COMPARISON OF A FEW METHODS FOR MEASURING THE DIFFUSION RATE OF PLASMA ACROSS A MAGNETIC FIELD

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Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Institut für Plasmaphysik GmbH und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

I. Theoretical considerations

In a magnetic field the rate of diffusion of plasma particles becomes anisotropic. In the direction parallel to the field the diffusion rate is almost unaffected, while the diffusion normal to the field, may be greatly reduced. In a weakly ionized gas the equations for the particle currents may be derived from the equations for the three fluid model given by SCHLÜTER [1], assuming that Coulomb collisions and inertia forces are negligible. This gives:

(1a) $$n_i v_{i\perp} = - D_{i\perp} \nabla_\perp n_i + n_i \mu_{i\perp} E_\perp$$

(2a) $$n_i v_{i\parallel} = - D_{i\parallel} \nabla_\parallel n_i + n_i \mu_{i\parallel} E_\parallel$$

(1b)
(2b) The same for electrons.

As particle currents are difficult to measure, we use the particle conservation equation to determine the diffusion from density variations:

(3a) $$\frac{\partial n}{\partial t} = - \nabla n_i \vec{v}_i - \alpha n_i^2 + \beta n_i$$

(3b) The same for electrons.

Furthermore we have $\nabla \cdot \vec{E} = \frac{e}{\varepsilon_0}(n_i - n_e)$ and in the cases under consideration $\nabla \times \vec{E} = 0$. Writing $\vec{E} = -\nabla \phi$ we thus obtain:

(4) $$\nabla^2 \phi = - \frac{e}{\varepsilon_0}(n_i - n_e)$$

In spite of all the simplifying assumptions, the equations (1a) to (4) form a set of non linear partial differential equations which - even in the most simple cases - cannot be solved without making more or less plausible assumptions. GOLANT [2] has found solutions for several experimental conditions. We will try to give an outline of his work:

1. If the walls are insulating, no electric currents may flow to the walls and one tries to obtain solutions where, in the whole volume, no electric currents are flowing in the directions of the density gradients. This means:

(5) $$n_i V_{i\parallel} = n_e V_{e\parallel} \quad , \quad n_i V_{i\perp} = n_e V_{e\perp}$$

It then follows from equations (1a), (1b) respectively (2a), (2b) in the case $n_i = n_e$ (quasineutrality) that:

(6a) $$E_{\parallel} = \frac{D_{i\parallel} - D_{e\parallel}}{\mu_{i\parallel} - \mu_{e\parallel}} \cdot \frac{\nabla_{\parallel} n}{n}$$

(6b) $$E_{\perp} = \frac{D_{i\perp} - D_{e\perp}}{\mu_{i\perp} - \mu_{e\perp}} \cdot \frac{\nabla_{\perp} n}{n}$$

and neglecting production and recombination in (3a,b) we find for the density the soluble equation:

(7) $$\frac{\partial n}{\partial t} = D_{a\parallel} \nabla_{\parallel}^2 n + D_{a\perp} \nabla_{\perp}^2 n$$

2. For metal walls there are no boundary conditions for the currents, however the potential of the wall is known. (A more detailed investigation for this case, especially in the presence of potential drops in boundary sheaths, is made by WHITEHOUSE and WOLLMAN [3].)
In analogy to equation (6) a solution based on the relation $E = \xi \frac{\nabla n}{n}$ is sought. (In contradiction to eq. (6) the constant ξ has the same value parallel and perpendicular to the magnetic field.) In this way we obtain for the density n, an equation in form similar to equation (7) but with different coefficients. Up to now there is no experimental proof for the correctness of this procedure, which is in any case questionable.*

These solutions given by GOLANT present the difficulty that the quantity $\frac{\nabla n}{n}$ goes to infinity near the walls.

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*One of us (Geissler) tries to obtain experimental information on these points (see page 12).

The calculations under 1. and 2. are not valid for plasma regions where external electrical fields and currents are responsible for plasma production. In principal they are made for thermal plasmas.

