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Abstract

For the purpose of supporting the experimental work in ionizing a hydrogen or deuterium gas by high current discharges, a simple model is presented which describes the ionization by ohmicly heated electrons. Simple scaling laws are introduced by normalizing the equations with respect to the filling density. This is possible if the considerations are restricted to a density region where binary collisions prevail and if homogeneity of the plasma is assumed. The results provide values for the minimum current density necessary to ionize the gas in a certain time and for a given filling density.

I. Introduction

In order to produce hot and dense plasmas in theta pinch experiments it is necessary to start out from a fully ionized and well conducting plasma. The conversion of the neutral gas into a plasma is furnished by so-called preheating discharges where an axial or an azimutal current is passed through the gas. Breakdown, as well as ionization, exhibit no major difficulties at filling pressures above fifty mTorr whereas ignition of the neutral gas and ionization becomes quite intricate at filling pressures of ten mTorr and below. One of the causes, of course, is that collisions become less frequent at low densities and losses become large due to fast diffusion to the walls.

Notwithstanding the fact that many and involved processes are dominant in the gas during breakdown and ionization, the essential parameters which can be altered in an experiment are only a few: the amplitude and duration of the voltage or current pulse applied to the discharge. The geometric dimensions as for instance the distance between the electrodes are regarded as fixed.

From a pragmatic point of view, the problem of how to ionize the gas is solved for a whole density regime if the gas can be successfully ionized at one distinct density or filling pressure, and provided the density scaling laws for voltage and current amplitude and time are known. However similarity laws do not necessarily exist over an extended density region because processes dominant at a certain density may be replaced by other processes at somewhat higher or lower densities. For example photorecombination is predominant at low densities, while three body recombination is dominant at high densities.

Nevertheless it is useful to look for similarity laws and simple models based on them to provide working hypotheses for the experimental work. In the following a discussion of the ionization of a hydrogen gas by thermal elelctrons is given. The energy source for the electrons is ohmic dissipation of electric currents flowing through the plasma. These considerations are intended to answer the question of what current density or what electric field strength are necessary to fully ionize a gas of an initially low degree of ionization in a given time. This problem can be normalized with respect to the density by introducing characteristic quantities and by assuming binary collisions prevail in the plasma. Because of the assumption of ohmicly heated electrons the results are applicable only if the electric field within the plasma is small compared to the critical field as given by Dreiser 1) or by two stream instability criteria. Thus the relations presented in the following are not valid for processes occurring during the phase of breakdown where electrons and ions are accelerated directly in the externally applied fields.

First a description of the equations and the assumptions leading to them is given. Next several solutions for different parameters are discussed. Finally the relevance of the results for real discharges is investigated and neglected effects such as the compression of the plasma and diffusion are discussed.

II. The Formulation of the Equations

The derivation of the magnetohydrodynamic three fluid equations have been given by several authors 2). Düchs 3) has applied the equations to the problem of theta pinch compressions in case of a not fully ionized plasma. His report has been taken as a basis for the formulation of the following equations.

A homogeneous hydrogen plasma consisting of electrons, ions and neutrals is considered. Molecules are not taken into account. A current is flowing through the plasma but Lorentz forces are neglected. Because there is no space dependence, according to the requirement of homogeneity of the plasma, all quantities are time dependent only and the plasma can be described by a set of normal differential equations. The requirement of quasineutrality yields

$n_e = n_i = n$. The equation of continuity of each component degenerates then to:

(1) $\frac{dn_o}{dt} = -\Delta n$

(2) $\frac{dn}{dt} = \Delta n$

Δn is the change of density by ionizing and recombining collisions. It will be defined below. The density of the neutrals is denoted by n_o.

The change of energy with time of each species of particles is given by:

(3) $n_o \frac{3}{2} \frac{dT_o}{dt} - \frac{3}{2} T_o \Delta n = \Delta E_o$

(4) $n \frac{3}{2} \frac{dT_i}{dt} + \frac{3}{2} T_i \Delta n = \Delta E_i$

(5) $n \frac{3}{2} \frac{dT_e}{dt} + \frac{3}{2} T_e \Delta n = \Delta E_e$

In order to abreviate, T_o, T_i and T_e are set for kT_o, kT_i and kT_e. The first term at the left side in all three equations is the rate of change of temperature. The second term left is the gain or loss of energy of one species by the gain or loss of particles of that species. ΔE_o, ΔE_i, and ΔE_e at the right stand for the rate of change of energy by collisions.

Introducing the rate coefficient for ionization S and the rate coefficient for recombination Q the change of density per sec and cm^3 is given by:

$$\Delta n = n_o n S - n^2 Q \qquad (6)$$

The rate of change of energy by collisions per sec and cm^3 for the neutrals comprises two terms for energy transfer from the ions by recombination and to the ions by ionization, and two terms accounting for the energy transfer by collisions with electrons and ions.

$$\Delta E_o = \frac{3}{2} T_i n^2 Q - \frac{3}{2} T_o n n_o S + + n n_o R(T_e - T_o) + n n_o Y(T_i - T_o) \tag{7}$$

Accordingly, but including a term accounting for the relaxation of energy from the electrons, the rate of change of energy by collisions for the ions can be written as:

$$\Delta E_i = \frac{3}{2} T_o n n_o S - \frac{3}{2} T_i n^2 Q + + \frac{3}{2} n \frac{T_e - T_i}{t_{eq}} - n n_o Y(T_i - T_o) \tag{8}$$

R and Y are the energy exchange rate coefficients for electron neutrals and ion neutrals in case of unequal temperatures. t_{eq} is the electron-ion relaxation time. In the expression for ΔE_e for the electrons finally appear the ionization energy χ_H and the joule heating term ηj^2:

$$\Delta E_e = -\frac{3}{2} T_e n^2 Q - \chi_H n n_o S - \frac{3}{2} n \frac{T_e - T_i}{t_{eq}} - n n_o R(T_e - T_o) + \eta j^2 \tag{9}$$

with η being the specific resistivity and j the current density. The rate coefficients are defined by the product of the cross-section of the process in question and the velocity of the colliding particle averaged over the distribution function, which is here supposed to be Maxwellian. If all the rate coefficients S, Q, R and Y are assumed to be independent of the density, which is done here until further comments are made, then all collision rates in equation (6), (7), (8) and (9) are proportional to n^2 or nn_o. Since the electron ion relaxation time t_{eq} as given by Spitzer is proportional to $\frac{1}{n}$, except for a weak dependence on the density via the Coulomb logarithm, the term for the ion electron relaxation in (8) and (9) is proportional to n^2. In order to eliminate the densities from equation (1) through (5) it suggests itself to introduce the degree of ionization given by $\alpha = n/n_H$ and n_H being the number of ions and hydrogen atoms $n_H = n_o + n$ per cubic centimeter, which is a constant because of the assumption of homogeneity. All that is needed, then, is to replace the real time t by $t = \tau / n_H$, the relaxation time by $t_{eq} = \tau_{eq}/\alpha n_H$ and finally the current density by $j = i \cdot n_H$ and the density drops out in all equations. The such normalized differential equations consist then of one "continuity" equation describing the change of the degree of ionization:

