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with Superposed Hexapole

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A toroidal theta-pinch discharge with superposed non-helical hexapole field is investigated. The characteristic data of the discharge are: major diameter 52 cm, minor inner diameter of the vacuum vessel 5 cm, maximum magnetic field between 20.5 and 25 kG, pulse time (quarter-cycle) 3.0 μ s, maximum temperature between 40 and 100 eV, maximum density between 1 and 3×10^{21} m $^{-3}$, beta values between 0.3 and 1.0. The plasma loss rates are determined by measuring particle velocity, temperature and plasma radius. The loss rates are compared with loss rates of models written up for ohmic, resistive losses, and beam emission. Measured loss rates are consistent with those expected from beam losses with a cusp slit-width of one ion gyro-radius. Above electron temperatures of 20 eV, resistive losses are negligible. Bohm diffusion is not consistent with the measurements, but is of the same order of magnitude. Measurements with "alternating hexapole" showed higher loss rates. The machine is now being converted into a $\ell = 3$ "fast stellarator" configuration, i.e. a toroidal theta pinch with superposed helical hexapole field.

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Abstract

A toroidal theta-pinch discharge with superposed non-helical hexapole field is investigated. The characteristic data of the discharge are: major diameter 52 cm, minor inner diameter of the vacuum vessel 6 cm, maximum magnetic field between 10.5 and 21 kG, rise time (quarter-cycle) 3.0 μ s, maximum temperature between 40 and 100 eV, maximum density between 1 and $3 \times 10^{16} \text{ cm}^{-3}$, beta values between 0.3 and 1.0. The plasma loss rates are determined by measuring particle density, temperature and plasma radius. The loss rates are compared with loss rates of models which account for cusp losses, resistive losses, and Bohm diffusion. Measured loss rates are consistent with those expected from cusp losses with a cusp slit-width of one ion gyro-radius. Above electron temperatures of 20 eV, resistive losses are negligible. Bohm diffusion is not consistent with the measurements, but is of the same order of magnitude. Measurements with "alternating hexapole" showed higher loss rates. The machine is now being converted into a $\ell = 3$ "fast stellarator" configuration, i.e. a toroidal theta pinch with superposed helical hexapole field.

I. Introduction

Attempts to force a theta-pinch plasma into a closed toroidal equilibrium configuration, without resulting azimuthal currents, have shown that the confinement time increases as the theoretically postulated plasma surface (M + S surface) is approached.¹⁾ Further improvement would depend on exact knowledge of the required external magnetic field and - due to the initial compression - of the required discharge vessel geometry. Another question concerns the stability in regions of unfavourable field curvature. Each problem is investigated experimentally and theoretically.²⁾ On the other hand, they may be avoided by superposing magnetic multipole fields on the M + S configuration. Preliminary experiments appeared to be promising. The lack of magnetic field closure inherent in these configurations, however, should lead to additional plasma losses of the mirror or cusp type.

Consequently, the superposition of such multipole fields on a purely azimuthal compression field was investigated. Hexapole fields appeared to be the most advantageous configurations in preliminary experiments with a plasma in the 10 eV region.³⁾

II. Theory

In a toroidal (non-helical) hexapole cusp geometry, neglecting the azimuthal field B_φ , particle losses may be described by a flux of particles of density n and mean thermal ion velocity v_i through six slits of width δ ; this model defines the loss width δ . The change of the line density N is then

$$\dot{N} = - 6 n v_i \delta. \quad (1)$$

Consequently, the loss rate γ is

$$\gamma = - \frac{\dot{N}}{N} = \frac{6 n v_i \delta}{n \pi r_p^2} = \frac{6 v_i \delta}{\pi r_p^2}. \quad (2)$$

In the case of $\beta = 1$ and zero resistivity η , δ should be of the order of an ion gyro-radius ρ_i ,⁴⁻⁶⁾

$$\rho_i = \frac{m_i v_i}{e B}, \quad (3)$$

and the loss rate is

$$\gamma_i = \frac{6 v_i \rho_i}{\pi r_p^2} = \frac{6 m_i v_i^2}{\pi r_p^2 e B} = \frac{18 k T_i}{\pi r_p^2 e B}. \quad (4)$$