As mentioned above it is interesting to compare the calculations made by WHITEHOUSE [3] for the case of a HF discharge with those made by GOLANT [2] for a decaying plasma.

3. Another treatment of the equations was given by SIMON [4]. Substitution of equations (1a) and (1b) in equations (3a) and (3b) yields

(8a) $$\frac{\partial n}{\partial t} = D_{i\parallel} \nabla_\parallel^2 n_i + D_{i\perp} \nabla_\perp^2 n_i - \mu_{i\parallel} \nabla_\parallel n_i E_\parallel - \mu_{i\perp} \nabla_\perp n_i E_\perp$$

(8b) The same for electrons.

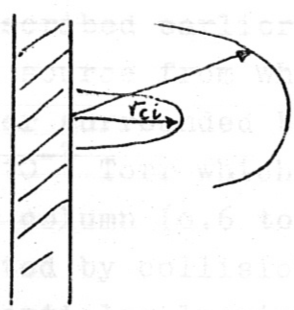
If conducting end plates are present, according to SIMON, a short-circuiting effect occurs in such a way that the term $\mu_\perp \nabla_\perp n E_\perp$ may be neglected. Equations (8a) and (8b) may be added and an equation of the form of eq. (7) is obtained with $D_{i\perp}$ instead of $D_{a\perp}$. In his calculations SIMON assumed a sinusoidal density profile in axial direction: $n(z)=n_o \cdot \sin \frac{\pi z}{L}$. With this assumption the diffusion equation is easily solved and the density is predicted to vary with radius as a modified Bessel function. For $r \gtrsim 1{,}5\,q$ this function is experimentally indistinguishable from a simple exponential $e^{-\frac{r}{q}}$. For q one finds

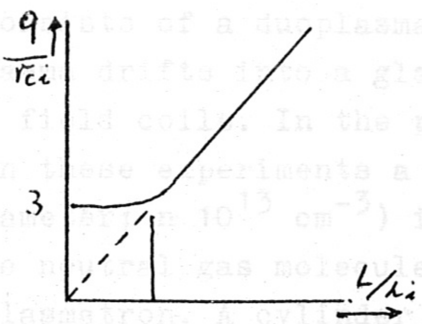
(9) $$q = \frac{1}{\pi} \frac{L}{\lambda_i} r_{ci}$$

The assumption $n(z)=n(o) \cdot \sin \frac{\pi z}{L}$, made for analytical simplicity, is not always realistic, very often the density decreases monotonically from the cathode and the term $D_\parallel \nabla_\parallel^2 n$ is positive instead of negative.

For every special case one has to know the real form of the axial density profile and try to find corresponding solutions of the diffusion equations.* In many cases one should make use of a Fourier series or of a sum of Bessel functions.

Moreover the validity of formula (9) for the e-folding length q is limited by the finiteness of the iongyro-radii. Rev. [5]





The fact that one finds $q \sim \frac{1}{B}$ is by no means a guarantee that collisional diffusion is operative, since the same dependence is expected from the iongyro radius effect.

In view of the many uncertainties, it seems necessary to measure the plasma density n as a function of z as well as of r, and in addition $E_{\parallel}$ and $E_{\perp}$ and the variation of temperature with radius. As far as we know such an extensive program was first elaborated by ROTHLEDER and ROSE [6] on a H.C.D. These measurements indicated that the diffusion was collisional. We have also started such measurements on our apparatus.

II. Measurements on a stationary plasma

Starting from eqs. (8a) and (8b) with $\frac{\partial n}{\partial t} = 0$, we tried to measure the functions $n(r,z)$ and $\vec{E}(r,z)$ in a cylindrical plasma volume

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* It may be remarked here that for the stationary case the radial density profile of a plasma column of infinite length is independent of $D_{\perp}$.

with well defined boundaries. By determining the proper derivatives of these functions at several points in space one may obtain a system of equations from which the diffusion coefficients may be derived. Unfortunately we did not obtain the desired results as may be seen from the following.