$$\frac{d\alpha}{d\tau} = \alpha(1-\alpha) S - \alpha^2 Q \qquad (1o)$$

and three energy equations for the three temperatures:

$$\frac{dT_o}{d\tau} = \frac{\alpha^2}{1-\alpha} Q (T_i - T_o) + \frac{2}{3} \alpha Y (T_i - T_o) + \frac{2}{3} \alpha R (T_e - T_o) \quad (11)$$

$$\frac{dT_i}{d\tau} = (1-\alpha) S (T_o - T_i) + \alpha^2 \frac{T_e - T_i}{\tau_{eq}} - (1-\alpha) Y (T_i - T_o) \quad (12)$$

$$\frac{dT_e}{d\tau} = \frac{2}{3} \cdot \frac{1}{\alpha} \cdot \eta \cdot i^2 - \frac{2}{3}(1-\alpha)\left(\chi_H + \frac{3}{2} T_e\right) S - (1-\alpha) \frac{2}{3} (T_e - T_o) R - \alpha^2 \frac{T_e - T_i}{\tau_{eq}} \quad (13)$$

where all the rate coefficients are functions of the temperatures, S and Q and R depend on the electron temperature alone, while Y depends on both T_i and T_o. For given initial conditions and a given normalized current pulse $i(\tau)$ these equations can be solved to yield α, T_e, T_i and T_o as functions of τ. The physical problem described is the ionization of a gas by a current pulse, that is, by Ohmicly heated electrons, without assuming equilibrium conditions for ionization and recombination processes as would be done if the Saha equation were used. The similarity laws valid for the problem described are evident from the normalization of the quantities: $t = \tau/n_H$, $j = i \cdot n_H$. The dependence of the degree of ionization on time for different filling densities n_H stays similar if the time scale is shortened or stretched such that $t \cdot n_H$ stays constant and if the current pulse is diminished or enhanced such that j/n_H remains the same function of τ . Then also the heating rate per electron stays the same function of τ as can be seen from $j/n_H = \alpha e v_D$, where e is the charge of an electron and v_D the electron drift velocity.

The assumption of the independence of the rate coefficients S and Q on density is questionable because of three body recombination at higher densities and because of the influence of the collision limit 4) which is a function of electron density, whereas the electron-atom and ion-atom rate coefficients are independent of the density. Bates, Kingston and McWhirter 4) have derived the collisional-radiative rate coefficients S and Q as functions of electron temperature and density. Griem 5) has given a good analytical approximation which is used here in a slightly simplified form.

In order to determine for which filling densities the similarity relations given above are valid it is sufficient to consider just the electrons to be heated, and to leave the ions and atoms cold, that is we assume that relaxation from the electrons to the ions is a slow process compared with the ionization of the gas. The two differential equations for this case are:

$$\frac{d\alpha}{d\tau} = \alpha(1-\alpha)\cdot S - \alpha^2 Q \tag{10}$$

$$\frac{dT_e}{d\tau} = \overbrace{\frac{2}{3}}^{6.25\cdot 10^{-4}}\frac{1}{\alpha}\eta i^2 - \frac{2}{3}(1-\alpha)(\chi_H + \frac{3}{2}T_e)\cdot S \tag{11}$$

Instead of using the current density $i = j/n_H$ it is advantageous to take the normalized electric field $w = i\eta = E/n_H$. In the next section examples of solutions are given of the rise of ionization degree for different densities n_H and for different constant field strengths w applied to the plasma. Before that can be done S and Q must be defined and proper dimensions introduced.

The temperature is measured in electron volts, the density n_H in 10^{16} particles per cc, and the time t in 10^{-6} sec. The characteristic time $\tau = n_H \cdot t$ is then one if the density n_H is 10^{16} cm^{-3} and if t = 1 μsec. Similarily the dimensions of the rate coefficients are chosen to be 10^{-10} cm^3/sec. The specific resistivity in Ω cm which consists of a term due to electron-ion interaction and of a term due to electron-neutral interaction, is then 6):

$$\eta = 5.25\cdot 10^{-3}\, T_e^{-3/2}\cdot \ln\Lambda +$$

$$+ \frac{1-\alpha}{\alpha}\cdot 3.55\cdot 10^{-7}\cdot\left(S + q\cdot 6.7\cdot 10^{7}\, T_e^{1/2}\right) \tag{15}$$

with the Coulom logarithm $\ln\Lambda = \ln\left(1.54\cdot 10^{2}\cdot \frac{T^{3/2}}{\sqrt{\alpha n_H}}\right)$

and the electron-atom collision cross-section 7):

$$q = \left(\frac{47.4}{T_e + 1.3} + 2.14\right)\cdot 10^{-6} \tag{16}$$

The ionization cross-section S can be divided into two parts one of which does not explicitly depend on the density and another one of which is proportional to the electron density.

$$S = S_1 + S_2 \tag{17}$$

These two terms can be written according to (5):

$$S_1 = 7\cdot 10^{2}\cdot\left(\frac{\chi_H}{T_e}\right)^{-1/2} \frac{m^2}{m^2-1}\cdot \exp\left[-\frac{\chi_H}{T_e}\left(1-\frac{1}{m^2}\right)\right] \tag{18}$$

$$S_2 = 1.99\cdot 10^{3}\,\frac{4m^2}{4m^2-1}\cdot \alpha n_H \exp\left[-\frac{\chi_H}{T_e}\left(1-\frac{1}{m^2}\right)\right] \tag{19}$$

m is the quantum number of the so-called collision limit which has to be evaluated by iteration from the following equation (5):

$$m = 1.66 \cdot (\alpha \cdot n_H)^{-2/17} \cdot \left(\frac{\chi_H}{T_e}\right)^{-1/17} \cdot \exp\left(\frac{4}{17} \frac{1}{m^3} \frac{\chi_H}{T_e}\right) \qquad (20)$$

The rate coefficient for recombination can also be divided into a term only weakly dependent on the electron density, which is that describing photo recombination, and one being proportional to the electron density describing three body recombination:

$$Q = Q_1 + Q_2$$

$$Q_1 = 5 \cdot 10^{-4} \left(\frac{\chi_H}{T_e}\right)^{3/2} \left[\sum_{l=1}^{m} \frac{E^*}{l^3} \exp\left(\frac{1}{l^2} \frac{\chi_H}{T_e}\right)\right] \qquad (21)$$

$$Q_2 = 1.4 \cdot 10^{-5} \cdot m^6 \left(\frac{\chi_H}{T_e}\right)^2 \cdot \alpha \cdot n_H \cdot \exp\left[\frac{\chi_H}{T_e (m+1)^2}\right] \qquad (22)$$

E^* is defined by the integral:

$$E^* = \int_{x_0}^{\infty} \exp\left(-\frac{x}{T_e}\right) \frac{dx}{x} \qquad (23)$$

and x_0 by:

$$x_0 = \chi_H \left(\frac{1}{l^2} - \frac{1}{(m+1)^2}\right) \qquad (24)$$

Analytical expressions for the rate coefficients have been used in preference to tables as given by Bates, Kingston and McWhirter [4]. The involved dependence of the rate coefficients on the electron temperature implies that there is no hope for finding analytical solutions of equation (1o), (11), (12) and (13) even for the simple case of a constant current density.

