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ABSTRACT

Measurements of mixing (nonlinear interaction) of two low level microwave signals in an isotropic positive column are described. One signal (650 MHz) excites a propagating electroacoustic wave. The other signal (1150 MHz) propagating along a "TEM" line interacts with the plasma wave whereby signals with the sum and difference frequencies are produced (and observed). The amplitudes of the scattered signals are in reasonable agreement with the values predicted by a simple theory involving an "equivalent" dielectric constant ($\varepsilon(\vec{r},t)$).

Nonlinear interaction of electromagnetic waves with plasmas is predicted when waves of large intensity are involved because of the nonlinear nature of the basic plasma equations. Also in the case of low intensity waves one expects nonlinear effects in inhomogeneous plasmas because of gradients in density and static electric fields. In this paper experimental observations of nonlinear effects are reported which result from periodic variations in the electron density. A longitudinal plasma or electroacoustic wave is produced which propagates along an isotropic plasma column and generates these density variations. A transverse electromagnetic wave whose frequency is different from that of the plasma wave then is allowed to interact with the plasma and the sum and difference frequencies (plasma wave and TEM wave) are observed. One may consider this to be a scattering of the TEM wave by the density variations and other nonlinear effects produced by the plasma wave. The frequencies of the waves involved are such that only the motion of the electrons need be considered. Similar experiments are described by Stern (1965) and Stern and Tzoar (1965). In these experiments, however, a standing wave with dipolar azimuthal dependency was excited. These dipolar waves are more strongly coupled to fields exterior to the plasma than the propagating plasma waves having azimuthal symmetry which are excited in the experiments to be described. Because of this coupling the waves of dipolar dependency resemble less closely "pure" electroacoustic waves than those having the higher symmetry.

The plasma is the positive column of a low pressure (neutral gas pressure $2 \cdot 10^{-3}$ mm Hg) mercury vapor discharge contained in a glass tube with a hot oxide cathode (Fig. 1a). The outside diameter of the discharge tube is 18.7 mm and the tube length is 84 cm. The discharge current was varied between 0 and 2 amp. producing average (over the tube cross section) plasma densities up to $2.4 \cdot 10^{11} (cm^{-3})$ (plasma frequency $(\omega_p^2)^{1/2} = 2.8 \cdot 10^{10}$ sec^{-1}. The impressed frequencies ranged from ca. 200 to 2000 MHz. The collision frequencies are believed to be an order of magnitude smaller than the impressed frequencies. The coupler used to excite the plasma waves is a coaxial reentrant cavity similar to those used with low frequency klystron oscillator tubes. The transmission line used to conduct the TEM wave was constructed using a S-Band waveguide as outer conductor and a 1 mm thick x 46 mm wide brass sheet as inner conductor (Fig. 1b). The ends of the outer conductor (wave guide section) were closed and the connections at each end to the inner conductor are coaxial. The system has a VSWR ≤ 2 for frequencies up to

1900 MHz. (VSWR measured before the hole for the discharge tube was bored through the "TEM" conductor.) The inner conductor was mounted closer to one side of the outer conductor (10 mm) than to the other (23 mm), see Fig. 1a, in order to produce a more intense E-field and small space of interaction with the plasma.

In the first measurements to be described the plasma waves were excited at 650 MHz with 20 mW signal and a signal of 1150 MHz, 20 mW, was transmitted through the "TEM" conductor. By properly tuning the coaxial reentrant cavity the reflection (of the 650 MHz signal) by the cavity was kept under 3 db over the range of discharge currents of primary interest. (The cavity tuning depends strongly on the discharge current because of capacitive loading of the gap.) The power reflected by and transmitted through the "TEM" conductor as a function of discharge current are shown in Fig. 2. The weak absorption and reflection maxima at about 0.2 and 0.3 amp. are due to the n = 0 mode as will be explained below. The strong absorption and reflection, i $\gtrsim$ 1 amp., is due to the surface wave; see Trivelpiece and Gould (1959), Schuman and colleagues (1960), Hahn (1939) and Ramo (1939).

In a homogeneous plasma of infinite extent one expects electroacoustic (or plasma) wave to propagate at frequencies in the vicinity ω_p. In a bounded, but otherwise homogeneous plasma the propagation of these waves takes place at a series of different frequencies with various radial and azimuthal mode numbers and is coupled to fields outside the plasma. (Compare propagation of electromagnetic wave in free space and in wave guides.) Because of the coupling to the region outside the plasma a further type of transmission exists also with various azimuthal mode numbers - this is often named the "surface wave". Due to the inhomogeneities which exist within the plasma, the propagation of these "primarily" electroacoustic modes can take place in the region of tenuous plasma near the boundary such that the oscillating frequency is less than the average plasma frequency $(\overline{\omega_p^2})^{1/2}$. Calculation and measurement of the dispersion relations for modes with symmetric, n = 0, and dipolar, n = 1, azimuthal dependencies are given by O'Brien, Gould and Parker (1965), O'Brien (1967) and Diament, Granatstein and Schlesinger (1966). In the experiment at hand the n = 0 mode, with various radial mode numbers, is excited because of the cylindrical symmetry of the coupler. An ω-β diagram of the transmission between two identical couplers (of the type shown in Fig. 1a) has been measured and is shown in Fig. 3. ω is the impressed frequency, 650 MHz, $\beta = 2\pi/\lambda_z$ where λ_z is the wave length along the plasma column of the transmitted signal and r_w is the radius of the plasma column. $(\overline{\omega_p^2})^{1/2}$, the average plasma frequency, was measured by observing the shift in the resonant frequency of a microwave cavity placed around the discharge, see Agdur and Enander (1962).

One may expect to find scattering (productions of signals at 1150 ± 650 MHz) at those values of anode current, i.e. $\overline{\omega_p^2}$, for which the plasma wave, at 650 MHz, can propagate. It has been observed in the transmission experiments from which Fig. 3 was obtained, that there is strong attenuation of the plasma wave for short wave lengths; $(\beta\, r_w)^2 \gtrsim 4$. For this reason and also because of "interference" which takes place when the wave length of the plasma wave is approximately equal to the spacing between the inner and outer conductor in the "TEM" line, one would expect strong scattering only in the region (values of $\overline{\omega_p^2}$) of small $(\beta\, r_w)^2$.

