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Abstract: A toroidal thetapinch discharge with superimposed hexapole field is investigated. The characteristic data of the discharge are: 12 kG maximum field, 4o eV maximum temperature and densities of about $1o^{16}$ cm^{-3}. The plasma loss rates are determined by measurements of particle density, temperature and plasma radius. The measured loss rates are compared with loss rates of models which account for cusp losses, resistive losses and others. Measured loss rates are consistent with those expected from resistive models. Furthermore the results prove that a toroidal thetapinch with superimposed hexapole field provides a stable toroidal high beta equilibrium configuration at least during the times investigated and in the temperature range from 1o to 4o eV.

Introduction

Attempts to force a thetapinch plasma into a closed toroidal equilibrium configuration, without resulting azimuthal currents, have shown that the confinement time increases as the theoretically postulated plasma surface (M+S-surface) is approached [1] . Further improvement would depend on exact knowledge of the required external magnetic field and - due to the initial compression - of the required discharge vessel geometry. Another question concerns the stability in regions of disadvantageous field curvature. Both problems are investigated separately experimentally and theoretically [2] . On the other hand they may be avoided by superimposing magnetic multipole fields over the M+S-configuration. Preliminary experiments appeared to be promising. The lack of magnetic field closure inherent in these configurations, however, should lead to additional plasma losses of the mirror or cusp type.
Consequently the superposition of such multipole fields over a purely azimuthal compression field was investigated. Hexapole fields appeared to be the most advantageous configurations [3] . Two toroidal thetapinch discharges, "Lupus" and "Spinne", with the same geometry of vessel and hexapole field cover a range of peak fields (crowbarred) from 1o to 2o kG and temperatures up to 1oo eV. In the present report plasma loss rates are determined for fields and temperatures up to 12 kG and 4o eV and are compared with those expected from theoretical models.

Apparatus

"Lupus" and "Spinne" are toroidal thetapinches with the same geometry R = 26 cm major and r = 3 cm minor radius. The coil of "Lupus" has eight turns and that of "Spinne" one turn. The field rise $\dot{B}$ is $1.3\cdot 1o^{9}$ up to $5\cdot 1o^{9}$ G/s. The hexapole field is generated by six conductors between the coil and the pyrex glass vessel. Some data of the discharge circuits are given in Table I. Each circuit is crowbarred and its energy is characterized by the voltages applied to the condensor bank, e.g. 8/12 kV means 8 kV at the thetapinch bank, 12 kV at the hexapole bank. Both circuits have approximately the same discharge frequency and are fired at the same time.

Measurements

The plasma radius is obtained from streak pictures (integral light) and from scanning the continuum intensity across the plasma. A streak picture and contour lines are shown in Figure 1. The values indicated at the contour lines give the fraction of light intensity relative to the spatial maximum at each time. As the plasma is partially hidden behind the hexapole conductors, the calculation of the plasma radius is subject to a relatively high probable error. Therefore during the first microseconds when the plasma radius changes very rapidly, plasma losses are not evaluated. At later times the plasma radius is nearly constant, and the error $\Delta r_p/r_p$ is estimated to be $^{+3o}_{-25}$ %.

The electron temperature of the "Lupus" plasma (Figure 3) is obtained from the ratio H_β /continuum. Viewing parallel to the plane of the torus the signal remains unaffected by the plasma drift in the case without hexapole field. The temperature on the axis is higher by a factor of two [4] as compared to the value of the electron temperature when the light is integrated along the line of sight. So in Figure 3 twice the value of the electron temperature averaged over the diameter is plotted. The plotted temperature is representative for the temperature of the centre of the plasma and could have been determined by applying the Abel inversion to a scan across the plasma.

The electron temperature of the centre of the "Spinne" plasma (Figure 1o) is obtained in three time intervals of the main discharge. The first part of the temperature curve is evaluated from the time of maximum intensity of line radiation of oxygen impurities (Table II) [5] using the rate coefficients given by Hinnov et al. [6] . Near the current maximum at t = 3 μs the temperature curve is continued under the assumption of adiabatic compression. For $t > 6$ μs twice the value of the temperature calculated from the H_β /continuum ratio is taken as above for the "Lupus" plasma. The error $\Delta T/T$ is estimated to be $^{+7o}_{-4o}$ %.

The relaxation time of proton-electron collisions is less than o.2 μs, hence the temperatures of ions and electrons are nearly equal.

The average electron density n is determined by absolute measurements of a continuum band 1oo Å wide, centered at 522o Å for the "Spinne" discharge and at 5285 Å for the "Lupus" discharge, and in the "Lupus" discharge also by H_β Stark broadening. If we denote the local electron density by n'(r), the line density by N and the minor radius of the vacuum vessel by r_o, the plasma radius r_p and an average electron density n are defined as follows:

$$N = n\pi r_p^2 = 2\pi \int_0^{r_o} n'(r)\, r\, dr, \qquad (1)$$

$$n^2 \cdot \frac{1}{2 r_p} \cdot 2 \int_0^{r_o} n'^2(r)\, dr. \qquad (2)$$

This definition of n is reasonable from a practical point of view, as the measured intensity of the continuum radiation is proportional to the integral in Equation (2). It leads to the following expression for r_p:

$$N^2 = \frac{(\pi r_p^2)^2}{r_p} \int_0^{r_o} n'^2(r)\, dr = 4\pi^2 \left(\int_0^{r_o} n'(r)\, r\, dr \right)^2$$

$$r_p = 2 \left[\frac{\left(\int_0^{r_o} n'(r)\, r\, dr \right)^2}{2 \int_0^{r_o} n'^2(r)\, dr} \right]^{1/3} . \qquad (3)$$

At some typical instants the integral $\int n'(r)\, r\, dr$ was evaluated by scanning the continuum intensity across the vacuum vessel and applying the Abel inversion - assuming of course rotational symmetry. Considering relevant density-profiles the value of r_p given by Equation (3) corresponds to the 20% contour line of the relative continuum-intensities (see Figure 1).

Figure 6 shows the measured electron densities in "Lupus" discharges of 12/0 and 12/12 kV thetapinch/hexapole bank voltages. Figure 13 shows the electron density of a "Spinne" discharge 8/12 kV measured at two positions. Figure 7 and Figure 14 show the respective line densities. The slopes used for the evaluation of plasma loss rates are indicated.

