I N S T I T U T F Ü R P L A S M A P H Y S I K

G A R C H I N G B E I M Ü N C H E N

Measurement of the anisotropy in the radial neutral gas pressure.

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Abstract

The rate of radial momentum transported by the neutral gas from a plasma beam to the walls of the container (i.e. the anisotropy in the radial pressure of the neutral gas) has been measured by the force exerted on the plate of a torsion balance. This force is plotted as a function of the neutral gas pressure and magnetic field strength. Some heuristic arguments are given to explain the experimental results.

Apparatus

The measurements were made in an experimental set-up called "CABINET" a detailed description of which is given in Ref. 1. It consists of a cylindrical steel vacuum vessel of 140 cm length and 20 cm diameter, surrounded by magnetic field coils, on top of which is the plasma source, a "duoplasmatron". The plasma drifts out of an emission aperture of 3 mm diameter and streams along the axis of the cylinder.

The anisotropy in the radial pressure of the neutral gas is measured by a torsion balance inserted into a vacuum vessel outside of the magnetic coils and communicating with the plasma chamber through an aperture of 3 cm diameter in the steel cylinder. The neutral molecules which have collided in the plasma beam may reach one plate of the balance, while the ions are held back by the magnetic field (Fig. I).

The angle of rotation of the torsion balance is measured by means of an optical system, as used in a galvanometer, The sensibility of this balance is 3.8×10^{-4} dyn/cm with a precision of 10 %.

Measurements are made in hydrogen with a pressure of about 10^{-3} Torr and with magnetic field strengths between 400 and 2000 gauss. The plasma density in the beam is of the order of 10^{13} cm^{-3} and the ion temperature is of the order of 10^3 ^oK, both depending on neutral gas pressure and magnetic field strength.

General considerations

If a plasma beam streams along the axis of a cylindrical vessel with a temperature T_i higher than the wall temperature T_n, we expect a net energy and momentum transport, by the neutral gas, from the plasma beam to the walls. It should then be possible to measure the rate of transport of this momentum by the force exerted on a torsion balance placed near the wall.

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1) F. Boeschoten, F. Schwirzke, Nucl. Fusion 2, 54 (1962)

A rough estimation of this force can be given on the basis of some crude hypothesis.

If we suppose that the ions in the beam and the neutral molecules have a Maxwellian velocity distribution at different temperatures, T_i and T_n respectively, we can calculate the mean energy gained by a neutral molecule in a collision with an ion:

$$\Delta \overline{\frac{1}{2} m_n v_n^2} = 4 \frac{m_n m_i}{(m_n + m_i)^2} k (T_i - T_n) \quad . \tag{1}$$

If we assume further that after a collision $\bar{v}_n = (8/3\ \pi)^{1/2}\sqrt{\overline{v_n^2}}$, as it would be for a Maxwellian velocity distribution, we obtain for the momentum:

$$\overline{\Delta m_n v_n} = (\frac{8km_n}{3\pi})^{1/2} \left\{ (\frac{8m_n m_i (T_i - T_n)}{(m_i + m_n)^2} + 3\ T_n)^{1/2} - (3\ T_n)^{1/2} \right\} . \tag{2}$$

With the same assumptions we can also calculate the collision rate per unit volume:

$$N_{n,i} = 2\ n_n\ n_i\ \sigma_{i,n}\ (\frac{2k}{\pi})^{1/2} \left\{ \frac{m_i T_n + m_n T_i}{m_i m_n} \right\}^{1/2} ,$$

where m_n, T_n, n_n and m_i, T_i, n_i are mass, temperature, and density of the neutral gas molecules and ions, respectively;

v_n = velocity of neutral gas molecules;

$\sigma_{i,n}$ = cross section for a collision between ions and neutral gas molecules;

k = Boltzmann constant.

The total momentum gained per unit volume and per second by the neutral gas is, therefore:

$$N_{n,i}\ \overline{\Delta m_n v_n} = 8\, n_n n_i \frac{\sigma_{i,n}}{\pi} kT_n \left(\frac{\mu T_i + T_n}{T_n}\right)^{1/2}$$

$$\left\{\left(\frac{8\mu}{3(1+\mu)^2}\frac{T_i - T_n}{T_n} + 1\right)^{1/2} - 1\right\}, \qquad (3)$$

where $\mu = m_n/m_i$.

Let us now consider an ideal cylinder of radius R and length dz around the plasma beam. If we suppose that the neutral gas molecules after a collision have still an isotropic velocity distribution and that there is no radial momentum transported by the neutral gas in the beam direction (which is reasonable if the plasma beam is long enough for all gradients in the beam direction to be negligible), then the radial momentum gained by the neutral gas per second in the volume of this cylinder: $M_\perp = \frac{3}{2}\overline{\Delta m_n v_n}\, N_{n,i}\, s\, dz$, s being the cross section of the beam must be equal to the rate of transport of radial momentum through the side surface $\sum = 2\pi R dz$ of the cylinder.

The flux of radial momentum per unit surface, i.e. the anisotropy in the radial pressure of neutral gas, at a distance R from the beam, will, therefore, be:

$$\Delta P_\perp = \frac{M_\perp}{\sum} = \frac{8}{3\pi^2}\frac{s}{R} n_n n_i \sigma k T_n \left(\frac{\mu T_i + T_n}{T_n}\right)^{1/2}$$

$$\left\{\left(\frac{8\mu}{3(1+\mu)^2}\frac{T_i - T_n}{T_n} + 1\right)^{1/2} - 1\right\} \qquad (4)$$

or because $n_n = P_n/(kT_n)$, P_n being the neutral gas pressure:

$$\Delta P_\perp = \frac{8}{3\pi^2}\frac{s}{R} n_i \sigma P_n \left(\frac{\mu T_i + T_n}{T_n}\right)^{1/2} \left\{\left(\frac{8\mu}{3(1+\mu)^2}\frac{T_i - T_n}{T_n} + 1\right)^{1/2} - 1\right\}. \qquad (5)$$

If we now assume, in Eq. (5):

$\mu = 1 - 2$; $s \approx 0.5\ cm^2$; $\sigma = 2 \times 10^{-15}\ cm^2$;

$T_n = 300^oK$; $P_n = 10^{-3}$ Torr ;

R = 30 cm (distance from the plasma beam to the balance);
and $n_2 \approx 10^{13}\ cm^{-3}$; $T_i \approx 10^{3o}K$;

which are reasonable values for our plasma, we obtain $\Delta P_\perp \approx 2 \times 10^{-4}$ dyn/cm^2 and on the 4 cm^2 plate of our balance a force $F \approx 10^{-3}$ dyn which is within the sensitivity range of our device.