A block diagram of the experimental arrangement used to detect the scattering is shown in Fig. 4. The noise figures of the 500 and 1800 MHz receivers are, respectively, 5 db and 7 db (without filters). Care was taken to ensure that the 1150 MHz signal would not overload or even reach the front end of the receivers in sufficient strength to produce spurious signals by mixing with the small amount of signal at the plasma wave frequency, 650 MHz, picked up in the "TEM" line. For some of the measurements the 650 MHz generator was modulated at 1 kHz and the detected output of the IF amplifier was processed through phase coherent amplifier, P.A.R. Type JB-5, in order to increase the sensitivity. In all cases, however, the magnitude of the scattered signal is corrected (for modulation losses).

The magnitude of the received signals at 500 and 1800 MHz is shown in Fig. 5 as a function of the discharge current ($\overline{\omega_p^2}$). Particular emphasis has been placed on the signals scattered by plasma waves propagating at discharge currents smaller than 200 ma because these waves (bands of propagation) embody more nearly the nature of plasma waves in an infinitely extended homogeneous medium. The band of propagation at discharge currents extending upwards from ca. 200 ma is strongly dependent on the geometry, boundary conditions, and medium surrounding the plasma column (see references given by "surface waves"). The amplitude of the scattered wave depends linearly on the amplitude of the plasma wave exciting signal and of the "TEM" signal.

One can compare the amplitude of the scattered signal as a function of discharge current with the bands of propagation of the n = 0 mode, at 650 MHz, Fig. 3. One observes strong scattering at discharge currents larger than 250 ma corresponding to the range of current where "surface wave" band of propagation has been observed. At smaller currents one observes a series of scattering maxima located around the values of 100 ma, 55 ma and 35 to 40 ma. These maxima can be associated with the bands of propagation of the 650 MHz signal corresponding to higher radial mode numbers. The fact that the amplitude of the scattered signal falls rapidly as one goes to higher radial mode numbers is to be expected from the data obtained from transmission measurements performed at 650 MHz. The amplitude of the signal transmitted between two coaxial couplers is shown in Fig. 6, which, of course, is the amplitude of signal represented in Fig. 3. The ratio of the amplitudes of successive bands (radial mode numbers differing by unity) excluding the "surface wave" is about 25 db for the transmission signal and about 15 db for the scattered signal. This will be further discussed.

From Fig. 5 it is obvious that the technique of observing the scattered signal can be used to detect the presence of bands of propagation with azimuthal symmetry at other plasma wave frequencies. Further, comparison of such measurements with direct transmission measurements (between two couplers) affords another means of corroberating the proposed hypothesis, that the cause for the scattering is indeed plasma waves. Such measurements are now to be described. In order to simplify the experimental observation of the scattered wave the frequencies of the two primary signals (driving the plasma wave coupler and "TEM" line) were chosen to differ by 30 MHz. The receiver, a 30 MHz amplifier, was then sensitive at the difference frequency. The receiver output was recorded as a function of discharge current, producing curves similar to Fig. 5. These curves, measured using many different frequency pairs, in turn were compiled and the maxima (excluding "surface wave") plotted as a function $r_w^2 / \overline{\lambda_d^2}$, Fig. 7. $\overline{\lambda_d^2}$ is the average Debye length defined as $\overline{\lambda_d^2} = (K\ T_e/m_e)/\overline{\omega_p^2}$; K is the Boltzman constant, T_e electron temperature and m_e electron mass. In a calculation by Parker (1963) it was shown that the electron number density as a function of radius in an isotropic collisionless positive column can be described by the parameter $r_w^2 / \overline{\lambda_d^2}$. As the propagation of the plasma waves in the column can be treated as an eigenvalue problem, compare wave guide propagation, it is obvious that the electron density profile is an important parameter - for a discussion of this see O'Brien (1967). A block diagram of the experimental arrangement used to observe the wave scattered at 30 MHz is shown in Fig. 7. Also shown in Fig. 7 is the locus of the cutoff points of the n = 0 bands of propagation as observed by transmission between two couplers.

Theory

In order to estimate the ratio of the magnitude of the scattered signal (at the sum and difference frequencies) to the magnitudes of the primary signals a quite simplified and approximate model will be used. The signals in the plasma produced by the plasma wave and TEM wave will be considered small and taken as perturbation quantities. The plasma wave, launched by the coupler, is assumed to propagate undisturbed by the TEM wave. The interaction of the TEM wave with the plasma wave, on the other hand, is assumed to be influenced by the propagating plasma wave. The ions are taken to be at rest. Collisions are neglected (collision frequencies are at least an order of magnitude smaller than the primary frequencies). Macroscopic (fluid) equations are taken to describe the electron gas. Certain relationships valid for an infinite homogeneous plasma will be used in conjunction with other relationships derived for the inhomogeneous plasma in question.

The mechanism for the interaction between the TEM wave and the plasma is represented by an effective dielectric constant which is approximately of the form $\varepsilon_o(1 - \omega_p^2/\omega_T^2)$; ε_o is the permittivity of vacuum and ω_T the frequency of the TEM wave. ω_p^2 ist proportional to the electron number density which is taken to have a time independent part, n_o, and a time dependent part, realpart n_1 exp $i\omega_E t$, stemming from the Electroacoustic (plasma) wave, frequency ω_E. (The exact expression for the effective dielectric constant for the problem at hand is given in eq.(15).) One may account for the perturbation to the "TEM" line caused by the plasma by replacing it with a capacitor having the effective dielectric constant. As this capacitor disturbes the "TEM" line, its presence will change the transmission (and reflection) characteristics of the line.

Further, as the effective dielectric constant varies in time, the capacitor will change the transmission (and reflection) properties at frequency ω_E and thus function as a modulator, modulating the TEM signal at ω_E. The modulated signal, when Fourier analysed, is seen to contain ω_T and the sum and difference frequencies, $\omega_T \pm \omega_E$.

Implicit in the above model is the approximation that interaction of the TEM wave with the plasma does not involve excitation of and radiation from further electroacoustic waves at ω_T. In the case to be discussed, $\omega_T/2\pi = 1.15$ GHz and $\omega_E/2\pi = 0.65$ GHz, the range of plasma density of interest, i.e. ω_p^2, is such that little coupling at ω_T between TEM and electroacoustic waves is present*). An effective dielectric constant for the plasma is obtained by linearizing the macroscopic fluid equation. The inclusion of a term varying at frequency ω_E represents a second order perturbation term. A complete consistent second order solution of the equations is not attempted, but rather a result is obtained, in a manner, similar to that in scattering theory, which calculates the currents varying at the frequencies $\omega_T \pm \omega_E$.