Electron density, electron temperature and magnetic field yield the $\beta = \frac{4\mu_0 n kT}{B_e^2}$ of the "Lupus" (Figure 8) and the "Spinne" (Figure 15) discharge. Whereas the β of the "Lupus" discharge falls rapidly to a rather low value, the β of the "Spinne" discharge is found to be near unity during the first six microseconds.

As only the relative change of electron density with time, $\dot{n}/n$, is of interest, most systematic errors (absolute calibration, influence of ff- and fb-radiation due to impurities) cancel out. From a comparison of $\dot{n}/n$ measured by Stark broadening and by continuum radiation the error $\frac{\Delta \dot{n}/n}{\dot{n}/n}$ is estimated to be 20%. According to the density measurements at two positions 4 mm apart (Figure 13) $\dot{n}/n$ is not very sensitive to a displacement of the plasma. Changes induced by varying β and/or changes of the pressure gradient as well as a change of the ionization stage of impurities can contribute to the error of $\dot{n}/n$ and of r_p. The discussion of this point leads to the conclusion that this error is covered by the limit given below.

The experimental loss rate is determined by

$$\nu_{exp} = -\frac{\dot{N}}{N} = -\left(\frac{\dot{n}}{n} + 2\frac{\dot{r}_p}{r_p}\right).$$

The error $\Delta\nu_{exp}/\nu_{exp}$ is mostly due to the uncertainty of $\dot{r}_p$ and is estimated according to the errors given (Table III) to be $^{+100}_{-50}$ %.

The results of the measurements and the related errors are summarized in Table III and plotted in Figure 16.

Theories

In a toroidal hexapole cusp geometry neglecting the azimuthal field B_φ, particle losses may be described by a flux of particles of density n and mean thermal ion velocity $\bar{v}$ through six slits of width δ. This model defines the loss width δ. Per unit length of the torus the change of line density N is then

$$\dot{N} = -6 n \bar{v} \delta.$$

Consequently the loss rate is

$$\nu = -\frac{\dot{N}}{N} = \frac{6 n \bar{v} \delta}{n\pi r_p^2} = \frac{6\bar{v}\delta}{\pi r_p^2}.$$

In the case of $\beta = 1$ and zero resistivity η, δ is of the order of the ion Larmor radius ρ_i [7,8,9]

$$\rho_i = \frac{m_i \bar{v}}{eB},$$

and the loss rate is

$$\nu_i = \frac{6\,\bar{v}\,\varrho_i}{\pi\, r_p^2} = \frac{6 m_i\, \bar{v}^2}{\pi\, r_p^2\, e B} = \frac{18\, k\, T_i}{\pi\, r_p^2\, e B}\,.$$

Taking into account the electric field in the plasma boundary layer, β turns out to be of the order of the electron Larmor radius ϱ_e [8,1o]

$$\varrho_e = \frac{m_e\, \bar{v}_e}{e B}\,,$$

and the loss rate becomes

$$\nu_e = \frac{6\,\bar{v}\,\varrho_e}{\pi\, r_p^2} \quad \frac{18\, k \sqrt{T_e T_i}}{\pi\, r_p^2\, e B} \sqrt{\frac{m_e}{m_i}}\,.$$

In Figure 16 both ν_i and ν_e are plotted versus temperature $T = T_e = T_i$ for the respective magnetic fields and plasma radii of our experiments. The magnetic field is of the order of 1o kG, the plasma radius is of the order of 1 cm.

If the azimuthal field B_φ is not neglected - still assuming $\beta = 1$ and zero resistivity - the loss rate is reduced by a factor of

$$\psi = \sqrt{1 + (B_\varphi / B_\vartheta)^2} = \frac{1}{\sin\alpha}\,,$$

where B_φ and B_ϑ are the respective magnetic field strengths near the plasma surface and α is the angle between the centre line of the plasma (vacuum vessel) and the direction of the magnetic field near the plasma surface. In our experiments ψ is about 1o on the plasma surface.

The respective loss rates are then

$$\nu_{i\psi} = \nu_i / \psi\,,$$

$$\nu_{e\psi} = \nu_e / \psi\,.$$

If there is a trapped azimuthal field B_p in the plasma region $r < r_p - \beta < 1$ but still assuming zero resistivity - the loss rate will be further reduced. According to this model losses are reduced by a factor of three for $\beta = 0.9$ and a factor of eight for $\beta = 0$ [9] . In our experiments there probably always is some trapped field and $\beta < 0.9$. Thus the loss rate due to a cusp slit width of an ion Larmor radius is reduced to

$$\nu_{i\beta} \approx \nu_i / 3\psi \approx \nu_e\,.$$

Electric fields in the boundary layer as quoted above have not been taken into account as yet.

According to Bickerton [11] the thickness of the current-carrying layer between the plasma ($\beta = 1$) and the magnetic field is

$$\delta_c = \sqrt{\frac{2\,\eta_\perp\,\tau}{\mu_0}}\,.$$

τ is the lifetime of the plasma that escapes along the field lines

$$\tau = \frac{\ell}{\bar{v}}$$

where ℓ is the length inside the vacuum vessel of field lines near the plasma surface. In our experiments ℓ is of the order of 5o cm.

In this case the particle loss is

$$\dot{N} = - \frac{2\pi r_p \delta_c \frac{n}{2}}{\tau} = -\pi r_p n \sqrt{\frac{2\eta_\perp}{\mu_o \tau}}$$

and the loss rate due to resistive cusp losses turns out to be

$$\nu_{r\vartheta} = -\frac{\dot{N}}{N} = \frac{1}{r_p}\sqrt{\frac{2\eta_\perp}{\mu_o \tau}} = \frac{1}{r_p}\sqrt{\frac{2\eta_\perp \bar{v}}{\mu_o \ell}} \; ; \; \eta_\perp = \frac{129 \ln\Lambda}{T^{3/2}} .$$

If there is a trapped azimuthal field B_p in the plasma region $r < r_p$, $\nu_{r\vartheta}$ will be further reduced as the plasma has to cross the field lines B_p by classical diffusion.