Finally, it should be noted that in case of equal electron, ion and neutral temperatures, two simple differential equations are again obtained from equation (1o) through (13) which are the same as equations (1o) and (14) except that the right side of equation (14) is to be multiplied by a factor $\frac{\alpha}{1+\alpha}$.

III. Some Solutions and the Validity of the Similarity Relations.

Because of the dependence of the rate coefficients for ionization and recombination on the density, and because of the weak dependence of the resistivity on the density, the normalization of the equations above could not be carried through completely. In order to obtain solutions, therefore, not only the initial values of the varibales and the function $i(\tau)$ or $w(\tau)$ have to be given, but still also a value for the filling density n_H. However the hope is that a solution for one value of n_H is representative for all solutions over an extended density interval. In order to check this it is sufficient to study a reduced set of equations, namely (10) and (14). That is, it is assumed that only the electrons are heated; energy transfer to ions and neutrals is neglected. This is a valid procedure because the filling density n_H affects the rate coefficients S and Q which are retained in (10) and (14) whereas it is of little influence on the other rate coefficients and transport coefficients.

In figure 1 a typical set of solutions for four different values of the normalized field w is given. Instead of a normalized current density $i(\tau) = j/n_H$ a constant electric field $w = E/n_H$ was superposed upon the plasma. This gives a more realistic picture than a constant current pulse and avoids too high drift velocities in the beginning of the discharge, when the degree of ionization is still small. The current $i(\tau)$ is then determined by the specific resistivity η, that is, by the electron temperature and the degree of ionization.

The actual field applied to the plasma is determined by $E = n_H \cdot w$ in volts per centimeter if n_H is inserted in 10^{16} cm^{-3}. The initial values for $\tau = 0$ were $\alpha = 0.01$ and $T_e = 0.9$ eV. The filling density was chosen to be $n_H = 0.066$ cm^{-3} in units of 10^{16} per cc as mentioned above. This density corresponds to a filling pressure of 10 mTorr. The time τ_o in which the gas is completely ionized and α reaches the value one depends strongly on the field applied or on the current passed through the gas. The electron temperature rises steeply at early times until ionization sets in, and stays approximately constant thereafter until ionization has been completed. The electron temperatures in figure 1 are in the flat part when effective ionization takes place for the cases w = 100, 70, 50 and 30: T_e = 14, 10.5, 8 and 5 electron volts approximately. These values are by a factor of 4 to 10 larger than what one is used to from equilibrium considerations, that is, when the ionization process proceeds slowly.

Examples for solutions for different densities n_H are given in figure 2 for a normalized field of w = 100. The values of n_H = 0.0166, 0.066, and 0.66 cm^{-3} correspond to 2.5, 10 and 100 mTorr filling pressure. The curves coincide indeed well enough that the normalization with respect to the density is justified. The solution for $n_H = 0.066$ cm^{-3} or 10 mTorr filling pressure can therefore be taken as representative for the density region $0.0066 \leq n_H \leq 0.66$ cm^{-3} or $1 \leq p_H \leq 100$ mTorr. The scaling fails at higher densities because of the stronger variation of the collision limit with the density as compared to lower densities.

In the case of lower electron temperatures, that is for smaller fields w, the dependence of the solutions on the density is more pronounced, as shown in figure 3.

The ionization time τ_0 deviates here at the high density $n_H = 0.66$ cm^{-3} already by about 25 percent from the time given by the solutions for the lower densities $n_H = 0.066$ cm^{-3} and $n_H = 0.166$ cm^{-3}. However within the scope of the model the curve for $n_H = 0.066$ cm^{-3} can still be regarded as to represent all solutions for w = 30 in the pressure region $1 \leq p_H \leq 100$ mTorr. The comparatively low electron temperature in the high density case ($n_H = 0.66$) is not a consequence of an enhanced recombination rate. The recombination rate is still negligible compared with the ionization rate. The latter, however, is larger because of lowering of the collision limit due to the higher density.

The normalized current densities for the solutions shown in figure 1 are displayed, in figure 4 as a function of α . The density parameter is $n_H = 0.066$ cm^{-3} or 10 mTorr for the solid curves. For w = 100 and w = 30 the curves $i(\alpha)$ for $n_H = 0.0166$ and 0.66 cm^{-3} are also shown dashed. The separation of the curves is due to the change of the resistivity with temperature. However, it should be emphasized that even if the filling pressure differs by a factor of 10 the normalized currents differ by not more than a factor of two. The values of $i(\alpha)$ can be therefore represented by the solid curves for the pressure range $1 \leq p_H \leq 100$ mTorr.

For estimates of the current densities necessary to ionize a hydrogen gas from 1 % to 100 % the relation shown in figure 5 is convenient where the values of i for $\alpha = 0.5$ or fifty percent ionization were taken and plotted over the ionization time. The deviations due to different filling pressures from 1 to 1oo mTorr are shown by the error bars.

The solutions presented were numerical solutions obtained using a computer. However, the curves in figure 1 imply that good estimates can be gained without solving the two differential equations. In the temperature and density region considered above, that is for $T_e \geqq 2$ eV and $1 \leqq p_H \leqq 100$ mTorr, the term describing the recombination in the continuity equation (1o) can be safely neglected to yield the equation:

$$\frac{d\alpha}{d\tau} = \alpha(1-\alpha) \cdot S \tag{25}$$

The numerical solutions showed further (fig. 1) that during the time of effective ionization the electron temperature stays approximately constant as in a boiling liquid. For a constant electron temperature the rate coefficient S can also be regarded as constant and equation (25) can be integrated:

$$S \cdot \tau_0 = \ln \frac{\alpha}{1-\alpha} \Big|_{\alpha=0.01}^{\alpha=0.7} \tag{26}$$

For the lower boundary the same value of $\alpha = 0.01$ as for the numerical solutions was chosen. A remark on this value and the electron drift velocity is made in chapter IV.