The basic equations are:

$$m\, n \frac{d\vec{v}}{dt} = - e\, n\, (\vec{E} + \vec{v} x \vec{B}) - \nabla p \tag{1}$$

$$\nabla \cdot (n\, \vec{v}) = - \frac{\partial n}{\partial t} \tag{2}$$

$$p = \gamma\, K\, T\, n \tag{3}$$

$$(\nabla\, x\, \vec{B})/\mu_o = \vec{j} + \varepsilon \frac{\partial \vec{E}}{\partial t} \tag{4}$$

$$\nabla\, x\, \vec{E} = - \frac{\partial \vec{B}}{\partial t} \tag{5}$$

$$\nabla \cdot \varepsilon_o \vec{E} = - e\, n \tag{6}$$

$$\nabla \cdot \vec{B} = 0 \tag{7}$$

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*) Current values between 30 and 100 ma are of primary interest, see Fig. 3, for which $\omega_{T_3}^2/\overline{\omega_p^2}$ varies between 5 and 1.5 and $r_w^2/\overline{\lambda_d^2}$ between 10^3 and $3 \cdot 10^3$. Little coupling to electroacoustic waves by these values has been observed, see Fig. 7.

Temperature, pressure, particle number density, mass and charge all refer to electron quantities (e is the magnitude of the electronic charge). The plasma is taken to be homogeneous, isotropic and at rest; that is $\vec{v}$, $\vec{E}$, $\vec{B}$, $\vec{j}$ are only perturbation quantities.

The familiar plasma wave solution results when one linearizes, takes the divergence of the force equation, and makes the ansatz that all first order quantities vary as $\exp i(\omega_E t - \vec{k}_E \cdot \vec{\zeta})$. It is,

$$\omega_E^2 - \omega_{po}^2 = k_E^2 c_s^2 \qquad \text{with } c_s^2 =: \frac{\gamma K T}{m} \text{ and } \omega_{po}^2 =: n_o e^2/(m_e \varepsilon_o) \tag{8}$$

The $\vec{E}_E$ vector is parallel the $\vec{k}_E$ vector (ζ axis).

Also the familiar TEM wave solution is immediately obtained on linearizing with the ansatz $\vec{E}_T = \vec{E}_{\zeta T} \exp i(\omega_T t - \vec{k}_T \cdot \vec{\xi})$. It is,

$$\omega_T^2 - \omega_{po}^2 = k_T^2 c^2 \tag{9}$$

with c = speed of light in vacuum. We note that $\vec{E}_T$($\|\zeta$ axis) is perpendicular to $\vec{k}_T$($\|\xi$ axis). In the case of the TEM wave, one may equivalently write the dispersion relation (9) as

$$\varepsilon_T = \varepsilon_o (1 - \omega_{po}^2/\omega_T^2) \quad \text{with } k_T^2 = \omega_T^2 \mu_o \varepsilon_T \,. \tag{10}$$

In a linear approximation the introduction of ε_T follows immediately by combining the terms $\vec{j} + \varepsilon_o \partial\vec{E}/\partial t$ into $\varepsilon_T \partial\vec{E}/\partial t$; right hand side of (4).

We now proceed to calculate the terms which vary at frequencies $\omega_T \pm \omega_E$. These quantities come from the product of terms with frequency ω_T and terms with frequency ω_E. Such terms are necessarily of second or higher (perturbation) order. Wishing to retain only second order terms we write the force equation in which we include all zero and first order variables.

$$m(n_o+n_{1E})\left[\frac{\partial}{\partial t}(\vec{v}_{1E}+\vec{v}_{1T}) + (\vec{v}_{1E}+\vec{v}_{1T})\cdot\nabla(\vec{v}_{1E}+\vec{v}_{1T})\right] = - e(n_o+n_{1E})\left[(\vec{E}_{1E}+\vec{E}_{1T}) + (\vec{v}_{1E}+\vec{v}_{1T}) \times \vec{B}_{1T}\right] - \nabla_{p1E} \tag{11}$$

The fact that n_{1T} and consequently p_{1T} are zero, which follows from the first order calculation, is expected as $\vec{v}_{1T}$ is parallel to $\vec{E}_{1T}$ which in turn are perpendicular to the propagation vector thus producing only a "side ways" motion of the electrons leading to no charge concentration. The two first order equations which result are:

$$m\, n_o\, i\omega_E\, \vec{v}_{1E} = -\, e\, n_o\, \vec{E}_{1E} - \nabla p_{1E} \quad \text{and}$$

$$m\, n_o\, i\omega_T\, \vec{v}_{1T} = -\, e\, n_o\, \vec{E}_{1T} \qquad (12a,b)$$

We now collect the sought-after 2 order terms:

$$n_{1E} i\omega_T \vec{v}_{1T} + n_o(\vec{v}_{1E}\cdot\nabla\vec{v}_{1T} + \vec{v}_{1T}\cdot\nabla\vec{v}_{1E}) = -\frac{e}{m}(n_{1E}\vec{E}_{1T} + n_o\vec{v}_{1E} \times B_{1T}) \qquad (13)$$

The current at the "mixed frequencies" is $j_{ET} = -\, e\, n_{1E}\vec{v}_{1T}; (n_{1T}\vec{v}_{1E} = 0)$.
In the appendix it is shown that under most conditions $|n_o\, \vec{v}_{1E} \times \vec{B}_{1T}| \ll |n_{1E}\, \vec{E}_{1T}|$. The $\vec{v} \times \vec{B}$ term will be neglected and, in order to simplify the $\vec{v}\cdot\nabla\vec{v}$ terms, the direction of the $\vec{E}$ and $\vec{k}$ vectors in a coordinate system will be chosen.

$$\vec{v}_{1E} \parallel \nabla p_{1E} \parallel \vec{E}_{1E} \parallel \text{z-axis};\ \vec{E}_{1E} = \{0, 0, E_{1E}\} \text{ with } \exp(i\omega_E t - ik_E z)$$

$$\vec{v}_{1T} \parallel \vec{E}_{1T} \parallel \text{z-axis};\ \vec{E}_{1T} = \{0, 0, E_{1T}\} \text{ with } \exp(i\omega_T t - ik_T x)$$

