# INSTITUT FÜR PLASMAPHYSIK

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On some Properties of the Radiation
near the Harmonics of the Electron
Gyro-Frequency in Plasmas

Über einige Eigenschaften der Strah-
lung im Bereich der Harmonischen der
Elektronen-Gyrofrequenz in Plasmen

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ABSTRACT:


The position of the lines emitted by an electron in a warm
plasma is calculated. From this result we deduce that the
anomalously high emission in a plasma is due in some
experiments to longitudinal waves, in some others to tran-
sverse waves. A mechanism for the coupling of the longitu-
dinal (and transverse) waves with the waves which go adiaba-
tically into vacuum is proposed. A reasonable number of su-
prathermal electrons can then account for the observed
radiation.

# 1. Introduction

One of the explanations for the high emission of a plasma
at the frequencies $\omega = n\omega_c$ ($n=1,2,..$) or near these is based
on the presence in the plasma of suprathermal electrons,
which should radiate longitudinal waves. The calculations
of Auer and Stone[1] and of Canobbio and Croci[2] are in
agreement with the experimental data, as far as the
dependence on $n$ of the amplitude of the radiated lines is
concerned, and could also account for the position and width
of the lines in a number of experiments (see [1]). A point
which is not yet clear is the transformation of the longitu-
dinal waves into transverse ones, because the condition

$$\frac{|grad\ \omega_p^2|}{\omega_p^2} \ll 1/\lambda$$    which is generally satisfied, seems to

cause the coupling coefficient to be too small.

Last but not least, Landauer's experiments, the first of
their kind, cannot be explained because in these the lines
are centered about $\omega = n\omega_c$, while the longitudinal waves
yield lines clearly shifted from the harmonics (see [1] and
later). We believe that a theory based on the instability of
electrostatic waves would be confronted with the same
difficulty: it is, moreover, improbable that the lines would
have the small width measured (1-2%).

Here we shall follow the method presented in [2] and show
that in the case of the transverse extraordinary waves the
radiation has the form of lines centered about $\omega = n\omega_c$ (the
shift being only of the order of $v_{th}^2/c^2$). We also present
a possible coupling mechanism of these waves and of the
longitudinal ones with the transverse waves, which have an
index of refraction such that $\lim_{\omega_p^2 = 0} N = 1$. Comparison with
the experiments shows that a reasonable number of supra-
thermal electrons can account for the emitted power observed.

## 2. Calculation of the emitted power

The power radiated by an electron per unit frequency interval in a uniform plasma with magnetic field $B_{os}$ is given by (see [2]):

$$-\frac{dE}{dt} = \frac{e^2}{\pi^2} \, \mathrm{Im} \left\{ \int_0^\infty \frac{d\omega}{\omega} \int d\bar{k} \sum_{m=-\infty}^{+\infty} \Pi_{\alpha\beta}^{(m)} \frac{D_{\alpha\beta}}{D} \, \delta(\omega - m\omega_e - k_{\shortparallel} v_{\shortparallel}) \right\}$$

where

$$\Pi_{\alpha\beta}^{(m)} \equiv \begin{pmatrix} m^2 \omega_e^2 \, J_m^2/k_\perp^2 & i\, m\omega_e v_\perp \, J_m J_m'/k_\perp & m\,\omega_e v_{\shortparallel} \, J_m^2/k_\perp \\ -i\, m\omega_e v_\perp \, J_m J_m'/k_\perp & v_\perp^2 \, J_m'^2 & -i\, v_\perp v_{\shortparallel} \, J_m J_m' \\ m\,\omega_e v_{\shortparallel} \, J_m^2/k_\perp & i\, v_\perp v_{\shortparallel} \, J_m J_m' & v_{\shortparallel}^2 \, J_m^2 \end{pmatrix} ;$$

the argument of the Bessel functions is $x = k_\perp v_\perp / \omega_e$;

$$\bar{k} = k_\perp \bar{e}_\perp + k_{\shortparallel} \bar{e}_3 \quad ; \qquad \bar{B}_0 = B_0 \bar{e}_3 \qquad ; \qquad v_{\shortparallel} \quad \text{and} \quad v_\perp$$

are the components of the test electron velocity; the plasma electron velocity distribution is assumed to be maxwellian with thermal speed $v_E$; $\omega_e$ is the electron gyrofrequency. $D_{\alpha\beta}$ is the adjoint of