Results

Fig. 1 to 4 show the rate $\Delta P_\perp / P_n$ as a function of the neutral gas pressure for different values of the magnetic field strength B. Fig. 5 to 9 show $\Delta P_\perp / P_n$ as a function of 1/B for different values of P_n.

The order of magnitude of $\Delta P_\perp$ is in agreement with our theoretical estimation. The experimental data may be described by an empirical equation (dotted lines).

$$\frac{\Delta P_\perp}{P_n} = A\ e^{-(h_1 + \frac{h_2}{B})P_n} , \qquad (6)$$

where $A = 1.9 \times 10^{-3}$; $h_1 = 0.64\ (dyn/cm^2)^{-1}$; $h_2 = 1.4 \times 10^2$ gauss $(dyn/cm^2)^{-1}$ are empirically deduced constants.

We may attempt to obtain a similar equation on the basis of our hypothesis. Setting in Eq. (5): T_n = const and $T_i/T_n >> 1$, we obtain:

$$\frac{\Delta P_\perp}{P_n} = \left(\frac{8}{3\pi^2}\ \frac{s}{R}\ \frac{\mu}{(1+\mu)}\ \frac{\sigma}{T_n}\right) \times n_i T_i = \text{const} \times n_i T_i , \qquad (7)$$

and if the ion temperature and density decrease along the z axis only by collisions with the neutral gas molecules, it is reasonable to set:

$$\frac{dT_i}{dz} = -\frac{\alpha}{\lambda_i} T_i \quad \text{and} \quad \frac{dn_i}{dz} = -\frac{\beta}{\lambda_i} n_i \quad ,$$

where $\lambda_i = 1/n_n\sigma_{n,i} = kT_n/P_n\sigma_{n,i}$ is the mean free path of the ions for collisions with neutral molecules and $\alpha = \alpha(z,B)$; $\beta = \beta(z,B)$ are coefficients which will be, in general, functions of z and B.

As we have done our measurements at a fixed $\bar{z} = 70$ cm, we may integrate dT_i/dz and dn_i/dz and substitute in Eq. (7);

we obtain:

$$\frac{\Delta P_\perp}{P_n} = -\left(\frac{8}{3\pi^2}\frac{s}{R}\frac{\mu}{(1+\mu)^2}\frac{\sigma}{\pi}\right) \times n_i^o T_i^o \, e^{-\left(\frac{\bar{\alpha}}{\lambda_i}+\frac{\bar{\beta}}{\lambda_i}\right)\bar{z}} \qquad (8)$$

or

$$\frac{\Delta P_\perp}{P_n} = \left(\frac{8}{3\pi^2}\frac{s}{R}\frac{\mu}{(1+\mu)^2}\frac{\sigma}{\pi}\right) \times n_i^o T_i^o \, e^{-\left(\frac{\bar{\alpha}\sigma}{kT_n}+\frac{\bar{\beta}\sigma}{kT_n}\right)\bar{z}\, P_n} \quad , \qquad (9)$$

where $\bar{\alpha} = 1/\bar{z}\int_0^{\bar{z}} \alpha(\tau)d\tau$; $\bar{\beta} = 1/\bar{z}\int_0^{\bar{z}} \beta(\tau)d\tau$, and n_i^o, T_i^o are the ion density and temperature at $z = 0$.

Eq. (9) shows a dependence of $\Delta P_\perp$ on the neutral gas pressure P_n which is consistent with the empirically found Eq. (6). Further, if we compare the value of $\bar{\beta}/\lambda_i$ deduced from the axial slope of the ion density measured by a Langmuir probe (a measurement made by Dr. G. Siller), we find that it agrees (by less than a factor 2) with the value of $\bar{\beta}/\lambda_i$ deduced from Eq. (6), if we set $\bar{\alpha} \ll \bar{\beta}$ and $\bar{\beta}/\lambda_i \approx (h_1 + h_2/B)P_n$. It is, however, not possible to make any more quantitative comparison of Eq. (8) with the experimental results, because the parameters contained in it are too poorly known in

CABINET's plasma. It seems, therefore, not worthwhile to continue these measurements on CABINET. However, such measurements could be a useful additional diagnostic tool in other cases: for example, to measure $n_i T_i$ in hot plasma experiments.

