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Some Theoretical Aspects of Plasma Heating by High Frequency Fields+

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ABSTRACT:

Theoretical work done at Garching on plasma heating by high frequency electromagnetic fields is reported. Three subjects are covered:

1. A time-dependent calculation of plasma density and electron kinetic temperature in an ECR-produced, mirror-confined plasma.

2. A relativistic, single-particle analysis of the maximum ECR energy in a mirror configuration.

3. Theoretical study of cyclotron waves propagating in a hot plasma cylinder (solution of the Vlasov equation).

1. Density and Kinetic Temperature of Electrons in an ECR-Produced, Mirror-Confined Plasma.

(by K. Wöhler and H.K. Wimmel)

The non-stationary high frequency heating of the electrons of a partially ionized hydrogen plasma in a mirror configuration is considered. The following simplifying assumptions are made:

a) The total number of ions plus neutral atoms is assumed constant in time, and the plasma and neutral densities are taken as spatially uniform within the total volume.

b) The electromagnetic power absorbed by the plasma is supposed to be zero for $t<0$ and equal to a given parameter P for $t>0$.

c) The electrons are assumed to be monoenergetic.

The particle and energy balance are described by the following equations+):

(1) $$\frac{dn}{dt} = \frac{1}{2}\langle \sigma_i v_e \rangle n n_o - \frac{n}{\tau_e}$$

(2) $$\frac{d}{dt}(n\varepsilon) = \frac{P}{V} - \frac{1}{2}\langle \sigma_i v_e \rangle n_o n W_i - \frac{n\varepsilon}{\tau_e}$$

(3) $$n_o = N - n$$

+) Due to an oversight the factor $\frac{1}{2}$ in eqs. (1), (2), (4) is not taken into account in the numerical results reported.

Combining (1) and (2) gives:

(4) $$\frac{d\varepsilon}{dt} = \frac{P}{Vn} - \frac{1}{2}(\varepsilon + W_i)\langle\sigma_i v_e\rangle n_o ,$$

which may be used instead of (2).

Here n and ε are the electron density and energy, n_o is twice the density of H_2 molecules, W_i is the ionization energy for molecular hydrogen, $\langle\sigma_i v_e\rangle$ is the ionization coefficient for ionization of H_2 by electrons of temperature ε , τ_e is the mean scattering loss time for electrons:

(5) $$\tau_e = f\,\varepsilon^{3/2}/n$$

with

$$f = 5 \times 10^4 (1 + \ln R)\ cm^{-3}\,sec^{-1}(eV)^{-3/2} .$$

The equations assume that the particle balance is determined by volume ionization of H_2 versus electron-electron scattering into the loss cone. With each electron one atomic ion is lost, and half a neutral hydrogen molecule appears as a result of instant wall recombination. Volume recombination is neglected.

The energy equation takes into account high frequency heating, energy loss by ionization, and energy loss by the scattering of electrons into the loss cone. Energy transfer to the ions is neglected.

The above equations were solved numerically for a hydrogen plasma with the initial values $n = 10^{-2} N$, $N = 10^{12}\ cm^{-3}$, $\varepsilon = 13\ eV$, and for power densities P/V from 0.2 to 6.0 kW/liter. From the numerical solutions new solutions may be constructed by the scaling

$$(6) \quad \begin{cases} t \to \alpha t\,, \quad N \to \alpha N\,, \quad P/V \to \alpha^{-2} P/V, \\ n/N \to n/N\,, \quad \varepsilon \to \varepsilon, \end{cases}$$

provided the assumptions made remain valid.

The main results are the following (Figs. 1 - 6):

a) For power densities below $\sim$ 0.25 kW/liter the density and temperature reach stationary values after about 0.2 msec. The asymptotic electron energy stays below $\sim$100 eV, the degree of ionization below $\sim$0.9.

b) Above this critical power density total ionization is approached after about 0.3 msec, and after that most of the power goes into heating. The electron energy then increases linearly with time according to

$$(7) \quad \varepsilon \lesssim Pt/NV.$$

With $N = 10^{12}\ cm^{-3}$ and $P/V = 1$ kW/liter this gives $\varepsilon \approx 6$ keV after 1 msec. After the transient time the neutral density is given by the equilibrium condition

$$(8) \quad n_0 \approx 2/\tau_e \langle \sigma_i v_e \rangle\,.$$

If a relative concentration of neutrals below $n_0/N = 2 \times 10^{-3}$ is desired (see below), this will be accomplished for energies $\varepsilon \geqq 4.8$ keV.

c) There is a high energy peak during the initial low density heating phase. The higher the power density is chosen, the longer is the duration of this initial period.

This calculation of ECR heating was carried out in 1964/65 in connection with an experimental project proposed by F. BOESCHOTEN [1]. In the experiment a combination of ion and electron cyclotron resonance heating was to be used in order to obtain the following plasma parameters; $n_e \sim 10^{12}$ cm^{-3}, $T_i \sim 4$ keV, confinement time $t_c \sim 5 \times 10^{-2}$ sec.

Other plasma parameters follow from the requirement that the plasma state be not perturbed by processes of charge exchange, electron scattering into the loss cone, or energy equipartition by ion-electron collisions during the confinement time after turning off the electromagnetic power. Thus for the mean time of charge exchange to be longer than t_c the condition $n_o < 2 \times 10^9$ cm^{-3} or $n_o/N < 2 \times 10^{-3}$ must be satisfied. In order for the mean electron-electron scattering loss time τ_e to be longer than t_c the condition $T_e \geqq 6$ keV must hold. (A mirror ratio of R = 3 was used.) Finally, for the equipartition time τ_{Eq} to be longer than t_c it should hold that $T_e > 125$ eV, which is a less stringent condition than that preceding.

Comparison with the results of the heating calculation shows the following. Steady state operation of ECR heating would give at best $n_o/N \gtrsim 0.1$ and $T_e \lesssim 100$ eV. The required plasma parameters of $n_o/N < 2 \times 10^{-3}$ and $T_e \gtrsim 6$ keV can be obtained by dynamical operation only. The power density necessary for reaching an electron temperature of $T_e \gtrsim 6$ keV in 1 msec is P/V $\gtrapprox$ 1kW/liter.

2. Relativistic Estimate of the Maximum ECR Energy in a Mirror Configuration.

(by H.K. Wimmel and K. Wöhler)

Experiments by DANDL et al. [2] have shown that an ECR plasma in a mirror configuration may consist of electron components of quite different energy. In the experiment [2] there exists a colder component of about 100 eV and a hotter component of about 100 to 200 keV. The power density* in this experiment was 0.1 kW/liter, the electron density $n_e = 3 \times 10^{11}$ cm^{-3}, the magnetic field in the mirror mid-plane $B_{min} = 2 \times 10^3\ \Gamma$, and the mirror ratio R = 3.

The aim of this investigation is to show that in a simple resonance heating model there exists a maximum particle energy beyond which no more electron cyclotron resonance occurs in the mirror field. One may go further and interpret this maximum resonance energy as a virtual energy limit beyond which only a small percentage of electrons can be accelerated. This maximum energy is estimated with simplifying assumptions, and the estimate is compared with the results of the Oak Ridge experiment [2] . The comparison is made on the assumption that the low energy peak (100 eV) is determined by collision processes as well, while the high energy peak (100 - 200 keV) results mainly from single particle interaction with the mirror field and the electromagnetic wave. Thus our single particle analysis is concerned with the high energy component.

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*The frequency of the applied microwaves was 10.6 Gc/sec.

