INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

ABSORPTION COEFFICIENT OF RUBY-LASER RADIATION IN FULLY IONIZED LIGHT ELEMENTS

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1. Introduction.

The question concerning the absorption coefficient of light in fully ionized gases of high density was raised in the discussion about heating up plasmas by the intense light of Lasers.[1)2)3)] Besides of these theoretical calculations it was experimentally found that intense light with E-fields higher than 10^7 V/cm could ionize gases in a resonanceless manner.[4)5)] The primary question following therefrom is in what manner the ionization takes place. Secondary, to interpret the properties of the plasma it is necessary to know the optical constants of high density plasma. Last not least it is necessary for plasma diagnostics by lasers[6)] to know the absorption of the laserbeam in the observed plasma.

There are three different ways to calculate the absorption coefficient of plasmas. First it was possible under the assumption of Kirchhoffs' law to calculate the absorption from the emission of bremsstrahlung. The difficulties for high densities arose when plasma effects should be derived from the quantum mechanical treatment. The second way is the calculation from the microscopic plasma theory where the difficulty is to introduce the impact mechanism. Dawson and Oberman[7)] used a way of self-consistent field method to introduce the impacts into the formalism of the Vlasov equation. The third way is to evaluate the optical constants of a plasma given by the macroscopic theory of the two-fluid-model.[8)] The last way can be treated without any additional assumptions from the linearized macroscopic equations. It will be shown that the macroscopic theory, which covers completely the whole interesting range of densities and temperatures, especially with regard to the plasma frequency, gives nearly the same values as the other theories in their special ranges of validity. Limitations of the macroscopic theory are only given by a not too small Debyeradius and the occurence of Boltzman statistics. The first two ways in principle can be expanded beyond this limitation, but it seems that for the moment it has not been necessary to carry out this rigorous task.

As the here given calculations are nonrelativistic, a limitation is given for high electron energies near to 10^5 eV. We shall calculate down formally to energies of 1 eV, though it is clear that fully ionized Plasmas of densities around 10^{21} electrons/cm^3 only exist at higher temperatures than 10 eV. Further we shall calculate Plasmas under the assumption of fully ionization for nuclear numbers from 1 to 10. These values may be taken as an approximation for the multiple but not fully ionized very heavy atom, neglecting the discrete emission and absorption processes of the transition from the n to the (n+1) ionization step. The different mass of such an +-Ion in comparison to the mass of a nucleus with number n is not important for the following discussion.

2. Calculation of Absorption Coefficient.

To calculate the optical constants of a plasma by the macroscopic theory it is suitable to start with the basic equations, given by Schlüter [8]. With the macroscopic velocity $\underline{v}$, the plasma density ϱ, the current density $\underline{j}$ and an external magnetic field $\underline{H}_0$ the equation of motion is in cgs.-units

$$(1) \qquad \varrho \frac{\partial \underline{v}}{\partial t} = \frac{1}{c} \underline{j} \times \underline{H}_0$$

when nonlinear terms are neglected; this is always possible if the amplitude of oscillations is small. In case of laser application with high power densities it is necessary to know, at what electrical fields of the light this linearization is still allowed. Hitherto fields of 10^{10} v/cm are far from realization. These fields correspond to a power density of 2.7×10^{17} Watts/cm^2. The electrical field of ruby laser light with a frequency $\omega = 2.7 \times 10^{15}\,\mathrm{sec}^{-1}$ of this power density would accelerate electrons during a half period of light oscillation to move on 3.8×10^{-10} cm. A Debye distance being specially small for our cases is 2.35×10^{-8} cm for an electron density $n_e = 10^{23}\,\mathrm{cm}^{-3}$ and a temperature of 10^2 eV. This seems to be applicable also for the above very strong fields of the linearized equation (1).

The second basic equation with the electrical field $\underline{E}$, the electron and ion masses m_e resp. m_i and the electron charge e is Ohm's law

$$(2) \qquad \frac{4\pi}{\omega_p^2} \left(\frac{\partial \underline{j}}{\partial t} + \nu \underline{j} \right) = \underline{E} + \frac{\underline{v}}{c} \times \underline{H}_0 - (m_i - m_e) \frac{1}{e} \frac{\partial \underline{v}}{\partial t}$$

Here is the plasma frequency

(3) $$\omega_p^2 = \frac{4\pi e^2 n_e}{m_e}$$

and the collision frequency with the atom number Z

(4) $$\nu = \frac{\omega_p^2 \pi^{3/2} m_e^{1/2} Z e^2 \ln\Lambda}{8\pi \gamma_E(Z)(2kT)^{3/2}}$$

with the averaged ratio of Debye distance to the impact parameter

(5) $$\Lambda = \frac{3}{2Ze^2}\left(\frac{k^3T^3}{\pi n_e}\right)^{1/2}$$

and Spitzer's [9] function $\gamma_E(Z)$ taking into account the electron-electron impacts in addition to the electron-ion impacts. For the electron-ion impacts here the propositions of the Lorentz gas are used, whose applicability is good enough [10], for in all following interesting cases the collision frequency is very much smaller than the light frequency.