The product $\vec{v}_{1E}\cdot\nabla\vec{v}_{1T}$ is then equal to zero and

$$\vec{v}_{1T}\cdot\nabla\vec{v}_{1e} = v_{1T}(-ik_E)\,\vec{v}_{1E}$$

From eq.(12b) one obtains $\vec{v}_{1T}$ in terms of $\vec{E}_{1T}$ and from eq.(12a), (4) (whereby $B_{1E} \equiv 0$), and (3) one obtains $\vec{v}_{1E}$ in terms of ∇n_{1E}. Using eq.(8) and taking into account the special orientation of the vectors one obtains for the magnitude of j_{ET}:

$$j_{ET} = \frac{\varepsilon_o\, E_{1T}\, \delta\omega_p^2}{i\, \omega_T}\left(1 - \frac{\omega_E}{\omega_T}\right) E_{1T} \quad , \text{whereby } \delta\,\omega_p^2 =: \frac{e^2\, n_{1E}}{m\, \varepsilon_o} \quad . \qquad (14)$$

j_{ET} is directed parallel to the z axis; we note, of course, that n_{1E} has the $\exp(i\,\omega_E t - i\,k_E\, z)$ dependency.

We now write $\nabla \times B/\mu_o = j + \varepsilon_o\, \partial E/\partial t$ and take $j = j_{1E} + j_{1T} + j_{ET}$ and $E = E_{1E} + E_{1T}$. Note the difference in the ansatz for j and E. Taking the phase of the E_{1E} and E_{1T} vectors to be zero at t = 0 and z = 0, we can combine the particle and displacement current ($\vec{j}_{1E} + \varepsilon_o \partial \vec{E}_{1E}/\partial t = 0$). With $\vec{j}_{1T} =: - e\, n_o \vec{v}_{1T}$ we obtain from (12b) $\vec{j}_{1T} = -(\omega_{po}^2/\omega_T^2)\, i\, \omega_T \varepsilon_o \vec{E}_{1T}$. Introducing these quantities, $\vec{j}_{1T}$ and $\vec{j}_{ET}$ into Maxwell's equation we have:

$$\nabla \times \vec{B}/\mu_o = \varepsilon_o \frac{\partial \vec{E}_{1T}}{\partial t} \left[1 - \frac{\omega_{po}^2 + \delta\, \omega_p^2 (1 - \omega_E/\omega_T)}{\omega_T^2} \right] =: \varepsilon_{ET} \frac{\partial \vec{E}_{1T}}{\partial t} \qquad (15)$$

The expression in the brackets is the relative dielectric constant $\varepsilon_{ET}/\varepsilon_o$ which, through the dependence of $\delta\, \omega_p^2$ on n_{1E} (eq.(14)), varies in space as exp (i $\omega_E t$ - i k_E z). We see immediately that if the frequency of the TEM wave, ω_T, is large compared with the frequency of the electroacoustic wave, ω_E, we have the simple result $\varepsilon_{ET} = \varepsilon_o (1 - \omega_p^2 (\vec{r},t)/\omega_T^2)$.

The next basic step in the calculation is to devise a model with which we can compute the effect of the varying dielectric constant on the wave transmitted through the "TEM" line. As the dimensions of the interaction region are small compared with the wavelength of the TEM signal, we may reckon with a "lumped parameter" equivalent circuit. The effect of the presence of the plasma column will be represented by a capacitor in the "TEM" line. The value of the capacitance varies by a small amount, at frequency ω_E, about a quiescient point. This variation produces a change in the transmission factor τ of the signal through the line. The result of this variation in τ , or amplitude modulation, can be resolved into it's attendent frequency components yielding the amplitudes of the signals at $\omega_T \pm \omega_E$. An approximation to the geometry of the capacitor will be discussed later.

The ADDITIONAL capacitive reactance between the inner and outer conductors presented by the plasma is represented by Z_A. The characteristic impedance (and terminating impedance) of the "TEM" line is Z_o (equal to 50 Ω). The transmission factor τ is the ratio of the amplitude of the signal transmitted on past the point Z_A to the signal incident on the point Z_A. (See any standard work on transmission theory.)

$$\tau = \left(1 + \frac{Z_o}{2\, Z_A}\right)^{-1} \qquad (16)$$

Because of the variation in Z_A, τ is assumed to change between the values of $\tau_o + \delta\tau$ and $\tau_o - \delta\tau$ in a sinusoidal manner. We therefore write,

$$y = A\tau_o (1 - \delta\tau \cos \omega_E t) \cos \omega_T t \tag{17}$$

where y is the transmitted signal and A the amplitude of the incident signal. y is then written as

$$y = A\tau_o \cos \omega_T t + \frac{A\ \delta\tau}{2} \cos (\omega_T + \omega_E)t + \frac{A\ \delta\tau}{2} \cos (\omega_T - \omega_E)t \tag{17a}$$

One often speaks of the terms or the r.h.s. of (17a) as the carrier, upper and lower side bands respectively. We shall be concerned with the ratio V_s of the amplitude of one side band to the amplitude of the carrier. V_s is then $\delta\tau/(2\tau_o)$. Assuming $\delta\tau \ll \tau_o$ we have,

$$V_s = \frac{d\,\tau}{d\,Z_A}\ \delta\,Z_A\ (2\tau_o)^{-1} \qquad (= \frac{\tau_o}{4}\ \frac{Z_o}{(Z_A)_o^2}\ \delta\,Z_A) \tag{18}$$

