Some Aspects of Superconducting Magnetic Energy Storage

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Some Aspects of Superconducting Magnetic Energy Storage

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This paper shall give some theoretical aspects of superconducting energy storage.

Energy density

Compared with electrostatic energy storage systems, say condensers, the magnetic field promises higher energy density and for the entire storage systems considerable smaller size and lower costs.

The density of the magnetic field energy is

$$e_m = \frac{1}{2} \mu H^2$$

This indicates that a feromagnetic core will be of advantage. However the advantage of iron becomes rather insignificant in the range of high fields, on the other hand a fast changing field in the iron core would cause remarkable eddy currents and hysteresis losses, so that we may forget iron for this consideration. On the first picture the magnetic energy density is plotted against the field strength. The pair of curves on the right-hand side is continued by that on the left-hand side if you go to smaller values of H. For these two curves you must multiply the vertical scale with 0.1.

In comparison the electrostatic field energy, given by

$$\frac{\varepsilon}{2} E^2$$

is limited by the dielectric strength of insulating materials. For $\varepsilon_r = 5$ and $E = 100$ kV/mm a value of

$$0.2 \text{ Joules/cm}^3$$

can be approached.

On this diagram the maximum value for foil condensers would be situated at a point, which corresponds to a magnetic field (without iron) of about 7 k Oe.

The energy density of the magnetic field is only limited by the magnetic forces and in the case of superconductors by the quenching characteristic of the wire material.

NbZr-wire, for instance, allows to produce fields up to 70 kOe, corresponding to an energy density of about 2o Joules/cm^3; this means 100 x the condenser value.

Other superconductors capable of producing higher fields are available and perhaps in the next future much higher fields than 100 kOe can be achieved.

The further consideration shall be confined to completely closed systems, where the entire magnetic field is contained in the coil. In this case an infinite coil producing a homogeneous field along the axis represents a configuration with the highest possible energy per unit volume.

Considering the energy density of such a coil one can distinguish between a net-energy density related only to the field volume outside the wire volume and a gross energy density related to the entire field volume inside and outside the wire.

So the field consists of two parts, the homogeneous field inside the inner radius of the coil and the field penetrating the conductor cross section, which decreases to 0 at the outermost layer of the coil.

The energy density related to the entire coil volume is given by the equation on the figure. It contains as parameters the field strength and the radius ratio $\frac{r1}{r2}$ Dividing this equation by $\frac{1}{2}\mu o\, H_{o}^{2}$ one obtains an expression only depending on $\frac{r1}{r2}$, which represents the gross energy density divided by net energy density of the homogeneous field on the axis.

On the second figure the upper curve corresponds to this ratio. If H^2 is substituted by the current density i multiplied with the space factor we obtain a relation between energy density and current density. The lower curve corresponds to this relation also divided by the net energy $\frac{1}{2}\,\mu\, H^2$.

If H represents the limiting factor of the coil construction it is advantageous to choose a ratio $\frac{r1}{r2}$ near 1; and if the current density is limited a maximum of the energy density can be obtained for $\frac{r1}{r2}$ near 0, that means, the entire coil volume must be filled with current carrying conductors.

For superconductors we find according to the quenching characteristic a mixture of these two limitations.

On figure three the gross energy density of an infinite NbZr superconducting coil is plotted against the ratio $\frac{r1}{r2}$ if a space factor of $\gamma = 0.4$ is assumed.

We obtain several curves characterized by different outer coil radius, this means corresponding values of total stored energy per unit coil length.

By increasing the outer diameter the optimum ratio $\frac{r1}{r2}$ increases simultaneously. In addition the maximum energy density increases to finally approach the value of the net energy. If always the upper critical field could be achieved the curves would follow the upper limiting curve and then decrease if the wire volume given by $r_2 - r_1$ would not be sufficient to produce the maximum field.

For low values of $\frac{r1}{r2}$ and low field strength, which is of course of no interest, perhaps we have to take into account the magnetization characteristics of the material.

The curves are calculated for uniform current distribution in the coil. To approximate the quenching characteristic not only in the innermost layer, it is possible to provide an increasing current density in two or more steps from inside to outside.

Stekly has shown that a toroid with a ratio between the small and the large torus radius of $\frac{a}{r} = \frac{1}{3}$ represents for finite systems the configuration with the highest possible energy, related to a certain field strength.

According to this optimum geometry on figure four, the estimated costs of one Joule are plotted against the storage energy. This figure is based on commercially available NbZr wire with a uniform current density of 3.10^4 A/cm^2, which corresponds to a usual degraded current, and a field on the axis of 60 kOe.

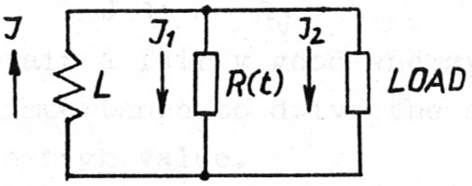
The diagram indicates that it would be of commercial advantage to use inductive storage systems beyond 100 k Joules.

This rather optimistic prediction depends of course on the realization of large superconducting coils with high field strength.

Perhaps the difficulties included in the construction of large coils would suggest lower values of $\frac{a}{r}$ than $\frac{1}{3}$.

Discharge characteristics

The discharge characteristics of magnetically stored energy can be described by means of a simple circuit.



While the inductor is charged, the current flows constantly in the left circuit, where R $(t) = 0$. There is no current flowing through the load.

Now discharge can be initiated by driving R (t) from 0 up to a certain value R. The inductor current is now split up into two transient currents J_1 and J_2. In order to obtain a good energy transfer to the load a rapid decrease of the current J_1 is required.

The switch has to open a superconducting circuit. One can assume that it consists of a superconducting wire, which will be driven into the normal state by means of an electrical or thermal triggering device. On the other hand one can perhaps imagine a mechanically driven switch, although it should be difficult to realize a disconnectable superconducting contact.

