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(in English)

Abstract

An experiment is described in which a linear Z-pinch is stabilized by means of a superimposed high frequency Theta pinch. Stability is obtained more easily than would be expected from Weibel's criteria. Future plans for using this method to build a dynamically stabilized screw pinch are discussed.

1. Introduction

As far as we know, any high β toroidal equilibrium is unstable. For this reason the problem of dynamic stabilization of such equilibria is becoming increasingly important and is being discussed more frequently. It has become customary in these discussions to quote the simple mechanical analogue of the inverted pendulum, which can be dynamically stabilized if certain conditions are fulfilled. It is usually not mentioned, however, that the opposite may also happen, i. e. a normal stable pendulum may be excited by a kind of resonance to absorb so much energy that it becomes unstable [1].

An unstable plasma configuration which is to be stabilized dynamically may in a certain sense be visualized as a collection of many pendulums (modes) with different frequencies which are partly stable and partly unstable. The conditions for stabilizing one of these modes may not stabilize other modes as well and may or may not destabilize modes initially stable.

In dynamically stabilizing a plasma configuration one may distinguish two cases. In the first and simpler case the magnetic field providing the equilibrium has a constant modulus. It merely changes its direction viz. the so-called alternating pinch. In the second case the modulus of the magnetic field varies with time, i. e. the magnetic pressure varies and the equilibrium surface oscillates.

Dynamic stabilization has already been the subject of many papers [2-15]. Berkowitz, Grad, and Rubin, for example, have treated the simple case of a plasma in a gravitational field supported by a rotating magnetic field, assuming infinite conductivity, a sharp boundary and incompressibility. Here the Kruskal-Schwarzschild equation of motion becomes a Mathieu equation, as in the case of the inverted pendulum. Because a continuum of wave numbers and frequencies have to be taken into account, one is passing through stable and unstable regions of the Mathieu stability diagram. (This is an example of what has been mentioned above in connection with the inverted pendulum.) Tayler has discussed the similar problem of the alternating pinch [6]. The main difference is that the gravitational forces are replaced by centrifugal forces. Assuming also infinitive conductivity, sharp boundaries and incompressibility, he again obtains the Mathieu equation. In the limit of infinite frequency of the rotating magnetic field he finds the pinch column to be unstable with respect to m = 0 and m = 1 modes if the dimensionless wave numbers kr_o (r_o being the pinch radius) are

$$0 < kr_o \leq 0{,}6 \qquad (m = 1)$$
$$0 < kr_o \leq 1{,}3 \qquad (m = 0)$$

He also treats the compressible case, for which he gets the same unstable modes, but with different growth rates. Thus, rotating fields of constant modulus cannot provide stability for the above mentioned configurations, at least not in the case of sharp boundaries. The reason for this is very simple and can easily be understood for the plasma in a gravitational field as treated by Berkowitz et al.
A given unstable mode can only be stabilized dynamically if it becomes stable momentarily at least for some phases of the rotating field, the stability then depends on the way in which stable and unstable phases are mixed during one period of the dynamic field. Let us now consider in the static field case a mode with given wave numer k and angle θ with the static field. For $\theta = \frac{\pi}{2}$ any k leads to

an unstable mode. With decreasing θ more and more modes with k sufficiently large become stable. But even at $\theta = 0$ sufficiently small k modes are unstable and these modes can certainly not be stabilized by a rotating field, because they are unstable for any direction of the field, i. e. unstable for any phase of the rotating field. One should mention, however, that rotating fields do have a stabilizing effect in the sense that the maximum growth rates are reduced. This is the situation if one assumes infinite conductivity and sharp boundaries. It has been noted [5] that finite conductivity might have a stabilizing influence due to the shear produced by the penetrating alternating field. Theoretically, little is known about this problem. An experimental study [10] did not achieve the desired stability, perhaps because the conductivity was too low thus giving insufficient shear.