Taking into account the electric field in the plasma boundary layer, which prevents the ions from penetrating into the magnetic field much farther than the electrons, δ turns out to be more of the order of the electron gyro-radius ρ_e ,^{5,7)}

$$\rho_e = \frac{m_e v_e}{e B}, \quad (5)$$

and the loss rate becomes

$$\gamma_e \approx \sqrt{\frac{m_e}{m_i}} \gamma_i. \quad (6)$$

If the azimuthal field B_φ is not neglected - still assuming $\beta = 1$ and zero resistivity - the loss rate is reduced by a factor

$$\gamma = \sqrt{1 + (B_\varphi/B_z)^2} \approx B_\varphi/B_z \approx 1/\sin \alpha, \quad (7)$$

where B_φ and B_z are the respective magnetic field strengths on the plasma surface and α is the angle between the centre line of the plasma (vacuum vessel) and the direction of the magnetic field on the plasma surface. In our experiments γ is about 10 on the plasma surface when the plasma radius

r_p is 1 cm. In vacuum the magnetic field strength of a hexapole increases as r^2 for small r (Fig. 3). If we assume that the hexapole field B_h on the plasma surface varies as r_p^2 , we get the following expression (for our experiments):

$$\psi = \frac{10}{r_p^2 / \text{cm}^2} . \quad (8)$$

The loss rate then is - assuming an ion gyro-radius as the slit width -

$$\gamma_{i\psi} = \frac{\gamma_i}{\psi} = \frac{18 \text{ k } T_i}{\pi r_p^2 eB} \approx \frac{1.8 \text{ k } T_i}{\pi eB \cdot 1 \text{ cm}^2} . \quad (9)$$

Let us now consider losses by classical diffusion for a hexapole field configuration with superposed azimuthal field B_φ . According to Bickerton⁸⁾ the thickness δ_c of the current-carrying layer between the plasma ($\beta = 1$) and the magnetic field is (T in $^\circ\text{K}$)

$$\delta_c = \sqrt{\frac{2 \eta_\perp \tau}{\mu_0}} , \quad \eta_\perp = \frac{129 \ln \Lambda}{T^{3/2}} [\Omega \text{ m}] . \quad (10)$$

τ is the lifetime of the plasma that escapes along the field lines

$$\tau = \frac{\ell}{v_i} , \quad (11)$$

where ℓ is the length inside the vacuum vessel of field lines that reach the plasma surface. If we had only hexapole field, then the field line reaching the plasma surface at $r_p = 1$ cm would have a length of about 5 cm inside the vacuum vessel. With superposed azimuthal field B_φ this line will be lengthened^e approximately by the above defined factor ψ and we have

$$\ell = 5 \psi [\text{cm}] \approx \frac{50}{r_p^2 / \text{cm}^2} [\text{cm}] . \quad (12)$$

For $r_p = 1$ cm the field line reaching the plasma surface has a length of about 50 cm inside the vacuum vessel. The particle loss by classical diffusion per unit length is

$$\dot{N} = \frac{2 \pi r_p \delta_c n/2}{\tau} = \pi r_p n \sqrt{2 \eta_1 / \mu_0 \tau}, \quad (13)$$

and the loss rate due to these resistive cusp losses turns out to be

$$\gamma_{r\psi} = - \frac{\dot{N}}{N} = \frac{1}{r_p} \sqrt{\frac{2 \eta_1}{\mu_0 \tau}} = \frac{2}{r_p} \sqrt{\frac{\eta_1 v_i}{\mu_0 \ell}} \approx \frac{0.3}{1 \text{ cm}} \sqrt{\frac{\eta_1 v_i}{\mu_0}}. \quad (14)$$

As to Bohm diffusion, let us assume a linear theta pinch of infinite length without additional hexapole. Plasma losses due to Bohm diffusion can be described by

$$\dot{N} = - n v_B 2 \pi r_p, \quad (15)$$

where the Bohm diffusion velocity is $v_B = 4 \times 10^{17} \nabla p / n B_m$ (MKSA units). In our experiments we can approximate ∇p by p/r_p , while B_m is a mean magnetic field strength near the plasma surface: $B_m \approx B_e (1 - \beta/2)$; B_e is the external field, $\beta = 2 \mu_0 p_0 / B_e^2$, and p_0 is the plasma pressure near the axis. The loss rate due to Bohm diffusion is then

$$\gamma_B = 4 \times 10^{17} \frac{4 \text{ k T}}{r_p^2 B_e \sqrt{1 - \beta/2}} \approx 2 \times 10^{18} \frac{\text{k T}}{r_p^2 B_e}, \quad (16)$$

which is of the same order of magnitude as $\gamma_{i\psi}$. For $r_p = 1$ cm we get $\gamma_{i\psi} \approx 2 \gamma_B$.