For our further consideration we may assume, that it is possible to drive R (t) up to certain value during a negligible short period.

To show the principle of magnetic energy discharges a discharge characteristic with an ohmic load is shown on the figure 5. After the switch resistence has approached its constant value the load current increases at once to an initial value given by the ratio between the resistances and then decreases simultaneously with inductor and switch current according to the time constant

$$\tau = L \left(\frac{1}{R} + \frac{1}{R_v}\right)$$

Load current and switch current are related such as

$$\frac{J2}{J1} = \frac{R}{R_v}$$

In order to obtain a fairly good energy transfer to the load it is of importance to drive the switch resistance to a comparable high value.

The efficiency of the discharge is characterized by the ratio between the integral of the ohmic losses, dissipated in the load and the energy originally stored in the inductor.

The matching condition for the highest obtainable power is given by $R = R_v$.

Figure 6 shows the principle discharge into a pure inductive load. The inductor current decreases to a final current given by $J_\infty = \frac{J_0}{1 + L_v/L}$

According to the initial conditions the load current increases during the same time to the same final value. Simultaneously the current in the switch path decreases to 0.

The lower the load inductance can be provided the higher the load current will increase.

The efficiency of the energy transfer is given by

$$\frac{\frac{Lv}{L}}{\left(1 + \frac{Lv}{L}\right)^2}$$

This ratio approaches a maximum value of 25% for the condition $L_v = L$. It is not possible, in the case of a pure inductive combination, to transfer more energy from an inductor to an inductive load than a quarter of the original stored energy. In this case the losses dissipated in the switch, given by $\frac{Lv}{L + Lv}$, are $\frac{1}{2}$.
We may notice that the part of the switch losses can be calculated independently of R itself.

The ratio between the losses in the switch and the transferred energy is

$$1 + \frac{L}{Lv} \geqq 1$$

and approaches a minimum value 1 for $L_v \gg L$.
These last results are also valid, if R (t) is considered to increase with a finite velocity.

In order to transfer magnetic energy to a inductive load with reasonable efficiency one has to use a transfer condenser with an energy capacity not very far from that of the magnetic energy storage itself.

On the following pictures some different discharge characteristics are shown, as they have been computed by means of the analogue computer Pace T R 10.

The first one shows the currents in the three paths of the circuit, if the load consists of an inductance and a resistance. For the first oscillogram the resistance on the load is 0, as you have seen just on the last diagram.

On the following oscillograms the coefficients are changed, so one can recognize the influence of different time constants resistances and inductances.

On the next figure the discharge characteristics are obtained from the same circuit, only R (t) been exponentially increased from 0 to a certain value during a finite period.

The time function of the switch resistivity is plotted below the current oszillograms. It is of importance to increase the resistance of the switch fast, compared with the discharge time.

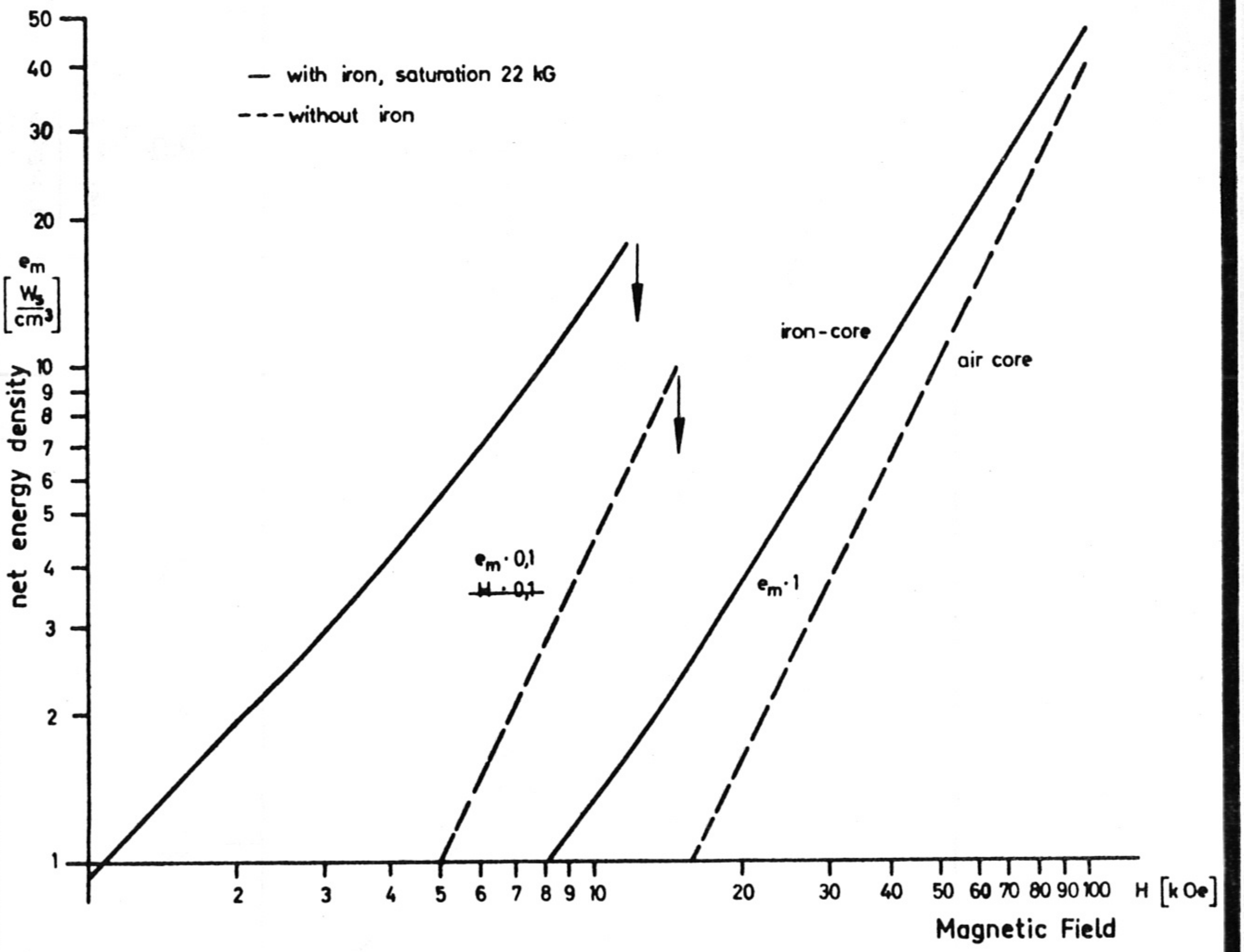
As you saw in the case of an inductive load magnetic energy systems seem to be advantageous only for resistive loads.