Considering the azimuthal field B_φ only (linear thetapinch of infinite length), plasma losses due to classical diffusion are described by $\dot{N} = -n\, v_\perp 2\pi r_p$. The resistive diffusion velocity is $v_\perp = -(\eta_\perp \nabla p)/B_m^2 \cdot \nabla p$ is estimated from our experiments to be $\nabla p \approx -p/r_p$. The intermediate value B_m^2 in the plasma boundary layer is $B_m^2 \approx B_e^2\,(1 - \beta/2)$, where B_e is the external field and $\beta = 2\mu_o\, p/B_e^2$. Thus the loss rate due to classical diffusion only is

$$\nu_{r\varphi} = -\frac{\dot{N}}{N} = \frac{2\pi r_p n \eta_\perp p}{\pi r_p^2 n r_p B_e^2 (1-\beta/2)} = \frac{2\eta_\perp}{\mu_o r_p^2 (2/\beta - 1)} .$$

In replacing $v_\perp$ by the Bohm diffusion velocity $v_B = (3.9 \cdot 10^{17} \nabla p)/n\, B_m$ (m kg s V A - Units) the corresponding loss rate is

$$\nu_B = 3.9 \cdot 10^{17} \frac{4kT}{r_p^2 B_e \sqrt{1-\beta/2}} ,$$

Which is nearly proportional to ν_e and of the same order of magnitude.

In the low beta case plasma confinement might be limited by free thermal expansion along the field lines intersecting the wall with a corresponding loss rate $\nu_{th} = 1/\tau = \bar{v}/\ell'$.

In our experiment field lines intersecting the region $r < r_p$ have an average length ℓ' of about 1 m inside the vacuum vessel.

Discussion

In Figure 16 measured loss rates (with error bars) are compared with various theoretical curves. The experimental data agree best with the resistive diffusion models characterized by $\nu_{r\varphi}$ and $\nu_{r\vartheta}$, though Bohm diffusion cannot be excluded. In the low beta case there is also good agreement with the model assuming free expansion along field lines characterized by ν_{th}.

To get further information on ν_{exp} our investigations are being extended to higher temperatures.

The results show furthermore that a toroidal thetapinch with superimposed hexapole field provides a stable toroidal high beta equilibrium configuration at least during the times investigated, and in the temperature range from 1o to 4o eV.

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		" Lupus "			" Spinne "		
		preion-ization	main	hexa-pole	preion-ization	main	hexa-pole
bank voltage U	(kV)	25	12	12	25	8	12
bank energy E	(kJ)	o.6	9	9	o.9	12	18
time to current maximum $\tau/4$	(μs)	1.4	1o	6.5	o.35	3.1	2.7
vacuum field B_{max}	(kG)						
at r = 3 cm		-	13	8.3	-	1o.3	7.o
at r = 1 cm		-	13	1.2	-	1o.3	1.o

Table I : Characteristic data of the electric circuits.

ionization stage	line (Å)	energy (eV) exit.	energy (eV) ioniz.	time of max. intensity (μs)
O^{II}	3749	26	35	0.40
O^{III}	2984	38	55	0.70
O^{IV}	3063	48	77	0.85
O^{V}	2781	72	113	1.55
O^{VI}	3811	82	137	2.1

Table II : Oxygen impurity lines used for the evaluation of the electron temperature (experiment "Spinne").

	"Spinne"				"Lupus"	
	S_1	S_2	S_3	error ± %	L	error ± %
t (μs)	4	8	12	-	10	-
r_p (cm)	0.92	0.85	0.78	+30 -25	0.97	+30 -25
T_e (eV)	40	21	13	+70 -40	5.4	+70 -40
B (kG)	9.1	6.3	5.2	10	12.5	10
n_e (10^{16} cm^{-3})	2.24	1.55	0.93	+30 -25	0.96	+30 -25
	0.87	0.66	0.36	+70 -40	0.027	+70 -40
$\dot{n}/n$ (10^4 s^{-1})	-7.0	-11.7	-13.5	20	-3.0	+40 -30
$\dot{r}_p/r_p$ (10^4 s^{-1})	-2.2	-1.8	-1.6	100	-	-
ν_{exp} (10^4 s^{-1})	11.4	15.3	16.7	+100 - 50	3.0	+100 - 50
$\nu_{r\varphi}$ (10^4 s^{-1})	5.9	11.0	11.3	+100 - 50	1.44	+100 - 50
$\nu_{r\vartheta}$ (10^4 s^{-1})	12.9	18.5	25.2	+100 - 50	-	-
ν_{th} (10^4 s^{-1})	-	-	-	-	3.9	+100 - 50

Table III: Results of the experiments and calculated loss rates ν_{exp}; $\nu_{r\varphi}$; $\nu_{r\vartheta}$ and ν_{th}.

Figure captions:

Fig. 1: Streak picture in integral light and continuum intensity contour lines of a discharge in "Spinne" at 4o μ hydrogen. To the left: schematic diagram of the torus with hexapole conductors; the thetapinch coil is not shown.

Fig. 2 through 8: Results of "Lupus" discharges at 4o μ hydrogen with and without hexapole field.

Fig. 9 through 15: Results of "Spinne" discharges at 4o μ hydrogen with hexapole field.

Fig. 16: Measured loss rates ν_{exp} with error bars. The theoretical curves are adjusted for the discharge parameters of our experiments and therefore are not straight lines. Best fit seems to be classical diffusion across the azimuthal field B_φ resulting in a loss rate $\nu_{r\varphi}$ (small circles) and resistive cusp losses resulting in a loss rate $\nu_{r\vartheta}$.