The resonance heating model assumes that stochastic heating of particles takes place only if and when there is resonance or near-resonance between the applied electromagnetic field and the gyro-motion of the particles. The particles are described by their kinetic energy and magnetic moment, and it is assumed that the loss cone is empty of particles. The calculation is relativistic and takes into account the relativistic mass increase and the relativistic Doppler effect due to the motion parallel to the magnetic field lines. One single field line is considered. The problem is greatly simplified by making one further approximation. We shall assume that the Doppler shift is that of a vacuum electromagnetic wave, i.e. the phase velocity is equated to c. This approximation is certainly realistic for very low density plasmas. But even for higher densities the error committed will be comparatively small for those particles which, due to the Doppler effect, have their resonance points in regions where the index of refraction does not deviate much from 1. In the laboratory system the relativistic resonance condition is then

(1) $$\omega \left(1 - \beta_{\parallel r} \cos \theta_r \right) = \frac{e B_r}{m c} \sqrt{1 - \beta^2} .$$

Here $\frac{\omega}{2\pi}$ is the applied frequency, B_r is the magnetic field at the resonance points, m is the rest mass, $\underline{\beta} = \underline{v}/c$, $\beta_{\parallel r}$ is the component of $\underline{\beta}$ parallel to $\underline{B}$, and $\theta_r = \sphericalangle (\underline{B}, \underline{k})$, both at resonance. By introducing the quantity $B_\omega = mc\omega/e$ one may transform (1) into

(1') $$B_r = B_\omega \left(1 - \beta_{\parallel r} \cos \theta_r \right) / \sqrt{1 - \beta^2} .$$

The mirror condition for a particle at a location of magnetic field strength B is given, also in the relativistic case, by the familiar inequality:

$$(2) \qquad \beta_{\parallel}^{2} \leqq (1 - B/B_{max})\,\beta^{2},$$

because the magnetic moment is $\mu = m v_{\perp}^{2}/2B(1-\beta^{2})$. [3] .

In the following we choose $\underline{k} \parallel \underline{B}$, e.g. $\cos\theta = \pm 1$. Equation (1') then becomes

$$(1'') \qquad B_r = B_\omega \left(1 \pm |\beta_{\parallel r}|\right)/\sqrt{1-\beta^2} \, .$$

The resonance fields B_r must lie in the interval

$$(3) \qquad B_{min} \leqq B_r \leqq B_{max} \, .$$

Because of (2), B_r must satisfy two additional inequalities, namely

$$(4) \qquad B_\omega \left(1 - \beta\sqrt{1 - B_r/B_{max}}\right)/\sqrt{1-\beta^2} \leqq B_r \leqq$$

$$\leqq B_\omega \left(1 + \beta\sqrt{1 - B_r/B_{max}}\right)/\sqrt{1-\beta^2} \, .$$

The problem is now to find, for given values of B_{min}, B_{max}, B_ω , the range of values of β (or of the kinetic energy) for which values of B_r exist that satisfy the four inequalities (3) and (4), and equation (1'').

Setting aside for the moment eq. (3), eqs. (1'') and (4) are equivalent to the following two inequalities to be satisfied simultaneously:

(5) $$a\beta^2 + b\beta + c \geqq 0$$

and

(6) $$a\beta^2 - b\beta + c \leqq 0,$$

with the definitions

(7) $$a = 1 - B_r/B_{max} + (B_r/B_\omega)^2$$

(8) $$b = 2\sqrt{1 - B_r/B_{max}}$$

(9) $$c = 1 - (B_r/B_\omega)^2 .$$

One has $a > 0$, $b \geqq 0$, and $a + c = 1 + b^2/4$. Resonance is altogether impossible if the common discriminant of (5) and (6) is negative, i.e. for $b^2 - 4ac < 0$. This excludes $B_r B_{max} < B_\omega^2$. Hence we now consider only

(10) $$B_\omega \leqq \sqrt{B_r B_{max}} \leqq B_{max} .$$

Elementary analysis shows that (5) and (6) are equivalent to

$$\operatorname{sign}(c)\cdot\left(b-\sqrt{b^2-4ac}\right)/2a \leqq \beta \leqq \left(b+\sqrt{b^2-4ac}\right)/2a. \tag{11}$$

The smallest lower limit possible for β is $\beta_{min} = 0$; this is realized for $c = 0$, i.e. for $B_r = B_\omega$, independently of B_{min}, B_{max}, B_ω. Further analysis shows that the maximum upper limit of β for given values of B_{min}, B_{max}, B_ω occurs for $B_r = \max(B_\omega, B_{min})$. If $B_{min} \leqq B_\omega \leqq B_{max}$, then β_{max} occurs for $c = 0$ and has the form:

$$\beta_{max} = \sqrt{1 - B_\omega/B_{max}}\,/\left(1 - B_\omega/2B_{max}\right). \tag{12}$$

The corresponding kinetic energy is then:

$$W_{max} = 2mc^2\left(B_{max}/B_\omega - 1\right). \tag{13}$$

If, on the other hand, $B_\omega < B_{min}$ then for $B_\omega \longrightarrow 0$ one has $W_{max} \longrightarrow \infty$, while there exists a low energy region $0 \leqq W \leqq W_{min}$ for which no resonance is possible.

In the resonance heating model which we use, one further assumption enters which may limit the significance of our results. It has been assumed - tacitly so far - that the high frequency electromagnetic wave fills the complete volume of the mirror field. This may now not be true due to strong absorption or to evanescence, e.g. for a cyclotron wave in the spatial region where $\Omega < \omega$. In this case, only those values of B_r are available which occur in that spatial region where the wave propagates, and resonance will occur in a smaller energy interval of the form

$$(14) \qquad 0 < W_{min} \leqq W \leqq W'_{max} < W_{max},$$

where W_{max} is given by (13).

So far we have considered $\underline{k} \parallel \underline{B}$. To conclude we consider also the case of $\underline{k} \perp \underline{B}$. This case is analytically much simpler because no longitudinal Doppler effect is involved. Therefore it may suffice to list the main results. The resonance field is now given by

$$(15) \qquad B_r = B_\omega / \sqrt{1-\beta^2} .$$

If $B_{min} \leqq B_\omega \leqq B_{max}$, then resonance occurs for energies

$$(16) \qquad 0 \leqq W \leqq mc^2 (B_{max}/B_\omega - 1).$$

Thus the maximum energy is one half that given in eq. (13). If $B_\omega > B_{max}$, no resonance occurs at all, and if $B_\omega < B_{min}$, then resonance occurs for energies

$$(17) \qquad mc^2 (B_{min}/B_\omega - 1) \leqq W \leqq mc^2 (B_{max}/B_\omega - 1).$$

When the wave propagates only through part of the plasma volume then B_{max} and B_{min} must be replaced by the extremal values of B in that part of the volume.

We may compare the estimate of eq. (16) with the results of the Oak Ridge experiment [2]. The applied heating frequency there corresponds to $B_{\omega} = 3.8 \times 10^{3}\ \Gamma$. Equation (16) then gives W_{max} = 290 keV, while the energy of the hot electrons in the experiment is, as mentioned, 100 to 200 keV. In view of the approximations made in our energy estimate, the agreement appears to be reasonable.

3. Cyclotron Waves of Axial Symmetry in a Hot Plasma Cylinder.

(by H. Grawe [4])

A hot plasma infinitely extended in one direction (z-coordinate) and bounded in the two other dimensions by a cylindrical surface of circular cross section is considered. There exists a stationary magnetic field $\underline{B}_o$ parallel to the z-axis. The unperturbed distribution function and $\underline{B}_o$ are assumed uniform in the cylinder, except for a thin surface layer. The gyroradii are assumed small compared with the diameter of the cylinder. The linear Vlasov equation and simplified boundary conditions are applied in order to derive a dispersion relation for waves of axial symmetry. This linear analysis of wave propagation in a cylindrical plasma may be useful for numerical computations of ICR or ECR heating even when the magnetic field or the plasma is slightly inhomogeneous.