Acknowledgments

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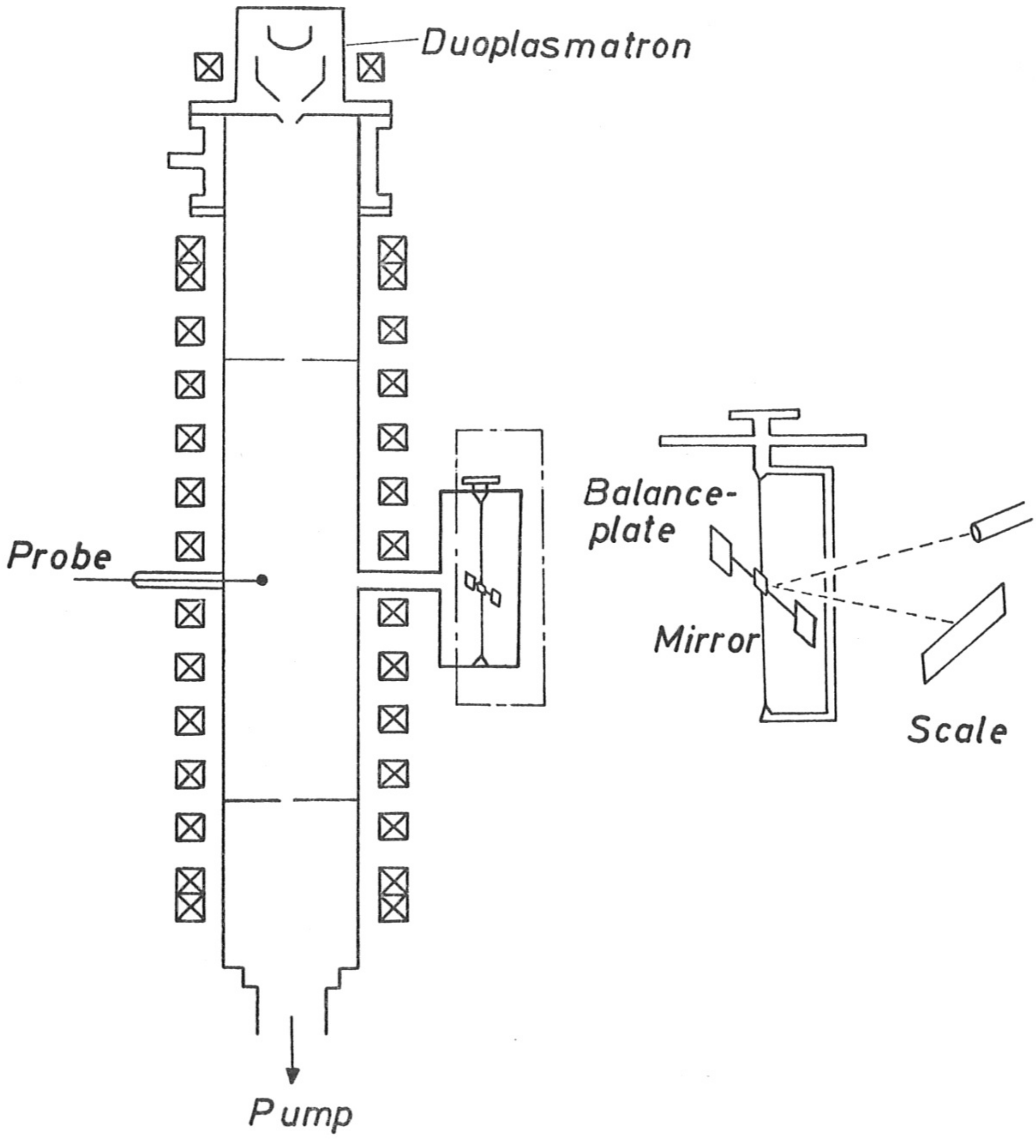