III. Experiments

The SPINNE device is a toroidal theta pinch with a major diameter of 52 cm. The vacuum vessel is made of pyrex glass and has a minor inner diameter of 6.0 cm, while the theta-pinch coil has a minor inner diameter of 9.2 cm. Between the vacuum vessel (minor outer diameter 7.0 cm) and the coil there are additional conductors for producing the non-helical hexapole field. There are six conductors, each consisting of six single wires (Fig. 1). Some data of the discharge circuits are given in Table I. Each circuit is crowbarred (Fig. 2) and its energy is characterized by the voltage to which the capacitor bank is charged. Both circuits have approximately the same discharge frequency and are fired at the same time.

Five sets of parameters were investigated (Table II): we varied the filling pressure of hydrogen (40, 30, and 20 mTorr) at a given bank energy (main bank 8 kV, hexapole bank 10 kV), and we varied the bank energy (main bank 8, 12, and 16 kV, hexapole bank 10, 14, and 18 kV) at a given filling pressure of 20 mTorr hydrogen. Lower filling pressures were not feasible because a lack of reproducible preionization in this region.

The plasma radius is obtained by photometric evaluation of streak pictures in integral light. The speed of the film used (Agfa Record) was not high enough to allow the use of filters for suppressing line radiation and just measuring continuum radiation. We had even to reduce the time resolution in order to get a reasonably good exposure. The streak camera used had a f-number of approx. 2, and the time resolution was about 0.5 μ s. Therefore, during the first microsecond, when the plasma radius changes rapidly, the calculated plasma losses are subject to relatively high possible errors. We evaluated mean loss rates for the

time from 2 to 4 μ s after the beginning of the main discharge, when the plasma radius tends to be constant. We used the 20 % intensity level of the streak picture in order to determine the plasma radius. During the time interval when the plasma radius is nearly constant, the error $\Delta r_p/r_p$ is estimated to be ± 20 %.

In Figs. 4 and 5 the plasma radius r_p is given for the five sets of parameters of Table II. As the time resolution is about 0.5 μ s, the measured plasma radius is about the same for all five sets of parameters for the first 0.5 μ s. The plasma radius is smallest near the current maximum at 3.0 μ s, this minimum plasma radius varies between 0.46 and 1.0 cm.

The mean electron densities n_e given in Figs. 6 and 7 are derived in a similar way as in Ref. 1, but the direction of view is now parallel to the plane of the torus through an opening in the inner hexapole conductor. This improvement was necessary in order to avoid an influence of the observed plasma motion parallel to the plane on the density measurement. Density profiles have not been evaluated, but a rough estimation from the streak pictures (perpendicular to the plane of the torus) shows that the density in the centre is 1.0 to 1.4 times the mean densities given in Figs. 6 and 7. The probable error of the mean density is approx. 20 % near its maximum and reaches 50 % for values near the beginning of the main discharge ($t = 0$). To get a better approximation for these early times, we assumed that the density at $t = 0$ is equal to the filling density (expressed in 10^{15} cm^{-3} : $n_0 = 1.32$ for 20 mTorr H_2 , $n_0 = 1.98$ for 30 mTorr, and $n_0 = 2.64$ for 40 mTorr).

The electron temperatures given in Fig. 8 are derived in a similar way to those of Ref. 1. The temperature curve up to $t = 2 \mu$ s is evaluated from the times of maximum intensity of oxygen impurity lines of successive stages

of ionization⁹⁾, for experiment (5) also the C V line at 2272 Å has been used. The necessary rate coefficients were taken from Lotz.¹⁰⁾ Near the current maximum from $t = 2$ to $t = 4$ μs the temperature curve has been extrapolated on the assumption that the temperature is nearly proportional to the magnetic field (adiabatic compression). The electron temperatures of Fig. 8 should be representative of the bulk of the plasma. Slightly higher temperatures may occur near the axis.

By adding known amounts of oxygen we verified that the oxygen impurity level in our discharges was between 0.2 and 0.8 %, depending, e.g., on the H₂ filling pressure and on proper "conditioning" of the vacuum-vessel surface by a sufficient number (usually at least 30) of discharges. We could not find any significant difference in oxygen impurity level, whether we used a pyrex or a quartz vacuum vessel, nor with or without a high frequency breakdown aid. This high frequency of 27 MHz from a 0.5 kW transmitter was necessary in order to achieve a reproducible breakdown by the preionization at 20 mTorr filling pressure. Oxygen is by far the most abundant impurity in our discharges.