It might be expected that the second method of dynamic stabilization, in which a rapidly changing magnetic pressure resulting in oscillation of the equilibrium surface is used provides better stability because stabilizing inertial forces are produced, at least temporarily. An interesting example is that discussed by Weibel [7, 8]. An infinitely long pinch column is confined by a magnetic field

$$\vec{B} = \left(0,\ F\frac{a}{r},\ G\right) \qquad (1)$$

and three cases are considered:

$$\text{I.}\quad F = \alpha\sqrt{2}\,B_o \cos\omega t, \qquad G = \sqrt{2}\,B_o \sin\omega t$$

$$\text{II.}\quad F = \alpha B_o, \qquad G = \sqrt{2}\,B_o \sin\omega t \qquad (2)$$

$$\text{III.}\quad F = \alpha\sqrt{2}\,B_o \cos\omega t, \qquad G = B_o$$

Therefore the plasma surface oscillates with an amplitude ε

$$r = a + \varepsilon(\varphi, z, t) \qquad \varepsilon \ll a \qquad (3)$$

Infinite conductivity and sharp boundary are assumed. Furthermore, the plasma is considered to be collisionless, with no trapped magnetic field, so that the particles are simply reflected at the surface (bounce model). At r = b there is an infinite conducting medium. Weibel's analysis then results in the following stability conditions:

$$0 < \alpha^2 < \frac{2a^2}{b^2-a^2} \qquad (4)$$

$$\frac{u}{a} \ll \omega \qquad (5)$$

where u is the square root of the mean square velocity of the ions. In equation (2) the parameter α describes the Z-pinch component as compared to the theta pinch component, and condition (4) can then be interpreted by saying that the Z-pinch component has to be weak enough (the closer the conducter at r = b comes to the plasma, the stronger may this component become). In this report we are interested in the case II as labeled above. This is a stationary Z-pinch combined with a rapidly oscillating theta-pinch supposed to stabilize the whole configuration. If we apply condition (4) to large compression ratio , e. g. $\frac{b}{a} = 10$, we find $\alpha < \frac{1}{7}$. This means that the fast theta pinch fields have to be much stronger than the Z-pinch fields. If one wants to stabilize a toroidal high β screw pinch [17] working above the Kruskal limit, i. e. the Z-pinch component is strong enough to separate the pinch well from tube walls [16] , the conditions just discussed, do not look very promising. Fortunately, the experimental results to be given in the next section change this picture. It seems that α values much larger than indicated by equation (4) may be obtainable. The reason is not yet clear. Certainly the "bounce model" used by Weibel is not applicable here because $\beta \neq 1$ and the temperature is too low in the experiments. This, however, should by itself not improve the situation because the MHD model which should be used instead tends to give even worse stability. The better stability is perhaps due to shear produced by the penetrating field.

Other experiments on dynamically stabilized Z-pinches have also been reported. In one case [11] high frequency hexapole fields have been used according to a proposal of Osovets [9] . In another experiment [12] a Z-pinch in a stationary longitudinal field has been stabilized by an additional high frequency Z-current. To achieve stability it was necessary to make the high frequency current approximately as large as the quasistationary current and the longitudinal field approximately as large as the azimuthal field. In both cases [11, 12] the frequency was of the order of 1 Mhz.

2. Experimental results on dynamic stabilization of a Z-pinch

a) Experimental arrangement

As already mentioned in the introduction, the aim of the present experiment is the dynamic stabilization of a quasi-static Z-pinch by a high frequency theta pinch. One advantage of this method is that the high frequency can be produced more easily due to the smaller inductivity of a theta pinch coil.

In planning the experiment it was necessary to choose the parameter so that instabilities (of m = 0 and m = 1 type) can be observed and also so that there is sufficient time to observe any decrease of the growth rates under the influence of the hf field. The arrangement is given in fig. 1.