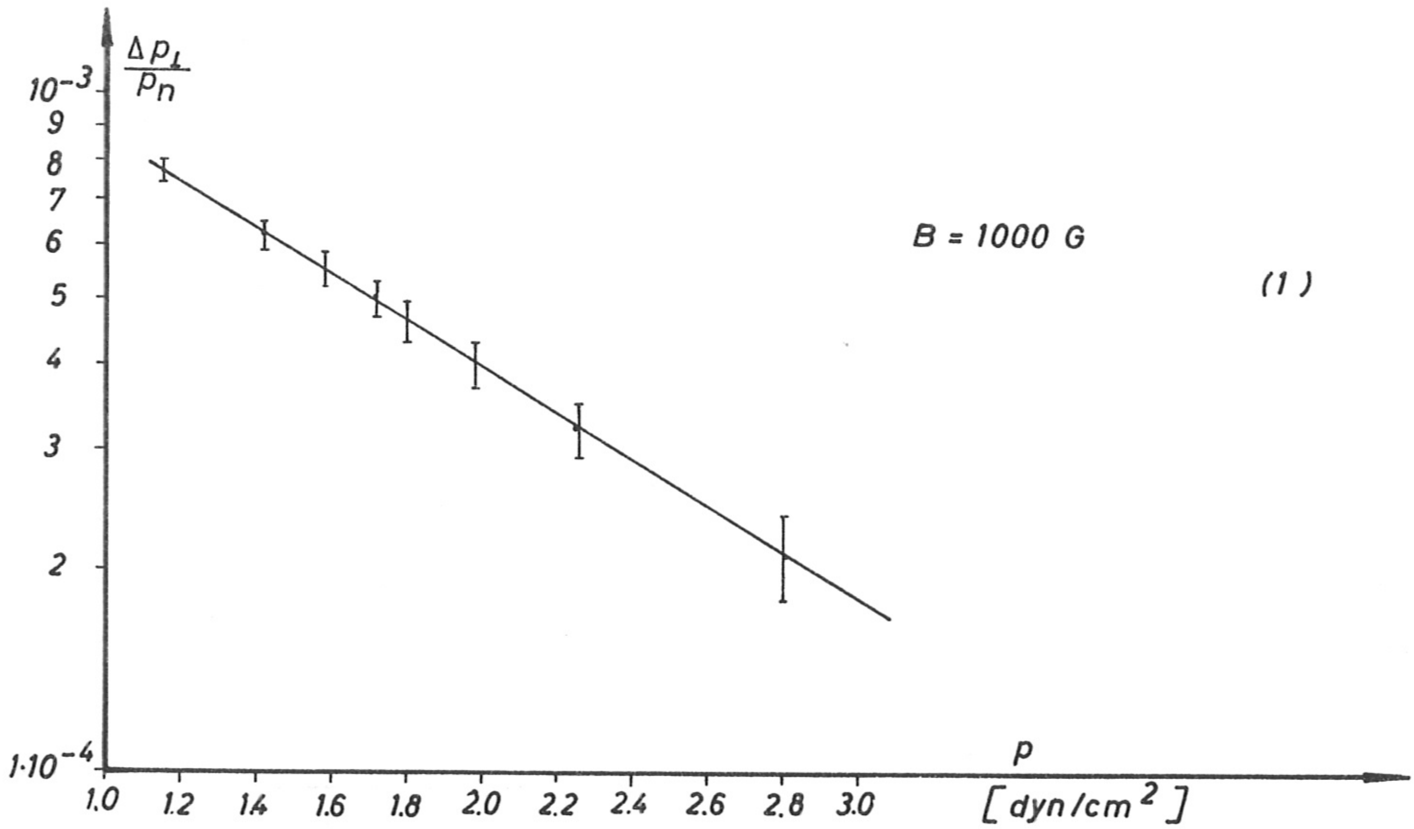
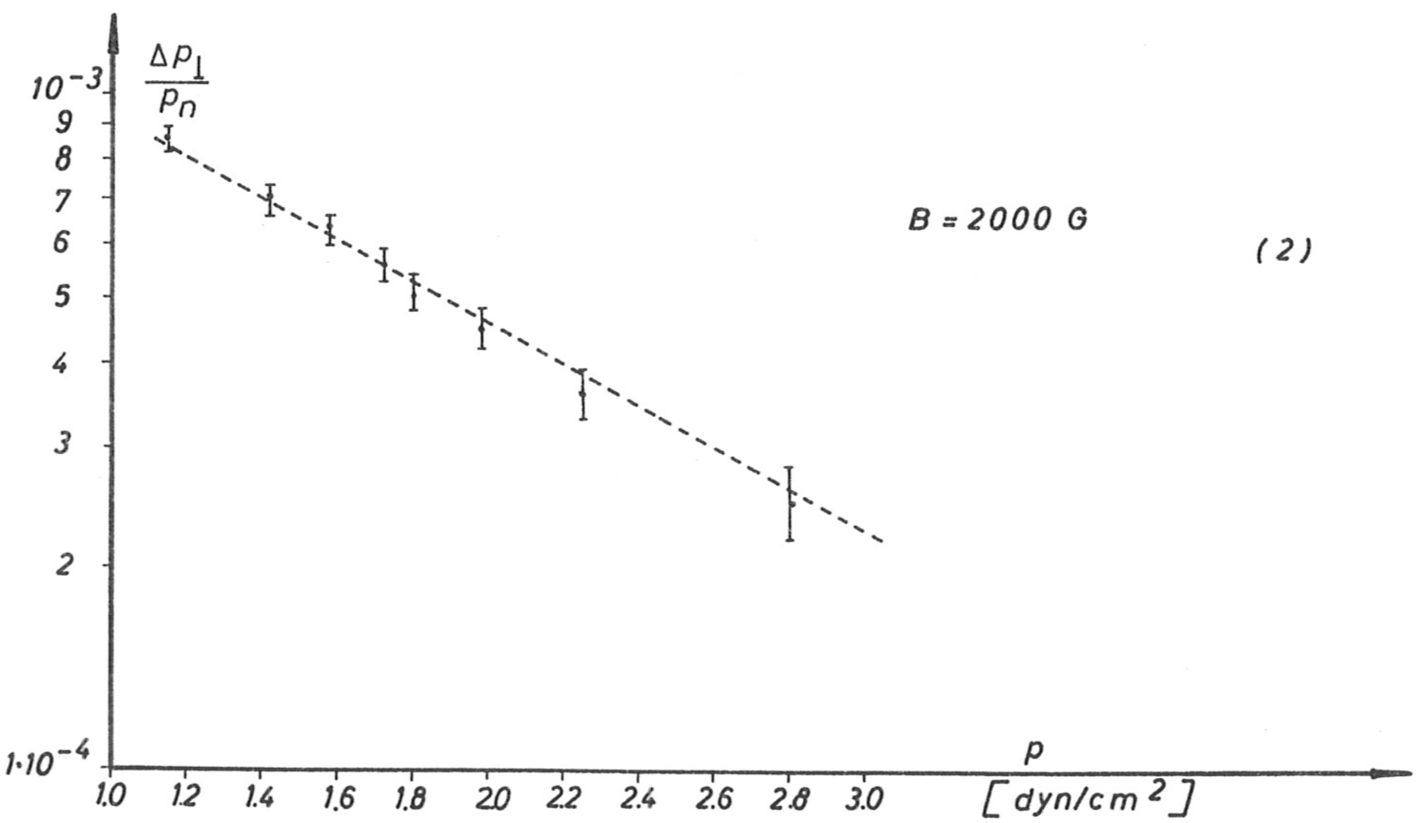
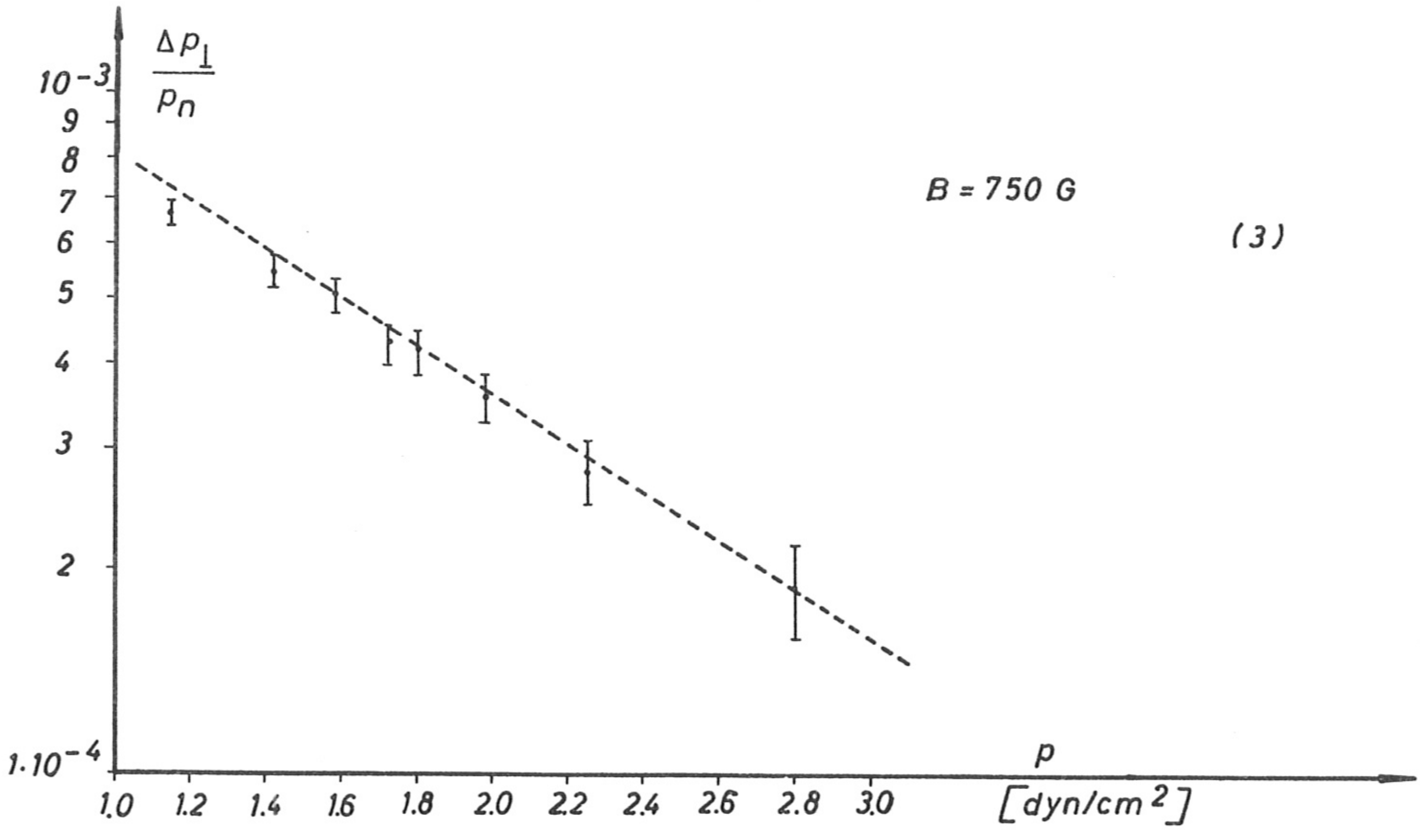