The linear Vlasov equation is solved for an infinite, homogeneous plasma by the method of characteristics. From this solution the electric current density can be derived. It has the following integral form:

(1) $$\underline{j}(\underline{r},t) = \left\{ \int \underline{\underline{K}}_1(\underline{r},\underline{r}') \cdot \underline{E}(\underline{r}')\, d\underline{r}' + \int \underline{\underline{K}}_2(\underline{r},\underline{r}') \cdot \underline{B}(\underline{r}')\, d\underline{r}' \right\} e^{-i\omega t},$$

where the tensor kernels $\underline{\underline{K}}_1$ and $\underline{\underline{K}}_2$ depend on the equilibrium parameters of the plasma and where an exponential time dependence of the perturbation has been assumed. Combination with Maxwell's equations yields a system of integro-differential equations. However, for axisymmetric waves of the form

(2) $$\underline{E}(\underline{r},t) = e^{ikz-i\omega t} \left\{ \tilde{E}_s J_1(k_\perp s)\, \underline{e}_s + \tilde{E}_\varphi J_1(k_\perp s)\, \underline{e}_\varphi + \tilde{E}_z J_0(k_\perp s)\, \underline{e}_z \right\},$$

where J_0 and J_1 are Bessel functions, it can be shown that $\tilde{E}_\nu$ and $\tilde{j}_\nu$ are related algebraically by a dielectric matrix $\varepsilon_{\mu\nu}$ (μ and ν = s, φ, z), e.g.:

(3) $$\frac{4\pi i}{\omega} \tilde{j}_\mu + \tilde{E}_\mu = \sum_\nu \varepsilon_{\mu\nu}(k_\perp, k, \omega)\, \tilde{E}_\nu .$$

This case is considered in the following. The dielectric matrix is calculated for distribution functions of the form

(4) $$f_0(\underline{v}) = g(v_z) \cdot h(v_\perp^2) .$$

The resulting expression for ($\varepsilon_{\mu\nu}$) is very similar to that obtained for plane waves with $k_x = k_\perp$, $k_y = 0$; this is shown in the following scheme:

$\mu \backslash \nu$	s	φ	z
s	ε_{xx}	ε_{xy}	$i\,\varepsilon_{xz}$
φ	ε_{yx}	ε_{yy}	$i\,\varepsilon_{yz}$
z	$-i\,\varepsilon_{zx}$	$-i\,\varepsilon_{zy}$	ε_{zz}

$\varepsilon_{\mu\nu}$

Similar to the case of plane waves an algebraic system of equations follows for the $\tilde{E}_\mu$, viz.:

(5) $$\sum_\nu L_{\mu\nu}\,\tilde{E}_\nu = 0$$

with

(6) $$(L_{\mu\nu}) = \begin{pmatrix} \varepsilon_{ss} - n_z^2 & \varepsilon_{s\varphi} & \varepsilon_{sz} + i n_\perp n_z \\ \varepsilon_{\varphi s} & \varepsilon_{\varphi\varphi} - n^2 & \varepsilon_{\varphi z} \\ \varepsilon_{zs} - i n_\perp n_z & \varepsilon_{z\varphi} & \varepsilon_{zz} - n_\perp^2 \end{pmatrix}$$

where $n_z = kc/\omega$, $n_\perp = k_\perp c/\omega$, $n^2 = n_\perp^2 + n_z^2$.
The dispersion relation, which would hold for the axisymmetric waves considered, but in an <u>infinite</u> plasma (without boundaries), has the form

(7) $$\det(L_{\mu\nu}) = 0 \;;$$

it is identical with that valid for plane waves, if $k_\perp$ is identified with k_x of the plane wave case with $k_y = 0$. This identity is plausible since it has already been demonstrated for a cold plasma by HOOKE and ROTHMAN [5]. The dielectric matrix for waves in a hot plasma cylinder was derived earlier by other authors, but with more restrictive conditions. Thus CHANDRASEKHAR et al. [6] assume $\omega^2 \ll \omega_{p\alpha}^2$ and $\omega^2 \ll \Omega_\alpha^2$ (α denotes the plasma components), while SIMON and THOMPSON [7] assume $\omega^2 \ll \Omega_\alpha^2$. In the present derivation it has merely been assumed that $n_0 \lambda_D^3 \gg 1$ and $\hbar \omega_{p\alpha} \ll \frac{3}{2} k T_\alpha \ll m_\alpha c^2$, i.e. Vlasov approximation and absence of quantum and relativistic effects. No condition was imposed on the frequency of the waves in the present derivation.

Equation (7) is not the complete dispersion relation because it does not take into account any boundary conditions for the waves. In order to formulate the boundary conditions one must define the exterior of the plasma cylinder. GRAWE takes the following example: The plasma cylinder extends radially from s = o to $s = s_0$; then there is a vacuum layer between $s = s_0$ and $s = S_0$, and at $s = S_0$ there is a metallic surface of infinite conductivity.

To simplify the analysis the dielectric matrix ($\varepsilon_{\mu\nu}$) is now expanded in powers of $k_\perp^2 a_\alpha^2$, where a_α are the gyroradii of the particles. It turns out that to lowest order in $k_\perp^2 a_\alpha^2$ Eq. (7) has generally three solutions $k_{\perp\nu}^2 (k, \omega)$, ($\nu = 1, 2, 3$) for fixed values of k and ω. Now the partial solution in the plasma cylinder region is characterized on the one hand by six independent field amplitudes $\tilde{E}_{s\nu}$ and $\tilde{E}_{z\nu}$ ($\tilde{E}_{\varphi\nu} = 1$ by normalization), which all are functions of $k_{\perp\nu}$, k , and ω and follow from Eq. (5), and on the other hand by three amplitudes A_ν (ν = 1,2,3). That partial solution in the vacuum region which satisfies the boundary conditions at the metallic surface ($s = S_0$) is characterized by only two amplitudes A_ν (ν = 4,5). In order to determine the A_ν , five linearly independent, linear boundary conditions are required at the surface of the plasma ($s = s_0$). The well-known jump conditions following from Maxwell's equations and from the first moments of the Vlasov equation, as derived, for instance, by CHANDRASEKHAR, KAUFMAN , and WATSON [8] , are used as boundary conditions. They may be written thus:

$$\sum_{n=1}^{5} a_{mn} A_n = 0 \quad ; \quad m = 1 \ldots 5 , \tag{8}$$

the a_{mn} being functions of k and ω . The final dispersion relation then reads

$$\det \left(a_{mn} \right) = 0 . \tag{9}$$

In general this can only be solved numerically. In the special case of the "magnetosonic regime" (where only first order terms in ω/Ω_α and $k_\perp^2 a_\alpha^2$ are taken into account) known results are reproduced analytically.

In conclusion, I shall mention only briefly two further results obtained by GRAWE [4] , [9] .

Firstly, single particles are considered in a static mirror field $\underline{B}_0$ with a superimposed transversal wave ($\underline{k} \parallel \underline{B}_0$, $\underline{E} \perp \underline{B}_0$). A simple resonance heating model is considered, i.e. the acceleration of particles is only calculated for near-resonance conditions. It is also assumed that the effective limitation of the resonance periods by the variation of the phase relation between the wave and the particles is brought about by the inhomogeneity of $\underline{B}_0$ alone. For $kL \gg 1$, where L is the characteristic length of inhomogeneity of $\underline{B}_0$ parallel to $\underline{B}_0$, it is shown that the power absorption per unit volume and unit time is independent of the inhomogeneity of $\underline{B}_0$. The reason for this is that on the one hand a single particle will be in resonance for a shorter time when the inhomogeneity is stronger, but on the other hand more particles will then come into resonance per unit volume and time.

Secondly, a numerical computation of axisymmetric electromagnetic wave propagation in a cold electron plasma cylinder enclosed by a metallic surface was made [9] for electron cyclotron waves ($\omega < \Omega_e$). There exist several modes, with different polarization properties. All modes possess a non-negligible longitudinal component of the electric field that can become very large compared with the transversal components. This is in contrast to the purely transversal polarization which is assumed in other investigations on ECR heating. The ratio of E_z to $E_\perp$ increases with increasing plasma density and for $\omega \rightarrow \Omega_e$. In Figs. 7 to 10 four examples of radial field distributions are shown.