Apparatus. The measurements were made in our "CABINET" which was described earlier [7]. It consists of a duoplasmatron as plasma source from which the plasma drifts into a glass cylinder surrounded by magnetic field coils. In the pressure range 10^{-3} Torr which we used in these experiments a luminous plasma column (o.6 to o.8 cm diameter; n 10^{13} cm^{-3}) is generated by collisions with the neutral gas molecules of the fast particles leaving the duoplasmatron. A cylinder was fixed in such a way inside the vacuum vessel that it could be adjusted relatively to the plasma column during the measurements (fig. 1*a*).

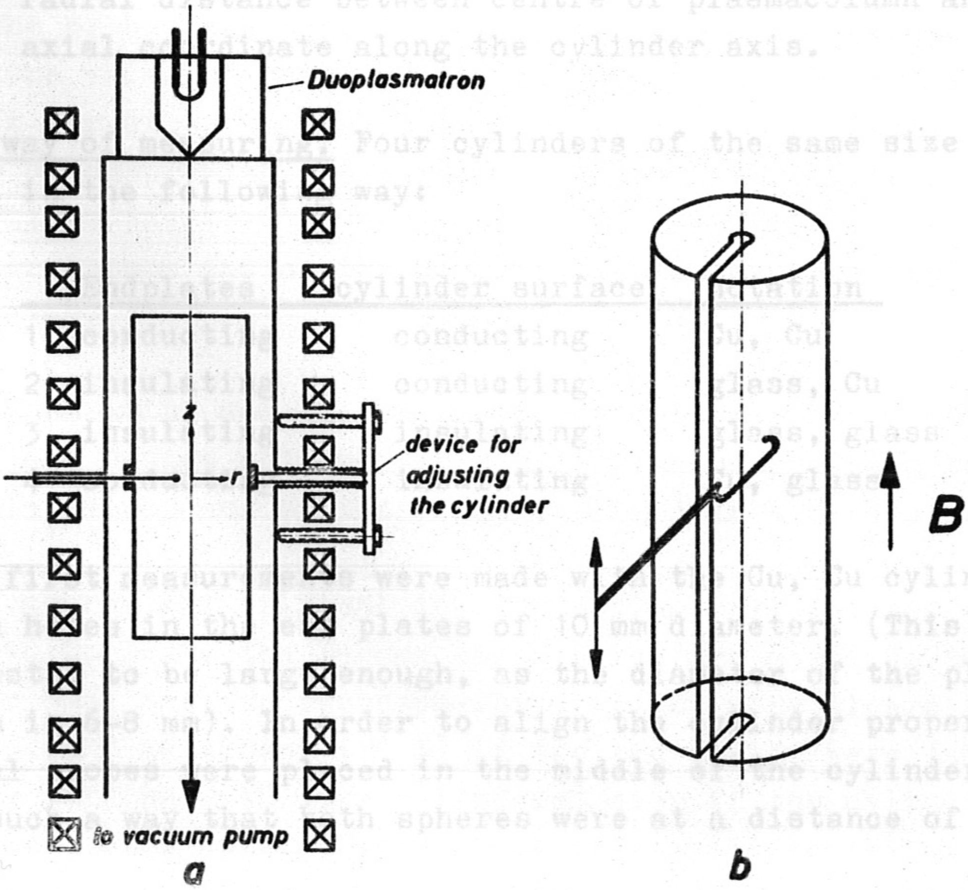

Fig.1 Schematic of the apparatus

Two movalbe spherical probes (diameter = 2 mm) were introduced into the cylinder through a slit along the cylinder surface, so that density and potential could be measured on the same diameter but at opposite sides of the plasma column. The probes could be moved parallel to the cylinder axis (fig. 1b). In order to bring these probes to different radial positions, it was necessary to interrupt the measurements and to open the apparatus. The measurements with these "axial probes" are supplemented by measurements with a "radial probe", this is a similar spherical probe which can be moved continuously in the radial direction (because of the construction of the apparatus, this is only possible in the middle of the cylinder z = 0).