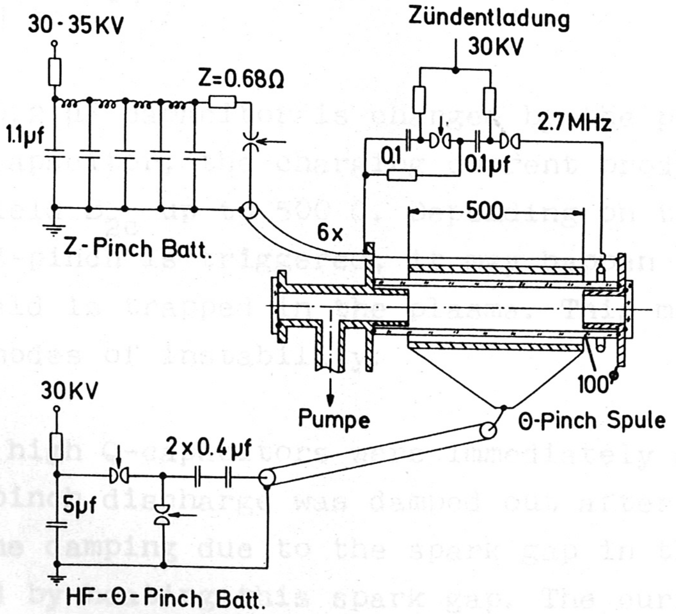

Fig. 1: Experimental set-up

The Z-pinch is produced in a quartzvessel 10 cm in diameter between two anular electrodes. The distance between the electrodes is 55 cm. The energy is stored in an artificial delay line. Its impedance is $Z = 0.68\,\Omega$. The voltages used range from 30 - 35 kV, leading to currents from 22 - 25 kA.

The rise time of the current is 1.2 µsec. and remains constant for about 10 µsec.

The data of the hf theta pinch circuit are as follows:

$C = 0.2\ \mu F \quad L_{total} = 220\ nH \quad L_{coil} = 18\ nH$ and

$\omega = 4.7 \times 10^{6} sec.^{-1}$

The capacitor in the hf circuit is charged by a pulse from a 5 µF capacitor which has a voltage of 30 - 35 kV. Thus an overvoltage by a factor 1.77 is produced corresponding to 53 - 62 kV. This gives theta pinch currents from 50 - 60 kA, max. B_z-fields of 1.2 - 1.4 kG, so that $\dot{B}_{z\ max.} = 5.6 \times 10^{9}\ G\ sec.^{-1}$.

While the 0.2 µF capacitor is charged by the pulse from the large capacitor, the charging current produces a magnetic field B_{zc} up to 500 G. Depending on the time at which the Z-pinch is triggered, it may happen that a part of this field is trapped in the plasma. This may influence the m = 0 modes of instability.

Because no high Q-capacitors were immediately available the theta pinch discharge was damped out after about 10 periods. The damping due to the spark gap in the hf circuit was reduced by heating this spark gap. The current necessary for this purpose was produced by the above mentioned 5 µF charging capacitor. Thus, the resistance of the circuit could be reduced by 14 %.

The experiment shows that the time delay between the two discharges is of decisive influence. To avoid too large a jitter in the breakdown of the Z-pinch, a weak preionization by a high frequency discharge of 2.7 Mhz and 60 kV between one electrode and an outer isolated ring is applied.

b) Results

The behaviour of the Z-pinch was observed by means of a 3-frame image converter camera with exposure times from 50 - 200 n sec.

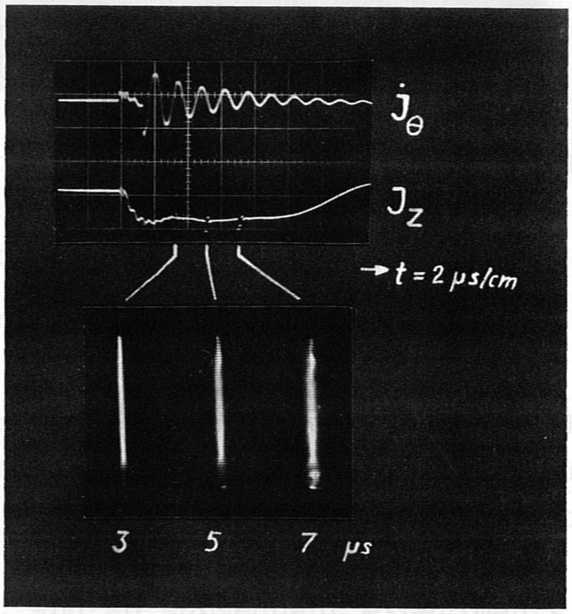

Fig. 2: Top: $\dot{J}_\theta$ and J_z, bottom: framing pictures 3,5,7,μsec.

Figure 2 gives an example of a stabilized discharge at 3, 5 and 7 μsec. after the beginning of the Z-pinch. The traces in the figure show the time history of $\dot{J}_\theta$ and J_z. Superimposed on J_z are signals indicating the times of exposure.

