Numerical Investigation of Collisionless Compression of a Plasma with Anomalous Friction

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Abstract

The effect of friction on the time development of the profile of a strong compression pulse propagating perpendicular to a magnetic field is studied in cylindrical geometry using two-fluid equations. The friction term roughly describes the action of a two-stream instability by limiting the electron velocity to a critical value. Depending on this critical value and a parameter δ (ratio of electron inertial skin depth to initial plasma radius) the steepening process during compression may or may not lead to a discontinuity. The time history of the profiles and conditions for the occurrence of the discontinuity are investigated for the case that no magnetic field is initially present in the plasma.

1. Introduction

The propagation of a strong disturbance perpendicular to a magnetic field in a plasma shows a nonlinear behaviour in two respects: in the course of propagation the profile of the disturbance will steepen and, under some conditions, reach a point where overtaking of fluid elements occurs[1-3]. On the other hand, already before overtaking occurs, the pulse may be unstable, thus giving rise to a turbulent dissipation of kinetic energy of directed motion [2,4-1o]. This latter process may smoothen the profile of the disturbance.

This report sets out to show the relationship between dissipation and macroscopic motion by numerical treatment of the fluid equations for a compressed plasma. It is assumed that ordered streaming of electrons in the pulse is dissipated by a two-stream instability. The nonlinear action of this instability is described only in a very crude and phenomenological way.

We were interested in the development in time of the compression dynamics, not merely in the final steady state, and so we solved the transient problem.

The compression geometry was that of a theta pinch, i.e. a cylindrical plasma in an axial magnetic field with a prescribed time varying value at the boundary of the plasma. Due to the finite extension of the plasma a geometric parameter

$$\delta = d/R_0 , \qquad d = c/\omega_{Pe}(n_0) \tag{1}$$

describing the ratio of electron inertial skin depth and initial plasma radius R_o enters the equations. Because

$$\delta^{-2} = \frac{4e^2}{m_e} n_o R_o^2 \pi = \frac{4e^2}{m_e} N = N / 8.87 \times 10^{11} ,$$

δ also measures the line density N $[cm^{-1}]$ of particles in the compressed cylinder. This parameter also can prevent the steepening process from reaching the overturning point: δ measures the diamagnetic screening of the magnetic field by the finite plasma. If δ is large there is little diamagnetism and the profile has no time to steepen to the critical point during the compression time. In fact, for very weak diamagnetism $\delta \gg 1$ one can find an analytic solution for the compression dynamics of a cold plasma without friction[11] which is homologous for all fluid elements of the cylindrical plasma and therefore, shows no steepening at all. One of the aims of the present numerical investigation was to study the influence of diamagnetism (δ^{-1}) on the steepening process.

2. Conditions of Investigation

The following work was restricted to the case where no initial magnetic field was present in the plasma when the external magnetic field B_e begins to rise from zero. The plasma was assumed to be cold at the beginning, i.e. $v_{th}(0) \ll (e/m_e)\,|E|\,t_o$ where $v_{th}(o)$ is the initial electron thermal velocity, E and t_o are respectively the induced electric field and a characteristic time to be defined later. (Because $E \approx \dot{B}_e d$ this condition is equivalent to $p(0) \ll (\dot{B}_e t_o)^2/8\pi$ where p(o) stands for the initial electron pressure.)

This case is in some respects singular, because for these conditions the Mach number in terms of steady state nonlinear wave theory[12-14] is very large and no steady state solution exists. On the other hand, this case is of special interest for experimentalists working with theta pinches because the acceleration of the plasma by the magnetic piston is most efficient.

As the Alfvén velocity cannot serve to define an appropriate time scale for our calculations, we choose the quantity

$$t_o = \dot{\omega}_h^{-1/2} , \qquad (2)$$

where

$$\dot{\omega}_h = e \dot{B}_e (m_i m_e)^{-1/2}$$

is the time derivative of the lower hybrid frequency formed with the initial time derivative of the external magnetic field. (Scaling factors for other quantities are shown in sections 3 and 4.)

It was assumed for the calculations that electrons suffered no collisions with ions. This can be achieved by making the rate of change of the external magnetic field so high that the electric field $E^* = E - uB$ which acts on the electrons compressed with velocity u is large compared with the critical runaway field E_c[15] except in a small time interval where E^* changes sign. Collisions of electrons and ions with neutrals were not taken into account either.

The initial density n_o of the plasma was assumed to be high enough to preserve quasineutrality during compression. This condition is fulfilled if the electron density is capable of building up an electric space charge field by charge separation big enough to drag the ions towards the axis in the characteristic time t_o.

As will be shown in section 3 the condition for quasineutrality can be expressed by

$$(m_i / m_e)\ \dot{\omega}_h / \omega_{Pe}^2 \ll 1 \qquad (3)$$

where the electron plasma frequency ω_{Pe} contains the initial density n_o.

In all calculations the plasma was assumed to fill homogeneously a cylinder of radius R_o and infinite length with density n_o. The magnetic field outside the cylinder was assumed to rise with time at a constant rate $\dot{B}_e$. The initial radial and azimuthal velocities u and v are assumed to be zero.

3. Compression without Friction. Scaling Relations

Compression dynamics for a cold plasma without friction in plane geometry has been studied for a finite initial magnetic field by Adlam and Allen [1] using a fluid model and for plane and cylindrical geometry by Auer, Hurwitz and Kilb [2,3] and Kilb [16] using a sheet model. To supplement the work of these authors we studied compression without inital magnetic field in a cylindrical geometry.

The finite initial radius of the plasma column introduces a parameter δ , which was defined in eq. (1), into the system of fluid equations. For the initial and boundary conditions described in the preceeding section, δ can be made the only parameter in the equations for compression dynamics by choice of appropriate normalising units. These units were chosen as follows:

Length: initial plasma radius R_o

Time: $t_o = \dot{\omega}_h^{-1/2}$ (see eq. (2))

Density: initial density n_o

Radial velocity: $u_o = R_o/t_o$

Azimuthal velocity: $v_o = (m_i/m_e)^{1/2}\, R_o/t_o$

Magnetic field: $B_o = \dot{B}_e \cdot t_o = (m_i/m_e)^{1/2}/(e t_o)$

where e is the electron charge, m_e and m_i are electron and ion mass respectively.

Electric field: $E_o = u_o\, B_o$

Fluid equations are expressed in Lagrange coordinates x and t, where

$$x = r_o^2/2$$

and r_o is the normalized initial distance of a fluid element from the axis. Using a dash for differentiation in respect of x and a point for a time derivation and omitting an indication for normalized quanities these equations are:

$$n\, r\, r' = 1 \quad (4)$$

$$\dot{r} = u \quad (5)$$

$$\dot{u} = \frac{v^2}{r} - vB \quad (6)$$

$$\dot{v} = -\frac{uv}{r} - E + uB \quad (7)$$

$$rB' = \delta^{-2} v \quad (8)$$

$$n(rE)' = -\dot{B} + r\, n\, u\, B'. \quad (9)$$

Two length scales could have been used for normalization purposes, i.e. d and R_o. We used the latter for sake of better comparison of cases with different δ . Because of this choice, not all quantities are limited to order 1: for $\delta \gg 1$, B' rises to order δ^{-2}, for $\delta \ll 1$, E is of order δ and v rises to order δ , and B' rises to order δ^{-1}.