Duoplasmatron
Probe
Balance-
plate
Mirror
Scale
Pump



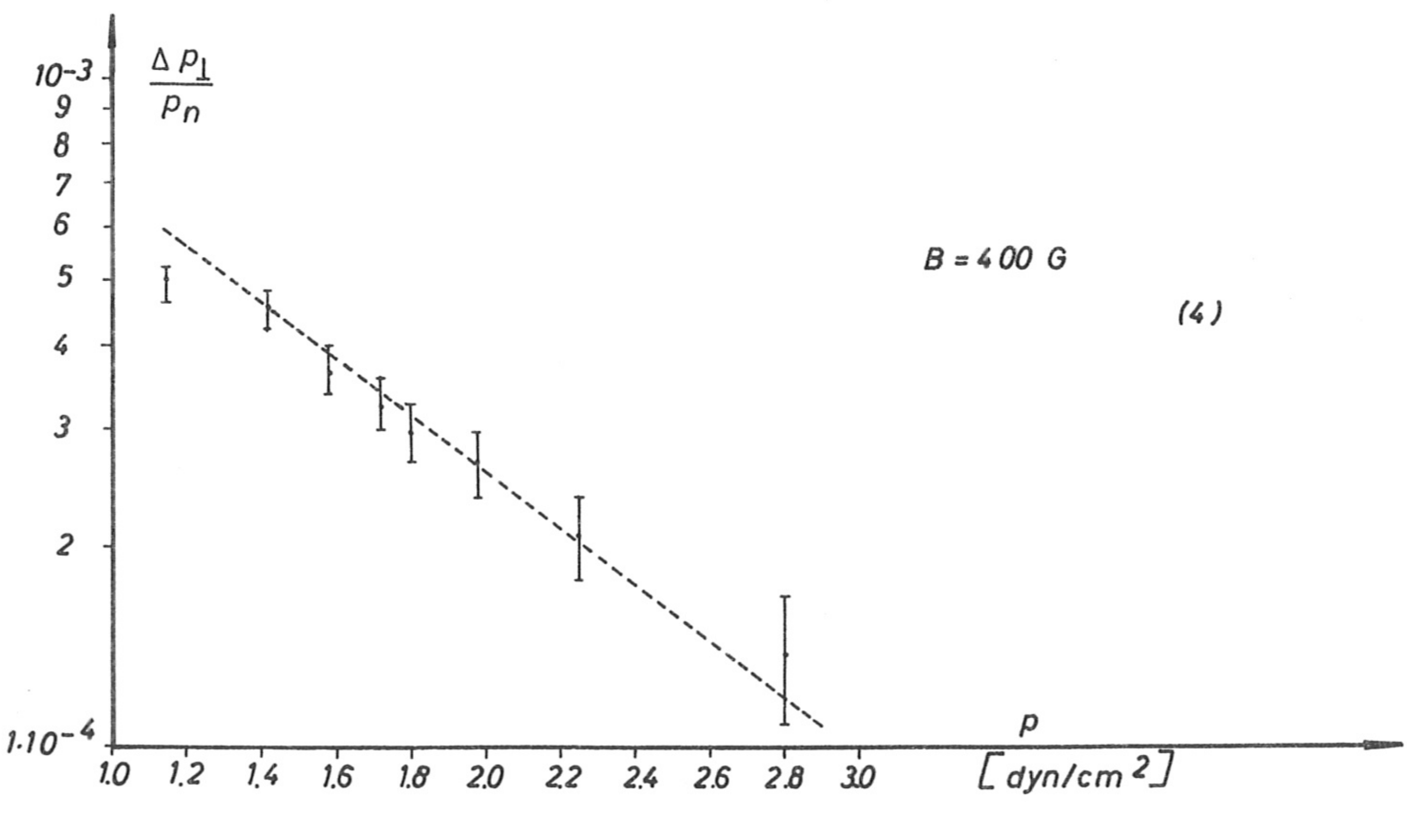
$\frac{\Delta p_\perp}{p_n}$
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
B = 1000 G
(1)
p
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
[dyn/cm^2]