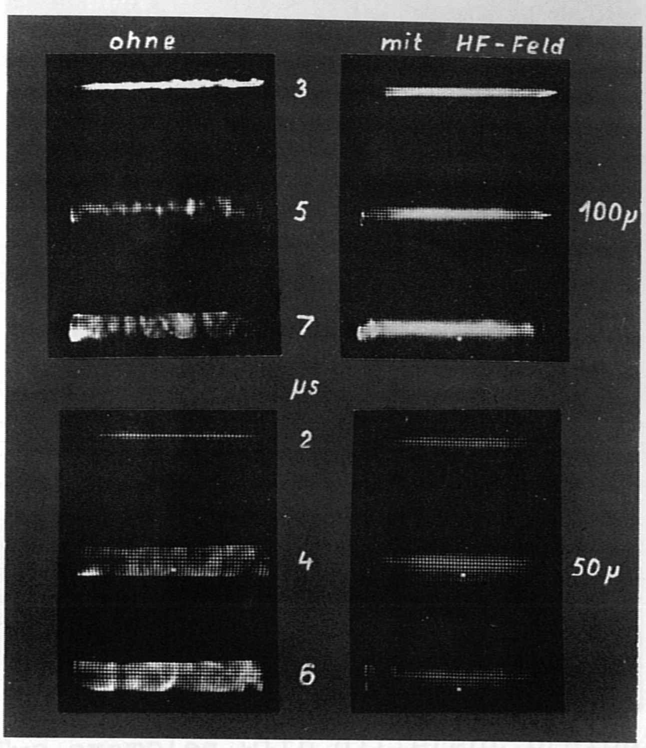

Fig. 3: Comparison without and with HF field
Top: 100 μ D_2
Bottom: 50 μ D_2
$B_{z\ max.}$: 1.2 kG

In figure 3 we compare discharges with and without stabilizing B_z-field. It is based on filling pressures of 50 and 100 μ D_2 and a maximum stabilizing field of 1.2 kG. For the 50 μ case, the light intensity is relatively weak, and so subsequent experiments were conducted at 100 μ.

To see if the preionization used (described above and shown in fig. 1) is sufficient, an additional preionizing discharge (5 kA, 1 μs) was tried. Figure 4 (stab. field increased to 1.4 kG) shows that this has no influence on the stability. Therefore, the additional preionization was again abandoned.

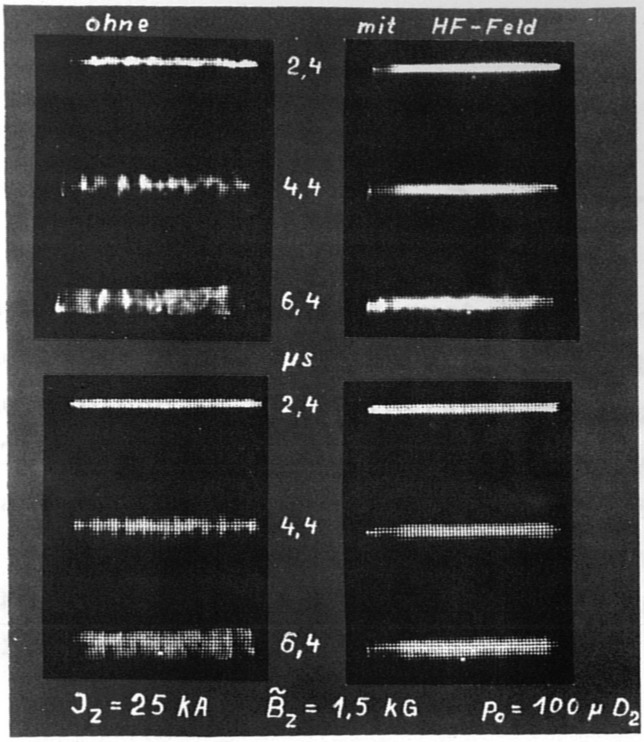

Fig. 4: Two examples with different preionization

Top: weak preionization

Bottom: strong preionization (5 kA, 1 μsec.)

As mentioned already, the time delay between the two discharges is very important. Figure 5 shows four discharges with different delays $\Delta t = t_\theta - t_z$.