The ionization time is obtained if proper boundary values are inserted:

$$\tau_0 \approx \frac{5}{S} \tag{27}$$

Usually a certain value for τ_0 is envisaged in an experiment and in order to reach this value the ionization coefficient has to be as large as given by equation (27). With a fixed value for S also the electron temperature is determined, which can be obtained using tabulated values $S(T_e)$ as given by Bates et al. [4] or taking the relations (18), (19) and (2o) above. The fact that the electron temperature is approximately constant in time allows one also to neglect the time derivative in equation (14). Setting for $\alpha(1-\alpha)$ an average value of o.2 yields then for the normalized current density:

$$i = \left(3.2 \cdot 10^2 \frac{1.5 \cdot T_e + \chi_H}{\eta} \cdot S\right)^{1/2} \qquad (28)$$

It remains to determine the resistivity from (15) after setting $\frac{1-\alpha}{\alpha} = 1$ and taking q from (16):

$$\eta = 5.25 \cdot 10^{-2} \cdot T^{-3/2} + 3.55 \cdot 10^{-7} \cdot \left(S + q \cdot T_e^{1/2} \cdot 6.7 \cdot 10^{-7}\right) \qquad (29)$$

The values for i obtained by this procedure are plotted in figure 5 in order to compare them with the values obtained by the numerical solutions. They compare favorably indeed and good estimates are possible for the minimum current density necessary in a hydrogen gas to ionize it in the required time.

Finally a few solutions for another extreme case are shown in figure 6. Here a sufficiently fast energy flow from the electrons to the ions and neutrals was assumed such that the temperatures of all components are equal at all times. This is easy to take into account by multiplying equation (14) by a factor of $\frac{\alpha}{1+\alpha}$ at the right side. The solutions for the electron temperatures show a slower rise because the neutrals and the ions must be heated. The degree of ionization stays low until the temperature becomes sufficiently high to ionize the gas in the time scale of the problem considered. Again the normalization is justified for the high field strength case (w = 1oo, 7o) as can be seen from the solutions for different filling densities. However for the lower values of the field w, (w = 3o), the solutions for different filling pressures start to separate.

If the neutrals stay cold and only the electrons and ions are heated, essentially the same curves are obtained as shown in figure 1, where the electrons have been regarded only. The curves only stretch out in time a little more; however, by less than a factor of two.

IV. A Discussion of the Applicability of the Model to Real Discharges.

In order to arrive at such a simple model as described above a series of assumptions had to be made the most relevant of which will be discussed here.

1) The assumption of Ohmicly heated electrons

One of the fundamental questions is whether the electrons are really heated by Ohmic dissipation or if they gain energy by direct acceleration in the external fields. At low electron densities and not too small electric fields the latter is certainly true. The complete motion of the particles in the electric and magnetic fields including space charge distortions, has then to be investigated. Usually this is referred to as the breakdown phase of a discharge 8). Chodura 9) has discussed the problem of breakdown in a theta pinch configuration in great detail and several experimental investigations are reported 10, 11). However soon after breakdown the electric field within the plasma falls below the critical field as defined by Dreiser 1) or, what is essentially the same, the electron drift velocity becomes less than the electron thermal velocity. The electrons are then heated by Ohmic dissipation. The latter inequality is easy to check in the formulation used here. The normalized current density i was defined by:

$$i = j/n_H = \frac{\overbrace{\alpha \cdot n_H}^{n_e} \cdot e\, v_D}{n_H} = \alpha \cdot e\, v_D \qquad (30)$$

with v_D being the electron drift velocity and e the electron charge. In the units used here, and with v_D in centimeter per second, the normalized current becomes:

$$i = 1.6 \cdot 10^{-3}\, \alpha \cdot v_D$$

For Ohmic dissipation the inequality

$$v_D \ll v_{th} = \left(\frac{T_e}{m_e}\right)^{1/2} \qquad (31)$$

must hold.

Using the normalized current densities instead of the velocities and denoting the critical current, which corresponds to an electron drift velocity equal to the electron thermal velocity by i_c, yields:

$$i \ll i_c = 6.7 \cdot 10^4 \cdot \alpha \cdot \sqrt{T_e} \qquad (32)$$

if T_e is inserted in electron volts. The current i_c has been evaluated for the temperatures of the solutions w = 100, w = 30 and n_H = 0.066 or 10 mTorr and compared in figure 7 with the actual current as obtained from the solutions of figure 1. In the low field strength case (w = 30) the electron drift velocity is at all times considerably smaller than the electron thermal velocity. In the high field strength case (w = 100) the drift velocity approaches the thermal velocity in the beginning but soon falls to lower values. However, i_c does not decrease to much less than half of i. Evidently in the pressure region considered a normalized field of about w = 100 is the upper limit for the applicability of the model. These considerations also justify the value of α = 0.01 as initial value. In the high field strength case the concept of Ohmic heating cannot be justified anymore for still lower initial values of α.

2) <u>The neglect of dissociation</u>

A severe assumption is to start out from an already dissociated hydrogen gas instead of taking H_2 molecules. However not all the rate coefficients of the processes and indeed not all the relevant excitation processes leading to dissociation, are sufficiently known to incorporate them in this model. Because of too many coupled processes it is not yet clear if the normalization with respect to the density can successfully be carried through also in this case. For fast heating of the electrons dissociation and ionization rise simultaneously because both have to be achieved by colliding electrons, in contrast to thermal dissociation and ionization. The time scale of ionization is then determined by the slowest rate coefficient involved.

3) The assumption of homogeneity

Because of the presence of the discharge tube walls temperature and density gradients exist and the assumption of homogeneity will in general not be justified. In addition the rather high current density necessary to ionize the gas brings about Lorentz forces which in turn lead to a compression of the plasma. Accordingly the main processes neglected are diffusion and compression of the plasma. In order to get a feeling for how these processes affect the similarity laws introduced in the second chapter the variation of the characteristic diffusion time and of the characteristic compression time can be compared with the characteristic ionization time of discharges which obey the similarity relations, that is if $i = j/n_H$ stays unchanged.

The time scale of the compression of a cylindrical plasma by axial currents according to the snow plow model 12, 13), for instance, is given by the normalized compression time:

$$\tau_{compr.} = \left(R \sqrt[4]{\varrho_0 \pi}\right)^{-1} \cdot \left(\frac{dI}{dt}\right)_{t=0} \cdot t \qquad (33)$$

with R being the tube radius, $\varrho_0 = \alpha \cdot n_H \cdot m_i$ the mass density of the plasma and $\frac{dI}{dt}$ the rise of current with time at $t = 0$. Following the prescription given by the similarity for ionization and introducing $i = I/F \cdot n_H$ and $\tau = n_H t$, with F being the area where the current flows, yields the proportionality by neglecting all constants:

$$\tau_{compr} \propto n_H^{3/4} \cdot \tau_{ion} \qquad (34)$$

The compression time scales indeed differently with the ionization time for varying densities, but the dependence on the density is not particularly strong.