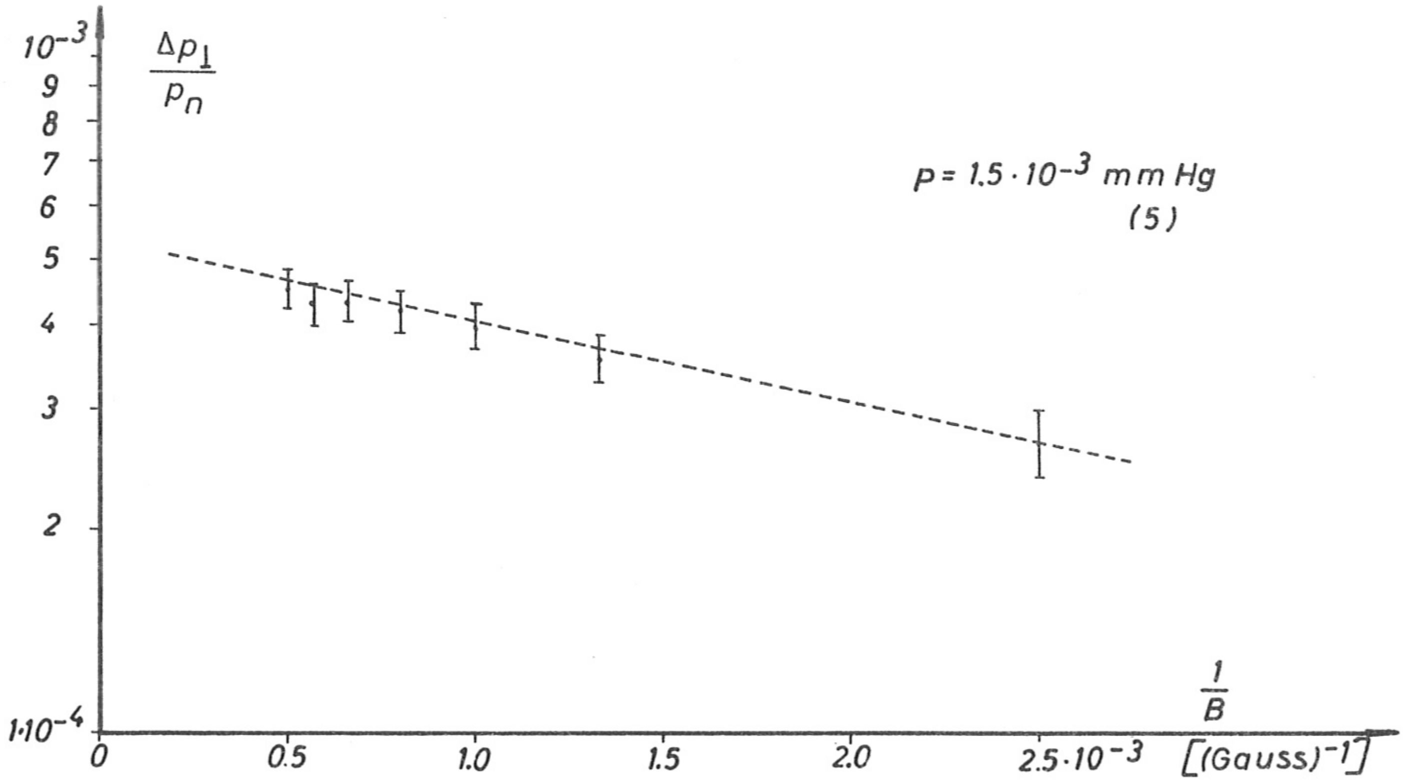
$\frac{\Delta p_\perp}{p_n}$
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
B = 2000 G
(2)
p
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
[dyn/cm^2]



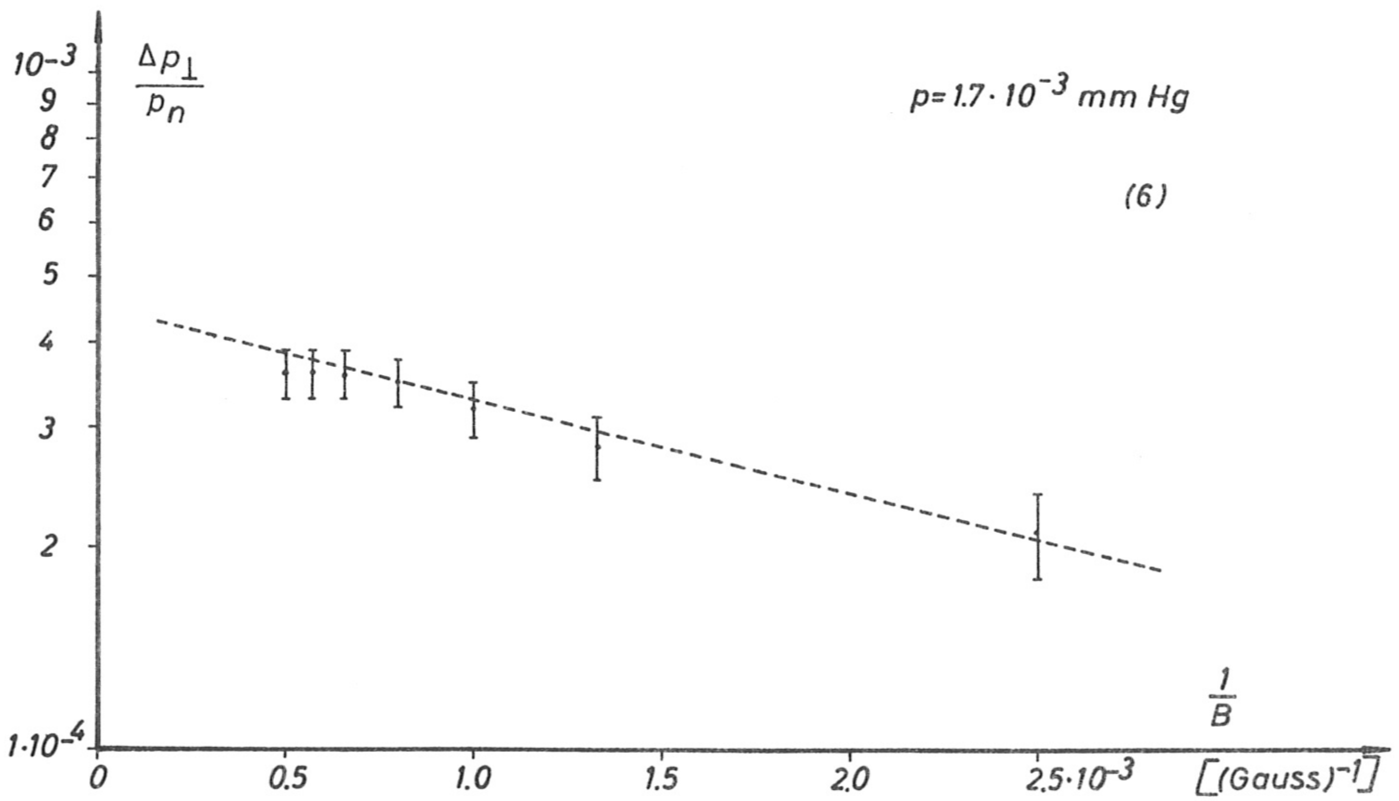
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
$\frac{\Delta p_\perp}{p_n}$
B = 750 G
(3)
p
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
[dyn/cm^2]