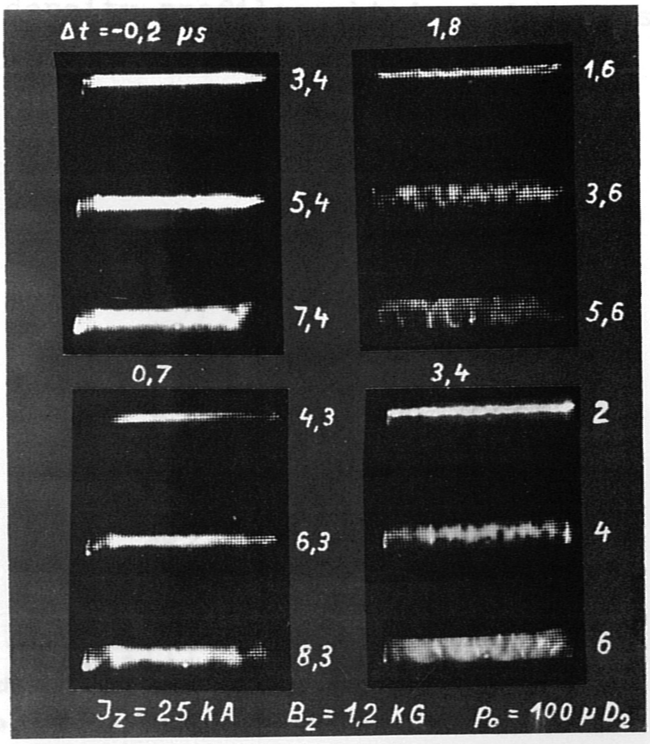

Fig. 5: Comparison of discharges for various Δt

The most stable behaviour is obtained if

$$0.4\ \mu\text{sec.} \quad < \quad \Delta t \quad < \quad 1\ \mu\text{sec.}$$

If Δt is larger, instabilities developed before the theta pinch set in and they cannot be suppressed again. If, on the other hand, the theta pinch is triggered before the Z-pinch an m = 1 instability appears very rapidly. The m = 0 instability is obviously stabilized in this case by the trapped longitudinal field. The reason for the m = 1 instability in this case is not yet understood. Perhaps the trapped field reduces the shear of the field inside the plasma.

c) Comparison with the criteria for stability

For the comparison with the stability criteria of Weibel (4) and (5) we have to know the temperature of the plasma. We assume equilibrium and use the electron densities measured end-on with a Mach-Zehner interferometer. The profiles obtained in this way can be used as long as the image converter pictures indicate a stable plasma. Figure 6 shows the density profiles obtained in one shot for different times.

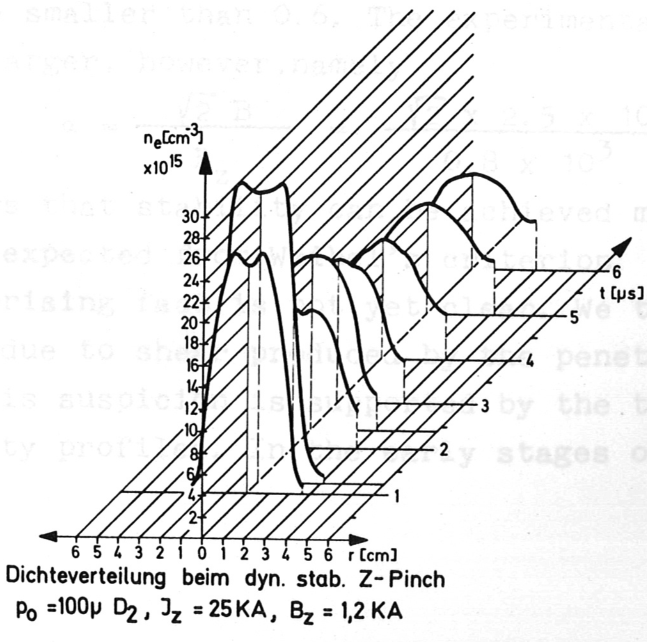

Fig. 6: Electron density profiles

The second profile represents the moment of maximum compression with a plasma radius a = 1.25 cm. At later times the plasma expands again to a = 2 - 2,5 cm. This expansion is due to ohmic heating, to the decrease of B_θ during the expansion, and to the damping of the stabilizing B_z-field.