As an example of approbriate application one can consider a load consisting of an inductance and a resistance, where the inductive energy of the load has been delivered from a condenser and during the second part of the discharge an inductor is used to keep the current constant.

Such a circuit looks like a power crowbar-system, where the power condensor is substituted by an inductor.

In conclusion I may point out, that a superconducting coil applied for magnetic field production will be usually protected against fast field and current changes, in order to avoid normal transition.

In the case of an inductor fast field changes are desired. One has to find out, how the inductor can be protected from a considerable amount of own ohmic losses, if transition occurs. For instance this is only possible, in the case of a resistive load, if the resistance of the load is much higher than the normal resistance of the inductor.



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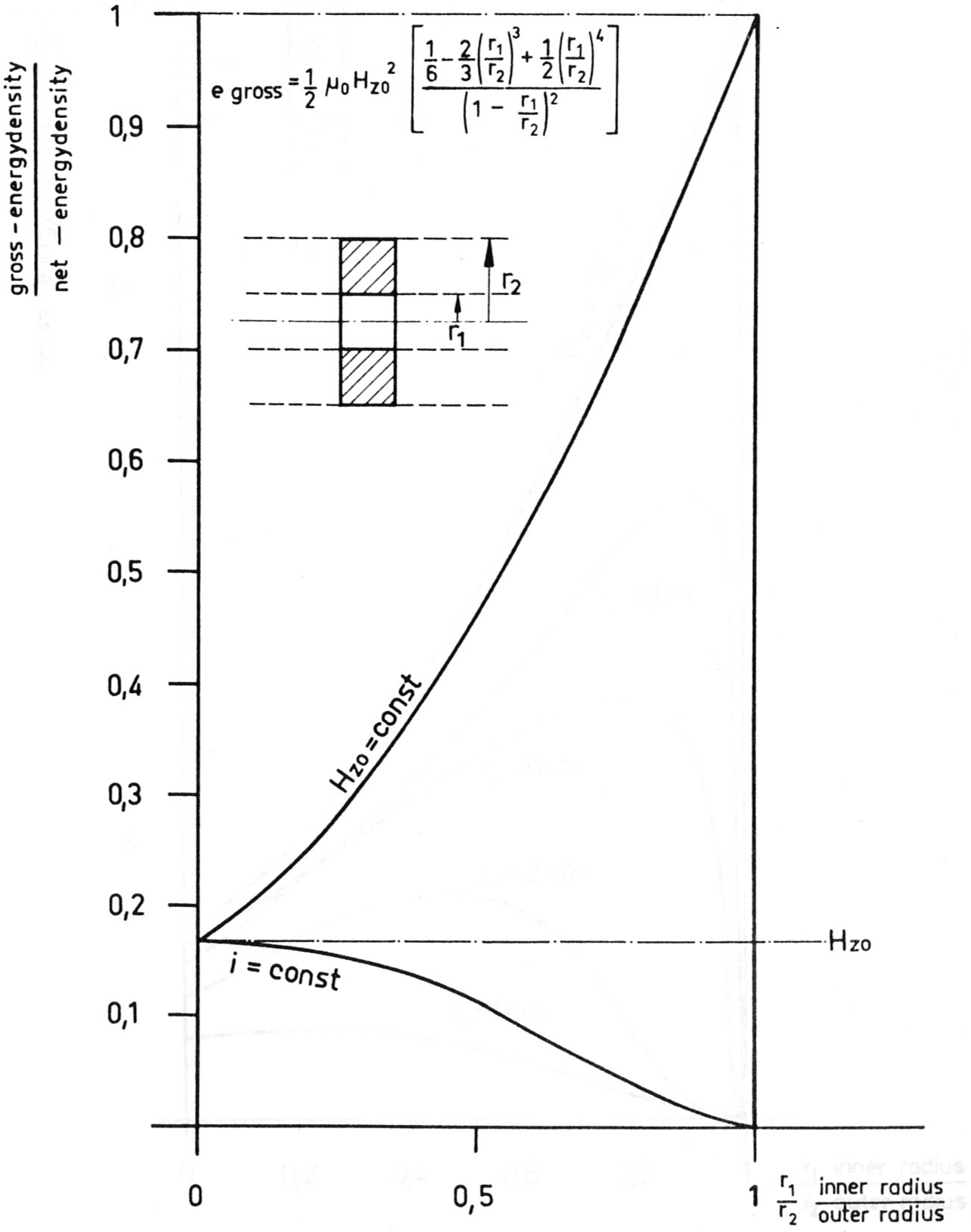
NET ENERGY DENSITY WITH AND WITHOUT IRON