$$\overline{D}_{\alpha\beta} \equiv \begin{pmatrix} (k - k_\perp)^2 \frac{c^2}{\omega^2} - \varepsilon_{11} & -\varepsilon_{12} & -k_{\shortparallel} k_\perp \frac{c^2}{\omega^2} - \varepsilon_{13} \\ \varepsilon_{12} & \frac{k^2 c^2}{\omega^2} - \varepsilon_{22} & -\varepsilon_{23} \\ -k_{\shortparallel} k_\perp \frac{c^2}{\omega^2} - \varepsilon_{13} & \varepsilon_{23} & (k - k_{\shortparallel})^2 \frac{c^2}{\omega^2} - \varepsilon_{33} \end{pmatrix}$$

$\varepsilon_{\alpha\beta}(\omega, \bar{k})$ is the dielectric tensor and $D \equiv \| \overline{D}_{\alpha\beta} \|$. Integration over $k_{\shortparallel}$ reduces to the substitution (in the $\varepsilon_{\alpha\beta}$'s):

$$\Delta = \frac{\omega - m\omega_e}{\omega} \frac{\sqrt{2}\, v_E}{v_{\shortparallel}} \quad ; \quad \zeta_s \equiv \zeta_s(m) = \frac{\omega - s\omega_e}{\omega - m\omega_e} \frac{v_{\shortparallel}}{\sqrt{2}\, v_E} .$$

Let us consider first the case $|v_\parallel| \lesssim v_t$. Then all the $\zeta_s(m)(s, m = 0, \pm 1, \ldots)$ are much smaller than 1. It follows (see [3]) that all the waves with $|N| \gg 1$, which are the only ones of interest here, are severely damped. When $|v_\parallel| \gg v_t$, let us write $\omega = (n + \delta)\omega_c$ with $|\delta| < 1/2$. Then not only the term $\zeta_n(n)$ is much larger than 1 (and therefore all the $\zeta_s(n)$ $(s = 0, \pm 1, \ldots)$), but also all the $\zeta_s(m)$ $(s = 0, \pm 1, \ldots)$ for which

$$\frac{v_\parallel}{v_t} \gg 1 + \frac{|m - n|}{|\delta|} .$$

It follows that the equation $D\{\zeta_s(n)\} = 0$ has real solutions (which correspond to non-damped waves), and that the same happens for all $D\{\zeta_s(m)\} = 0$ for which

$$1 + \frac{|m - n|}{|\delta|} \ll \frac{v_\parallel}{v_t} .$$

We can therefore neglect the terms $m \neq n$ if $|\delta|$ is sufficiently small, that is, if the frequency is sufficiently near a harmonic. We shall see that the energy is mostly radiated at frequencies such that only the term $m = n$ can be important, for reasonable values of $v_\parallel / v_t$. From now on, we shall therefore consider only this term.

When $D(\mu, \omega) = 0$ $\left(\mu = \frac{k_\perp^2 v_t^2}{\omega_e^2}\right)$ has real solutions, $\varepsilon_{13}$ and $\varepsilon_{23}$ are zero, and the same happens for $D_{13}$ and $D_{23}$.

It follows that

$$\mathcal{E}_n \equiv \left(-\frac{dE}{dt\,d\omega}\right)_{m=n} = \frac{e^2 \omega_e^2}{\omega\,v_t^2\,|v_\parallel|} \sum_i \left[\Pi_{\alpha\beta}\left(\frac{v_\perp}{v_t}\sqrt{\mu_i}\right)\frac{D_{\beta\alpha}(\mu_i)}{|D'(\mu_i)|}\right]_{k_\parallel = \frac{\omega - n\omega_c}{v_\parallel}}$$
$$(\beta, \alpha) \neq (1,3), (2,3)$$

where the sum is extended over the real solutions, $\mu_i$, of $D(\mu, \omega) = 0$. Averaging over the following distribution of the fast electrons

$$f_0 = \frac{a}{2\pi v^2} \, e^{-v_\perp^2/2v^2} \, \delta(v_{\shortparallel} - v_{\shortparallel}^0) \tag{1}$$

we obtain for the terms (1,1) and (2,2) (the others are analogous):

$$\langle \mathcal{E}_n^2 \rangle = \frac{e^2 w_e^2 a}{\omega' v_t^0} \sum_n \left[ \frac{D_{11}}{|D'|} \frac{n^2}{\mu_2} \, \Gamma_n \left( \frac{v^2}{v_{t^2}} \mu_2 \right) + \frac{D_{21}}{|D'|} \left( \frac{n^2}{\mu_2} \Gamma_n - 2\mu_2 \frac{v^4}{v_t^4} \Gamma_n' \right) \right]_{k_{\shortparallel} = \frac{\omega - n\omega_e}{v_{\shortparallel}^0}}$$