A source of particle losses is diffusion toward the walls. Assuming ambipolar diffusion with its diffusion coefficient inversely proportional to the filling density n_H the characteristic diffusion time compares with the characteristic ionization time as:

$$\tau_{Diff} \propto n_H^{-2} \tau_{ion} \tag{35}$$

The discharge tube dimensions were kept unchanged of course. The proportionality in relation (35) implies that the diffusion time is getting shorter towards lower densities as compared to the ionization time. That is, diffusion can become dangerous at low pressures.

4) <u>A remark on the electron ion relaxation time</u>

In chapter III examples of solutions for the differential equations (1o) and (14) in which negligible energy transfer from the electrons to the ions and to the neutrals, were given. The reason for this was to have a rather simple and clear model for checking the validity of the similarity relations. The current density necessary to ionize the gas in a given time should therefore be regarded as a minimum value. If the assumption of negligible relaxation is satisfied can be checked in comparing the relaxation time with the ionization time. The normalized electron ion relaxation time in the units used above is given by the expression:

$$\tau_{eq} = 3.23 \cdot 10^{-3} \frac{T_e^{3/2}}{\alpha} \qquad (36)$$

where the Coulomb logarithm has been set equal to $\ln \Lambda = 10$. This time has to be compared with the times in figures 1, 2 and 3. Except for the case w = 100 the relaxation time is short compared with the ionization time and the ions are heated as the electrons. This however, will not slow-down the ionization considerably. The error made in neglecting the heating of the ions is less than a factor two in the ionization time.

5) A preliminary comparison with experimental data.

Results from a preheating discharge by a short, one microsecond long current pulse have been reported by Eberhagen et al. [14]. Although the behaviour of such discharges is quite complicated because of rapid compressions early after breakdown, it could be seen that effective ionization occurred only in those regions of the discharge where the conditions as described by the solutions of chapter III are at least met for the current density and for the electron temperature. Unfortunately the experiment to date has not permitted a check of the similarity stated above because of too little freedom in varying the parameters.

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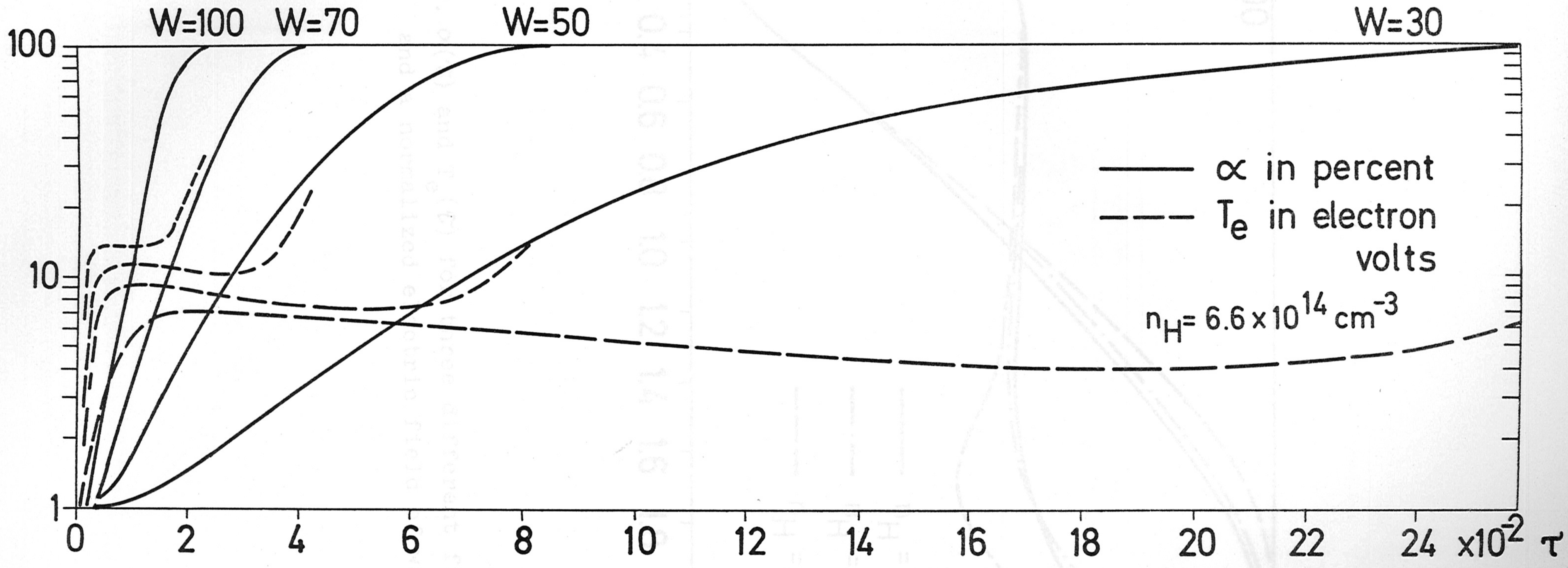

Fig.1. The rise of ionization $\alpha(\tau)$ and the electron temperature $T_e(\tau)$ for various electric fields w. The normalized time τ is in 10^{10} sec cm^3. That is the real time t is obtained in microseconds if τ is divided by the filling density n_H (in units of $10^{16} cm^{-3}$).