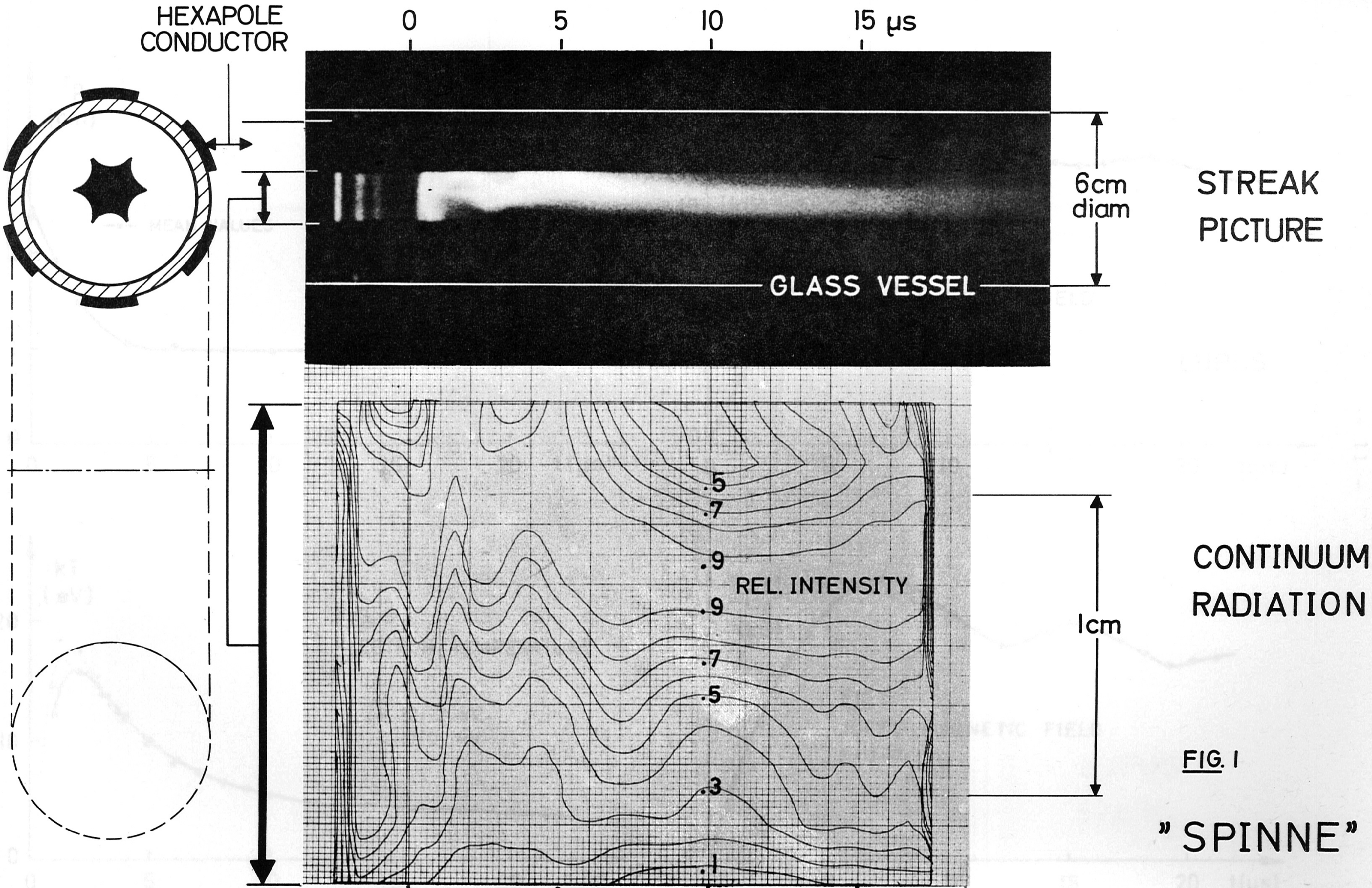

FIG. 1

"SPINNE"