R E F E R E N C E S

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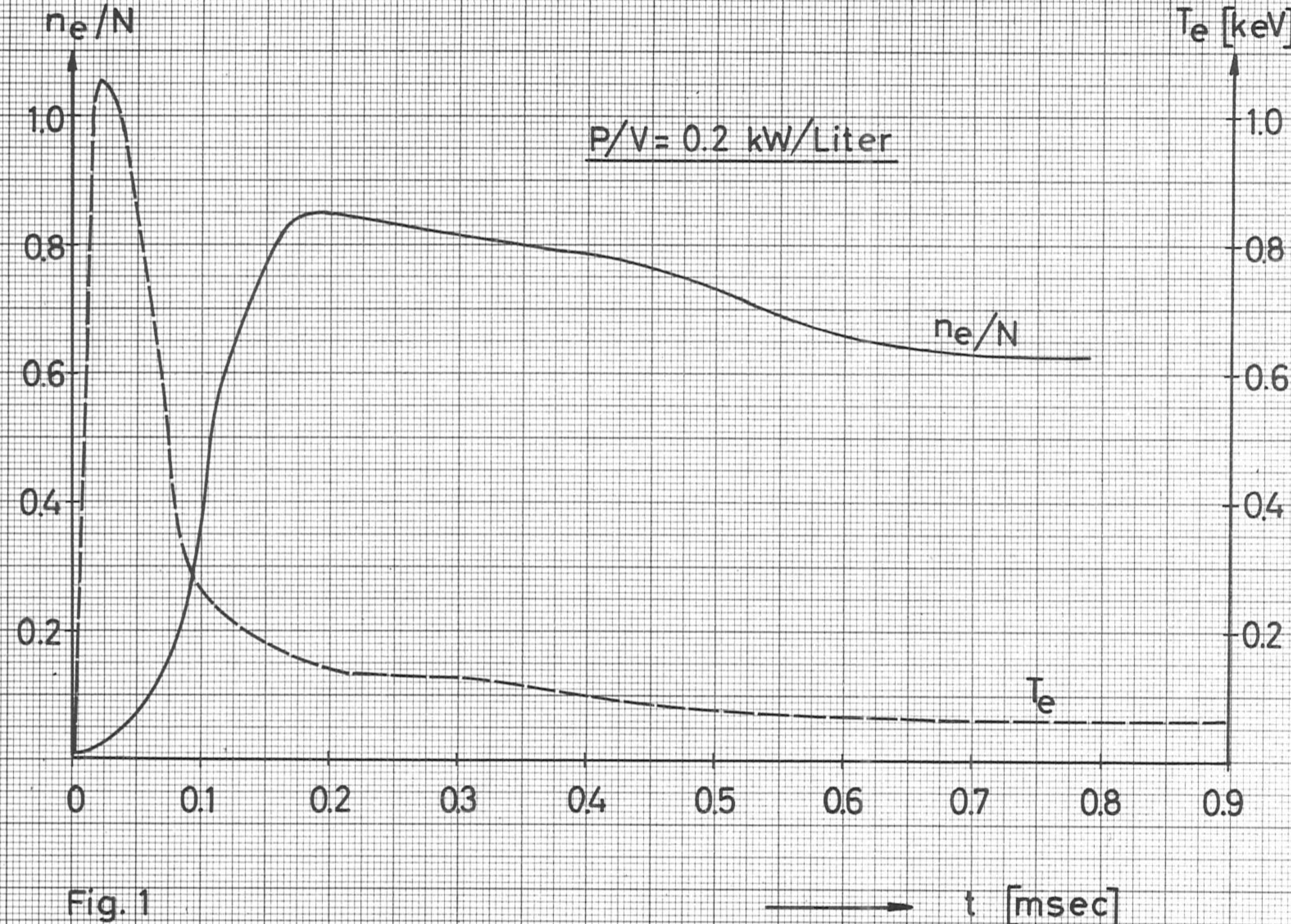
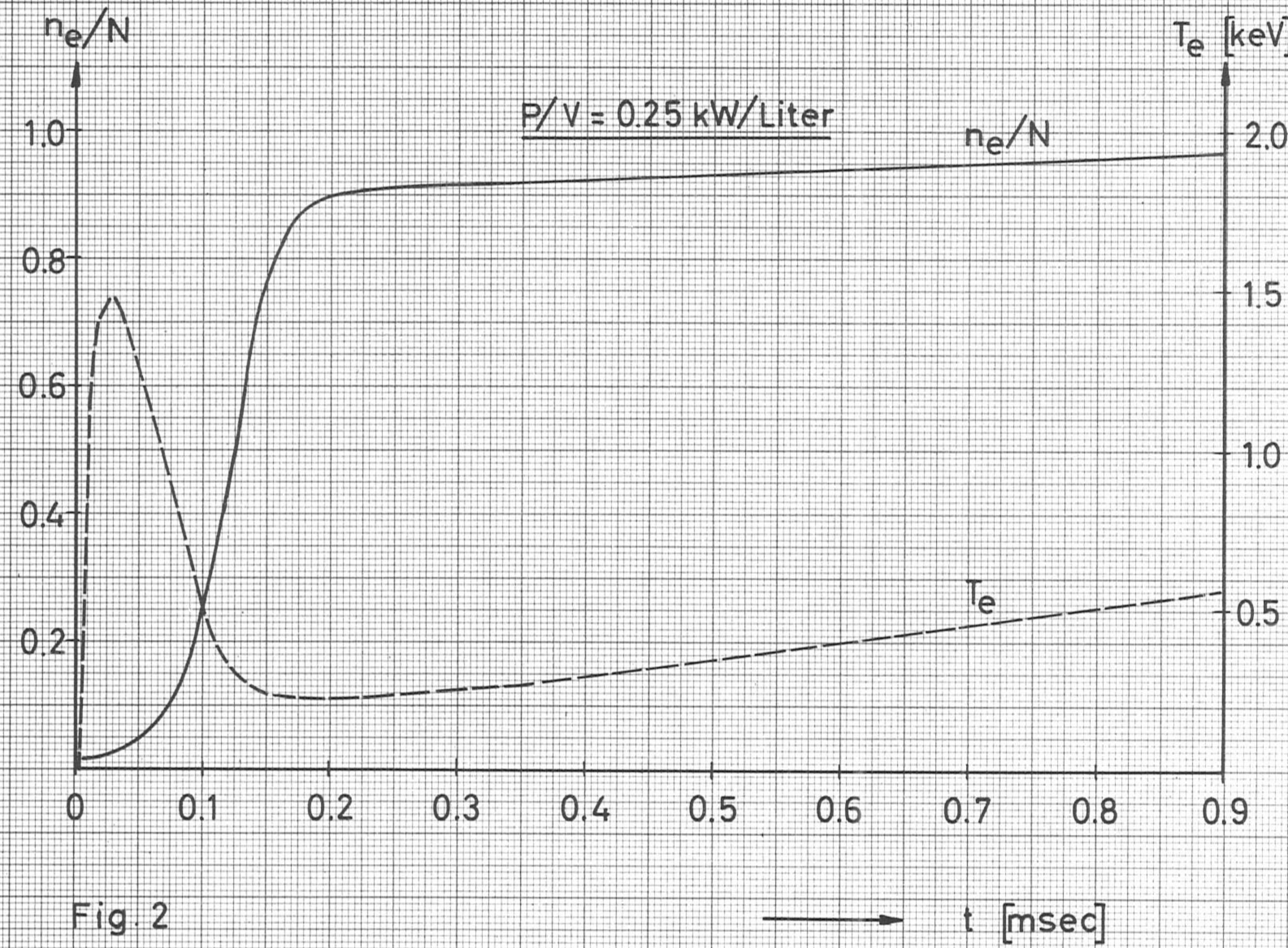
<u>Figs. (1) to (6) :</u> Electron density and temperature versus time for $N = 10^{12}\ cm^{-3}$ and power densities from 0.2 to 6.0 kW/liter. Initial values : $n = 10^{-2}N$; $\varepsilon = 13$ eV.