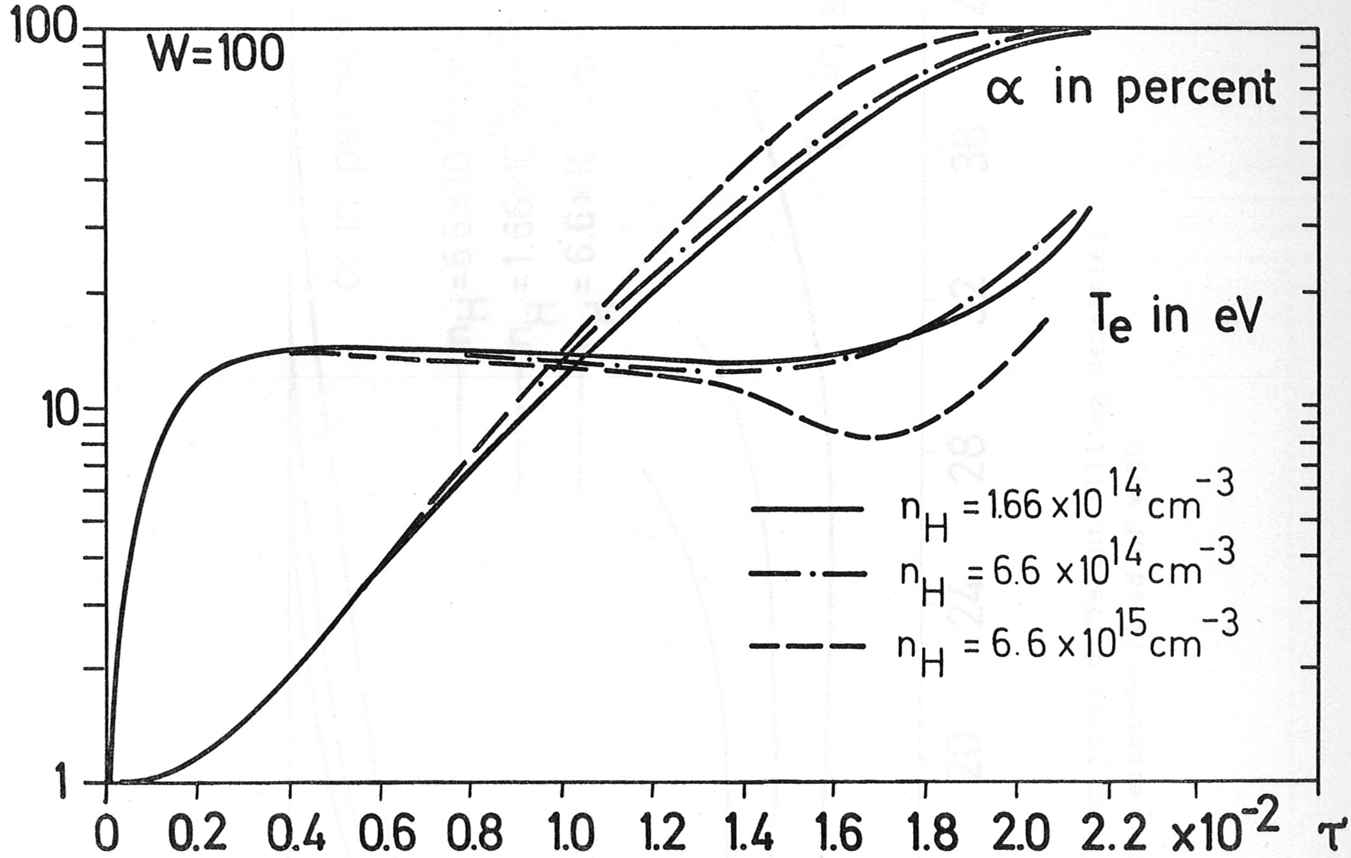

Fig.2. $\alpha(\tau)$ and $T_e(\tau)$ for three different filling densities and a normalized electric field of w=100.

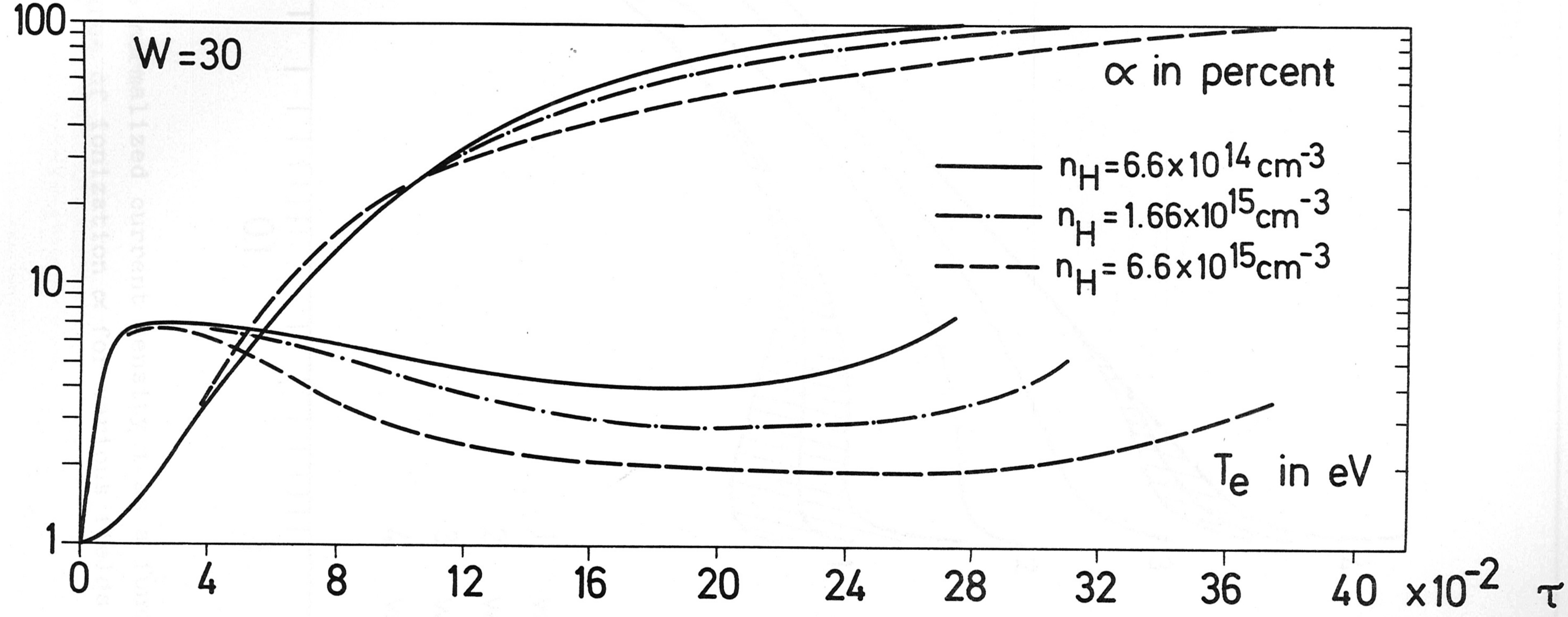

Fig.3. $\alpha(\tau)$ and $T_e(\tau)$ for three different filling densities and a normalized electric field of w=30.