We will use the following notations: (see fig. 1a,b)

S_I the axial probe at the side of the slit

S_{II} the axial probe at the opposite side of the slit

S_{III} the radial probe

r radial distance between centre of plasmacolumn and probe

z axial coordinate along the cylinder axis.

The way of measuring. Four cylinders of the same size were used in the following way:

	Endplates	cylinder surface	notation
1	conducting	conducting	Cu, Cu
2	insulating	conducting	glass, Cu
3	insulating	insulating	glass, glass
4	conducting	insulating	Cu, glass

The first measurements were made with the Cu, Cu cylinder, with holes in the end plates of 10 mm diameter. (This was expected to be large enough, as the diameter of the plasma beam is 6-8 mm). In order to align the cylinder properly the axial probes were placed in the middle of the cylinder (z = 0) in such a way that both spheres were at a distance of 30 mm from the axis $z(S_I) = z(S_{II})$, $r(S_I) = r(S_{II})$. During the alignment we encountered the first difficulties in our program: the two identical probes in equivalent positions registered plasma densities (as measured from the ion saturation current) of more than one order of magnitude difference and even the whole shape of the probe characteristics was different. After reversing the direction of the magnetic field, probe S_I showed the same characteristic as probe S_{II} did before, and vice versa. We tried to adjust the cylinder in such a way that the characteristics measured by the two probes were the same. It turned out that the ion current received by a probe depends very strongly on the alignment of the cylinder and it was impossible to bring both probe characteristics even to approximate agreement. This was only possible after the diameter of the holes in the end plates was increased up to 25 mm.*

But even then a very small change in the direction of the cylinder axis (e. g. of only 0.1^{o}) brought back the asymetry in the probe readings. Generally the adjustment is possible for one value of the magnetic field strength B, but is lost again for other values of B. Thus one can only obtain a kind of compromise where the probe characteristics are as nearly as possible in agreement over the whole B range - better than a factor-two-accuracy in ion current may not be expected.

For insulating end plates the adjustment was still more difficult and became only possible after the plasma beam between duoplasmatron and cylinder was enclosed in a conducting tube.

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* Of course the original idea of having well defined end plates is lost, the larger these holes are made. On the other hand the two probe characteristics were always in "complete" agreement (10 %) if the end plates were taken off (thus working with open cylinders) even if the cylinder was tilted over a considerable angle relatively to the axis.

Experimental results. The measurements were made in hydrogen with a gas pressure of $p = 2.10^{-3}$ Torr, with a plasma density in the column of about 10^{12} cm^{-3} and with magnetic fields of 1,2,3 and 4 Kilo Gauss. As mentioned before, the cylinder was adjusted for $r(S_I) = r(S_{II}) = 30$ mm. In the following figure 2a,b,c,d the ion saturation currents of the probes are shown as functions of space coordinates.