The (mean) electron densities of Figs. 6 and 7 and the (mean) electron temperatures of Fig. 8 have been combined to calculate a mean beta value, assuming $T_e = T_i$:

$$\beta = 2 \mu_0 n_e k T_e / B_a^2, \quad (17)$$

where B_a is the external B_φ field for $R_0 = 26$ cm. The hexapole field B_{θ} is small near the plasma surface compared with B_{ϕ} and has been neglected here. The probable error of the mean beta values of Fig. 9 is $^{+30}_{-23}$ %, as far as the accuracy of n_e , T_e , and B_a is concerned.

In Fig. 10 the results of calculations of the relaxation time τ for energy exchange between electrons and protons are given. These relaxation times are generally smaller than 0.5 μ s, and so near the current maximum at $t = 3 \mu$ s the electron and ion temperatures will be nearly equal. The relaxation time is derived from Spitzer¹¹⁾ to be (T_e in eV, n_e in cm^{-3}):

$$\tau = 1.6 \times 10^8 T_e^{3/2} / n_e \ln \Lambda. \quad (18)$$

IV. Discussion

Figure 9 shows smoothed mean beta values derived by formula (17) with mean electron densities and mean electron temperatures. These mean beta values would be higher at least for the first two microseconds if we took into account that the ion temperatures are higher than the electron temperatures owing to the more effective heating of the ions by the first imploding shock wave. Beta values in the centre of the plasma may be even higher for the following reason: the density and the temperature in the centre are higher than their mean values. Thus the beta values in Fig. 9 are to be understood as lower limits of the real beta value in the centre of the plasma.

Nevertheless, some general conclusions can be drawn from Fig. 9:

- a) Other parameters being unchanged the beta values increase with decreasing filling pressure: experiments (1), (2), and (3).
- b) With increasing bank energy the calculated beta values are lower for the first two microseconds: experiments (3), (4), and (5).

Conclusion (a) might be attributed to less trapped field at lower filling pressures due to higher initial temperatures, while we tend to attribute conclusion (b) to the more effective heating of ions during the first microsecond.

In Figs. 11 to 15 the measured line densities are given for the five different experiments together with a best fit straight line in order to evaluate the experimental loss rate. The line density is $N = \pi r_p^2 n_e$ (Figs. 4 to 7). The loss rate indicated for each graph has a probable error of $^{+30}_{-23}$ %, this error being due mainly to uncertainties in the plasma radius. Nevertheless, the ratio of two loss rates of different experiments is more accurate, because most systematic errors will tend to cancel. The probable error of this ratio should be less than 10 %. This means that the loss rate of experiment (1) is certainly lower than the loss rate of experiment (2) and this again is certainly lower than the loss rate of experiment (3), though (1) and (3) are within the stated error limits of (2).

Figs. 11 to 15 seem to indicate that the respective loss rates are constant not only near the current maximum ($t = 3 \mu s$), but in the relatively wide range between $t = 1$ and $t = 5 \mu s$, though the temperature varies appreciably.

In Fig. 16 we compare the measured loss rates with the anticipated losses from a line cusp with a cusp slit width of an ion gyro-radius given by formula (9). As this formula predicts a proportionality between the loss rate γ_{iy} and T/B , we chose to plot γ versus T/B . For our experiments, formula (9) can be written (T in eV, B in kG):

$$\gamma_{iy} = 5.7 \times 10^4 T/B. \quad (19)$$

while the experimental points tend to give

$$\gamma = 4.6 \times 10^4 \text{ T/B} . \quad (20)$$

Within experimental error, formulae (19) and (20) are in good agreement.

In Fig. 17 we tried to compare the measured loss rates with the losses predicted by the Bohm diffusion formula (16). The loss rate by Bohm diffusion should be proportional to $T/B r_p^2$. The measured loss rates are of the same order of magnitude as those predicted by Bohm diffusion, but the loss rate of experiment (5) is only 1/3 of the theoretical value. This is beyond experimental error and thus Bohm diffusion following formulae (15) and (16) can be excluded for experiments (4) and (5).

A comparison with losses to be anticipated by classical diffusion, i.e. formula (14), shows that these losses will be negligible for temperatures above 20 eV compared with the measured loss rates.

In addition to the experiments described here, we tried another magnetic field configuration which is in some respects similar to the proposed Scyllac configuration¹²⁾ of the Los Alamos group. Along the circumference of the torus there are four feeding points for the hexapole wires, which are normally connected in such a way that the current through the hexapole wires is virtually closed around the torus. In our "Alternating Hexapole" experiments we connected the wires in such a way that the polarity of the hexapole is reversed at each of the four feeding points. The measured loss rates (expressed in 10^5 s^{-1}) then were compared with those of the "normal" hexapole: (1) 2.1 against 1.8, (2) 2.5 against 2.3, and (3) 3.0 against 2.6. Experiments (4) and (5) were not made with Alternating Hexapole.