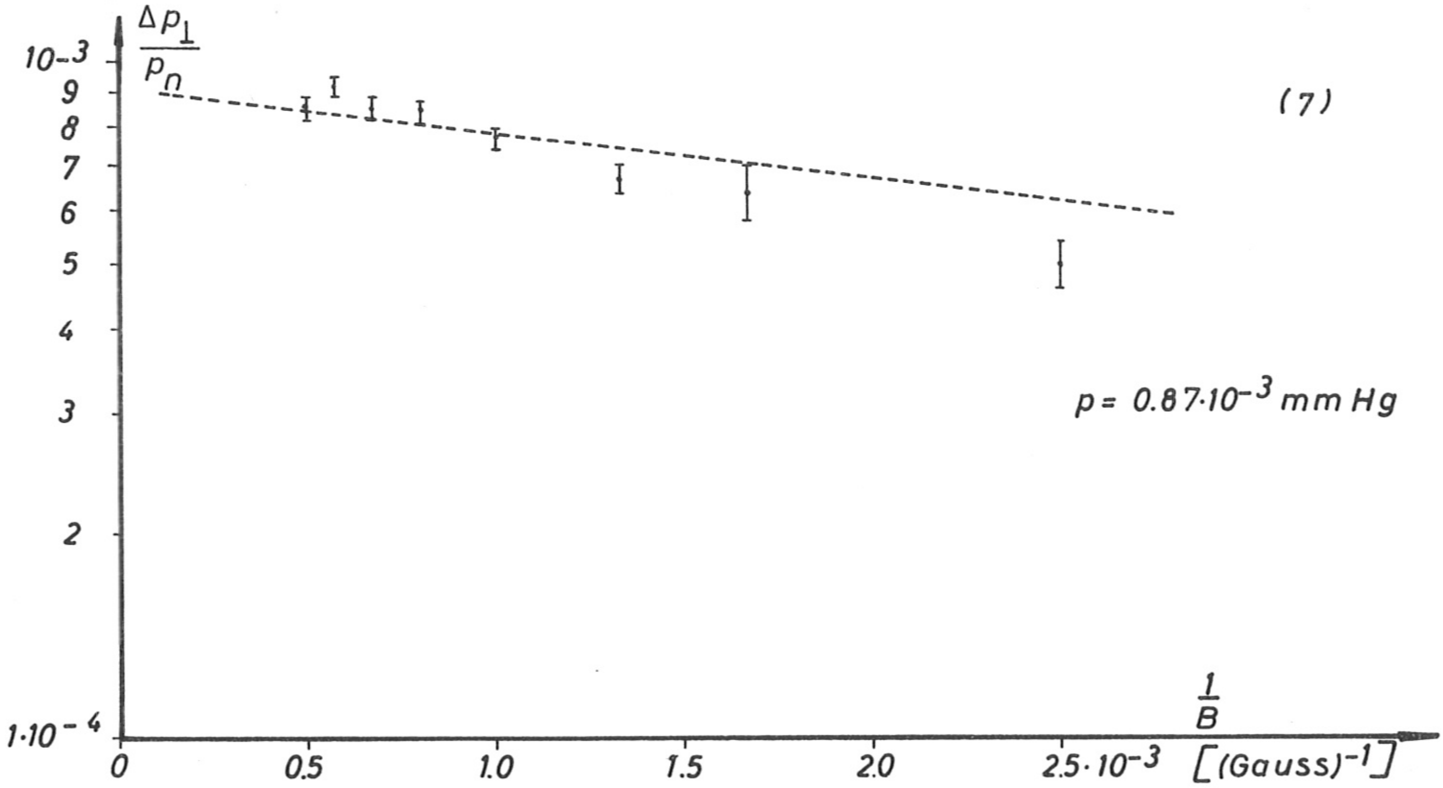
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
$\frac{\Delta p_\perp}{p_n}$
B = 400 G
(4)
p
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
[dyn/cm^2]



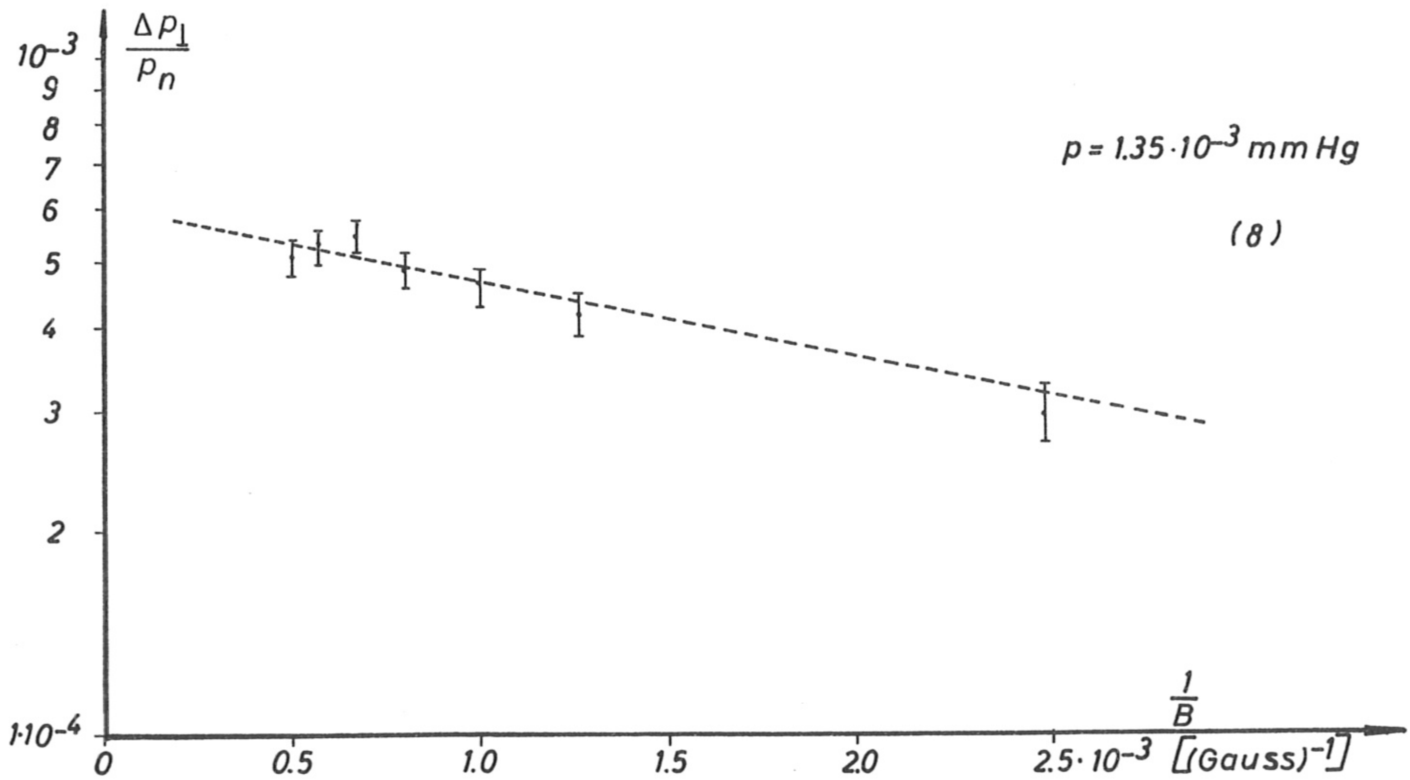
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
$\frac{\Delta p_\perp}{p_n}$
$p = 1.5 \cdot 10^{-3}$ mm Hg
(5)
0
0.5
1.0
1.5
2.0
$2.5 \cdot 10^{-3}$
$\frac{1}{B}$
[(Gauss)$^{-1}$]