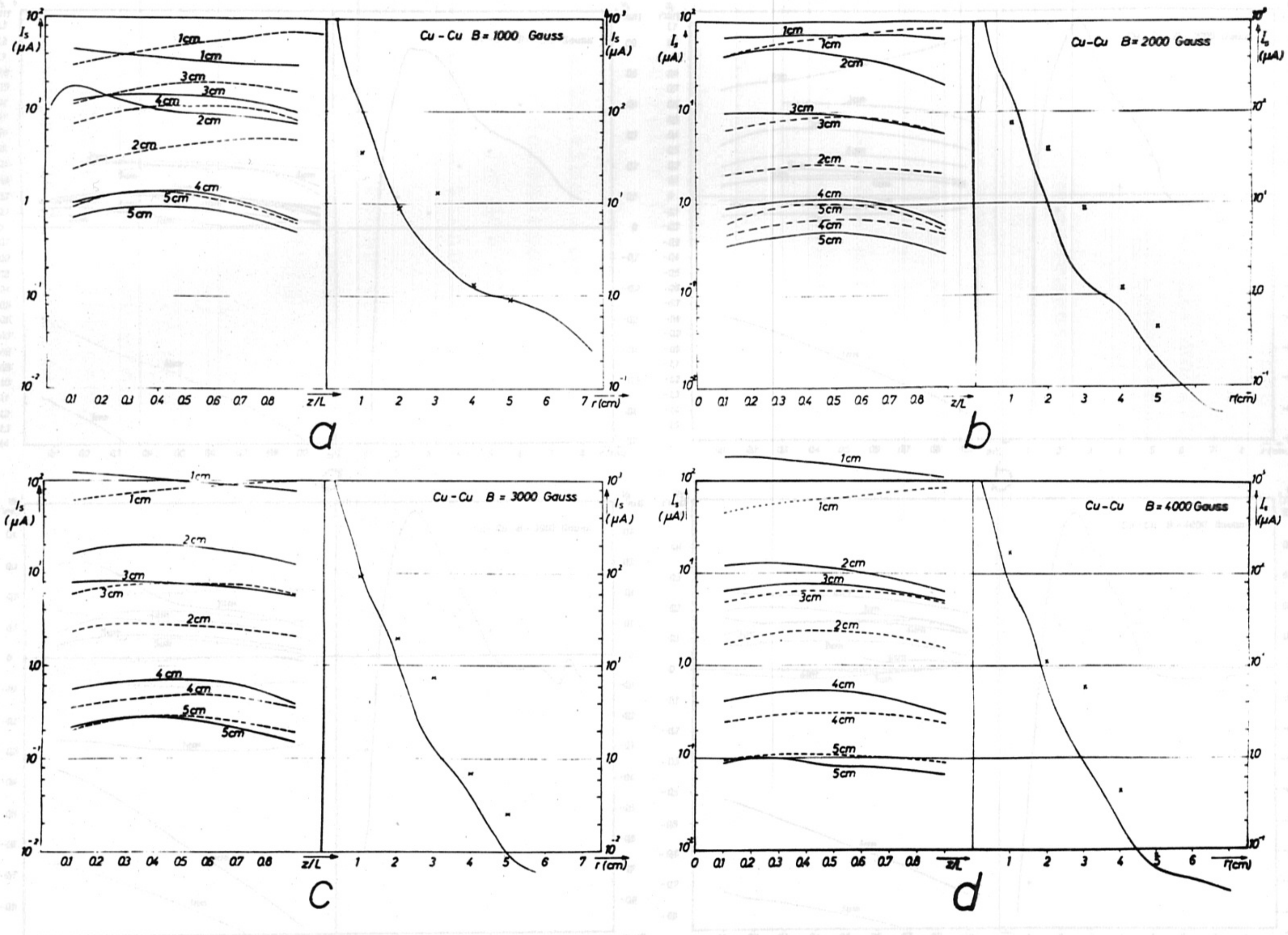
It is obvious that the desired symmetry is not present for all radial positions of the probes S_I and S_{II}. On the right hand side the ion saturation current to S_{III} is shown as function of radius. The ion saturation current to S_I is also shown for those points where the paths of S_I crosses that of S_{III}. (Between the measurements with S_I and S_{III} it is necessary to open the vacuum system - so we have in these points a criterium for the reproducability of the measurements).

If the electron temperature is approcimately independent of position it will be possible to replace ∇V_p by ∇V_f (V_p = plasma potential, V_f = floating potential). The results of the measurements of floating potential are shown in fig. 3a,b,c,d. Comparison of the measurements made with probes S_I and S_{III} indicates the extremely poor reproducability of these potential measurements. (Especially if the floating potential is smaller than one Volt.)

The lack of rotational symmetry and the difficulties we encountered in the measurement of the electrical field, gave us the impression that it was senseless to continue the measurements in this way.

III. Measurements in the afterglow

The measurements of the diffusion coefficients in a decaying plasma is based on equation (7). Recently one has taken recombination effects more precisely into consideration



L = 45cm

Fig. 2. The ion saturation current in the Cu, Cu cylinder. The left hand side shows the dependence on z, measured by probe S_I: ——— and S_{II}: - - - . On the right the radial dependence is shown.