<u>Figs. (7) to (10) :</u> Radial electric field distribution of an electron cyclotron wave propagating in a cold plasma cylinder surrounded by a metallic wall, for $\omega_p^2/\omega^2 = 0.5$ and 0.9; $\Omega_e/\omega = 1.2$. The plasma radius is chosen $s_0 = 20$ cm. Normalization: $E_z = 1$ for $s = 0$.



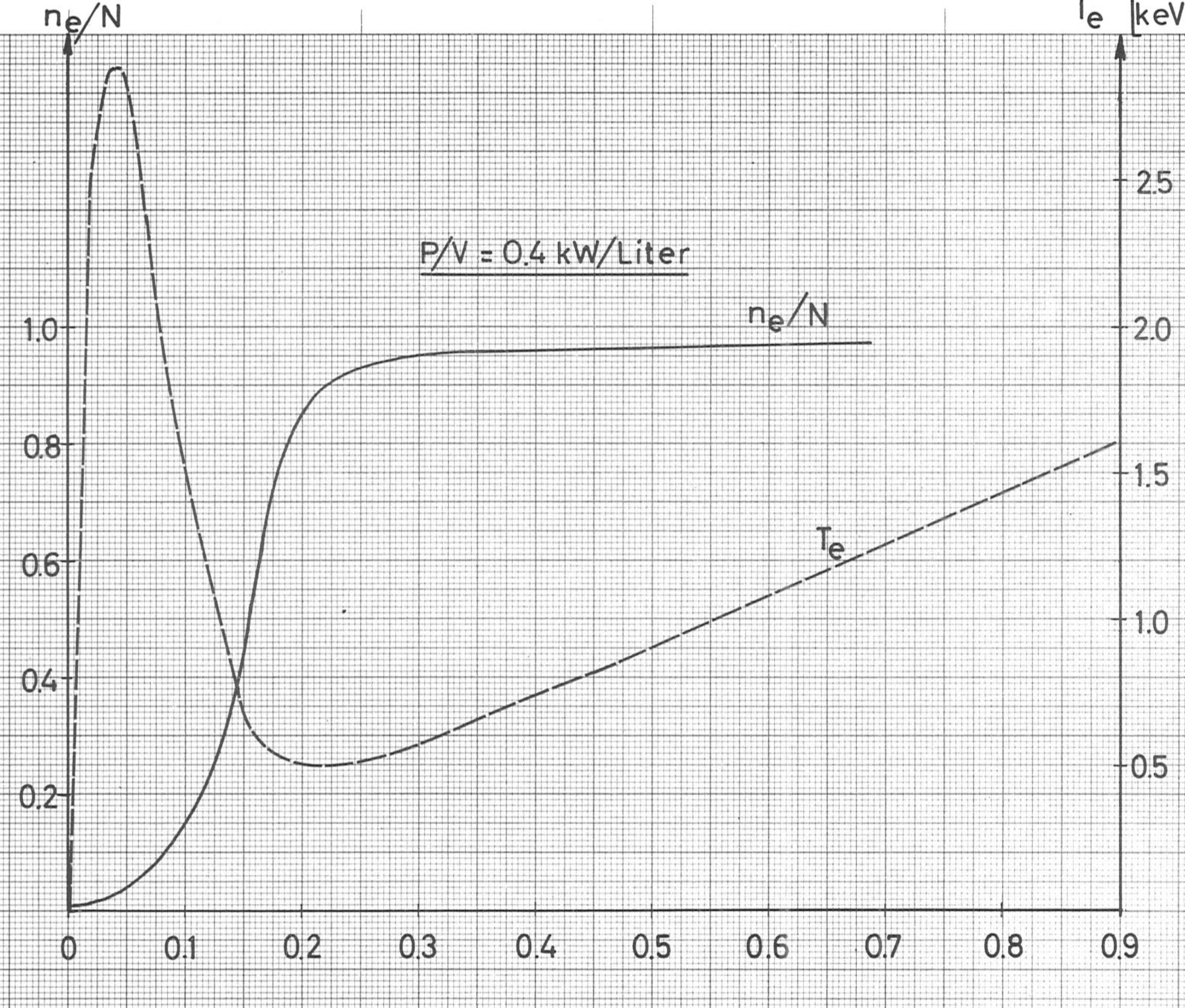
n_e/N
T_e [keV]
P/V= 0.2 kW/Liter
1.0
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
n_e/N
T_e
t [msec]

Fig. 1



n_e/N
T_e [keV]
P/V = 0.25 kW/Liter
n_e/N
T_e
1.0
0.8
0.6
0.4
0.2
2.0
1.5
1.0
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
t [msec]

Fig. 2



n_e/N
T_e [keV]
2.5
2.0
1.5
1.0
0.5
1.0
0.8
0.6
0.4
0.2
P/V = 0.4 kW/Liter
n_e/N
T_e
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
t [msec]