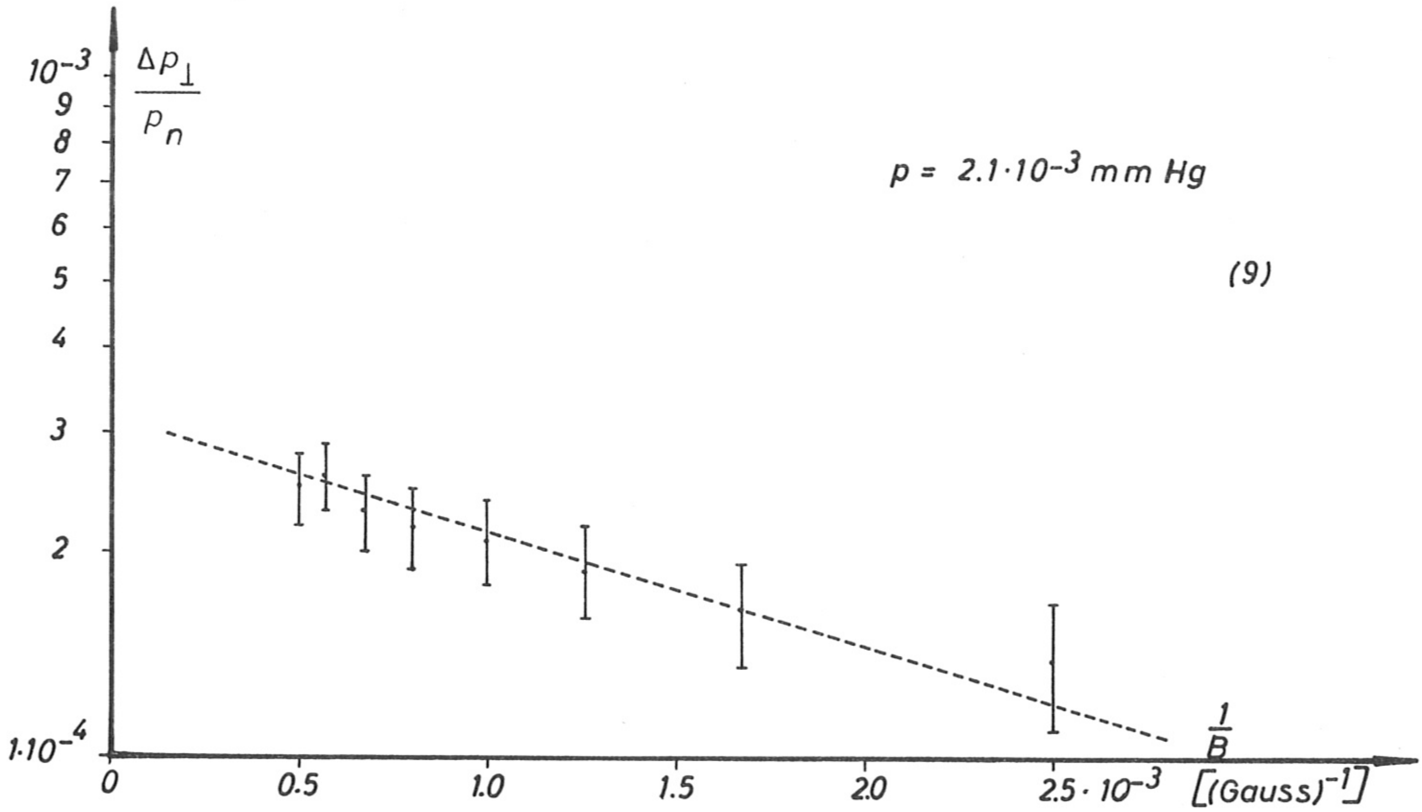
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
$\frac{\Delta p_\perp}{p_n}$
$p = 1.7 \cdot 10^{-3}$ mm Hg
(6)
0
0.5
1.0
1.5
2.0
$2.5 \cdot 10^{-3}$
$\frac{1}{B}$
[(Gauss)$^{-1}$]



$\frac{\Delta p_\perp}{p_n}$
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
(7)
$p = 0.87 \cdot 10^{-3}$ mm Hg
$\frac{1}{B}$
0
0.5
1.0
1.5
2.0
$2.5 \cdot 10^{-3}$
[(Gauss)$^{-1}$]



$\frac{\Delta p_\perp}{p_n}$
10^{-3}
9
8
7
6
5
4
3
2
$1 \cdot 10^{-4}$
$p = 1.35 \cdot 10^{-3}$ mm Hg
(8)
$\frac{1}{B}$
0
0.5
1.0
1.5
2.0
$2.5 \cdot 10^{-3}$
[(Gauss)$^{-1}$]



$\frac{\Delta p_\perp}{p_n}$
10^{-3}
9
8
7
6
5
4
3
2
$1\cdot10^{-4}$
$p = 2.1\cdot10^{-3}$ mm Hg
(9)
0
0.5
1.0
1.5
2.0
$2.5\cdot10^{-3}$
$\frac{1}{B}$
[(Gauss)$^{-1}$]