The machine is now being converted into a $\ell = 3$ "fast stellerator" configuration, i.e. a toroidal theta pinch with superposed helical hexapole field.

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Table I: Data of the batteries used.

Preionization Bank:

Energy	0.9 kJ	(5)
Voltage	25 kV	
Capacitance	3 μ F	20
Ringing frequency	700 kHz	

Main Bank:

Energy	58 kJ	21
Voltage	18 kV	
Capacitance	360 μ F	18
Rise time $\tau/4$	3.0 μ s	58
Magnetic field $B_{\max}$	24 kG	93
Field rise B_0	10^{10} G/s	3.5
Crowbar time	30 μ s	0.46

Hexapole Bank:

Energy	58 kJ
Voltage	18 kV
Capacitance	360 μ F
Rise time $\tau/4$	2.7 μ s
$B_{\max}$ at $r = 3$ cm*	16 kG
Crowbar time	30 μ s

* Between conductors.

Table II: Parameters of the Experiments.

Experiment No.	(1)	(2)	(3)	(4)	(5)
Filling pressure (mTorr H ₂)	40	30	20	20	20
Main Bank					
Voltage (kV)	8	8	8	12	16
Energy (kJ)	11.5	11.5	11.5	26	46
B _{max} (kG)	10.5	10.5	10.5	16	21
Hexapole Bank					
Voltage (kV)	10	10	10	14	18
Energy (kJ)	18	18	18	35	58
T _e max (eV)	38	50	62	76	93
n _e max (10 ¹⁶ /cm ³)	1.6	1.6	1.4	2.5	3.5
r _p min (cm)	0.95	0.86	0.80	0.60	0.46
Loss rate (10 ⁵ /s)	1.8	2.3	2.6	2.1	1.9

Fig. 1. Arrangement of vacuum vessel, hexapole conductors, and main coil. Each hexapole conductor consists of six single wires. The inner diameter of the vacuum vessel is 6 cm.

Fig. 2. Magnetic field of the main coil for $R_0 = 26$ cm (solid curve), and magnetic field of the hexapole conductors for $r_0 = 3$ cm between conductors (dashed curve). Main bank voltage 8 kV, hexapole bank voltage 10 kV.

Fig. 3. Magnetic field B_r of the hexapole conductors (without main field) versus minor radius r . Upper curve: undistorted field in vacuum (between conductors); lower curve: field distorted by a bore of 1.5 cm diameter in the main coil. Hexapole bank voltage 10kV.

Fig. 4. Plasma radius versus time as derived from streak pictures for experiments (1) and (2).

Fig. 5. Plasma radius versus time as derived from streak pictures for experiments (3) to (5).

Fig. 6. Mean electron densities of experiments (1) and (2) versus time.

Fig. 7. Mean electron densities of experiments (3) to (5) versus time.

Fig. 7. Mean electron densities of experiments (3) to (5) versus time.

Fig. 8. Mean electron temperatures versus time as derived from impurity radiation up to 2 μs . Temperatures at later times are extrapolated: $T_e \propto B$.

Fig. 9. Smoothed mean beta values derived from the mean electron density and the mean electron temperature, assuming $T_e = T_i$.

Fig. 10. Relaxation times τ for energy exchange between protons and electrons versus time t .

Fig. 11. Line density of experiment (1) versus time. The best-fit straight line gives a loss rate $\gamma = 1.8 \times 10^5/\text{s}$.

Fig. 12. Line density of experiment (2) versus time. The best-fit straight line gives a loss rate $\nu = 2.3 \times 10^5/\text{s}$.

Fig. 13. Line density of experiment (3) versus time. The best-fit straight line gives a loss rate $\gamma = 2.6 \times 10^5/\text{s}$.

Fig. 14. Line density of experiment (4) versus time. The best-fit straight line gives a loss rate $\gamma = 2.1 \times 10^5/\text{s}$.

Fig. 15. Line density of experiment (5) versus time. The best-fit straight line gives a loss rate $\gamma = 1.9 \times 10^5/\text{s}$.

Fig. 16. Loss rates of experiments (1) to (5) versus T/B . The upper curve $\nu_{i\psi}$ is derived from formula (9) or (19). The lower curve is a best fit, parallel to the upper curve.

Fig. 17. Loss rates of experiments (1) to (5) versus $T/B r_p^2$. The curve ν_{Bohm} is derived from formula (16).