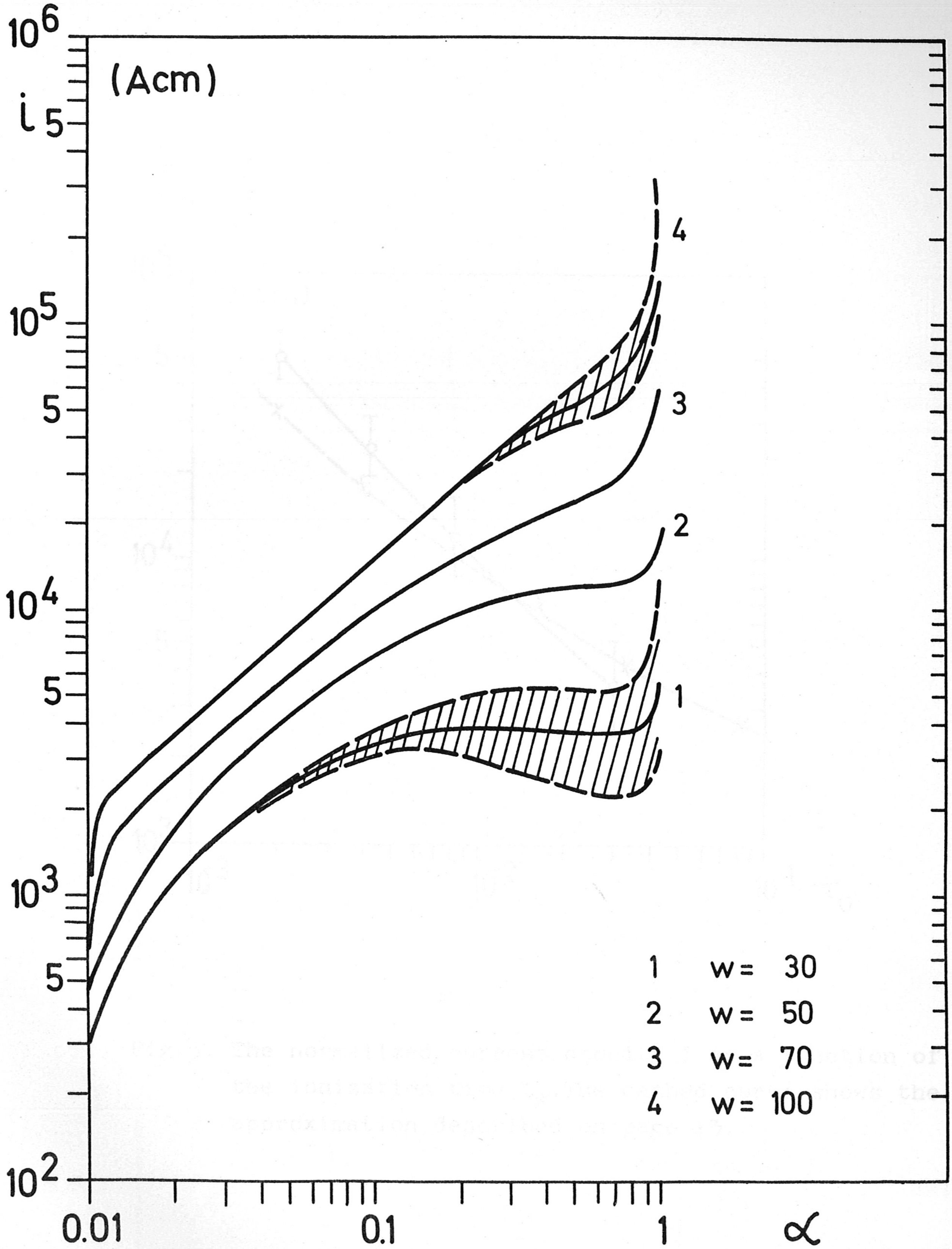

Fig.4. The normalized current density i as a function of the degree of ionization α for various fields w.

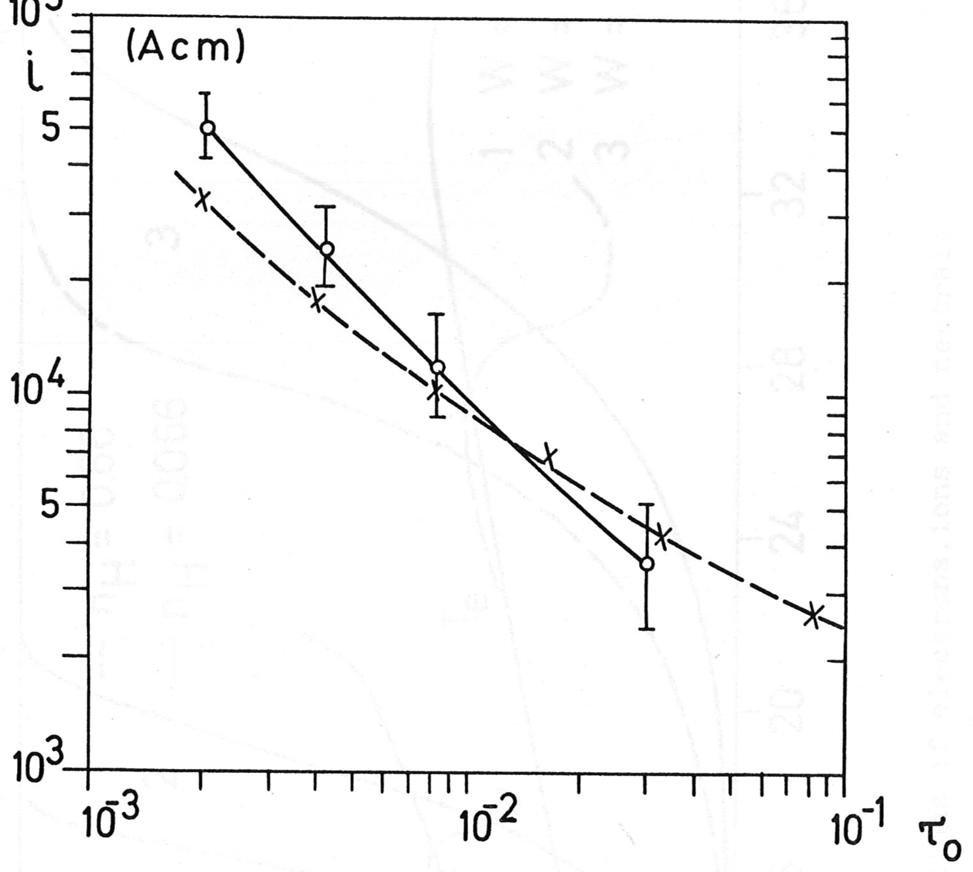

Fig.5. The normalized current density i as a function of the ionization time τ_o. The dashed curve shows the approximation described on page 13.