M007
1964



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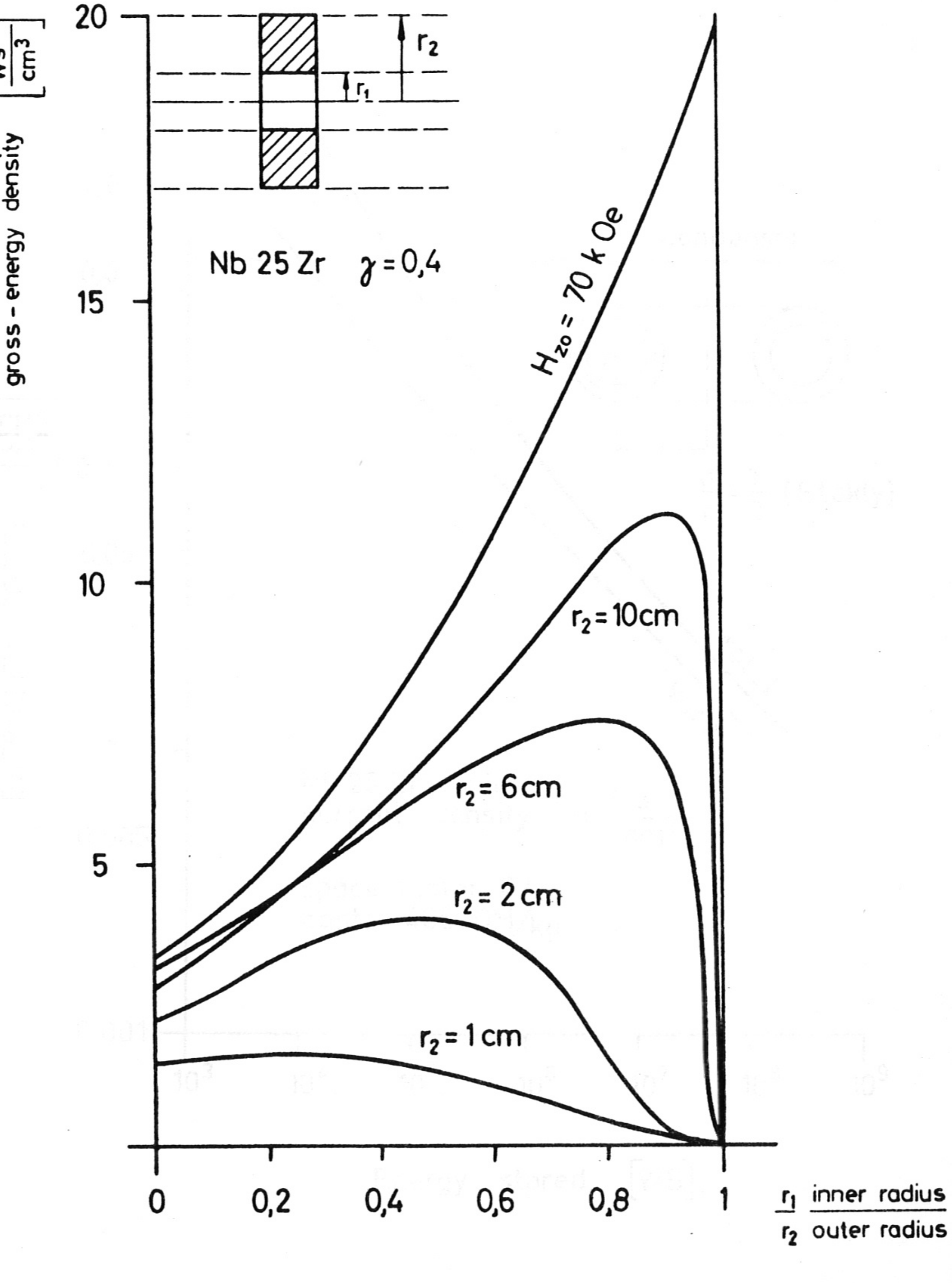
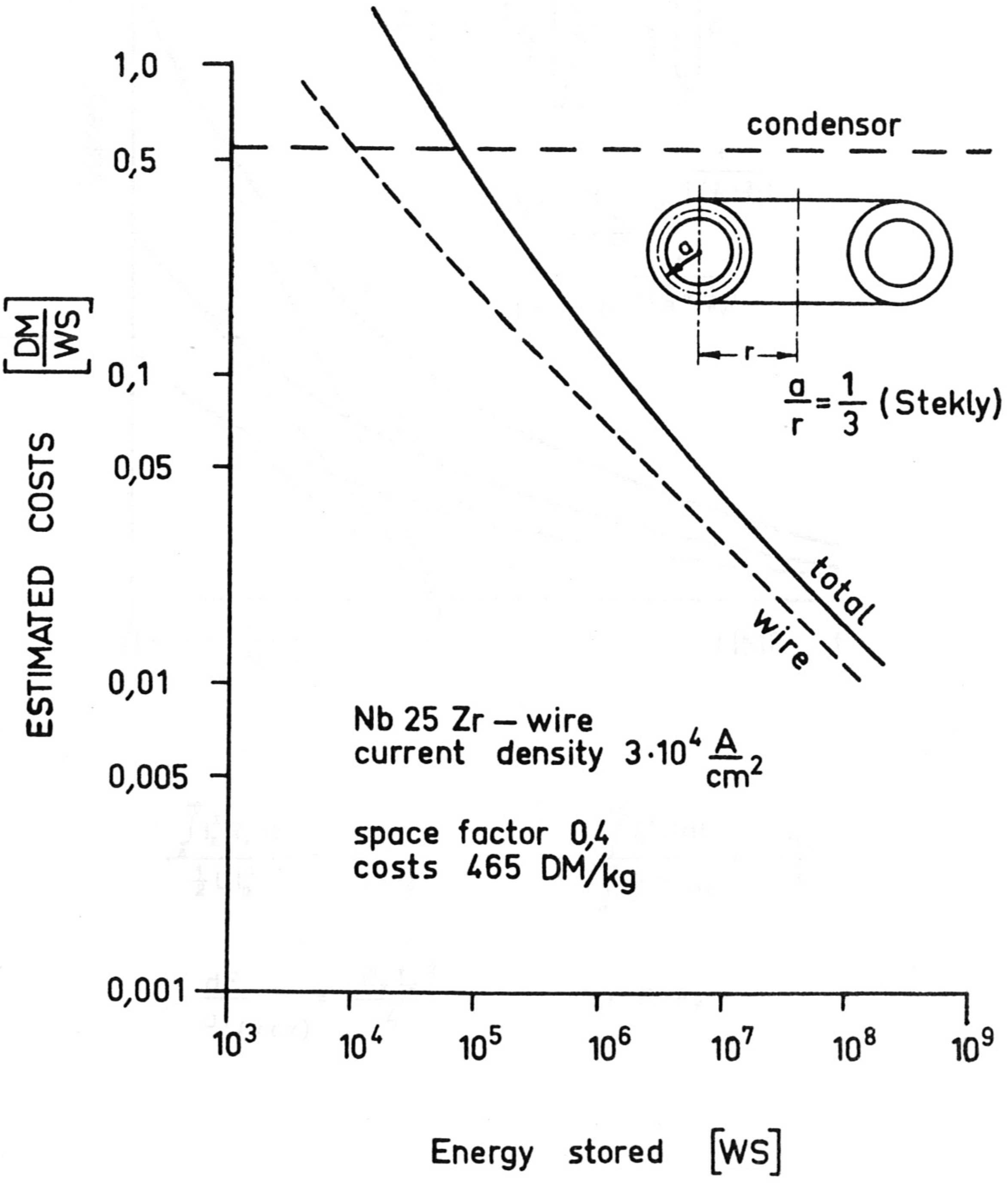
ENERGY DENSITY
OF INFINITE COILS