The β-value of the plasma can be assumed to be close to unity. The B_z-field is much smaller than the B_θ -field and can be neglected for the equilibrium. Thus we find:

$$T_i = T_e = T = \frac{Jz^2}{4\pi a^2 n k} \qquad (6)$$

For times between 2 and 3 μsec., with a = 2 cm, $J_z = 2.5 \times 10^4$ A = 2.5×10^3 emu, n = 1.2×10^{16} cm^{-3} one gets T = 7.5×10^4 K. In this case the thermal velocity of the ions is u = 2.5×10^6 cm $sec.^{-1}$. The growth rate of instabilities is then of the order 10^6. This has to be compared with the stabilizing frequency

$$\omega = 4.7 \times 10^6 \gg 10^6$$

This means that the frequency condition (5) is fulfilled. This, however, is not the case for the other condition (4).

With a coil radius b = 5 and a plasma radius a = 2 cm α should be smaller than 0.6. The experimental value of α is much larger, however, namely

$$\alpha = \frac{\sqrt{2}\, B}{B_z} = \frac{\sqrt{2} \times 2.5 \times 10^3}{0.8 \times 10^3} = 4.4 \qquad (7)$$

This shows that stability can be achieved more easily than would be expected from Weibel's criterion. The reason for this surprising fact is not yet clear. We think that it might be due to shear produced by the penetrating stabilizing field. This suspicion is supported by the time behaviour of the density profiles. In the early stages of the discharge the profiles show a steeper wing than in the later stages, which indicates field diffusion. In future experiments this problem will be investigated by magnetic probe measurements.

3. Conclusions

The experiment described in the preceding section indicates that it is not too difficult to stabilize a Z-pinch by a rapidly oscillating theta pinch. We do not see any reason why this should not also be possible for the toroidal Z-pinch. An interesting example of a toroidal equilibrium configuration is the so-called screw-pinch [17], which is a toroidal theta pinch with a superimposed Z-pinch. The magnetic field configuration is the same as in the Tokomak experiment. The difference is that in the first case one is working beyond the Kruskal limit, in the other case below the Kruskal limit. Therefore, a screw pinch beyond the Kruskal limit is unstable, because the Z-pinch is too strong as compared with the theta pinch. On the basis of the experimental results just described, we believe that these instabilities can be avoided by suitable dynamical stabilisation.

For this reason we want to continue the present linear experiments in toroidal geometry as well, i. e. we want to build a dynamically stabilized screw pinch. As a first step the feasibility of this idea should be tested with a relatively modest experiment which does not aim at very high temperatures.

The parameters which we want to achieve in this first experiment are as follows:

Radius of the torus	$R \approx 30$ cm
Radius of the θ-pinch coil	$b \approx 6$ cm

The heating of the plasma is produced by a quasistationary theta pinch with a rise time of about 2 µses. and a magnetic field of about 18 kG. For this purpose we need a bank with a stored energy of about $W_\theta = 60$ kJ. The plasma temperature to be expected under these conditions depends on the filling pressure and range from 100 eV (50 µ D_2) up to about 300 eV (10 µ D_2).

To produce an equilibrium sufficiently far from the tube wall according to the Shafranov condition [16] , a sufficiently strong Z-pinch is needed. If we allow the plasma column to find its equilibrium not more than about 3 cm outside the centre of the tube the Z-pinch current has to be $J_z \approx 60$ kA for a plasma radius of $a = 1$ cm. The current becomes smaller for larger radius. The rise time must be the same as that of the θ-pinch, namely about 2 µsec. The stored energy necessary for this purpose is about 20 kJ. For example the Isar IV theta pinch bank is suitable for the power supply of the J_θ and J_z currents. The bank is equipped with passive crowbar switches. Furthermore it should be very flexible so that parameters can easily be changed, e. g. the different magnetic fields can be programmed in several ways, as has been done by Ohkawa et al. [18] .

With the above given parameters we go beyond the Kruskal limit by about a factor of 20. Thus, we need an additional high frequency theta pinch for dynamical stabilization.

Extrapolating the experimental results discussed above one needs a variation of the B_z field of 4 kG oscillating with $f = 10^6$ sec.$^{-1}$. This can be produced by an additional bank with a stored energy of about 6 kJ.

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