The function $\Gamma_n(x) \equiv e^{-x} I_n(x)$ has the following form for $|x| \ll 1$ :

$$\Gamma_n(x) \approx \left( \frac{x}{2} \right)^n \frac{1}{n!} \left( 1 + \left( \frac{x}{2} \right)^2 \frac{1}{n+1} \right) e^{-x} \, .$$

It has a flat maximum for $x \approx n^2$ , where it assumes the value $\Gamma_n \approx 1/n$ , then decreases as $1/\sqrt{x}$ .

The average of the Bessel functions can, however, have a different dependence on $n$ , with other $f_0$ . For example, with $f_0 \propto \delta(v_\perp - v_\perp^0)$ the first term goes as $n^{-2/3}$ , and with a more realistic $f_0$ it would probably behave in a yet more favourable way.

The factor $D'(k_r, \omega)$ can be zero, in the limit of no absorption. Then the group velocity

$$v_g = -2 \frac{v_t^2}{\omega_e^2} \frac{\frac{\partial D}{\partial \mu}}{\frac{\partial D}{\partial \omega}}$$

is also zero.

Taking account of the small imaginary part of $\mathcal{L}_n$

$$\delta \equiv \sqrt{\pi} \, e^{-v_{\shortparallel}^2/v_t^2} \, \frac{|k_{\shortparallel}|}{k_{\shortparallel}} \tag{2}$$

which would be responsible for negligible Landau damping, we can write for the terms $D_{11}$ and $D_{22}$

$$\langle \mathscr{E}_{11} \rangle = \frac{e^2 \omega \, d}{v_{11}^0} \sum_{\nu} \left[ \frac{D_{11}}{|D'_{min}|} \frac{\Gamma_n}{\mu_r} + \frac{D_{22}}{|D'_{min}|} \left( \frac{\Gamma_n}{\mu_r} - \frac{2\mu_r}{n^2} \frac{v^4}{v_{E^4}} \Gamma_n' \right) \right]_{k_i = \frac{\omega - n\omega_s}{v_{11}^0}}$$

Before we calculate $|D'_{min}|$ , we shall determine the frequencies and wavelengths where $v_{f} = 0$.

We remember that, under the condition $|\eta| \ll \frac{c^2}{4\nu^2 v_c^2}$ , where $\nu = \omega / \omega_e$ ,

$$\eta = 1 + \frac{\nu^2}{\mu} \frac{\omega_p^2}{\omega^2} \left( 1 + \nu \int_0^{\pi} e^{\mu \cos t} \sin \nu t \, dt + \nu \cos \nu \pi \int_0^{\infty} e^{-\mu \cosh t - \nu t} dt \right)$$

$$\eta \approx \begin{cases} 1 - \frac{\omega_p^2}{\omega^2} \frac{\nu^2}{\nu^2 - 1} - 3 \frac{\omega_p^2}{\omega^2} \frac{\mu}{\nu^4} & (|\mu| \ll \nu^2, \text{ and } \nu \gg 1 \text{ for the third term}) \\ \\ 1 + \frac{1}{k_\perp^2 \lambda_D^2} \left( 1 - \frac{\nu^2}{\mu} \right) & (|\mu| \gtrsim \nu^2 \text{ or } |\mu| \gg 1) \end{cases}$$

the dispersion relation separates into the following three equations

$$\begin{cases} b \Gamma_\nu = \frac{\omega_p^2}{\omega^2} (\eta^2 - \eta) + \frac{\nu^2}{\mu} \\ \\ b \Gamma_\nu = -\frac{\omega^2}{\omega_p^2} \frac{D_1}{D_2} \\ \\ b \Gamma_\nu = -\frac{\omega^2}{\omega_p^2} \frac{D_o}{D_2} \end{cases}$$

(3)

(4)

where

$$b = -\frac{\pi \nu}{\tan \pi \nu}$$

$$D_o = \frac{(\eta - 1)^2}{\nu^2} + \eta \left( \eta^2 - \eta + 2 \frac{\mu}{\nu^2} (\eta - 1) \right)$$

$$D_1 = 2(\eta - 1)\beta + \frac{\nu^2}{\mu} \left( \eta^2 - 2\eta + 2 \frac{\mu}{\nu^2} (\eta - 1) \right) + 2\eta \beta \mu$$

$$D_2 = \nu^2 \left( \beta^2 + 2\beta - \frac{\nu^2}{n^2} \right)$$

$$\beta = \Gamma_L' / \Gamma_\nu \ .$$

(see [3]).