Also two time scales are inherent to the problem: the one used for normalization here is essentially the acceleration time or overtaking time described below, the other is the so called pinch time[17] for complete compression of the plasma. This latter time is for $\delta \gg 1$ of the same order as the former as is seen from eq. (15). For $\delta \ll 1$ it exceeds the overtaking time by a factor of $\delta^{-1/2}$ and than falls beyond the scope of the description used here.

The length and time scales define the unit of u. v_o is larger by the square root of mass ratio because only electrons are involved in azimuthal motion. The unit of magnetic field is chosen in a way that

$$B_o/t_o = \dot{B}_e,$$

the prescribed rate of change of the outer magnetic field, so that the dimensionless boundary condition is

$$B_e = t. \tag{10}$$

Using the dimensionless eqs. (4) to (9) the following schematic picture for the time development of compression may be drawn:
At the beginning electrons and ions are at rest. As soon as the external magnetic field begins to rise the magnetic field penetrates the plasma in a skin sheath of thickness δ without retardation because displacement currents were neglected. This can be seen from eqs. (7), (8) and (9) for u = 0. From eq. (9) it is seen that E has (in dimensionless units) a value of the order δ^k (k = 1 for $\delta \ll 1$ and 0 for $\delta \gg 1$) at the beginning. Now electrons and ions are accelerated in the electromagnetic field. Electrons gain their velocity from the induced electric field according to

$$v \approx -E\, t \approx \delta^k\, t. \tag{11}$$

Ions are accelerated in the radial direction by a space charge field E_r, which transmits the Lorentz force vB acting on the electrons to the ions. From eqs. (6) and (11)

$$\dot{u} \approx -vB \approx -\delta^k t^2. \tag{12}$$

Since vB, and hence the space charge field, is largest at the outer edge of the plasma and falls off after a distance δ^k from the rim, the marginal fluid element will after a time t_d overtake its predecessor, thus raising the density at the edge to infinity. Overtaking will occur approximately when

$$|\dot{u}(t_1)|\, t_1^2 \approx \delta^k.$$

Thus the dimensionless overtaking time t_1 is of order:

$$t_1 \approx 1$$

and their real value is

$$t_d = t_1 \ t_o \approx \dot{\omega}_h^{-1/2} \qquad (13)$$

independent of δ.
Space charge field E_r (normalized by $v_o B_o$) rises until the overtaking time to the order

$$E_r \approx -\delta^k \ t_1^2 \approx -\delta^k \qquad (14)$$

varying over a sheath δ^k. From this the condition of quasi-neutrality (3) may be derived.

Finally the dimensionless pinch time t_2 for $\delta \ll 1$ may be defined by a condition that the ions by acceleration in the sheath must gain a velocity of order

$$|u\ (t_2)| \approx 1/t_2 \ .$$

From eq. (6)

$$u^2 \approx vB \ .$$

v is given by eq. (8) $\quad v \approx \delta^2 B/\delta.$

Thus $\qquad |u| \approx \delta B$

and $\qquad t_2 \approx \delta^{-1/2}$

and its real value

$$t_c = t_2 t_o \approx \delta^{-1/2} \ \dot{\omega}_h^{-1/2} \ . \qquad (15)$$

For $\delta \gg 1$

$$t_2 \approx t_1$$

and

$$t_c \approx \dot{\omega}_h^{-1/2} .$$

To investigate the time development of the overtaking process the partial differential equations for the compression (4) to (9) were solved numerically. A net of staggered mesh of 50 points distributed continuously in x was used, first solving the implicit equation for the magnetic field and then the equations for the radius and radial velocity simultaneously. The programme could be checked against the analytical solution found earlier for $\delta \gg 1$[11].

Profiles of density and magnetic field at different times are shown in Fig. 1 for $\delta = 0.2$. For $t_d = 1.80 \, \dot{\omega}_h^{-1/2}$ overtaking occurs, and n goes to infinity. Also the Eulerian derivation $\frac{\partial u}{\partial r}$ becomes infinite while $\frac{\partial v}{\partial r}$ stays finite (Fig. 2). After the time t_d the velocities will become double-valued as function of r (or the field quantities double-valued in x) which was treated not yet. We found overtaking for all the investigated δ^{-2} between 0.2 and 1000. Fig. 8 shows overtaking times t_d and the plasma radius at this time. The overtaking time t_d scales with $\dot{\omega}_h^{-1/2}$ and is only a very weak function of δ. For $\delta \gg 1$, t_d tends towards the compression time of a completely non-diamagnetic plasma; i.e., $t_d = 2.82 \, \dot{\omega}_h^{-1/2}$[11].

4. Compression with Friction

The speepening process discussed in the preceding section is of course essentially altered if any dissipation process transforms energy of ordered motion into internal energy. Sagdeev[4] and Auer, Hurwitz and Kilb[2] first pointed out that two-stream instabilities excited by the large velocities of electrons relative to the ions in the front of a compression wave may constitute such a dissipation mechanism.

Because the minimum relative velocity for the onset of two-stream instabilities (depending on the ratio of ion to electron temperature) is always smaller than the electron thermal velocity v_{th}, a sufficient condition for the onset of these instabilities during compression is according to (11)

$$v_{Max} \approx v_o \delta^k \gg v_{th}(0) \tag{16}$$

where $v_{th}(0)$ is the initial electron thermal velocity.

The instability is not affected by the magnetic field as long as[2]

$$\omega_{Pe} \gg \omega_{ce} = e B / m_e \tag{17}$$

which is already guaranteed by condition (3). The maximum growth rate is assumed to be at least an order of magnitude larger than the reciprocal characteristic time $1/t_o$ to ensure that the development of the instability can keep up with the compression.

The description of the dissipation in terms of fluid quantities, which is adopted here, is only very crude. It was originally introduced by Adlam and Holmes[18] for the estimation of the skin depth of an unstable layer and used later also by Bardotti, Cavaliere, and Engelmann[19] for numerical calculations of steadystate shock profiles. (A similar friction term in steady-state shock calculations was used by Tverskoi[2o], shock profiles with continuously acting friction were studied by Morawetz[21] and Sagdeev[5]).