Fig. 3

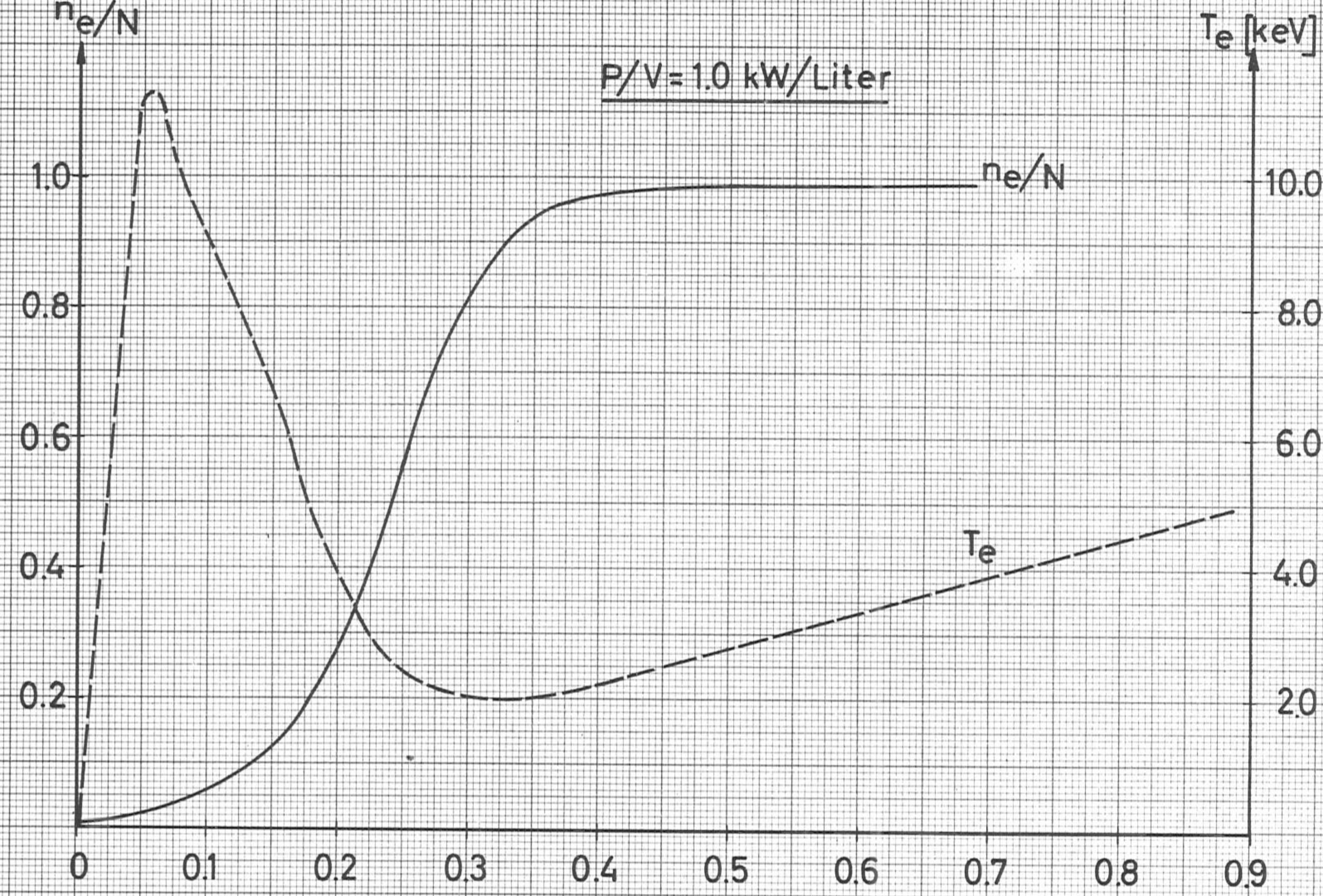


Fig. 4

A 4 210 x 297 mm

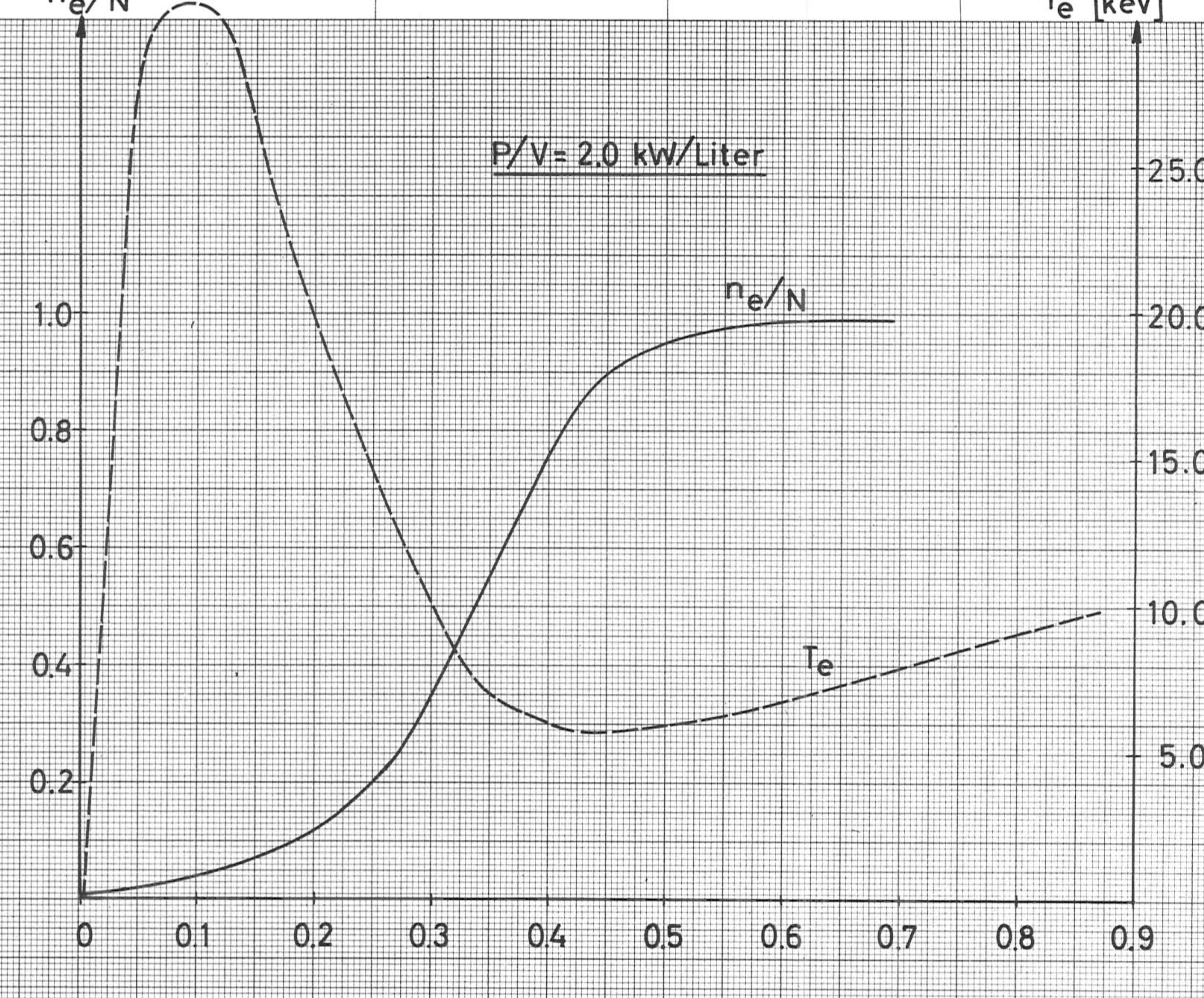
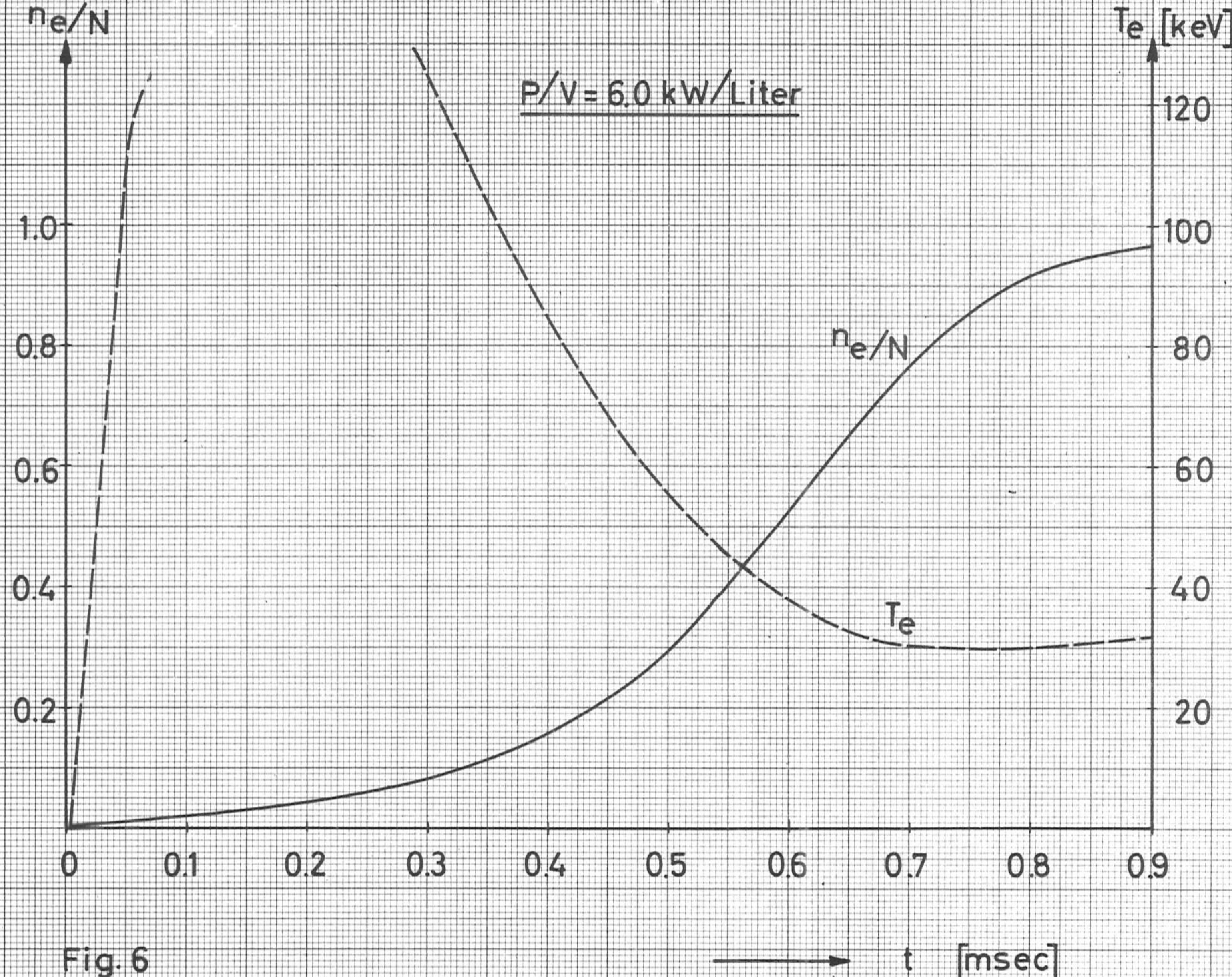