M008
1964



IPP Abt. Technik	ENERGY DENSITY OF NB ZR.- INFINITE COILS	**M005** 1964



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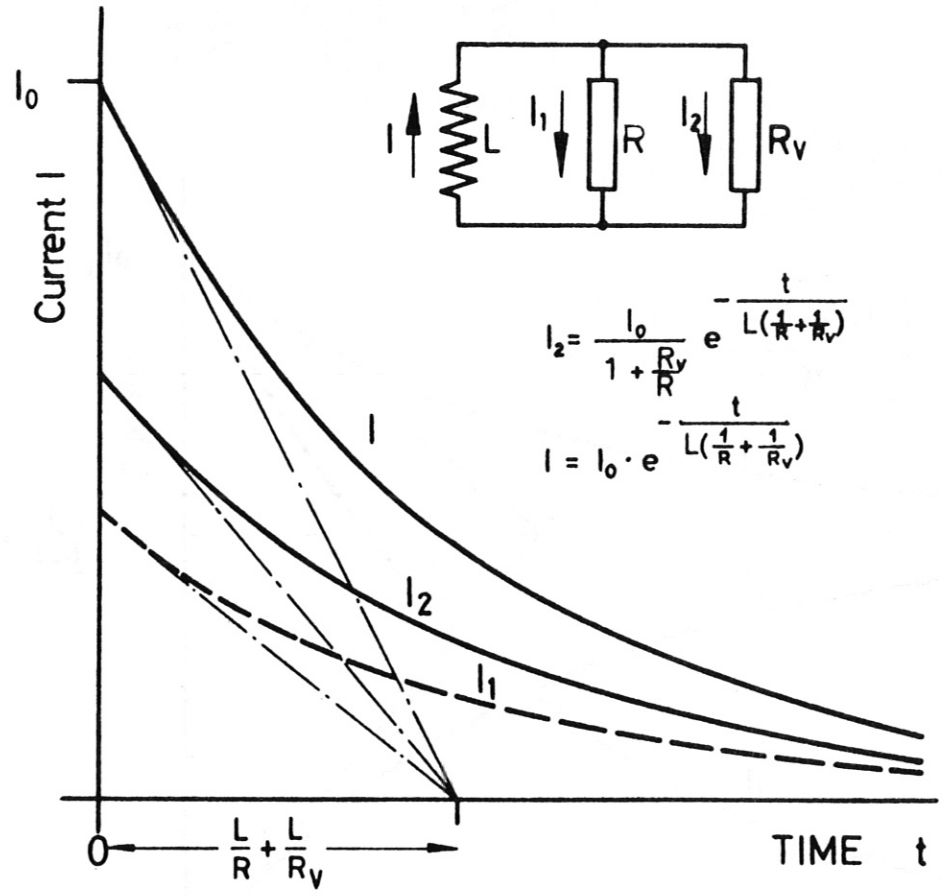
ESTIMATED COSTS OF SUPERCONDUCTING ENERGY - STORAGE

M 006
1964



$$I_2 = \frac{I_0}{1+\frac{R_v}{R}}\, e^{-\frac{t}{L(\frac{1}{R}+\frac{1}{R_v})}}$$

$$I = I_0 \cdot e^{-\frac{t}{L(\frac{1}{R}+\frac{1}{R_v})}}$$

$$\frac{\int_0^\infty I_2^2 R_v\,dt}{\frac{1}{2} L I_0^2} = \frac{1}{1+\frac{R_v}{R}} \qquad \frac{\int_0^\infty I_1^2 R\,dt}{\int_0^\infty I_2^2 R_v\,dt} = \frac{R_v}{R}$$