It is assumed that there exists a critical velocity

$$v_c = \beta \, v_{th} \qquad (18)$$

where v_{th} is the r.m.s velocity of unordered motion of electrons and β is a fixed parameter. If the azimuthal velocity of the electrons v is smaller than the critical velocity the friction force g is assumed zero:

$$g = 0 \quad \text{if} \quad |v| < v_c. \qquad (19)$$

As the velocity of the electrons tends to exceed the critical velocity the friction force sets in and forces the electron velocity to stay just at the critical limit. Thus, as long as

$$(v\,\dot{v})_{g=0} - (v_c\,\dot{v}_c)_{g=0} > 0 \qquad (2o)$$

which is equivalent to

$$g\,v > 0$$

the magnitude of v shall be

$$|v| \equiv v_c.$$

On the other hand, it is assumed that the work done by the electrons against the friction force g is put locally into their internal energy $\mathcal{E}_{th}$ with an efficiency α (eq. 23), thus raising the critical limit of the electron velocity by raising their velocity of unordered motion.

On this assumption the friction force in dimensionless form is given by:

$$g = -\frac{\gamma}{1+\gamma}\left[E - uB + \frac{uv}{r} - \frac{1}{2}\, n\, v\, (ru)'\right]. \qquad (21)$$

$$\gamma = 1/(\alpha \beta^2).$$

For this value of g it is assumed that the work against friction force raises internal energy only in two degrees of freedom perpendicular to the magnetic field, $\mathcal{E}_{th} = p$, and that v_{th} is connected with $\mathcal{E}_{th}$ by

$$\mathcal{E}_{th} = n\, m_e\, v_{th}^2 / 2.$$

Some arguments for this treatment of anomalous friction were given by Bardotti, Cavaliere and Engelmann[19] on the basis of the quasilinear theory[22,23] of the two-stream instability. It can be shown for our case that the energy density $\mathcal{E}_{fl}$ of the fluctuating electric field necessary to produce the macroscopic friction force g by diffusion of electrons in the velocity space towards lower velocities is of the order[19]

$$\mathcal{E}_{fl} \approx \gamma\, (\omega_{Pe}\, t_o)^{-1}\, \mathcal{E}_{th}. \qquad (22)$$

Thus for $\alpha \approx \beta \approx 1$ and $t_o \gg \omega_{Pe}^{-1}$ this is consistent with the assumption of weak turbulence.

In order to use a fluid description given below it must be guaranteed that the energy dissipated by the instability is fed into the plasma locally and that the dissipation process can keep up with the formation of the pulse; i.e., dissipation length and time must be small compared to characteristic values d and t_o respectively. Besides this it must be sure that electrons may not transport energy gained by dissipation or by compression over distances comparable to d by free flight. Because no collisions are allowed electrons are bounded only by magnetic field, thus electron gyration radius r_{ge} should be smaller than the characteristic length in the compression pulse.

Introduction of the friction term alters the choice of normalizing units. Instead of t_o of eq. (2) it should be used

$$t_o' = (1+\gamma)^{1/4} \dot{\omega}_h^{-1/2} \qquad (2')$$

as time scale now.

Other units are now
Length: again R_o
Radial velocity: $u_o' = R_o/t_o'$
Azimuthal velocity: $v_o' = (1 + \gamma)^{-1/2} (m_i/m_e)^{1/2} R_o/t_o'$
Deceleration by friction: $g_o = v_o' / t_o'$
Electron pressure: $p_o = n_o m_i R_o^2/t_o'^2$
Magnetic field: $B_o = \dot{B}_e t_o'$.

Using this normalization and the friction term of eq. (21) the following equations are obtained besides the unchanged eqs. (4), (5) and (9):

$$\dot{u} = (1+\gamma)^{-1} \frac{v^2}{r} - vB - rp' \qquad (6')$$

$$\dot{v} = -\frac{uv}{r} - E + uB - \frac{1}{2}\frac{\gamma}{\gamma+1} nv (ru)' \qquad (7')$$

$$rB' = (1+\gamma)^{-1} \cdot \delta^{-2} \cdot v \tag{8'}$$

$$\dot{p} = -2\,p\,n\,(r\,u)' + \alpha \frac{\gamma}{\gamma+1}\,n\,v\left[\dot{v} + \frac{1}{2}\,n\,v\,(r\,u)'\right] \tag{23}$$

again $\quad B_e = t$

at the boundary.

The characteristic sheath thickness is broadened by friction to

$$\delta' = (1+\gamma)^{1/2}\,\delta \tag{1'}$$

and the induced electric field enlarged to

$$E \approx \delta'^{\,k} \qquad \begin{matrix} k = 1 \\ k = 0 \end{matrix} \quad \text{for} \quad \begin{matrix} \delta' \ll 1 \\ \delta' \gg 1 \end{matrix}$$

thus

$$v \approx \delta'^{\,k}\,t .$$

An estimation for overturning time t_1' may be derived analogously as in section 3 by the condition

$$|u(t_1')|\,t_1'^2 \approx \delta'^{\,k}$$

from which

$$t_1' \approx 1$$

and

$$t_d' \approx t_0' = (1+\gamma)^{1/4}\,\dot{\omega}_h^{-1/2} . \tag{13'}$$

Likewise the dimensionless pinch time is derived to

$$t_2' \approx \delta'^{-\frac{k}{2}}$$

and

$$t_c' \approx \delta^{-\frac{k}{2}} \dot{\omega}_h^{-\frac{1}{2}} \cdot (1+\gamma)^{(1-k)/4} \qquad (15')$$

independent of γ for $\delta' \ll 1$.
As is seen from (6') to (23) for $\gamma \gg 1$ the whole set of equations (and, for $\alpha = 1$, also the conditions for onset and stop of friction) do not depend from γ and δ each but only from the combination $\gamma\delta^2$. Therefore, in cases where the friction force is switched on at the beginning already, the behaviour of the compression pulse during the action of friction should be the same for all cases with equal $\gamma\delta^2$.

The introduction of the friction force into the fluid equations causes a finite electron temperature. This considerably alters the character of the profiles, as was shown for a stready-state plasma by Hain, Lüst and Schlüter[24], by Banos and Vernon[25], by Bergold[26] and in addition for the piston problem by Morton[27].

Figs. 3 and 4 show the time behaviour of density and magnetic field for two typical cases, choosing the same friction parameter $\gamma = 1$ but different values of initial density (i.e. different δ). For low density the discontinuity in the case without friction is removed. Curves show fairly smooth behaviour (Fig. 3): the density pulse does not steepen until just before compression maximum, magnetic field is nearly homogeneous except for the last curve.

In contrary to this behaviour, for higher initial density (the same as in Fig. 1) and thus stronger diamagnetism of the plasma (Fig. 4) the density pulse has enough time to develop a discontinuity, i.e. a peak with infinite steep inner flank but finite height. As was the case for $\gamma = 0$ magnetic field and azimuthal electron velocity stay steady but radial velocity and electron temperature exhibit a discontinuity in their derivatives (Fig. 5).