40 μ H_2; 8/12 kV

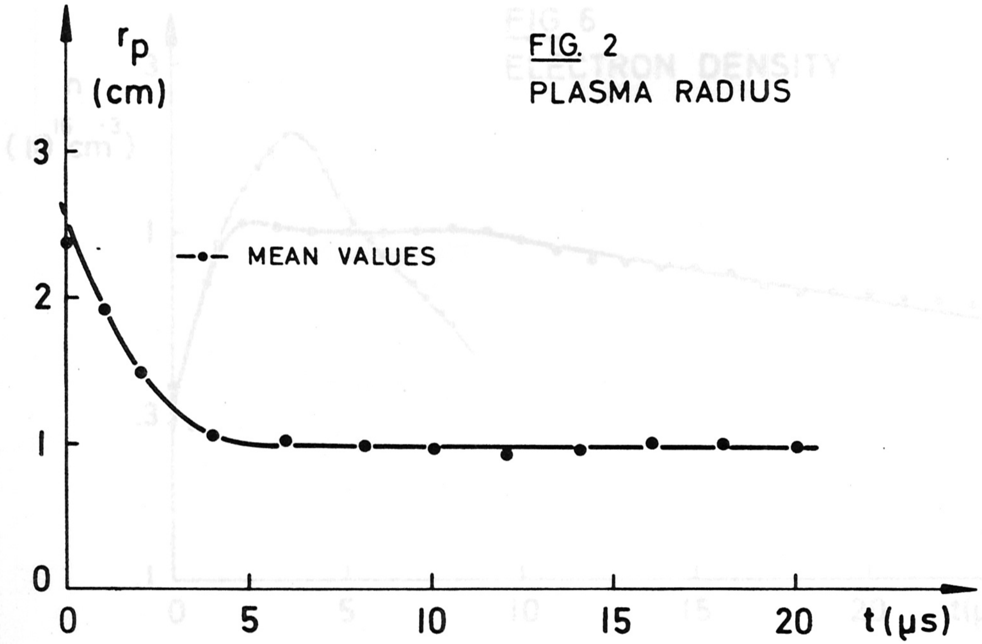

FIG. 2
PLASMA RADIUS

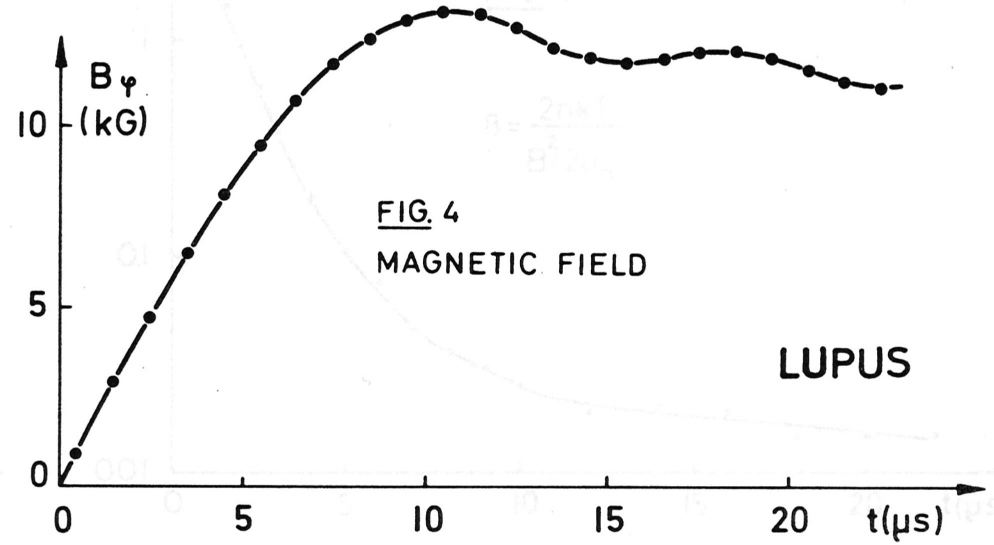

FIG. 4
MAGNETIC FIELD

LUPUS

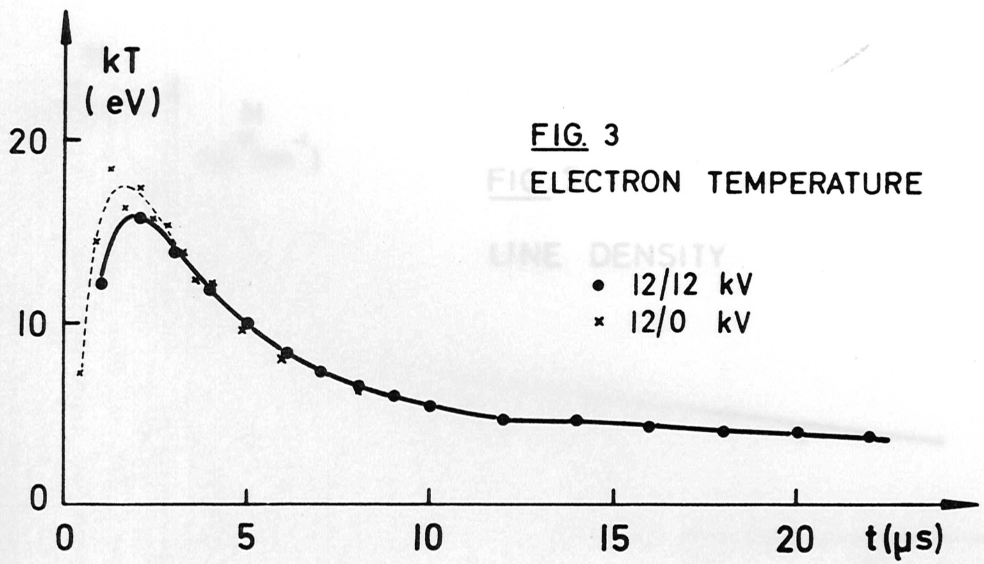

FIG. 3
ELECTRON TEMPERATURE

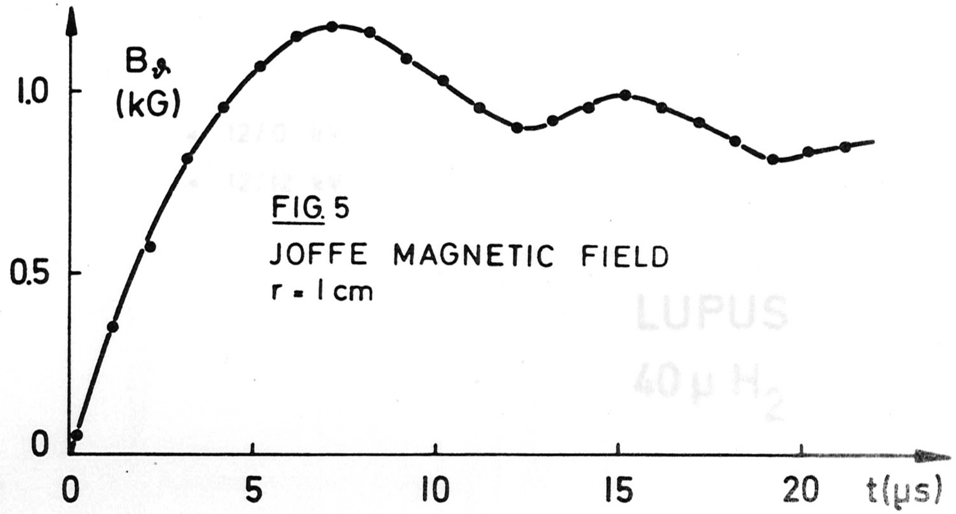

FIG. 5