$$\frac{dE}{dt}_{(max)} = \frac{R_v I_0^{\,2}}{4} \qquad \text{for } R=R_v$$

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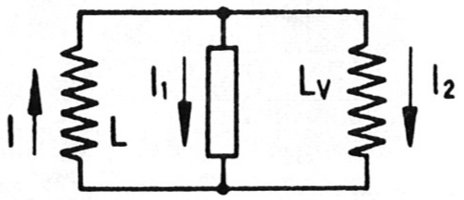
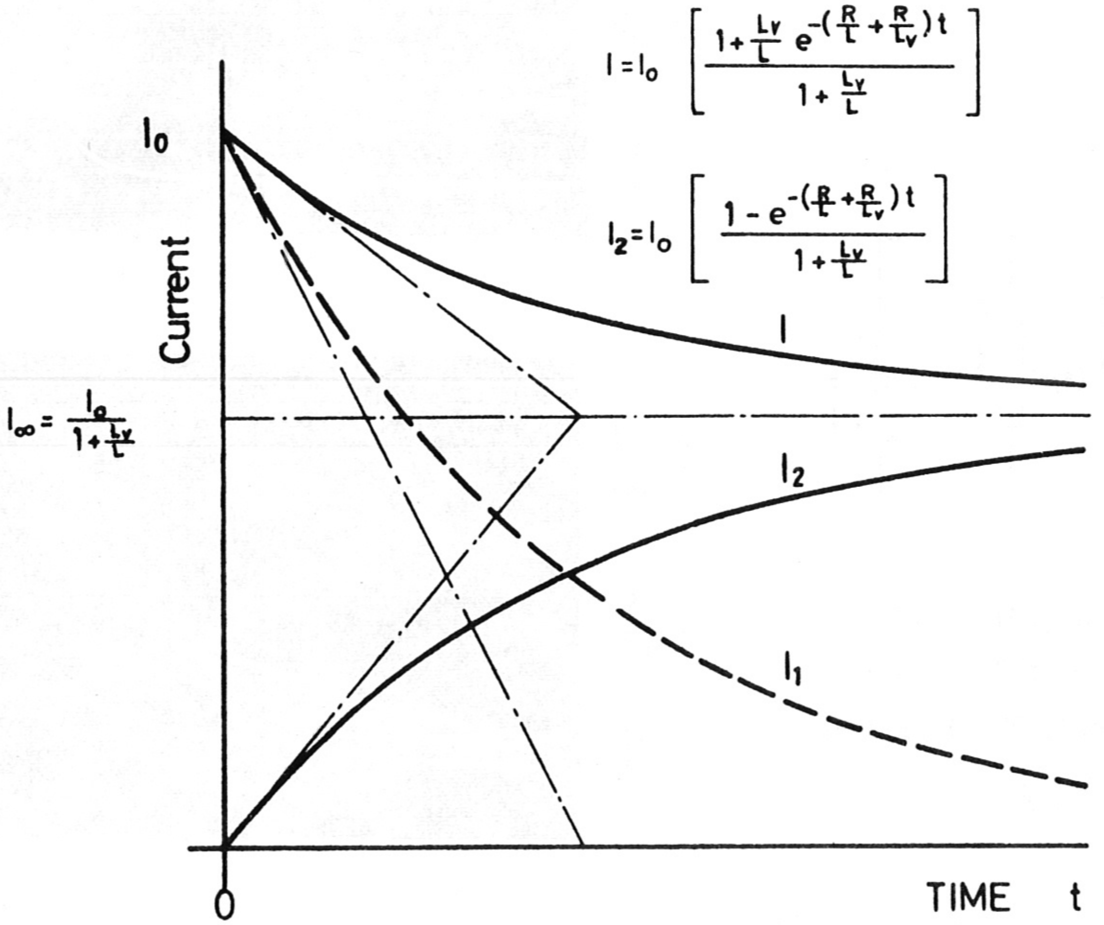
DISCHARGE CHARACTERISTIC
OHMIC LOAD

M 004
1964





$$I = I_0 \left[\frac{1 + \frac{L_v}{L} e^{-\left(\frac{R}{L} + \frac{R}{L_v}\right) t}}{1 + \frac{L_v}{L}} \right]$$

$$I_2 = I_0 \left[\frac{1 - e^{-\left(\frac{R}{L} + \frac{R}{L_v}\right) t}}{1 + \frac{L_v}{L}} \right]$$

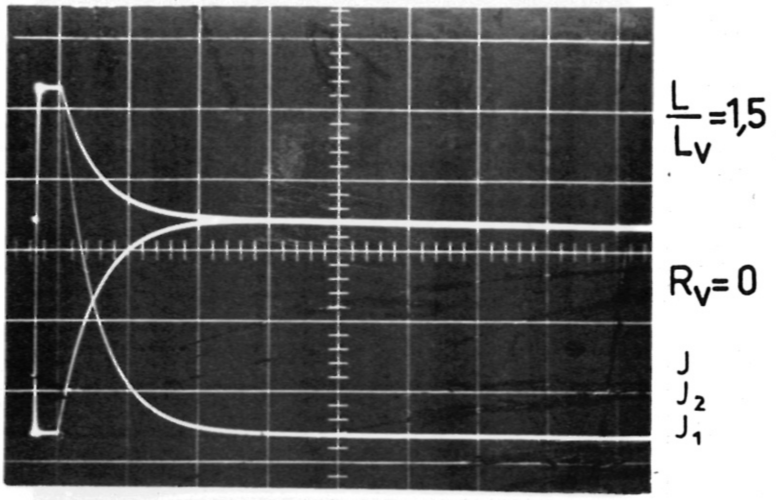
$$I_\infty = \frac{I_0}{1 + \frac{L_v}{L}}$$

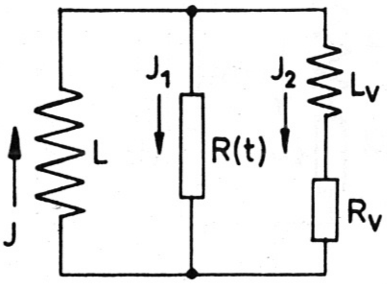
$$\frac{L_v I_{2\infty}^2}{L \cdot I_0^2} = \frac{\frac{L_v}{L}}{\left(1 + \frac{L_v}{L}\right)^2} = \frac{1}{4} \text{ (for } \frac{L_v}{L} = 1\text{) max.}$$