In order to decide on the value for Z_A and $\delta\,Z_A$ we must combine the ε_{ET} expression, (15), calculated for plane electroacoustic (and TEM) waves and the given cylindrically symmetric plasma waves propagating in a column. To this end we must estimate the size of our equivalent capacitor. Fig. 8 shows a sketch of the interaction region with a representation of the E-field of the TEM wave. As the strength of this field diminishes rapidly for decreasing values of radius (inside the plasma column), the interaction will be confined mainly to the peripheral region of the column. We then take the value of the equivalent capacitor to be $\pi\, r_w^2\, a\, \varepsilon/h$ whereby h is the distance between the inner and outer conductor and a is a number between 0 and 1 representing the ratio of the effective area of interaction to the cross section of the plasma column. (A reasonable value for a is perhaps 0.3.) Implicit in the above formula is that ε is constant throughout the interaction region which implies that the wavelength in axial direction of the plasma wave is at least 2 h or greater. Also the wave length in radial direction should be the order of $r_w/2$ and not have a node <u>within</u> the interaction region. This requirement is fulfilled by the "surface wave" mode ($\overline{\omega_p^2} > 7\cdot10^{19}$) and by the next mode ($\overline{\omega_p^2} \cong 3\cdot10^{19}$); the latter mode being of primary interest. It has an approximate node at r_w which is permissible. (The bands with higher radial mode numbers ($\overline{\omega_p^2} \lesssim 2\cdot10^{19}$) have one or more nodes in the interaction region whereby the scattering effect of increased particle density on one side of the node is at least in part cancelled by the reduced density on the other side. We therefore would expect less scattering from bands having large radial mode numbers - in accorcance with observation.)

The total capacitance presented by the presence of the plasma is C with $C = C_A + C_v$ whereby C_v is the capacitance were a vacuum present and C_A the additional capacitance caused by the plasma.

$$C_A = \frac{\pi r_w^2 a}{h} (\varepsilon_{ET} - \varepsilon_o)$$

$$C_A = \frac{\pi r_w^2 a \varepsilon_o}{h} \left[- \frac{\omega_p^2}{\omega_T^2} - \frac{\delta \omega_p^2}{\omega_T^2} (1 - \frac{\omega_E}{\omega_T}) \right] \tag{19}$$

We have for the static and varying parts of C_A respectively

$$C_{Ao} = \frac{\pi r_w^2 a}{h} \varepsilon_o \left(- \frac{\overline{\omega_p^2}}{\omega_T^2} \right) \qquad \delta C_A = \frac{\pi r_w^2 a}{h} \varepsilon_o \left[- \frac{\overline{\delta \omega_p^2}}{\omega_T^2} (1 - \frac{\omega_E}{\omega_T}) \right] \tag{20a,b}$$

whereby $\overline{\omega_p^2}$ is the average taken over the cross section of the column and $\overline{\delta \omega_p^2}$ the average of the <u>magnitude</u> of $\delta \omega_p^2$ (taken over the column).

With the expressions $Z_A = (i \omega_T C_A)^{-1}$, $\delta Z_A = (\partial Z_A / \partial C_A) \delta C_A$ and eq. (16), (18), and (20) we arrive at the following for V_s:

$$V_s = - \frac{1}{4} \frac{Z_o}{Z_o + 2 Z_A} \frac{\overline{\delta \omega_p^2}}{\overline{\omega_p^2}} (1 - \frac{\omega_E}{\omega_T}) \tag{21}$$

Where in (21) $Z_A = (i \omega_T C_{Ao})^{-1}$ is to be understood.

Introducing the dimensions $r_w = 7.8$ mm, h = 10 mm and $\omega_T = 2\pi\ 1.15 \cdot 10^9\ \sec^{-1}$ we calculate $Z_A = i\ 4.26 \cdot 10^{22} / (a\ \overline{\omega_p^2})$. The magnitude of V_s (which is the quantitiy observed in these experiments) is only slightly changed by neglecting Z_o compared with $2 Z_A$ when say $|2 Z_A| \geqslant 3 Z_o$. For a = 0.3 this is fulfilled for $\overline{\omega_p^2} < 1.6 \cdot 10^{20}\ \sec^{-2}$ which in turn is true for discharge currents less than 0.4 ampere. As our primary interest is in the range of currents 0 - 100 ma we use this approximation. Further restricting ourselves to the magnitude of V_s, inserting $\omega_E = 2\pi\ 650 \cdot 10^6\ \sec^{-1}$ we obtain

$$|V_s| = 6.3 \cdot 10^{-23}\ a\ \overline{\delta \omega_p^2} \tag{22}$$

The next step is to estimate the value of $\overline{\delta \omega_p^2}$. We must deduce this quantity, that is de facto n_{1E}, from the knowledge of the energy which we put into the electroacoustic wave and other parameters. The power put into the plasma wave is P (watts) which travels in the positive and negative "Z directions". The time averaged energy flux $\overline{N}^t = m\, v_\varphi c_s^2 n_1^2 / (2 n_o)$ for a plane electrostatic wave is taken from Denisse and Delcroix (1963). $\overline{N}^t$, energy area^{-1} sec^{-1}, is related to P by $\overline{N}^t$ 2A = P where A is the cross sectional area where the wave propagates. Solving for $\delta \omega_p^2$ (eq.(14)) we obtain

$$\delta \omega_p^2 = \frac{e}{m c_s} \left(\frac{P \omega_p^2}{\varepsilon_o A v_\varphi} \right)^{1/2} . \tag{23}$$

We must now make some estimates regarding the quantities A, v_φ, and ω_p^2 as we do not have the propagation of a plane wave in the experiment. (The application of relationships valid for plane waves to the inhomogeneous plasma is of course an approximation in itself.) The range of discharge currents where the plasma waves at ω_E propagate is ca. 30 to 100 ma, see Fig. 3. In this range r_w^2/λ_d^2 is between 1 and $3.6 \cdot 10^3$. Reference to the n_1 and n_o curves of Parker, Nickel and Gould (1964) show that, at these parameter values, the cross sectional area of the column where the waves propagate is about one half the physical cross section and that the values of ω_p^2 in this region is also about one half of $\overline{\omega_p^2}$. The plasma waves which propagate can be considered to be the sum of two interfering waves each of which propagates (primarily) in a radial direction however with a component of $\vec{k}$ in the z direction. The radial distribution of n_1 can have many nodes such that the n_1 contribution to $\delta \overline{\omega_p^2}$ from one point can cancel the contribution from a point at a different radius. To avoid this complication we shall deal only with that radial mode having one node interior to the plasma (at cutoff), see O'Brien (1965). For low phase velocities in the z direction there can exist many nodes (in the z direction) within the equivalent capacitor at one time instant. This would lead to interference and a reduction in the net scattered signal. We therefore require that v_φ be at least so that $\lambda_z \gg 2$ h, ($v_\varphi \gg 2 h \omega_E/(2\pi)$). We take the equality for further calculations. The mode with the desired radial dependency is that one occurring at about 100 ma, see Fig. 3. Inserting the appropriate values of ω_p^2 ($= \overline{\omega_p^2}/2 = 3.3 \cdot 10^{19}/2$), A ($= \pi r_w^2/2$), $c_s = 1.34 \cdot 10^6$ m/s ($T_e = 4 \cdot 10^4$ °K) and $P = 20 \cdot 10^{-3}$ W (power delivered to the plasma wave coupler), we obtain $\delta \omega_p^2 = 7.2 \cdot 10^{17}$ sec^{-2}. Introducing this into eq.(22) and taking a ≅ 0.3 we calculate

