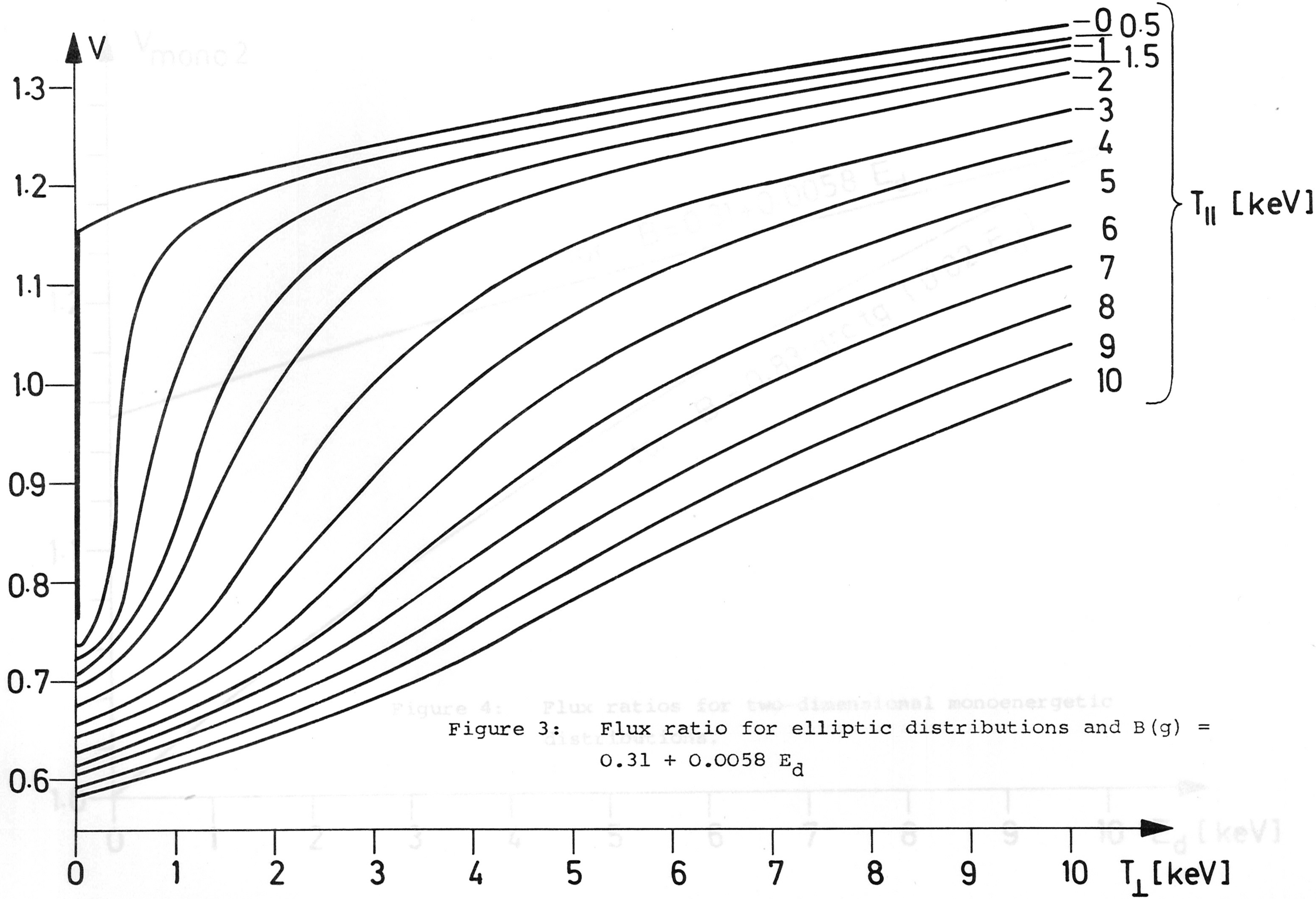

Figure 3: Flux ratio for elliptic distributions and B(g) = 0.31 + 0.0058 E_d