Of course the mechanism for steepening is the same as in the case of no friction: outer fluid elements are pushed inward stronger due to diamagnetism of the plasma but while the density of a cold plasma without friction rises infinitely at the outer edge (Fig. 1) the density of a plasma with finite temperature cannot, because the rising pressure gradient pushes the plasma away from the peak toward the axis as well as toward the periphery and prohibits overtaking of fluid elements. As the density maximum moves inward the inner flank of the density pulse becomes steeper than the outer one and finally may peak in a sharp discontinuity of finite height. Calculations then break down because no artificial viscosity was used.

Of course the development of the discontinuity in the pronounced form of the last curve in Fig. 4 is not physically because the condition for application of the fluid description mentioned before; i.e., electron gyro-radius small against characteristic length in the pulse, is violated already earlier. Fig. 4 shows the size of the gyro-radius r_{ge} for conditions in the peak. After the applicability of fluid description ceases, only a microscopic treatment of the warm plasma may proceed further.

Finally calculations were done also for a larger friction force, i.e. $\gamma = 25$. Fig. 6 shows the time development of profiles for the same initial density as Fig. 4 but the same $\gamma \delta^2$ as Fig. 3. In this case magnetic field moves in relatively faster than in the case of Fig. 3. Thus the magnetic field at the axis forms an obstacle which the plasma must stream against, forming a two-step distribution of density (for t = 4.2 and $4.6 \dot{\omega}_h^{-1/2}$) which resembles a bow shock.

Fig. 7, for four times the initial density of Fig. 6, shows profiles immediately after turn-over from continuous to discontinuous compression. Here also the magnetic field reveals peaked distribution for $t = 4.8\ \omega_h^{-1/2}$. For still higher densities the time behaviour of the compression pulse is similar to that of Fig. 4 in a stretched time scale: the density peak is built up in a magnetic field which decreases monotonically towards the axis.

Discontinuities in the course of compression were found for:

$$\gamma = 1 \qquad \delta^{-2} \gtrsim 2$$
$$\gamma = 25 \qquad \delta^{-2} \gtrsim 75.$$

Fig. 8 shows dimensionless time and radius at which discontinuities appear. Curves of overturning time t_d seem to show a flat minimum.

5. Conclusions

The preceding calculations are unsatisfactory because the friction term used here is founded by estimations[19] only, and, for this regime of electric field strength, not by exact calculations. But if γ shows up to be an appropriate quantity, comparison of calculated profiles with measured ones should give some indication on critical velocities at turbulent dissipation.

Another interesting point would be to study the connection between occurence of a discontinuity during compression and ion heating. The overturning of the compression pulse must constitute a strong mechanism for ion heating and, because occurence of the former depends on the strength of electron dissipation, both are coupled.

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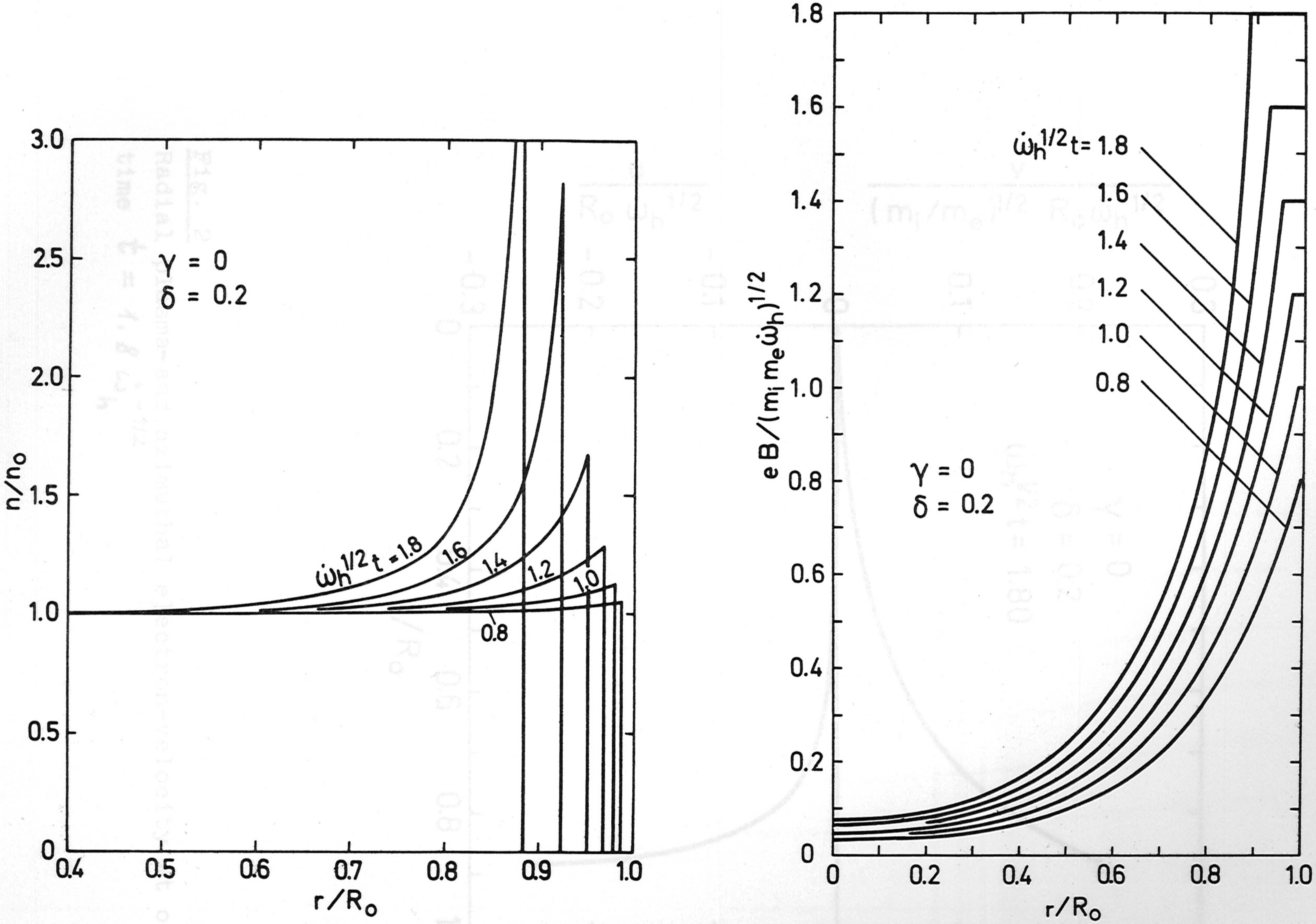


Fig. 1 a,b

Development of the profile of density and magnetic field in time without initial magnetic field and without friction. The magnetic field at the plasma boundary rises linearly with time. Times are normalized by the time derivative of the lower hybrid frequency $\dot{\omega}_h$