$$|V_s| = 1.36 \cdot 10^{-5}$$

The observed values of $|V_s|$ are respectively $0.4 \cdot 10^{-5}$ and $0.25 \cdot 10^{-5}$ for 500 and 1800 MHz. This agreement is surprisingly good considering the nature of the approximations and the experimental accuracy in the signal level determination, ca. 3 db.

It should be pointed out that in addition to geometric approximations, "homogeneous" wave theory, fringing fields etc., no account of the damping of the plasma wave has been made.

Also it is interesting to see the origin of the factor $1 - \omega_E/\omega_T$ in equations (14) and (15). From eq.(13) it is seen that the "1" comes from the $n_{1E}\ E_{1T}$ term (r.h.s.) and the "ω_E/ω_T" from the $(v \cdot \nabla)v$ term. For the case of very low frequency density fluctuations i.e. $\omega_E \ll \omega_T$, the expression for ε reduces to the commonly used form. The ω_E/ω_T term, however, indicates the presence of another source of non-linear interaction of the same perturbation order.

<u>Appendix</u>

We wish to show that $|n_o \vec{v}_{1E} \times \vec{B}_{1T}| / |n_{1E} \vec{E}_{1T}| \equiv A$ is much less than one for most of the measurements. $\vec{v}_{1E}$ is perpendicular to $\vec{B}_{1T}$ in our experiment. From eq.(5) we obtain $B_{1T} = k_T E_{1T}/\omega_T$ and from eq.(2) $n_o v_{1E} = \omega_E n_{1E}/k_{1E}$ which when introduced yield:

$$A = (\omega_E/k_E)\ (k_T/\omega_E) = (v_{\varphi E})/v_{\varphi T} = \left(\frac{\omega_T^2 - \omega_p^2}{\omega_E^2 - \omega_p^2}\right)^{\frac{1}{2}} \frac{\omega_E}{\omega_T} \frac{c_s}{c}$$

When $(\omega_E^2 - \omega_p^2)/\omega_E^2 > 1/10$, $A \ll 1$ because $c \approx 300\ c_s$.

This condition is fulfilled under most of the experimental conditions.

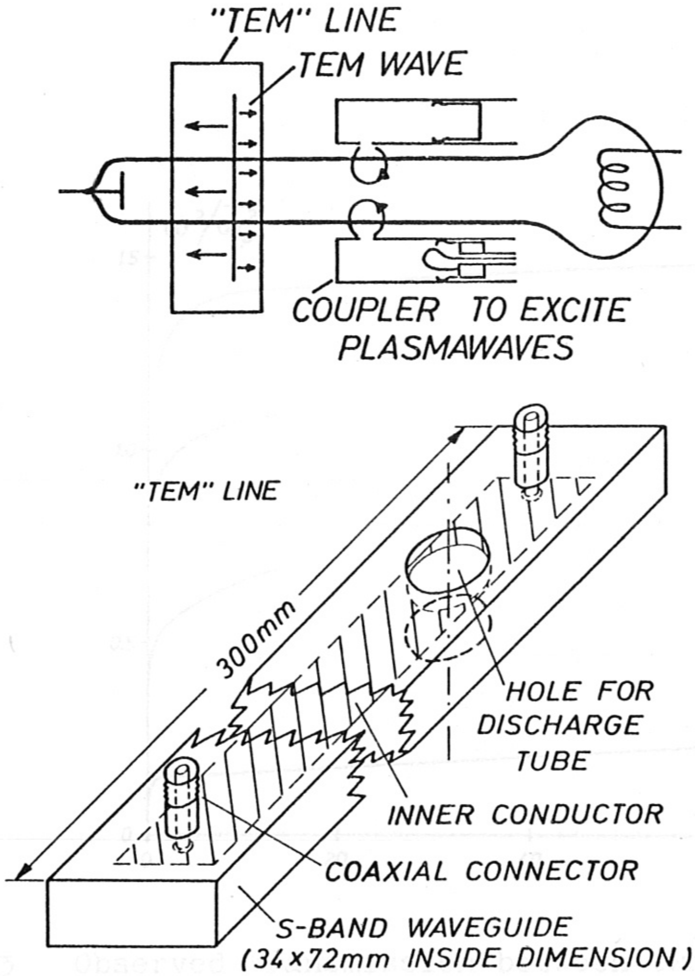

Fig. 1 (a) Apparatus (b) Sketch of "TEM" line.

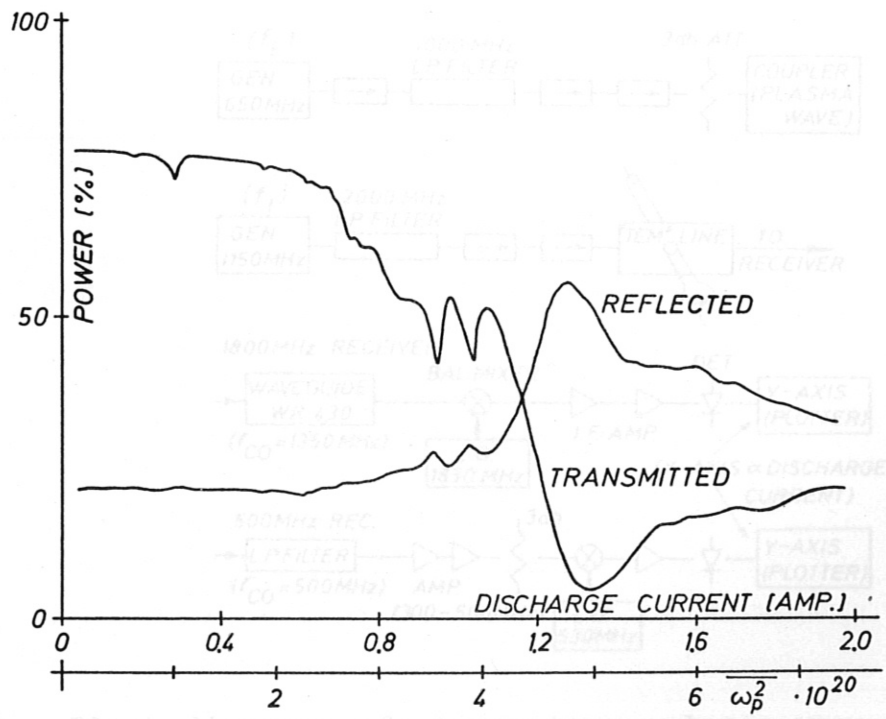

Fig. 2 1150 MHz signal reflected from and transmitted through the "TEM" line as a function of discharge current.