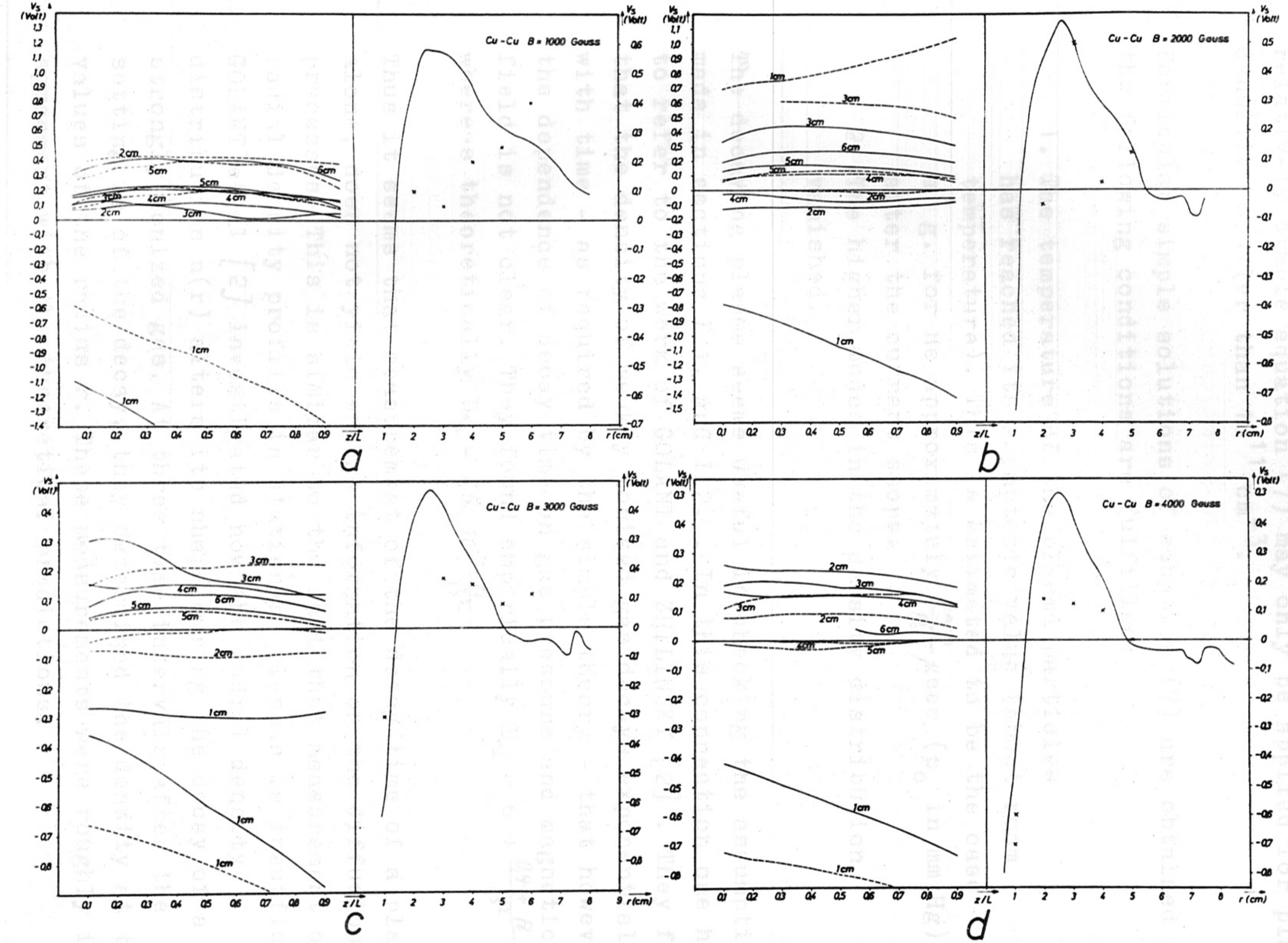


Fig. 3. The floating potentials in the Cu, Cu cylinder. On the left the potentials of S_I: —— , S_{II}: – – – . On the right the potential of S_{III}.