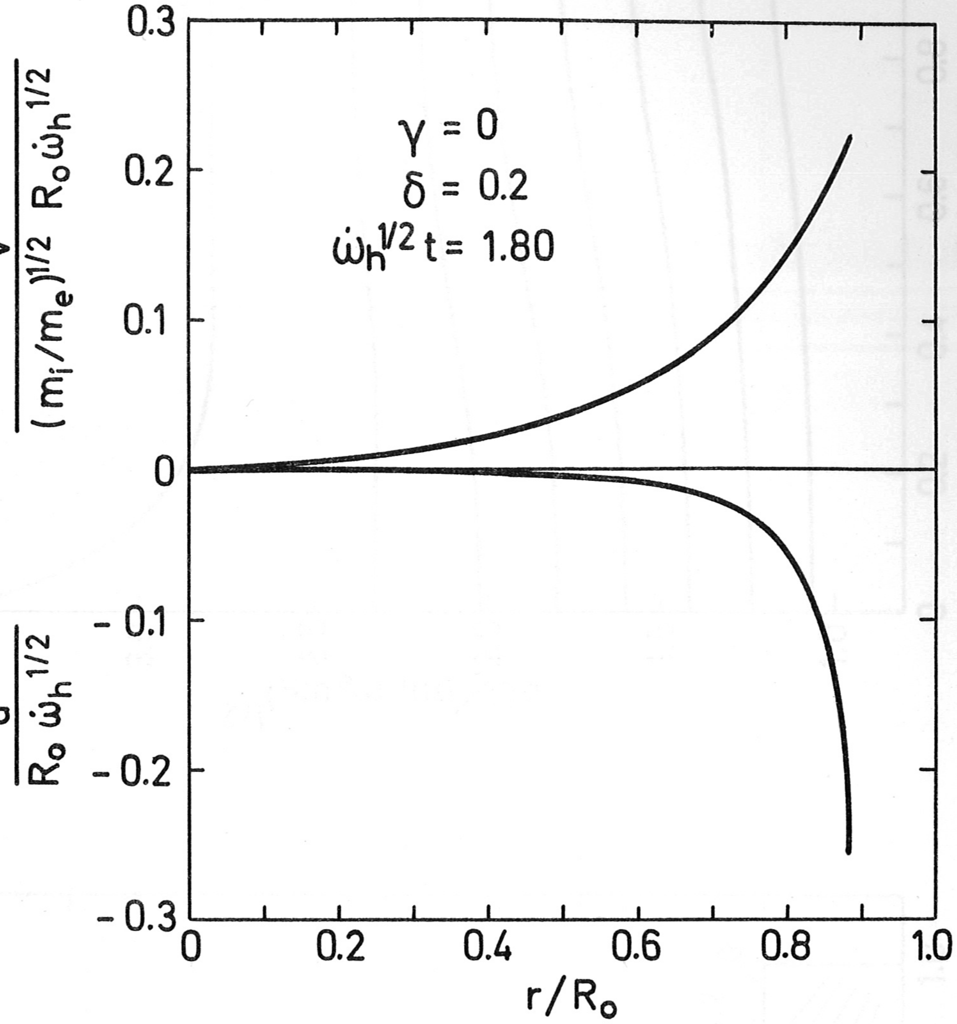


Fig. 2

Radial plasma- and azimuthal electron-velocity at overtaking time $t = 1,8\ \dot{\omega}_h^{-1/2}$