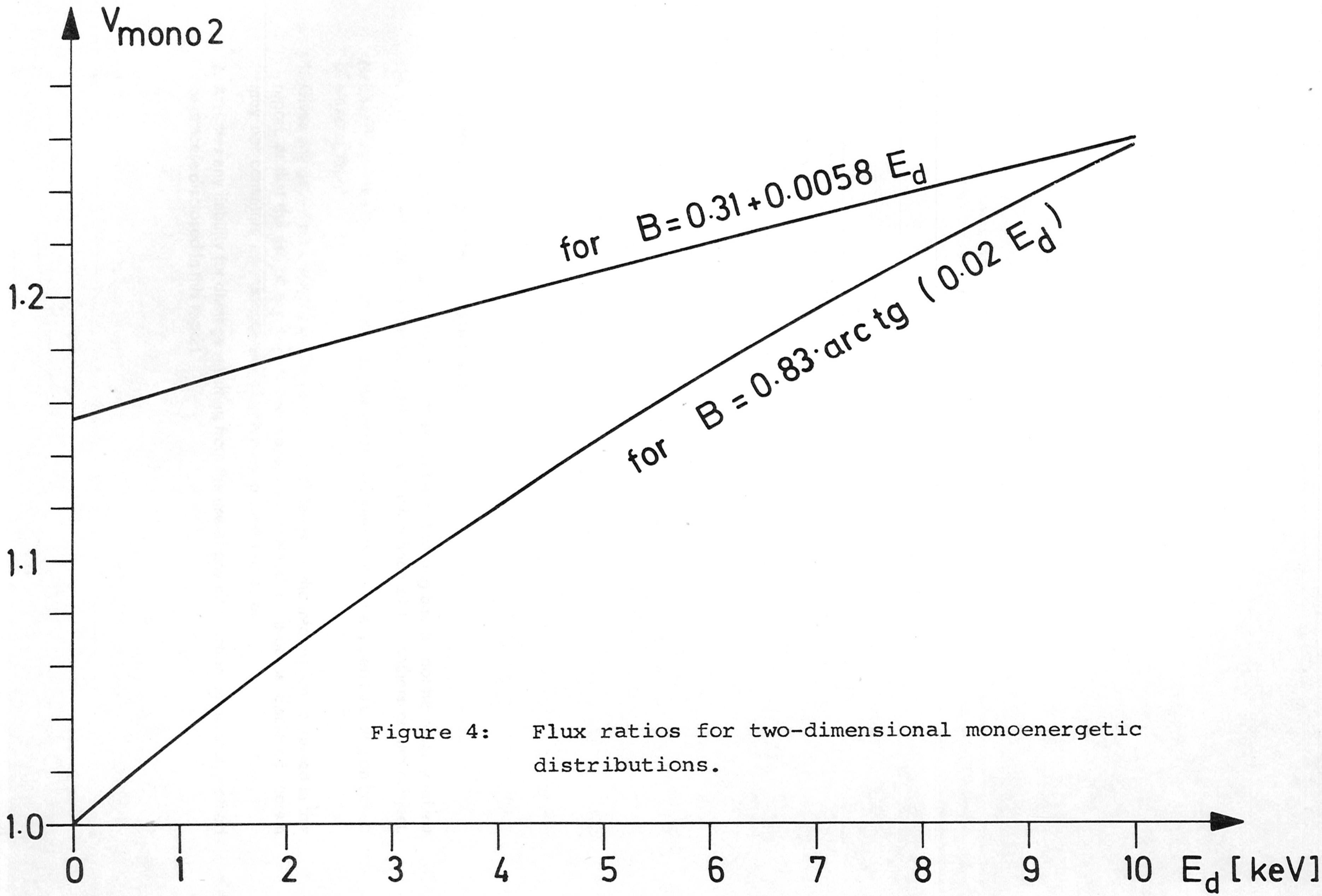

Figure 4: Flux ratios for two-dimensional monoenergetic distributions.