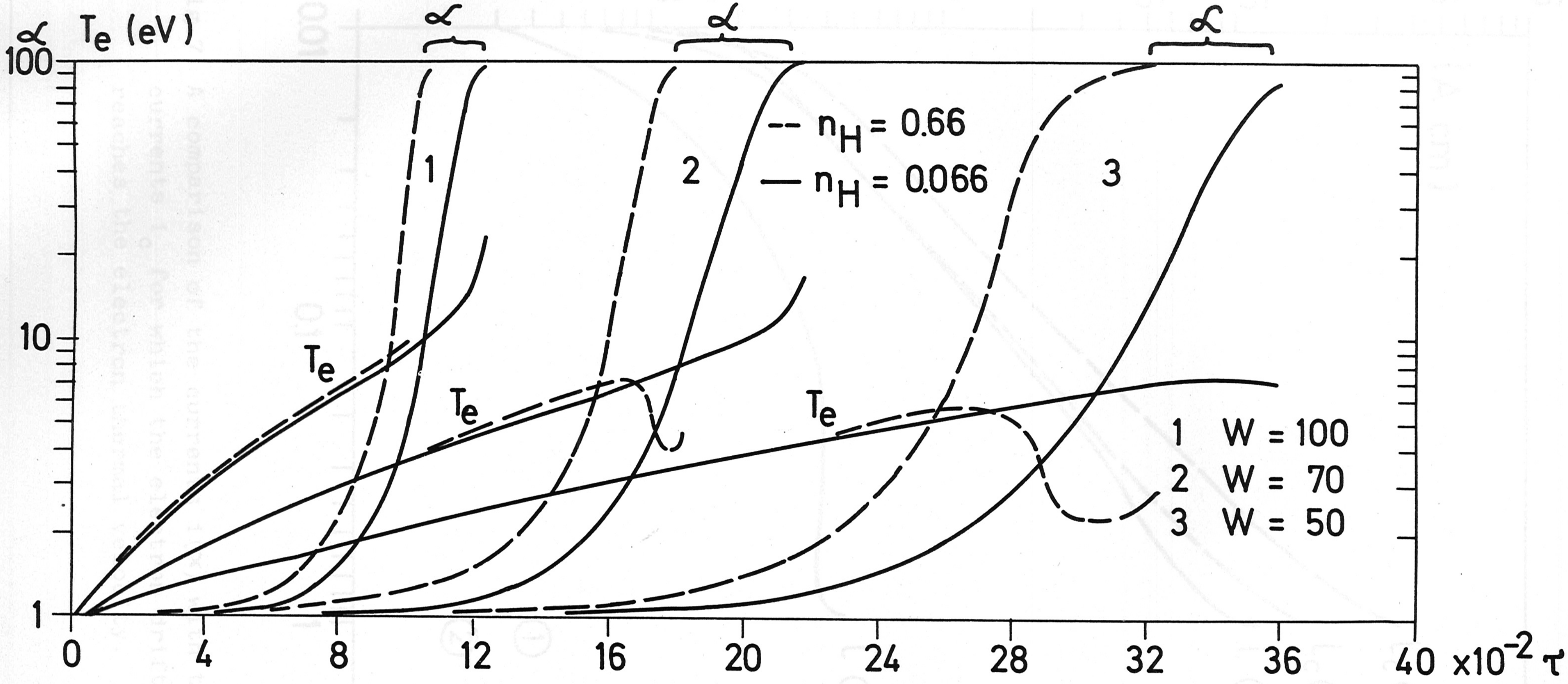

Fig.6. Examples for solutions if electrons,ions and neutrals are equally heated.

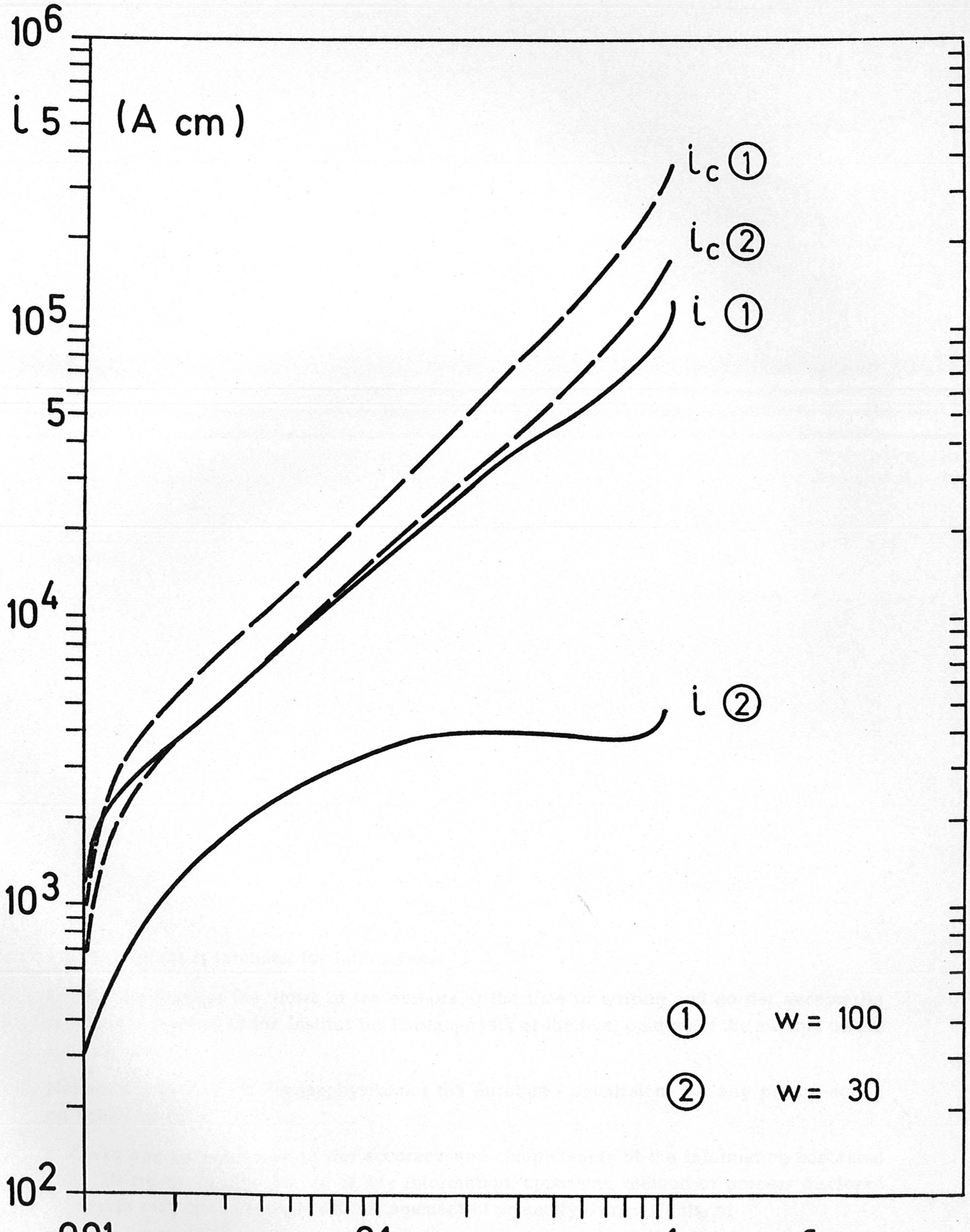

Fig.7. A comparison of the currents $i(\alpha)$ with the critical currents i_c for which the electron drift velocity reaches the electron thermal velocity.