JOFFE MAGNETIC FIELD
r = 1 cm

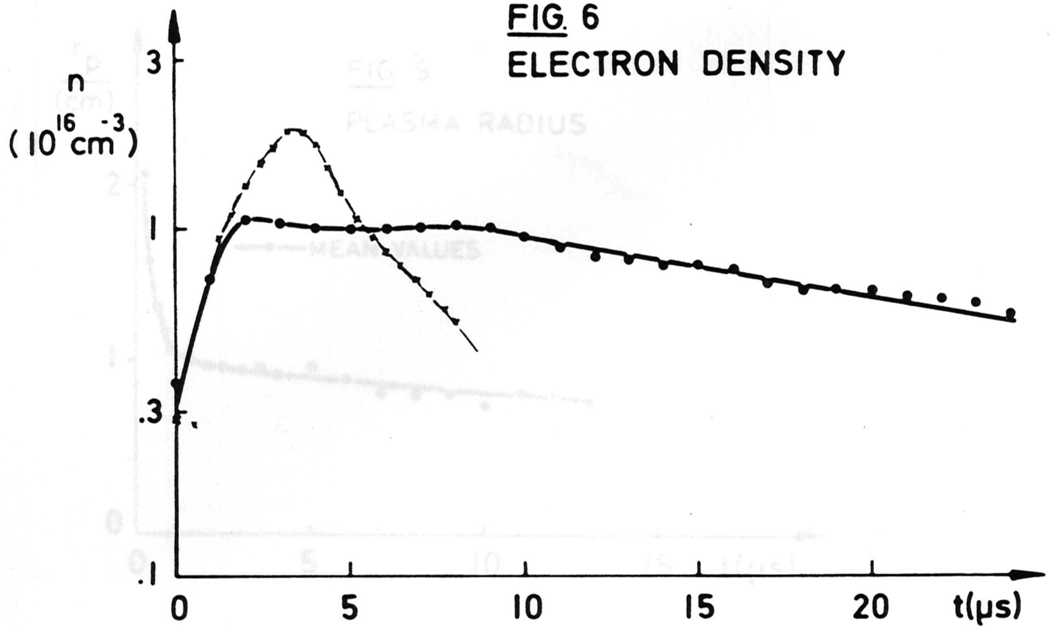

FIG. 6
ELECTRON DENSITY

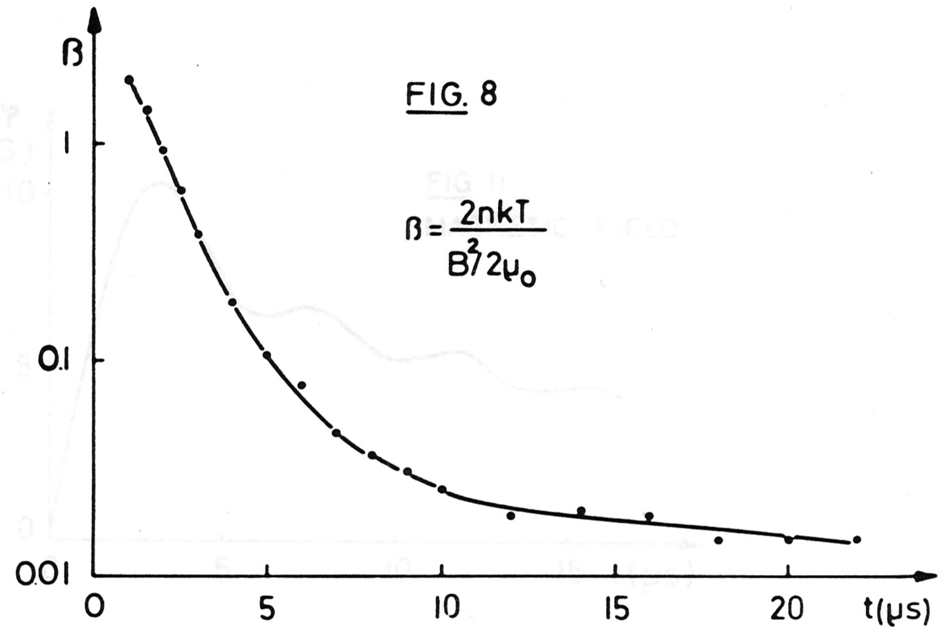

FIG. 8

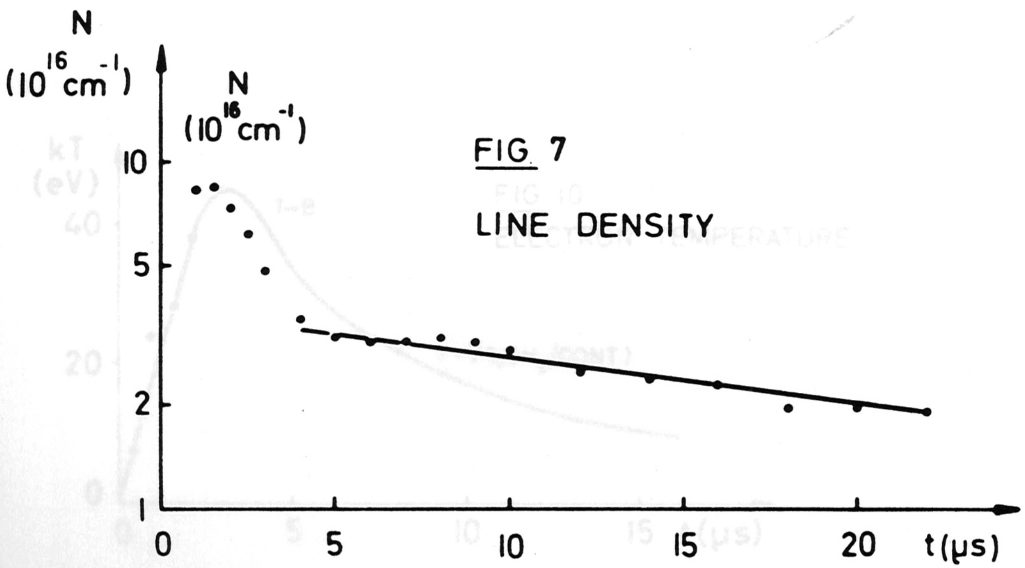

FIG. 7
LINE DENSITY

× 12/0 kV
• 12/12 kV

LUPUS
40 μ H_2

SPINNE

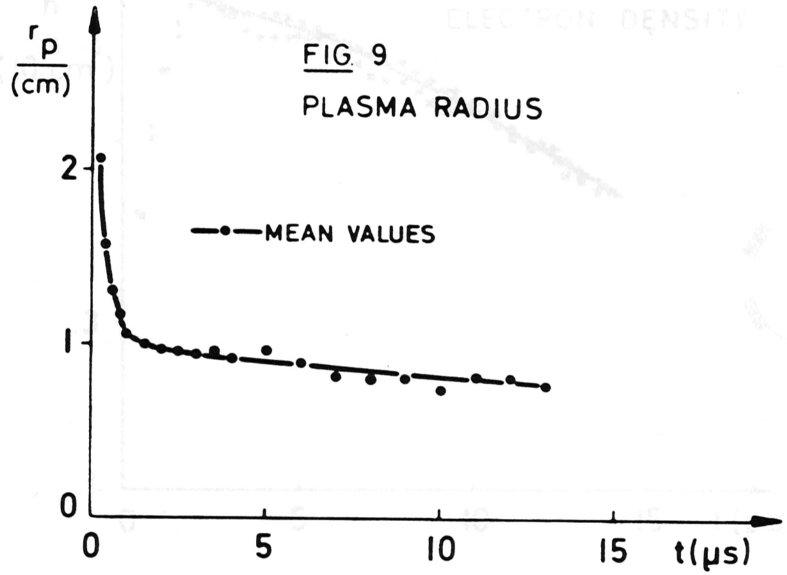