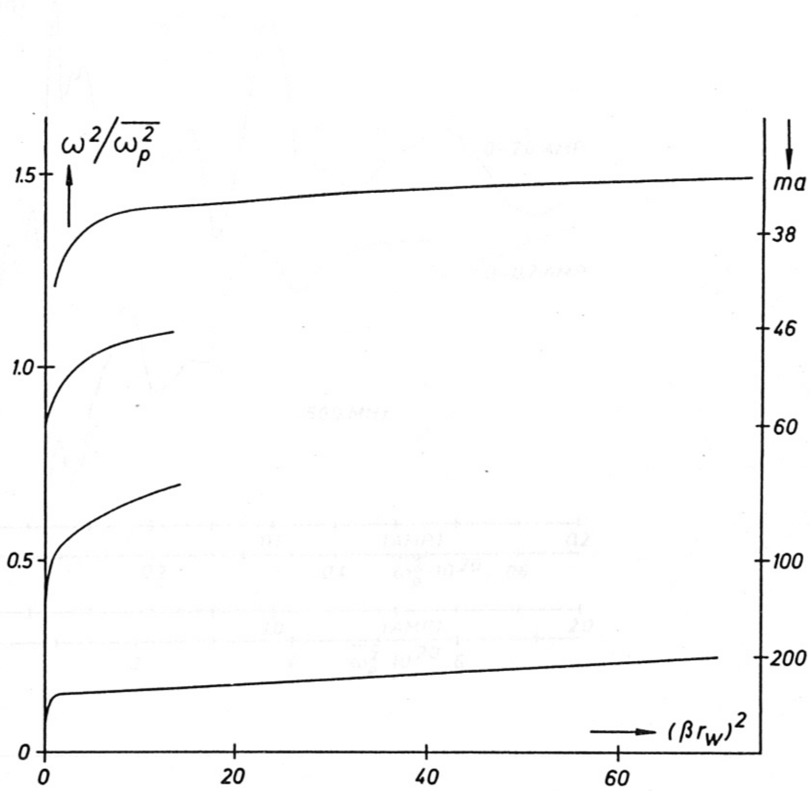

Fig. 3 Observed transmission between two plasma wave couplers at 650 MHz.

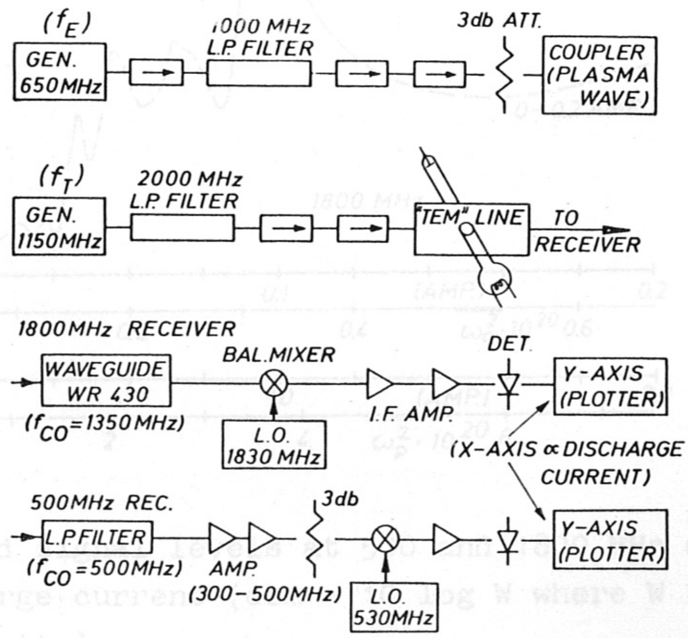

Fig. 4 Block diagrams of: generators; plasma wave, 650 MHz, and TEM wave, 1150 MHz, and receivers; 1800 MHz and 500 MHz.