Fig. 5

A 4 210 x 297 mm



n_e/N
T_e [keV]
P/V = 6.0 kW/Liter
1.0
0.8
0.6
0.4
0.2
120
100
80
60
40
20
n_e/N
T_e
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
t [msec]

Fig. 6

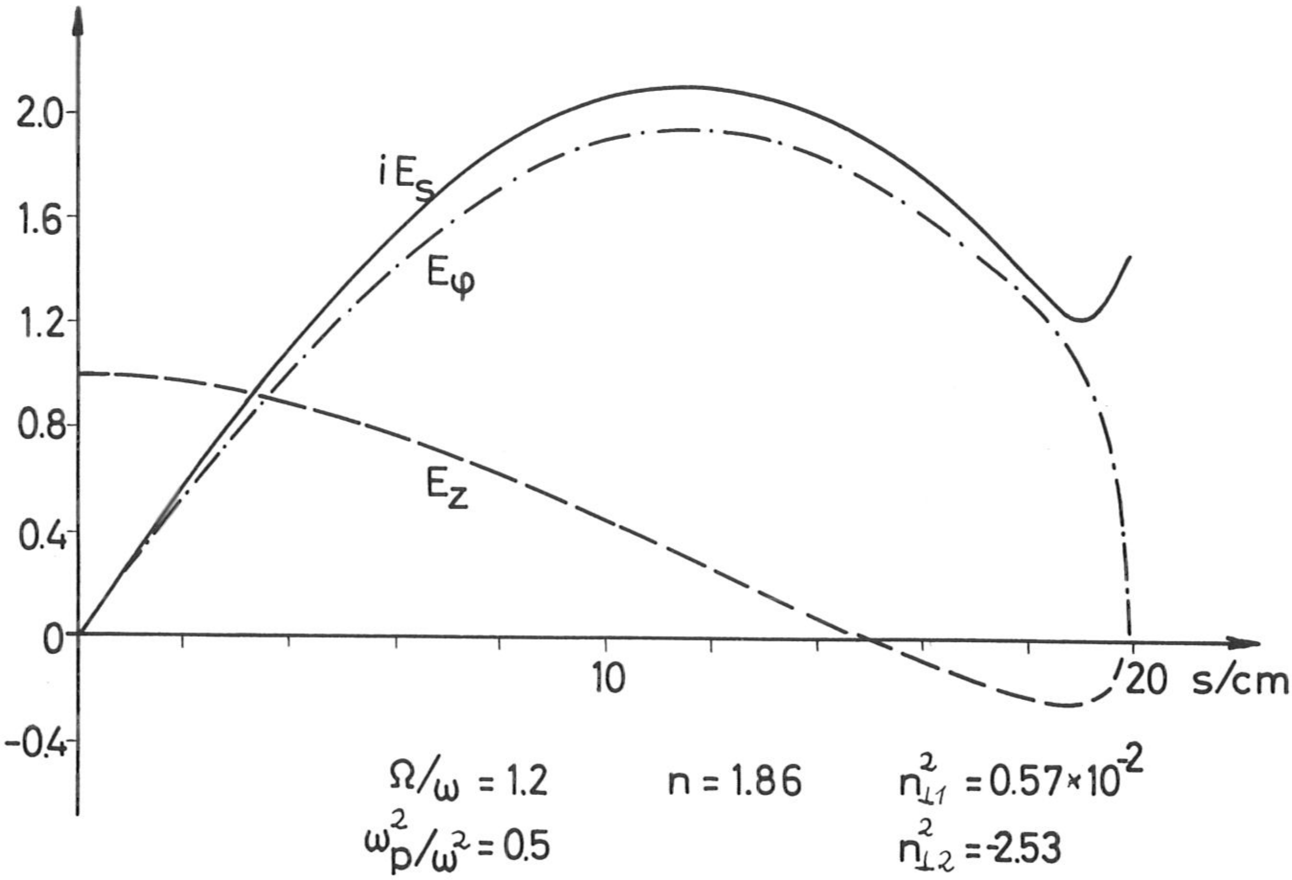

2.0
1.6
1.2
0.8
0.4
0
-0.4
10
20 s/cm
i Es
Eφ
Ez
Ω/ω = 1.2
n = 1.86
n²⊥1 = 0.57×10⁻²
ωp²/ω² = 0.5
n²⊥2 = -2.53

Fig.7

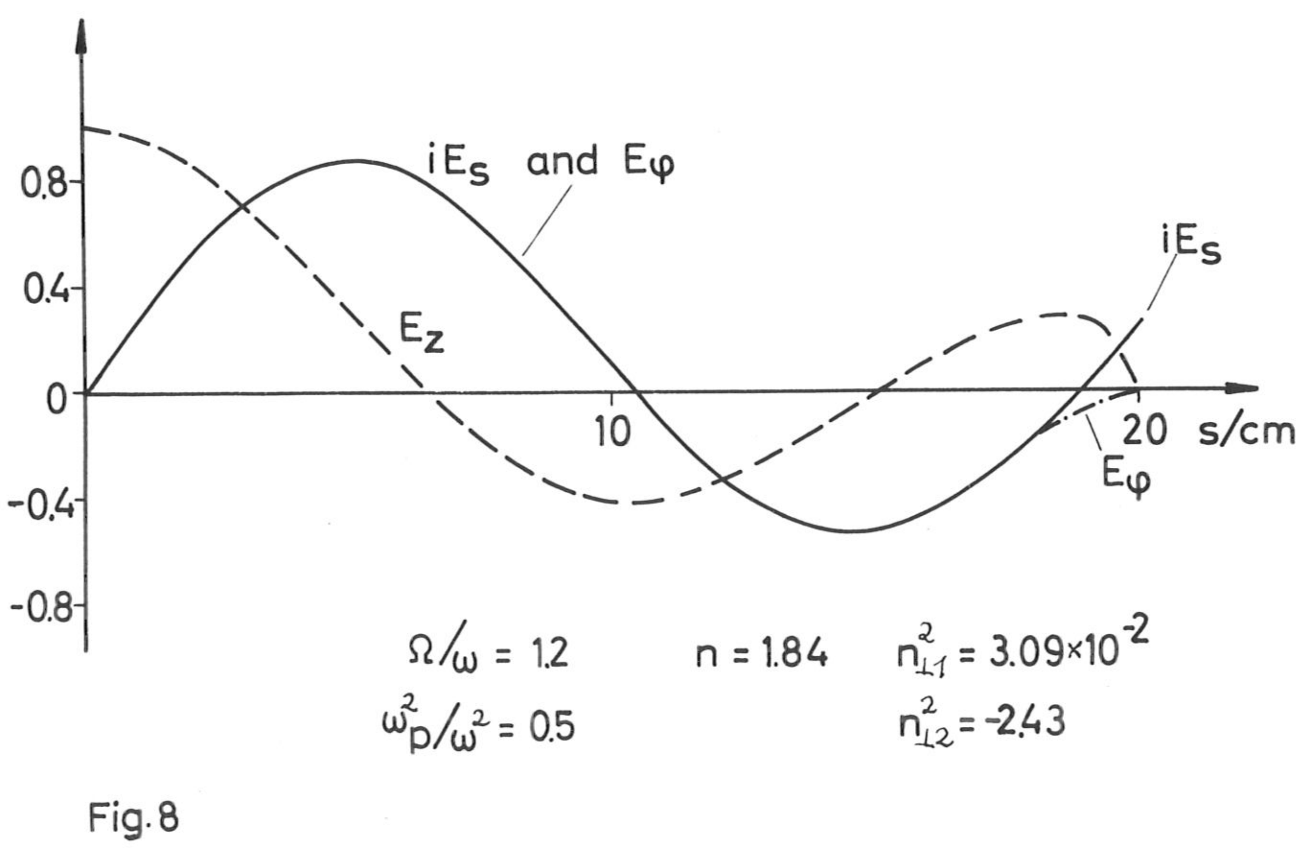

0.8
0.4
0
-0.4
-0.8
10
20 s/cm
iEs and Eφ
Ez
iEs
Eφ
Ω/ω = 1.2
n = 1.84
n²⊥1 = 3.09×10⁻²
ωp²/ω² = 0.5
n²⊥2 = -2.43