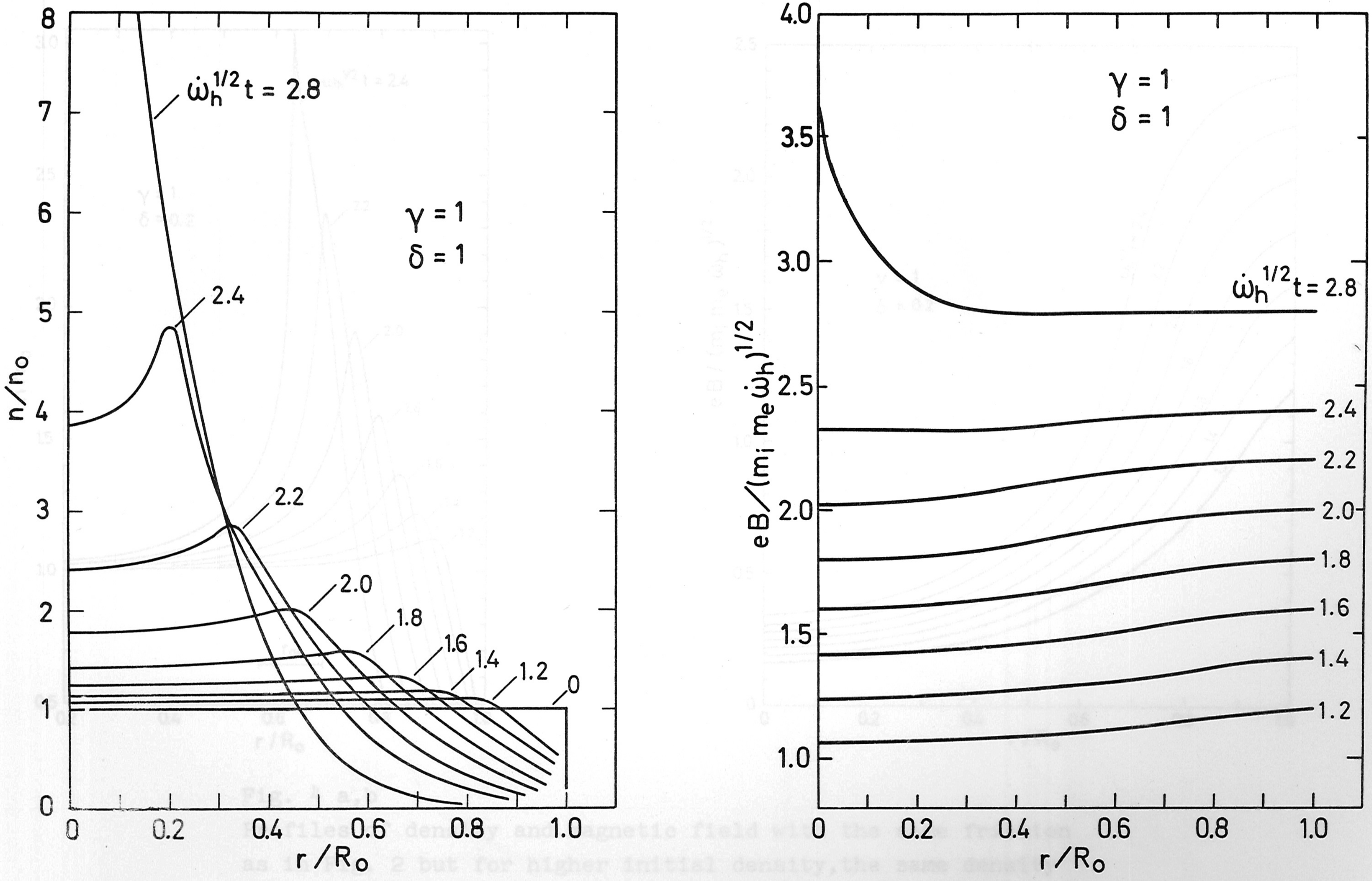
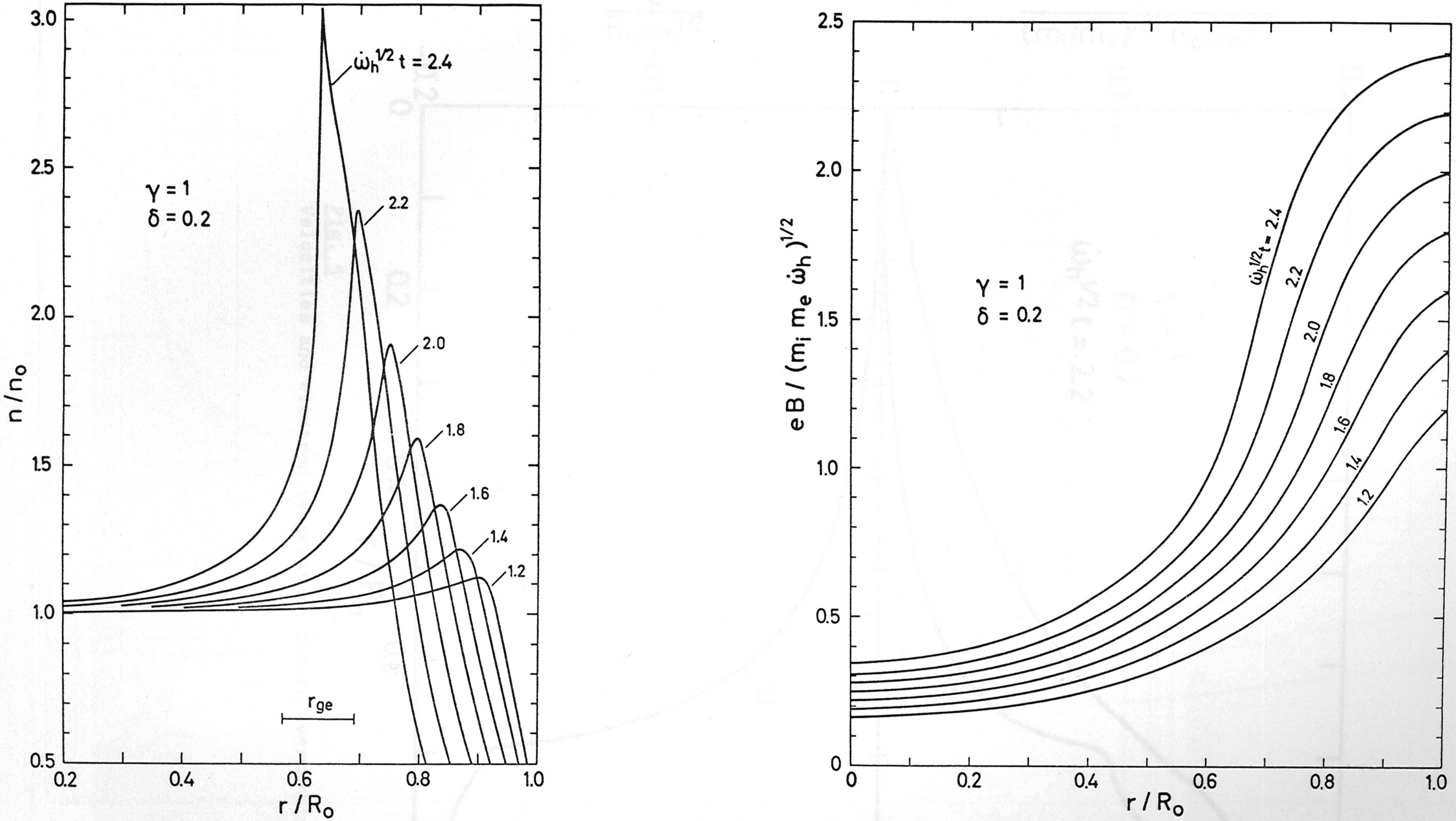


Fig. 3 a,b

Profiles of density and magnetic field with friction for low initial density.



<u>Fig. 4 a,b</u>

Profiles of density and magnetic field with the same friction as in Fig. 2 but for higher initial density, the same density as in Fig. 1.