The first equation corresponds to transverse ordinary waves. Then $\nu_{\tilde{g}}$ can be zero for $\lambda = \frac{r_L}{n}$ and $\frac{|\nu - n|}{\nu} \approx \nu \frac{v_t^2}{c^2} \frac{\omega^2}{\omega_p^2}$. However, with the small imaginary part (2), $\left| \frac{D_{33}}{D'} \right|$ is smaller than $\left| \frac{D_{22}}{D'} \right|$ and $\left| \frac{D_{11}}{D'} \right|$ (calculated for the transverse extraordinary and the longitudinal waves, respectively) by a factor $\nu^2 \frac{v_t^2}{c^2}$ and can therefore be neglected. The second equation corresponds to quasi-transverse waves:

$$\left| \frac{E_y}{E_x} \right| = \left| \frac{D_{12}}{D_{11}} \right| = \left| - \frac{\varepsilon_{12}}{\varepsilon_{11}} \right| \gg 1 \ ;$$

$$\text{for} \quad \lambda \approx \frac{r_L}{n} \ , \qquad i \frac{E_x}{E_y} \approx \frac{\eta - 1}{\nu} \frac{v_t^2}{c^2} \ .$$

(5)

The condition $\nu_{\tilde{g}} = 0$ is satisfied for

$$\lambda \approx \frac{r_L}{n} \qquad \text{and} \qquad \frac{|\nu - n|}{\nu} \approx \nu \frac{v_t^2}{c^2} \frac{\omega_p^2}{\omega^2} \ .$$

(6)

The third equation corresponds to quasi-longitudinal oscillations because

$$\left| \frac{E_x}{E_y} \right| = \left| \frac{D_{12}}{D_{11}} \right| \gg 1 \ ; \qquad \text{for} \quad \lambda \approx \frac{r_L}{n} \ , \qquad i \frac{E_x}{E_y} \approx \nu N^2 \ ;$$

(7)

Using the approximations given in [3] for $\beta = \Gamma_L'/\Gamma_\nu''$, one finds for the waves (4) a $\lambda$ somewhat smaller, but of

the order of $\frac{r_L}{h}$ and $\frac{|\tan \pi \nu|}{\pi \nu} \approx \frac{1}{\nu |\eta|}$.

The maximum of the radiation is therefore clearly shifted from the harmonics.

A particularly interesting case is that of the quasi-longitudinal waves in the neighbourhood of the hybrid frequency $\omega^2 = \omega_p^2 + \omega_e^2$. When $\eta$ is so small that $|\eta \nu^2 / N^2| \ll 1$, it follows from eq. (3) that

$$\lambda \approx \frac{r_L}{n} \qquad ; \qquad \frac{|\tan \pi \nu|}{\pi \nu} \gg 1 \tag{8}$$

in agreement with (7). The lines are then centered about $\omega = (m + 1/2) \omega_e$. In order to determine now $|D'_{min.}|$ we calculate the modification of the solutions $\mu_r$ of $D = 0$, due to the presence of the small imaginary part $\delta$. From (3) and (4) (we do not consider the transverse ordinary waves) it follows that the modified solutions are approximately

$$\mu_r + i \sqrt{1 + i\delta} \; \mu_r = (1 + i \sqrt{i\delta}) \mu_r$$

for both transverse extraordinary and longitudinal waves. Developing now in a Taylor series the function $D'(\mu_r (1 + i\sqrt{i\delta}))$ we obtain

$$|D'_{min.}| \approx \frac{|\eta|}{\nu^2} \sqrt{\delta}$$

for the longitudinal waves (9)

$$|D'_{min.}| \approx \frac{c^2}{v_t^2} \frac{\sqrt{\delta}}{\nu^2}$$

for the transverse extraordinary waves.