FIG. 9
PLASMA RADIUS

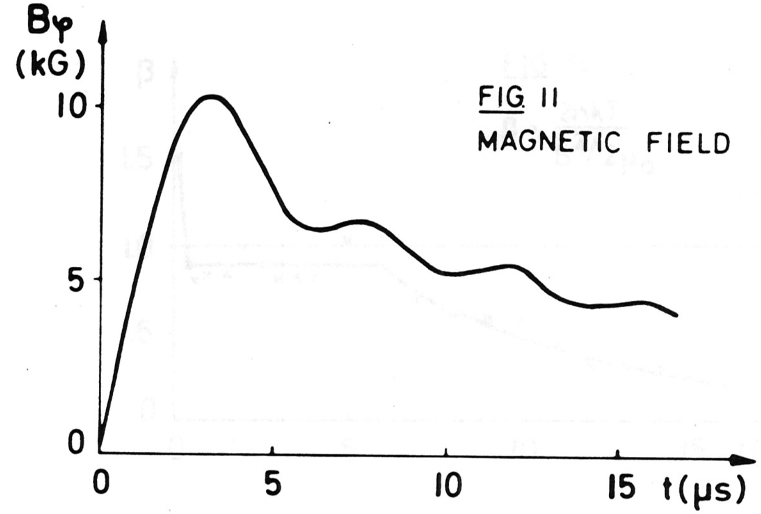

FIG. 11
MAGNETIC FIELD

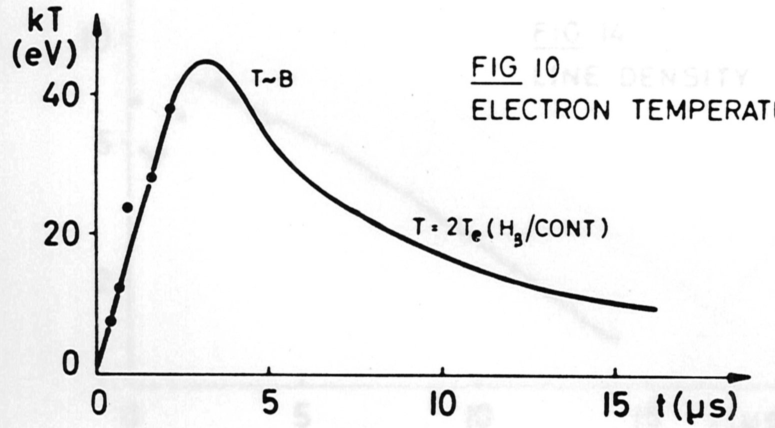

FIG 10
ELECTRON TEMPERATURE

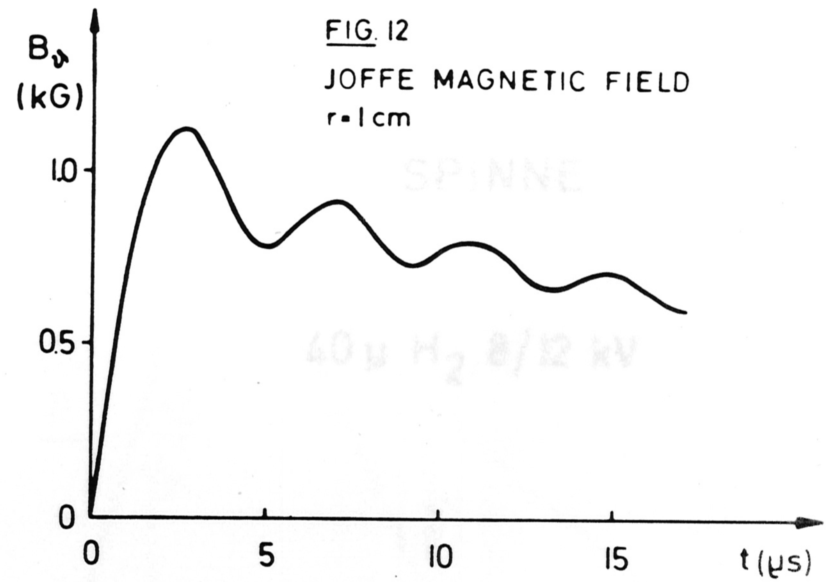

FIG. 12
JOFFE MAGNETIC FIELD
r = 1 cm

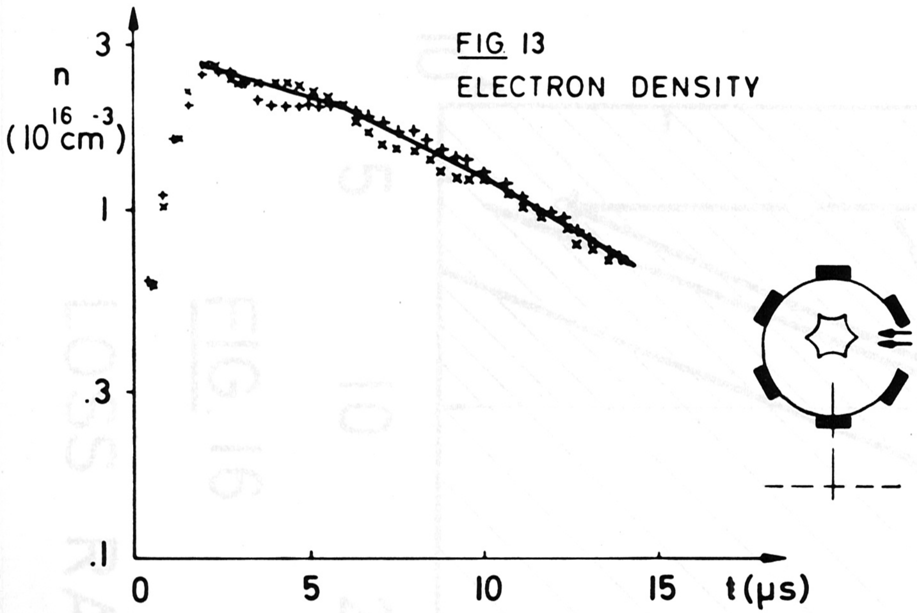