Fig.8

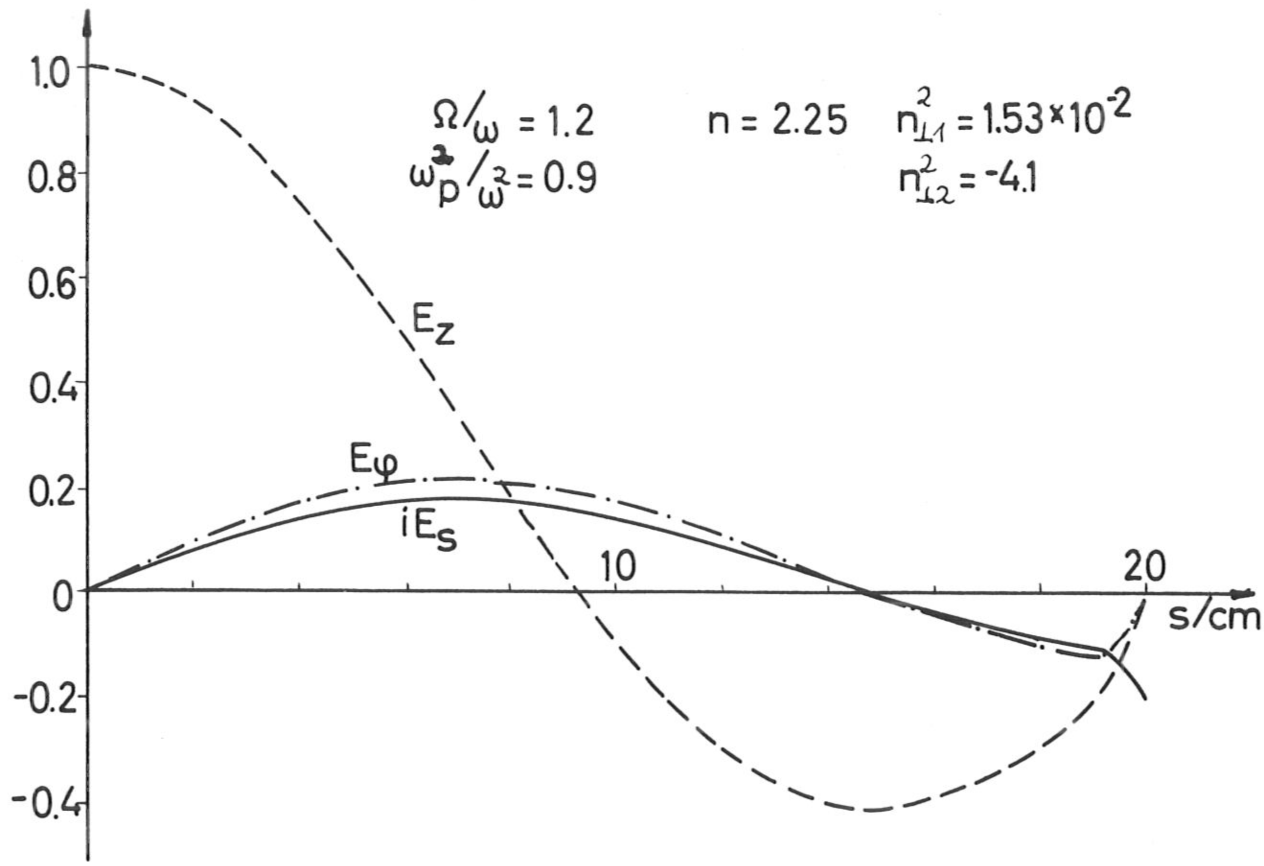

1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
Ω/ω = 1.2
ω²p/ω² = 0.9
n = 2.25
n²⊥1 = 1.53×10⁻²
n²⊥2 = -4.1
Ez
Eφ
iEs
10
20
s/cm

Fig. 9

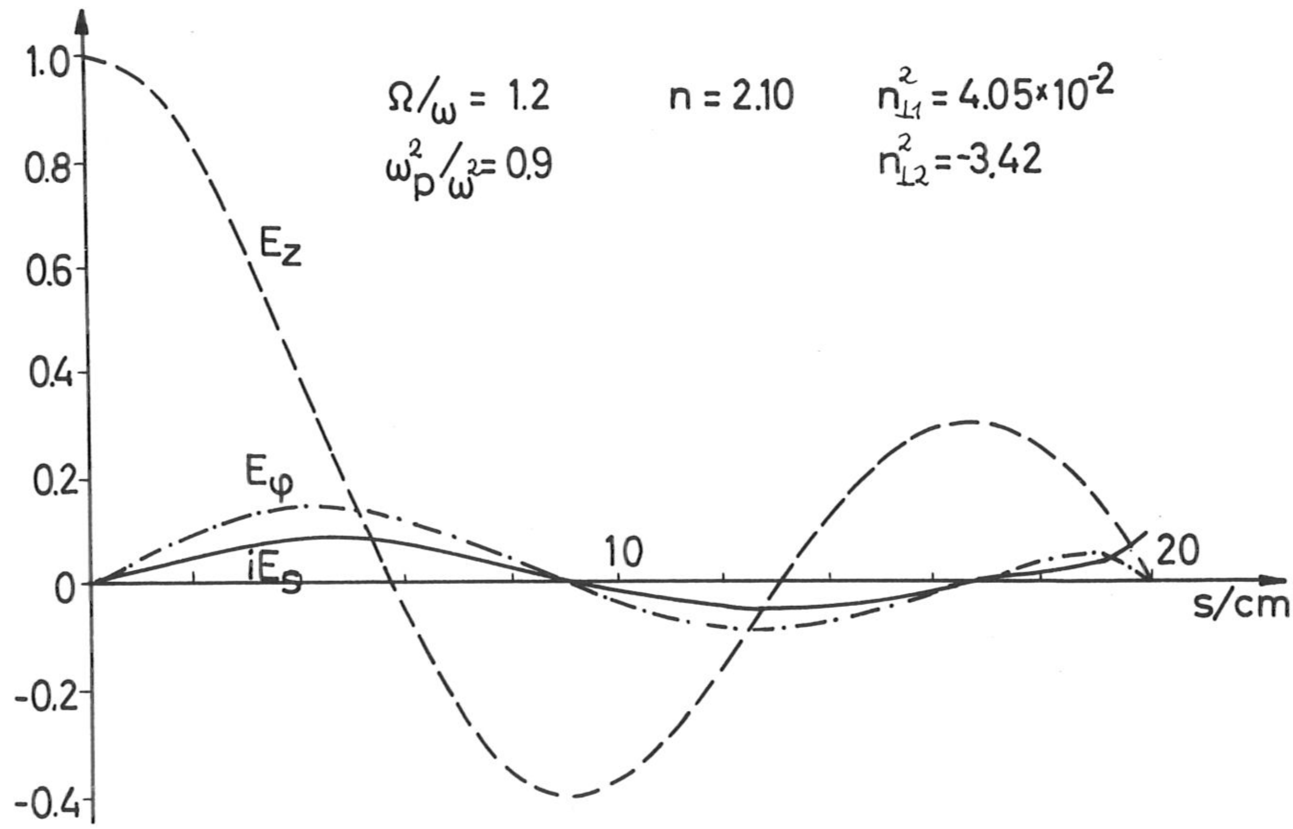

1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
Ω/ω = 1.2
ω²p/ω² = 0.9
n = 2.10
n²⊥1 = 4.05×10⁻²
n²⊥2 = -3.42
Ez
Eφ
iEs
10
20
s/cm

Fig. 10