In a non-uniform plasma these formulae are approximately valid as long as in their Taylor expansion at the density

which satisfies (7) or (8), the first term is the domi-
nant one. In this way it can be shown that (9) are appro-
ximately valid in the interval

$$\tilde{\omega}_p^2 < \omega_p^2 < \tilde{\omega}_p^2 + \Delta \omega_p^2$$

where (10)

$$\Delta \omega_p^2 \approx \delta \omega_p^2 .$$

The energy emitted per unit time per unit frequency by
electrons uniformly distributed with density $d$ in a
region of width $\ell$ is: (one can, of course, neglect the
small variation of the density over a Larmor radius)

$$\omega_c \frac{\ell}{v} < \mathcal{E}_n > d\, r_L = d \ell < \mathcal{E}_n > .$$

Choosing for $\ell$ the length defined by (10), we obtain

$$d \frac{\Delta \omega_p^2}{\text{grad } \omega_p^2} < \mathcal{E}_n > = \delta d \frac{\omega_p^2}{\text{grad } \omega_p^2} < \mathcal{E}_n > \equiv E_n .$$

The imaginary part of D appears now in the numerator.
From eqs. (5) and (7) it follows that the most important
term for the transverse waves is that proportional to
$D_{22}$ ; for the longitudinal waves it is the one
proportional to $D_{11}$ .


We can therefore write

$$E_{n,L} \approx \frac{e^2 \omega}{v_\parallel^o} d \frac{\omega_p^2}{\text{grad } \omega_p^2} \frac{c^2}{v_t^2} \frac{v^2}{|\eta|} \sqrt{\delta} \frac{\overline{i_n}}{\mu_r}$$

$$E_{n,T} \approx \frac{e^2 \omega}{v_\parallel^o} d \frac{\omega_p^2}{\text{grad } \omega_p^2} v \sqrt{\delta} \frac{\overline{i_n}}{\mu_r} .$$

$E_{n,L}$ is larger than $E_{n,T}$ by a factor $c^2/v_t^2$.

However, it will be shown that only a fraction $v_t^2/\nu c^2$ of

$E_{n,L}$ gives rise to a Poynting vector, the rest
corresponding to polarization losses. In fact, from the
Maxwell equations it follows for the electric displacement

$$D_r = - 4\pi i\, F^{-1} \left\{ \mathcal{E}_{rs} \frac{D_{st}}{D} \frac{j_t}{\omega} \right\}$$

where $F^{-1}\{\cdot\}$ indicate the inverse Fourier transform, and

$$\vec{j} = e\ F\{\vec{v}(x,t)\}.$$

Taking for simplicity $v_{\shortparallel} = 0$ , at the time where $\vec{v} \equiv v_x$ we obtain

$$D_x \approx 4\pi i\ F^{-1}\{\tfrac{j_x}{\omega}\}$$

that is,

$$\frac{\partial D_x}{\partial t} \approx -4\pi e\ v_x ,$$

where the terms neglected are of the order

of $\left|\dfrac{D_{\perp}}{D_{\shortparallel}}\right| \approx \dfrac{v_t^2}{v^2 c^2}$ (from (6)).

From the fact that the energy emitted is $-e\ \vec{v}\cdot\vec{E}$ , it follows that the divergence of the Poynting vector is of

the order $\dfrac{v_t^2}{v c^2} < \mathcal{E}_{n,L} >$ and that the largest part of

the emitted energy is due to polarization losses.

The ratio of the Poynting vectors connected with $< \mathcal{E}_{n,T} >$ and $< \mathcal{E}_{n,L} >$ is therefore

$$\frac{S_{n,T}}{S_{n,L}} \approx v.$$

With $\omega = 6.14 \times 3.5 \times 10^{10}\ s^{-1}$ , $v_{\shortparallel}^0 = 3 \times 10^8\ cm\ s^{-1}$, we obtain: and $c^2/v_t^2 = 2 \times 10^5$

$$E_{n,T} \approx 1.5 \times 10^{-11}\ \frac{\sqrt{\delta}}{n}\ d\ \frac{\omega_p}{grad\ \omega_p}\qquad [erg\ cm^{-2}].$$

## 3. The transmission coefficient

We shall develop here an idea touched on in [3].
The only wave which has an index of refraction such that

$$\lim_{\omega_p^2 = 0} N = 1$$

is the transverse "macroscopic" one, with

$$N^2 \equiv N_+^2 = \eta - \frac{(\eta-1)^2}{\nu^2} .$$

(for $\eta$ one has to take the limit $|\mu| \ll 1$ ).