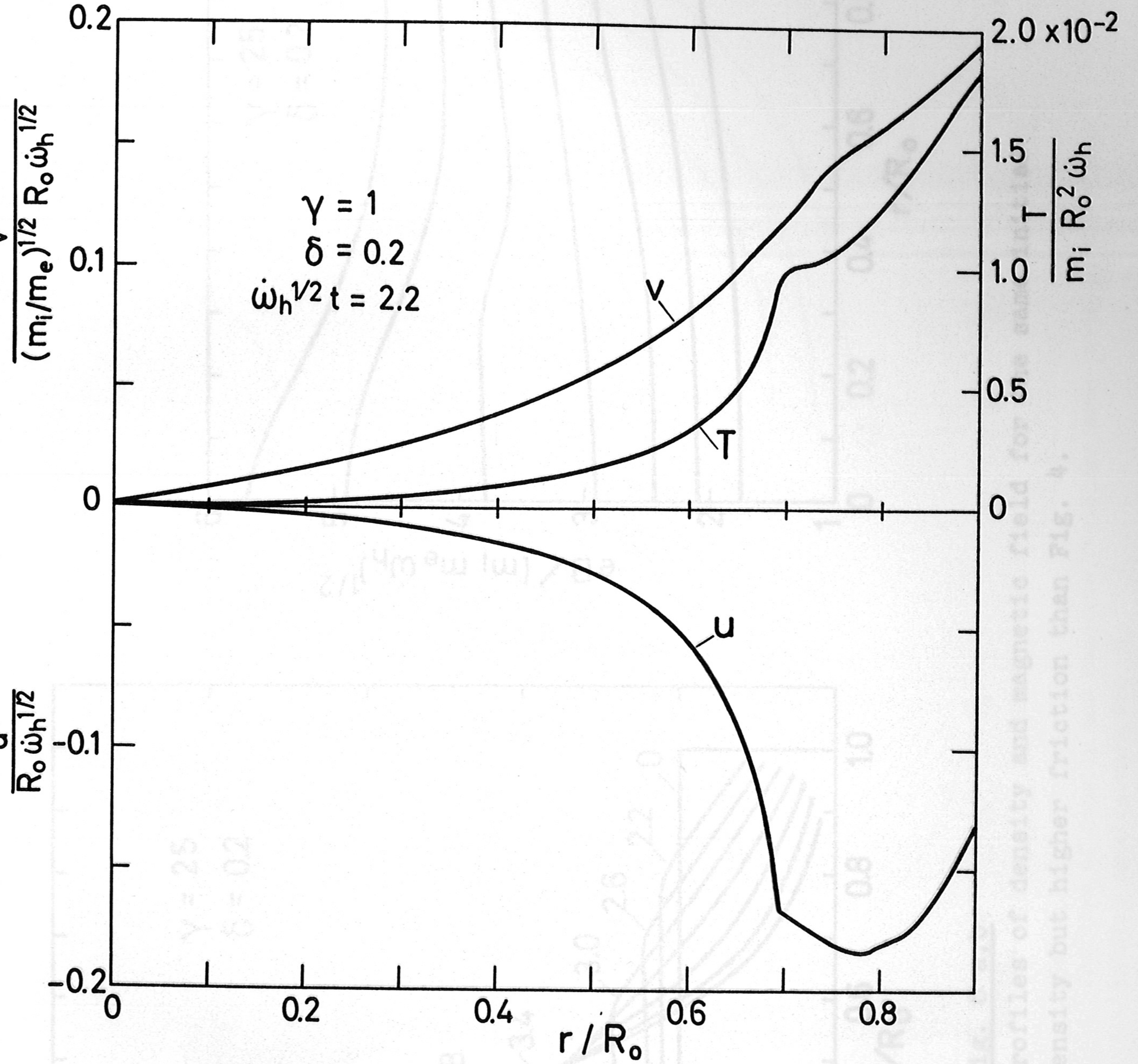


Fig. 5

Velocities and electron temperature before overturning.

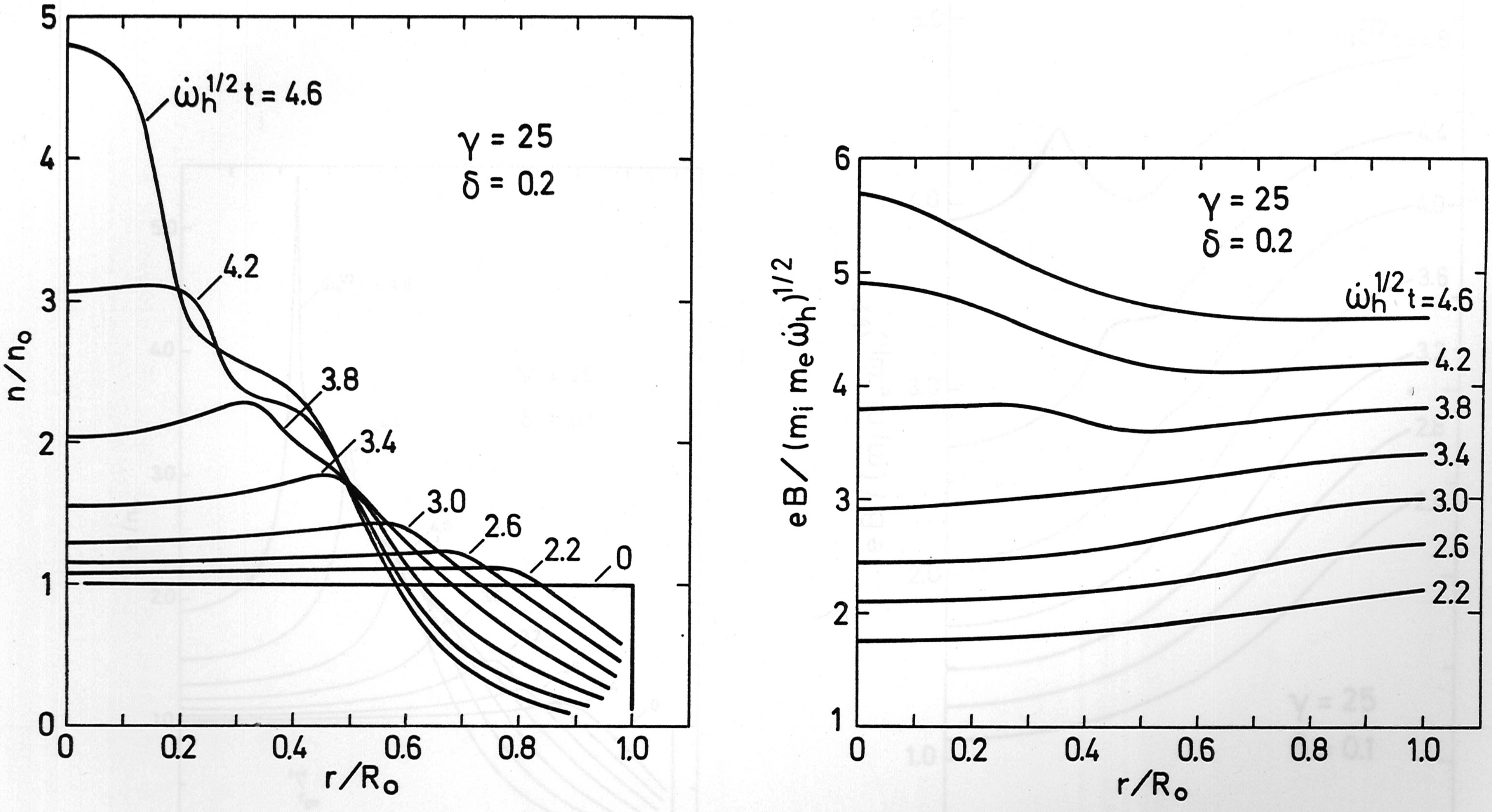


Fig. 6 a,b

Profiles of density and magnetic field for the same initial density but higher friction than Fig. 4.