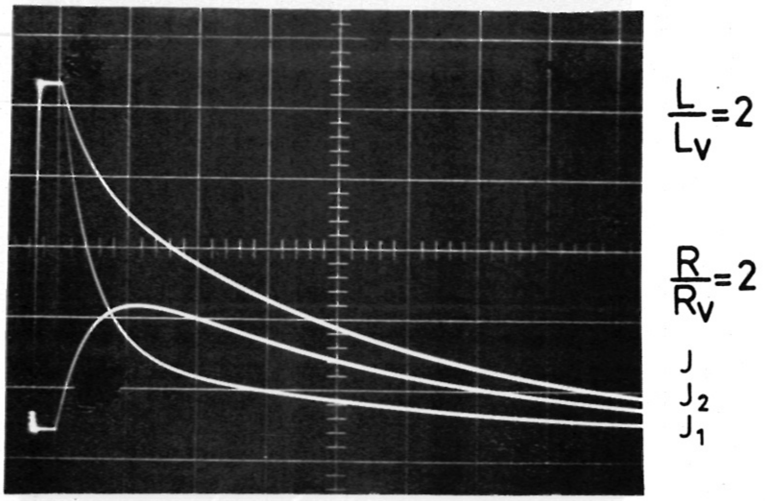
$$\frac{\int_0^\infty R I_1^2 \, dt}{\frac{1}{2} L I_0^2} = \frac{L_v}{L + L_v} = \frac{1}{2} \text{ (for } \frac{L_v}{L} = 1\text{)}$$

$$\frac{\int_0^\infty R I_1^2 \, dt}{\frac{1}{2} L_v I_{2\infty}^2} = 1 + \frac{L}{L_v} \geq 1$$

IPP Abt. Technik	DISCHARGE CHARACTERISTIC INDUCTIVE LOAD	M003 1964





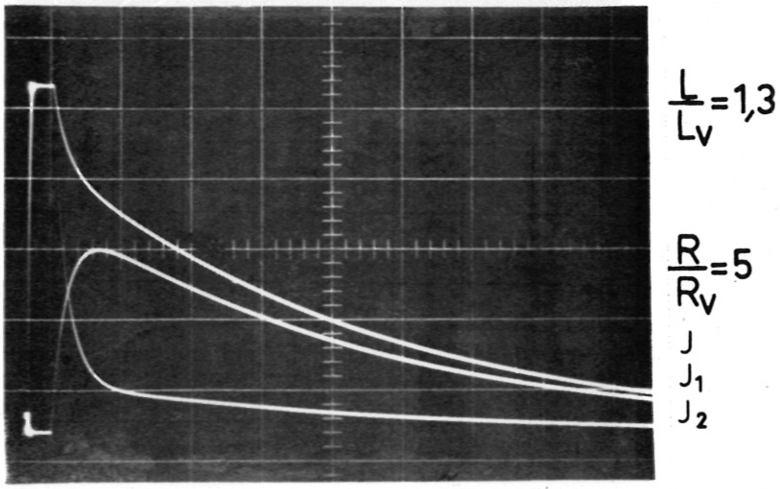


for $t<0$: $R(t)=0$

for $t>0$: $R(t)=R$

INITIAL CONDITIONS

$J_{(t=0)}=J_0$: $J_{(t=0)}=0$



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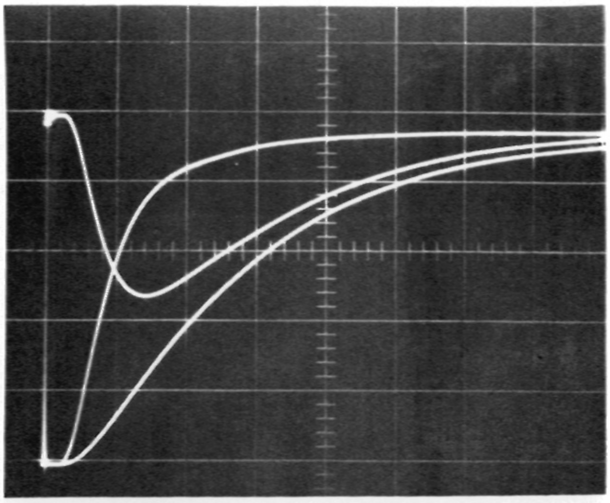
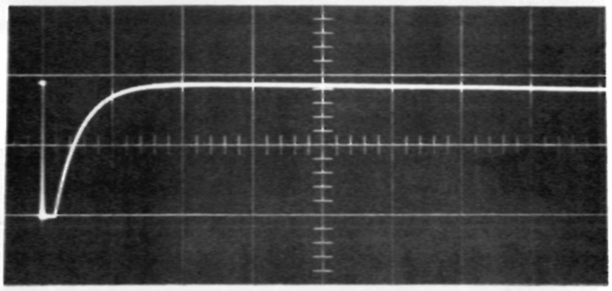
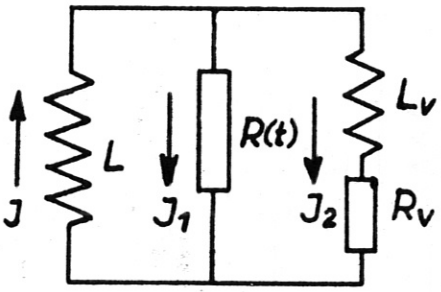
COMPUTED DISCHARGE CHARACTERISTICS