$N_-$ is real if

$$\omega_p^2 < \omega (\omega - \omega_e) \tag{11}$$

and in the neighbourhood of the hybrid frequency, namely for

$$\omega (\omega + \omega_e) > \omega_p^2 > \omega^2 - \omega_e^2 . \tag{12}$$

The longitudinal and transverse waves (3-4) have a real index of refraction only for densities larger than a critical density where $v_g = 0$ , as eqs. (3) and (4) show.

Because the emitted radiation has a wavelength $\lambda \approx \frac{r_L}{n}$ and the density gradients in a plasma are generally such that

$$\frac{|grad \; \omega_p^2|}{\omega_p^2} \ll \frac{1}{r_L} ,$$

we are usually allowed to use the WKB method. The emitted waves will therefore propagate back and forth in a plasma column defined by the density corresponding to $v_g = 0$ for the considered frequency. In the neighbourhood of $v_g = 0$ however, the WKB method is no longer valid, because then

$$\frac{dk}{dx} = - \frac{\omega_e^2}{2 v_t^2} \; \frac{\frac{\partial D}{\partial x}}{\frac{\partial D}{\partial \mu}} \quad \propto \frac{1}{v_g}$$

becomes infinite. At these frequencies (where at the same time the radiation reaches its maximum) we can expect good coupling with the other waves.

Because in the neighbourhood of the density where

$v_g = 0$ the equation $\left|\frac{\partial n}{\partial x}\right| \lambda \gg 1$ is satisfied, we

can consider this region as infinitely thin and treat it as a surface separating two mediums with different indices of refraction.

The only waves which can be coupled by this mechanism
with the "macroscopic" one are those with a frequency
such that $\nu_{\not{y}}$ is zero for densities in the interval (11)
(not in (12), because the condition $\quad k \lambda_0 < 1$

for $\lambda \approx \frac{r_L}{n}$ corresponds to $\omega_p^2 > \omega^2$ . Note also that

in the interval of $\omega_p^2$ about $\omega^2 - \omega_c^2$ , where
$|\eta \nu' N^2| << 1$ , the "macroscopic" wave goes into a
quasi-longitudinal one (see [3])).

That is, those with

$$\frac{|\nu - n|}{\nu} \approx \nu \frac{v_E^2}{c^2}$$

for the transverse waves, and

$$\left|\frac{\tan \pi \nu}{\pi \nu}\right| \geq \frac{\nu - 1}{\nu}$$

for the longitudinal waves.

Let us now take the plane $x = 0$ as the surface where $\nu_{\not{y}} = 0$.

Then if we assume that for $x < 0$ there is only an incident
and a reflected quasi-longitudinal (or transverse extra-
ordinary) wave, and for $x > 0$ a "macroscopic" transverse
wave, it follows from the Maxwell equations that the
y-component of the "macroscopic" electric field in the
second medium is approximately equal to the y-component of
the incident electric field, because $N_-$ is much smaller
than the indices of refraction which follow from (7) and
(8).

For the contribution to the Poynting vector in the second
medium from the quasi-longitudinal and the transverse
waves respectively, in the case where $f_0$ is given by (1),
we can then write

$$S_{n,L} \approx \frac{e^2 \omega}{v_\parallel^0} d \frac{\omega_p^2}{\text{grad} \, \omega_p} \frac{1}{|\eta|} \frac{\sqrt{\delta}}{\nu} \Gamma_n N_- \frac{v_E}{c}$$

$$S_{n,T} \approx \frac{e^2 \omega}{v_\parallel^0} d \frac{\omega_p^2}{\text{grad} \, \omega_p} \sqrt{\delta} \, \Gamma_n N_T \frac{v_E}{c}$$

The passage of the energy through the region where $N_T^2 < 0$
(that is, where $\omega(\omega - \omega_e) < \omega_p^2 < \omega^2 - \omega_e^2$ )      should not
cause difficulties, since the dependence of $Im\{N_T\}$
on $\omega_p^2$ is such that the absorption in this region
should be negligible . A density of the fast electrons
of the order $10^7 \, cm^{-3}$ is enough to explain the
observed power.

We conclude that experiments like those of Landauer,
where the lines are very thin and centered about $\omega = n\,\omega_e$
cannot be explained in terms of longitudinal waves. We
believe that they cannot be explained either in terms
of a possible theory of electrostatic instability, for
the same reason. The radiation due to transverse waves,
on the contrary, seems to correspond to Landauer's
experiments.

It is probable that in [4] both kinds of waves have been
observed.

## Acknowledgments

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