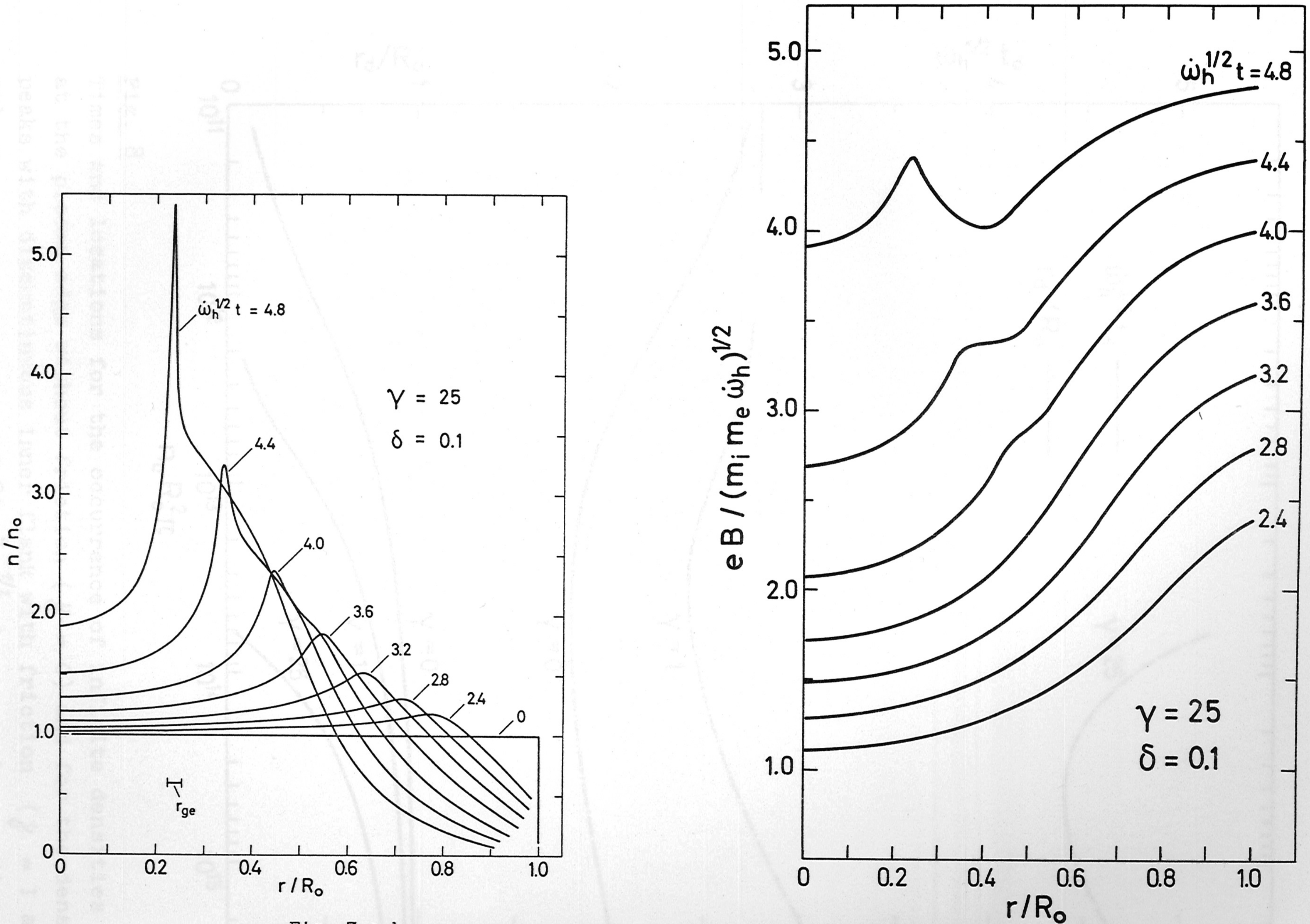


Fig. 7 a,b

Profiles of density and magnetic field for same friction but higher initial density than Fig. 6.

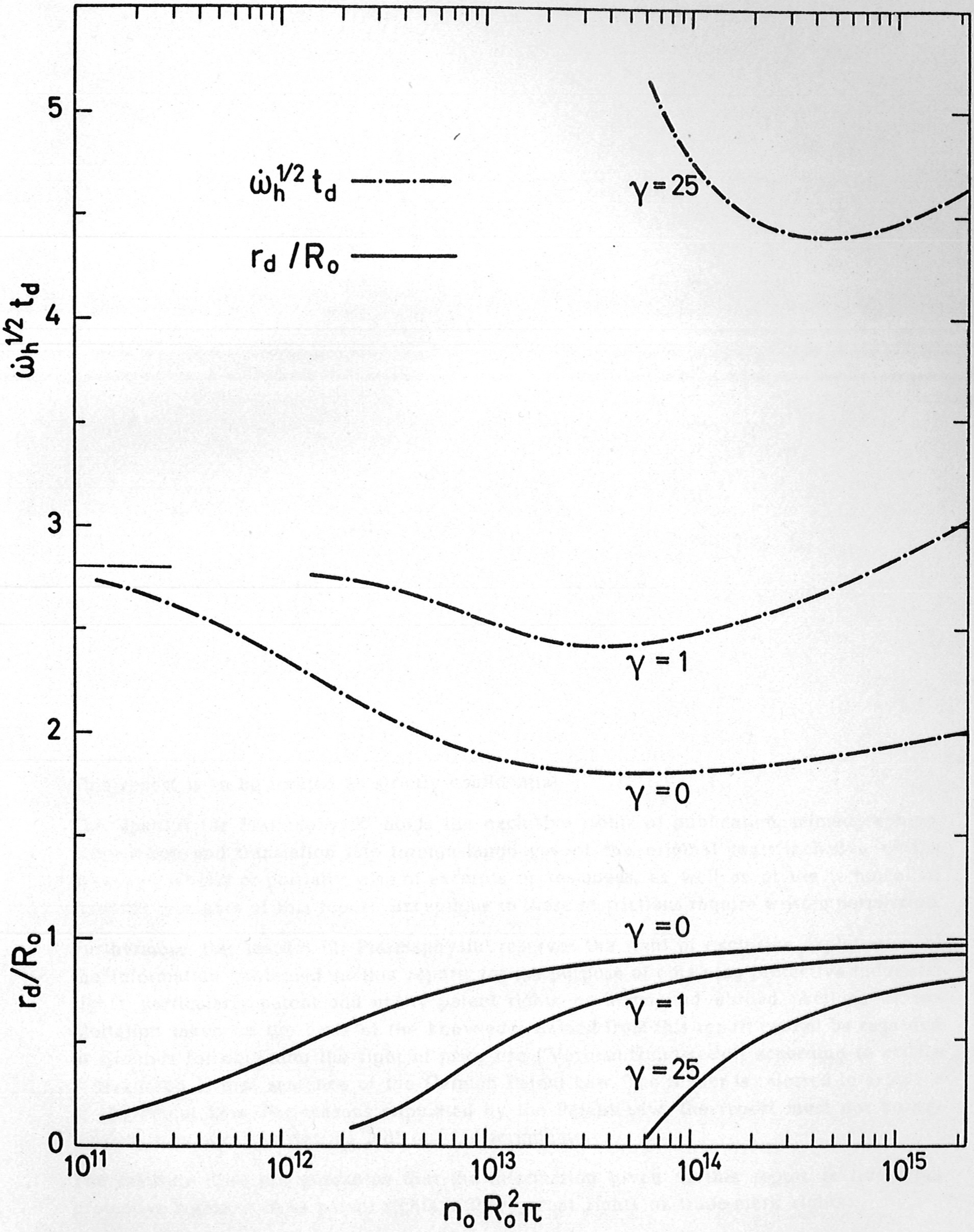


Fig. 8

Times and locations for the occurrence of infinite densities at the plasma edge without friction (γ = 0) and for the density peaks with discontinuous inner flank with friction (γ = 1 and 25). Compression time $t_c = 2.82\ \dot{\omega}_h^{-1/2}$ for a nearly non-diamagnetic plasma ($\delta \gg 1$) without friction is indicated by dashed line.