M 009
1964





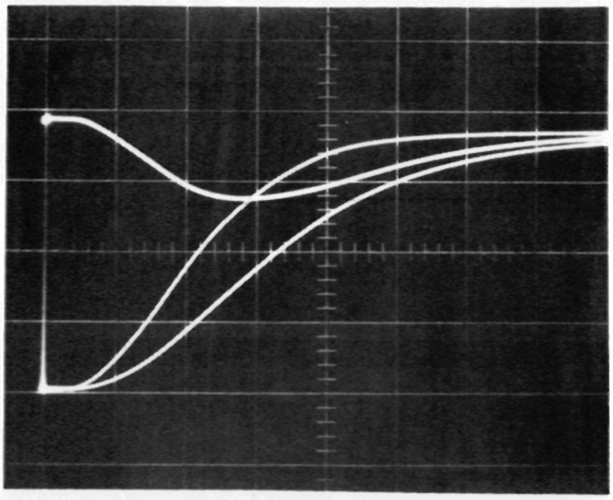


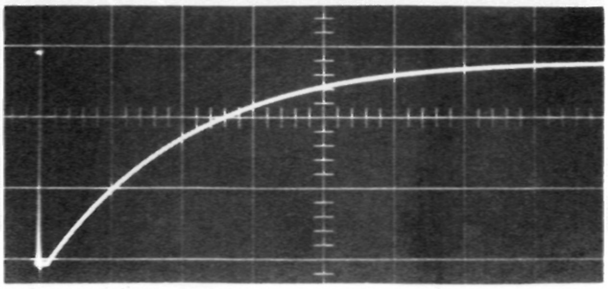
FOR $t<0$: $R(t)=0$

$t>0$: $R(t)=R_\infty(1-e^{-\frac{t}{\tau}})$

INITIAL CONDITIONS

$J_{(t=0)}=J_0\;,\;J_{2(t=0)}=0$





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COMPUTED DISCHARGE CHARACTERISTICS

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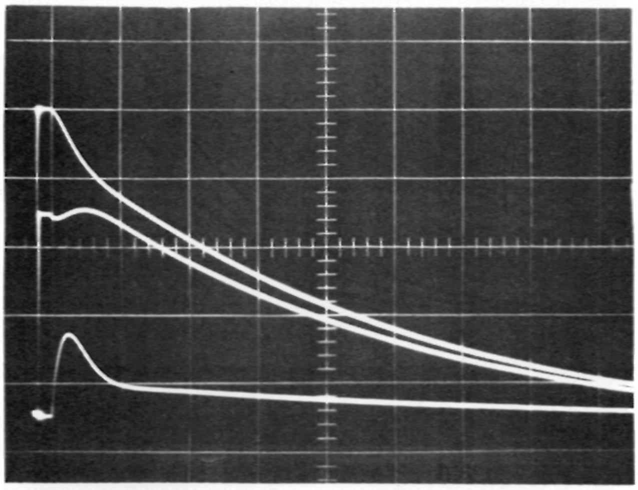

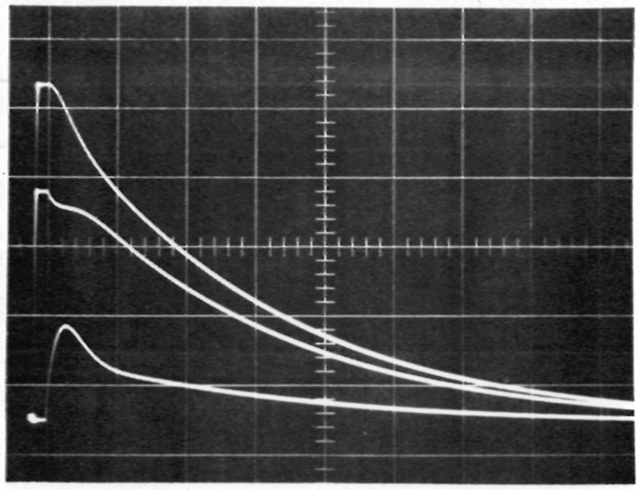

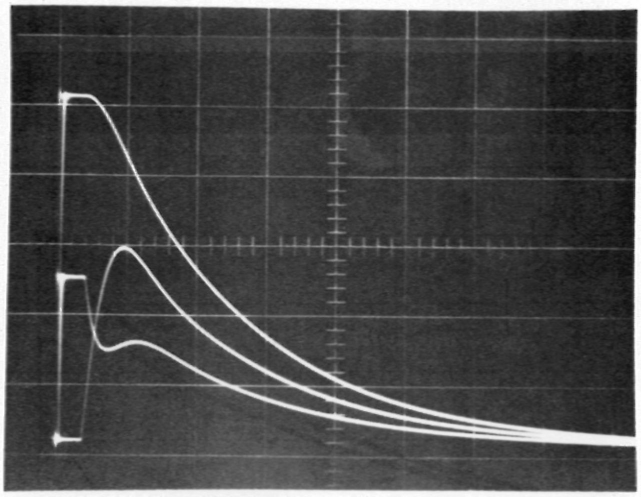

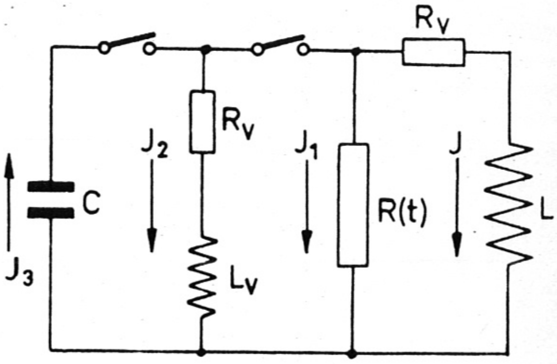

for $t < 0$: $R(t) = 0$
for $t > 0$: $R(t) = R$

INITIAL CONDITIONS

$J_{(t=0)} = J_0$; $J_{(t=0)} = J_{20}$
$n_{c(t=0)} = 0$

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COMPUTED DISCHARGE CHARACTERISTICS

M 010
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