(e. g. ALESKOVSKY and GRANOVSKY [8]) and it seems that the relatively simple equation (7) may only be applied for plasma densities smaller than 10^{11} cm^{-3}.

Particular simple solutions of equation (7) are obtained if the following conditions are fulfilled:

1. The temperature of the plasma particles has reached its asymptotic value (about room temperature). This is estimated to be the case e. g. for He approximately $\frac{150}{p_0}$ μsec (p_0 in mm Hg) after the current stops.
2. The higher modes in the density distribution vanished.

The decaying plasma seems useful in checking the assumption made in sections I 1. and I 2. . In this connection one has to refer to the work of GOLANT and ZHILINSKY [2] . They found that the density of weakly ionized gas decays exponentially with time - as required by the simple theory - that however the dependence of decay time on gas pressure and magnetic field is not clear. They found empirically $D_{a\perp} = 6 + \frac{0,4 + p_0}{B^2} \cdot 10^8$ whereas theoretically $D_{a\perp} = 7,5 \cdot 10^7 \frac{p_0}{B^2}$.

Thus it seems that measurement of the decay time of a plasma alone, does not yield enough information on the diffusion processes. This is similar to the fact that measurement of radial density profiles in stationary plasmas is insufficient. GOLANT et al [2] investigated how the radial density distribution n(r) alters its shape during the decay of a strongly ionized gas. At three time intervals after the setting in of the decay, they determined the density at three values of the radius r. These measurements were roughly in agreement with the theoretical predictions.

Till now such measurements have not been made for weakly ionized gases, which are particularly under consideration in this report. We hope that an exact knowledge of the function n(r,t) would give conclusive information concerning the validity of the theoretical assumptions. Especially the dependence of the quantity $\frac{\nabla n}{n}$ should be determined experimentally for different boundary conditions (if possible). Following this idea we are making measurements on a decaying plasma in our apparatus "Cabinet".

The plasma beam has been interrupted either mechanically or electrically. The density has been measured by probes. We are making these measurements for both conducting and insulating walls. In the case of conducting walls we use segmented cylinders (like ROTHLEDER [6]) so that the currents to different parts of the wall may be measured during the decay. The results so far were qualitatively in agreement with the collisional theory.

IV. Measurement on diffusion waves

A further method of measuring the diffusion coefficients lies more or less in between measurements on decaying, and on stationary plasmas. By changing the density periodically ($n = n_0 \sin \omega t$) one may obtain a simple solution of equation (7). The easiest way to explain this method is by assuming that n is a function of only one space coordinate x. The solution of (7) becomes in this case:

$$n(x,t) = A \cdot e^{-\sqrt{\frac{\omega}{2D}}\,x} \cos\left(\omega t - \sqrt{\frac{\omega}{2D}}\,x\right)$$

This formula shows that both measurement of phase and of attenuation of the diffusion waves may be used to determine the diffusion coefficient. GOLUBEV [9] worked out this method in further detail and already published some experimental results.

V. References

[1] SCHLÜTER, A. Z. Naturforschg. 6a, 73 - 78 (1951)

[2] GOLANT, V.E. Usp. Fiz. Nauk 74, 377 - 44o (1963)
engl. Sov. Phys. Uspekhi Sept.- Oct. (1963)

[3] WHITEHOUSE, D.R., WOLLMAN, H.B.
Phys. Fluids 6 1470 - 1478 (1963)

[4] SIMON, A. An Introduction to Thermonuclear Research
Pergamon Press London (1959)

[5] BOESCHOTEN, F. J. Nucl. Energy C, July - Aug. (1964)

[6] ROTHLEDER, S.D. Thesis MIT (1962)

[7] BOESCHOTEN, F. , SCHWIRZKE, F. Nucl. Fusion 2,54 (1962)

[8] ALESKOVSKY, Yu. M. , GRANOVSKY, Sov. Phys. JETP 16
887 (1963)

[9] GOLUBEV, V.S. Sov. Phys. JETP 16, 1399 (1963)