FIG 13
ELECTRON DENSITY

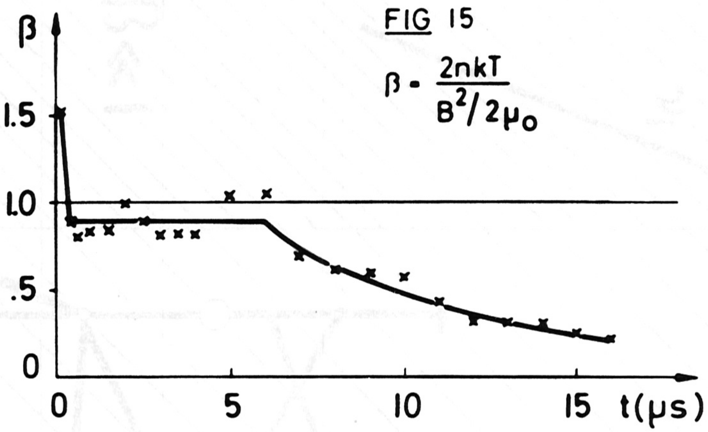

FIG 15
$\beta = \frac{2nkT}{B^2/2\mu_0}$

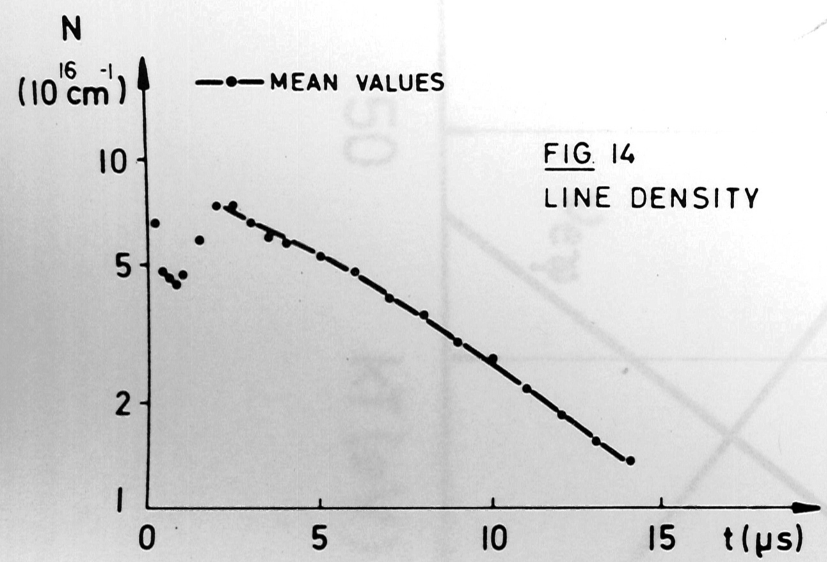

FIG 14
LINE DENSITY

SPINNE

$40 \mu \; H_2; 8/12$ kV

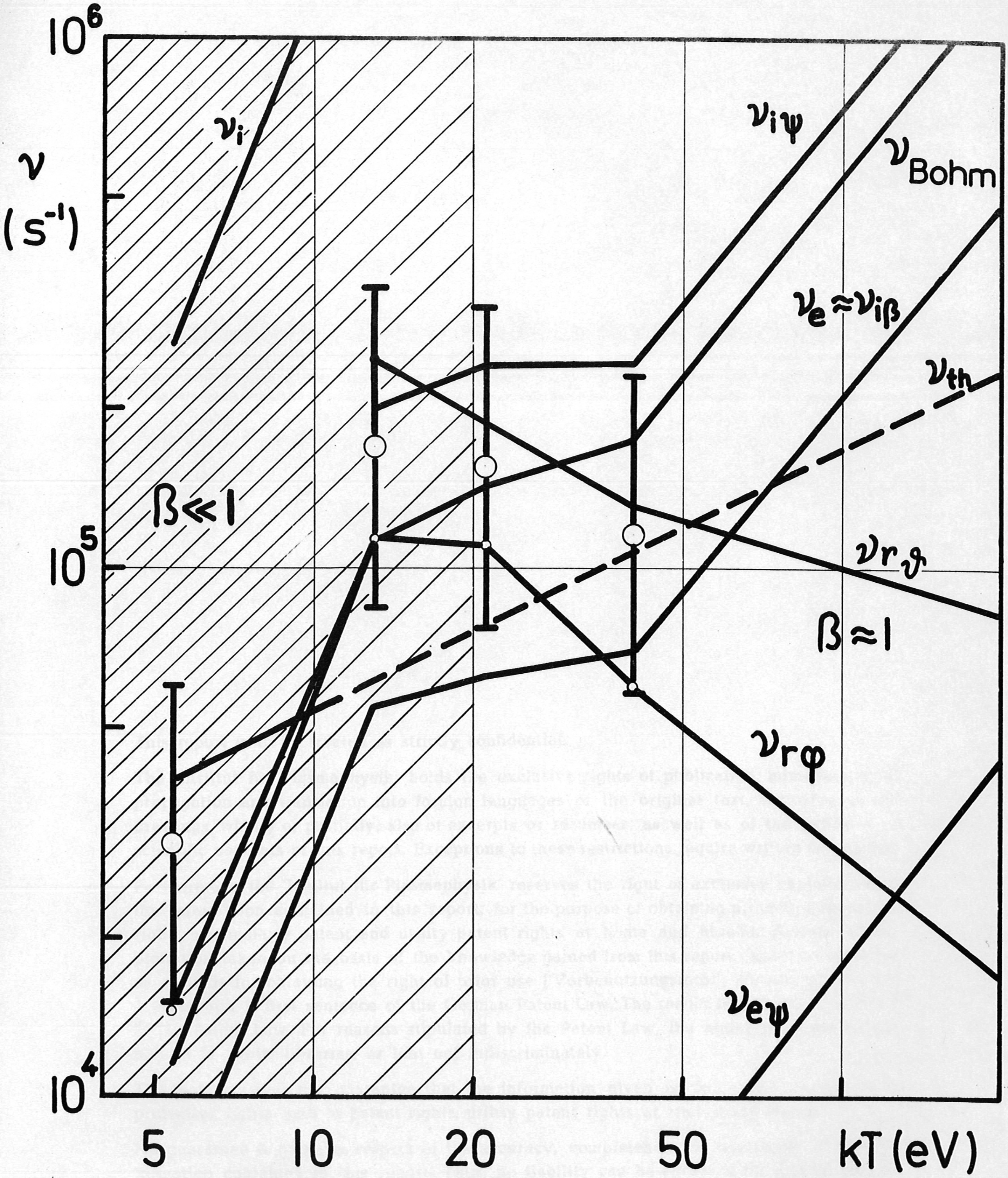

FIG. 16

LOSS RATES