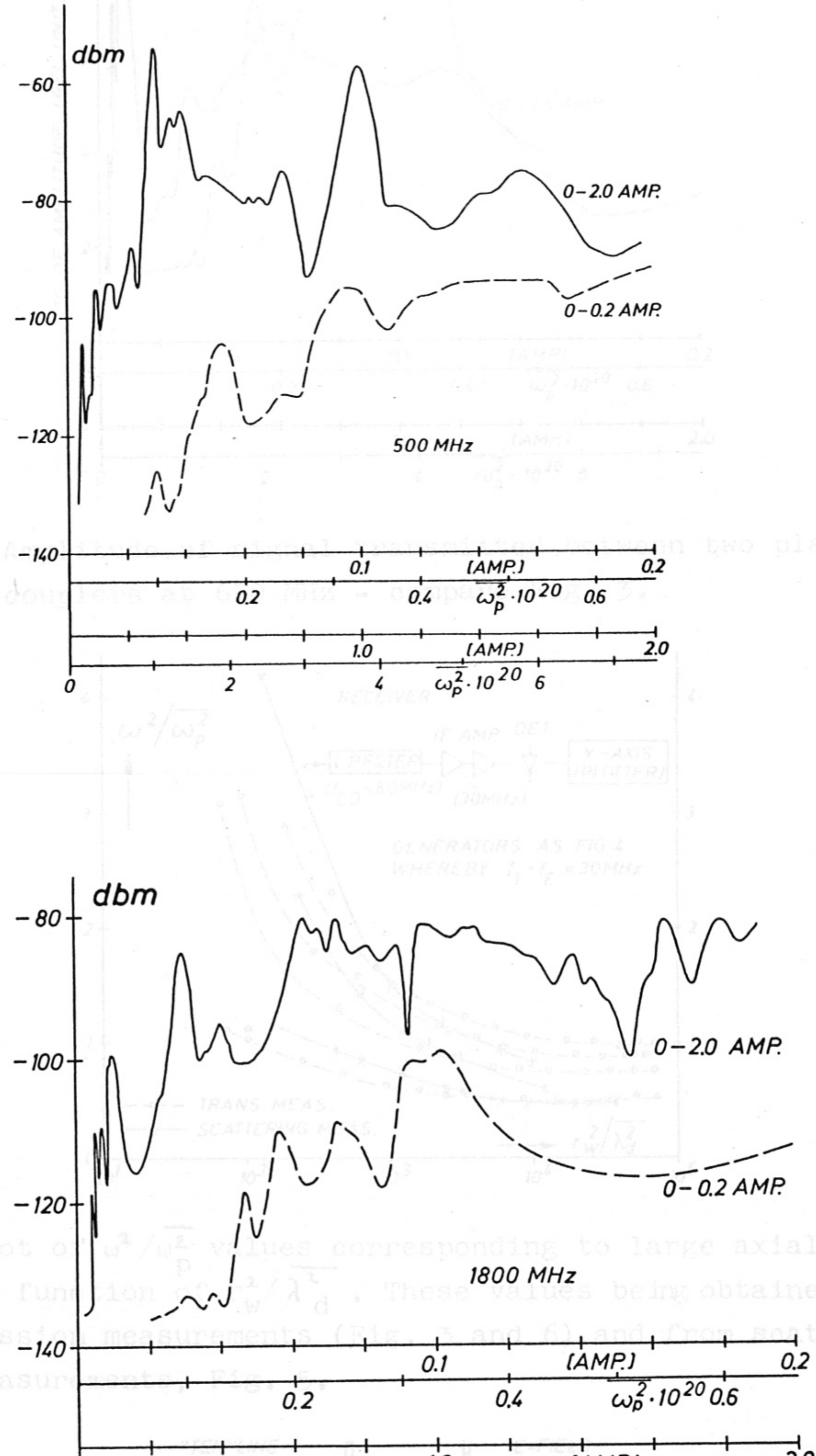

Fig. 5a,b Received signal levels at 500 and 1800 MHz as a function of discharge current (dbm = 10 log W where W is the power in milliwatts).

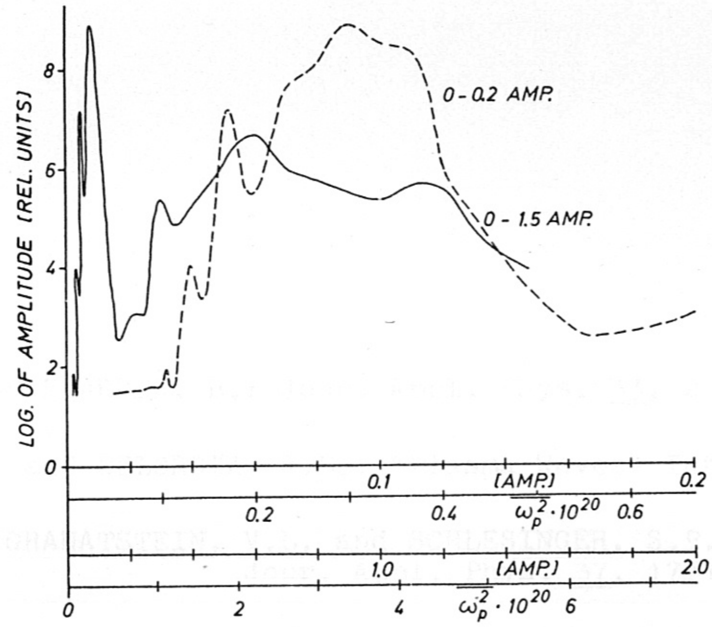

Fig. 6 Amplitude of signal transmitted between two plasma wave couplers at 650 MHz - compare Fig. 3.

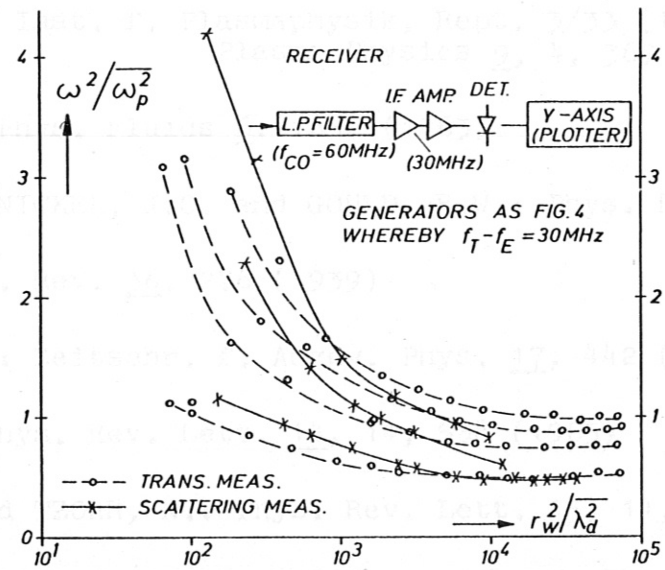

Fig. 7 Plot of $\omega^2/\overline{\omega_p^2}$ values corresponding to large axial wavelengths as function of $r_W^2/\overline{\lambda_d^2}$. These values being obtained from transmission measurements (Fig. 3 and 6) and from scattering measurements, Fig. 5.

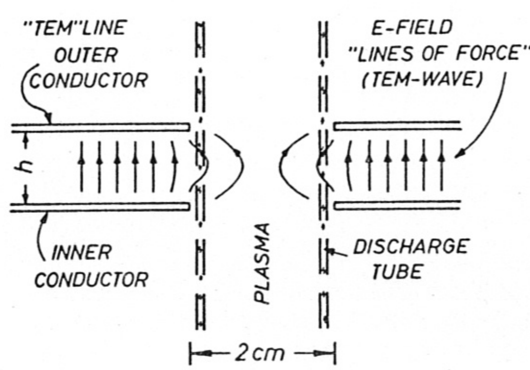

Fig. 8 Sketch of interaction region with electric field indicated.

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