When the external magnetic field H_0 is zero - the question up to what limitation this is allowed, should not be discussed here - the equations (1) and (2) lead by the ansatz

(6) $$\underline{E} = \underline{E}_r e^{i\omega t}$$

with the factor $\underline{E}_r$ being independent from time to

(7) $$\frac{\partial \underline{j}}{\partial t} = \frac{\omega_p^2}{4\pi} \frac{1}{1 - i\frac{\nu}{\omega}} \underline{E}_r$$

This gives together with the Maxwellian equations

(8) $$\Delta \underline{E}_r + \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2} \frac{1}{1 - i\frac{\nu}{\omega}}\right)\underline{E}_r = 0$$

The factor of ε_r is the square of the complex dielectric constant. Twice the imaginary part of this is the absorption coefficient K, which describes the decrease of energy with the depth x corresponding to $\sim exp(-Kx)$. From (8) is

$$(9) \qquad K = 2\frac{\omega}{c}\sqrt{\frac{1}{2}\left[\sqrt{\left(1-\frac{\omega_p^2}{\omega^2+\nu^2}\right)^2+\left(\frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2+\nu^2}\right)^2}-\left(1-\frac{\omega_p^2}{\omega^2+\nu^2}\right)\right]}$$

If $\ln \Lambda$ would be put constant, K would be a function of the 12th degree in n_e and 18th degree in T.

3. Range of Validity and Results.

The formula (9) is only valid when Λ corresponding to (5) is higher than 1. Therefore must be with $[T] = eV$ and $[n_e] = cm^{-3}$

$$n_e < \frac{9k^3}{4Z^2e^4} T^3 = 2.42\times 10^{20} T^3 \tag{10}$$

Furthermore it is necessary that Boltzmann statistics hold true. This is surely the case if the particle energy equivalent to the temperature of the plasma is higher than 10-time of the Fermi energy ζ_0 at T = 0. As the Fermi energy decreases to zero with increasing temperature, this limit in any case gives the quite sure validity of Boltzmann statistics. With Planck's constant h is

$$\zeta_0 = \frac{h^3}{2m_e} \left(\frac{3n_e}{8\pi}\right)^{2/3} \tag{11}$$

and with T in eV and $[n_e] = cm^{-3}$ we get the condition

$$T > 10\,\zeta_0 = 3{,}65\times 10^{-14} \tag{12}$$

As the moment of equienergetic ions is higher than that of electrons the condition (12) gives the existence of Boltzmann statistics for all particles with higher mass than m_e.

In figures 1 to 10 the absorption coefficient (9) when light of the frequency of a ruby laser falls in ($\omega = 2.71\times 10^{15}$ sec) is given for the fully ionized elements with nuclear number 1 to 10 depending on the temperature with the atom density as parameter. The curves are full drawn for the case that the condition of Boltzmann statistics with respect to (12) is valid and simultaneously the condition (10) is holding. In the case that Boltzmann statistics are not surely given but condition (10) is valid the curves are dashed. The continuation of the physically correct full drawn curves into the dashed region is only to get a wider survey.

From Fig. 1 to 10 it is to be seen that by the electron density $n_{eo} = Zn_{io} = ZN = 2.32 \times 10^{21}\,cm^{-3}$ ($N \sim$ number of atoms/cm^3) for which the plasma frequency is equal to the light frequency a significant difference is given for higher and lower densities. At n_{eo} for temperatures higher than 10^2 eV the value of $\partial K/\partial n_e$ becomes extremely high according to the singular nature of the K-function near the density connected with a plasma frequency equal to the light frequency.

In the case $\omega_p(n_e) \ll \omega$ and $\gamma \ll \omega$ in K under the second square root the first squared term is so much higher than the second squared term that a binomial progression leads to the approximation formula

$$K = \frac{4\pi^4}{\gamma_E(Z)} \frac{Z^2 n_e n_i e^6 \ln\Lambda}{c(\omega^2+\nu^2)(2\pi m_e kT)^{3/2}\left(1-\frac{\omega_p^2}{\omega^2+\nu^2}\right)^{1/2}} \tag{13}$$

4. Comparison with Dawson's Results.

Dawson's and Oberman's treatment of the absorption coefficient on the basis of the microscopic plasma theory [7] led to a value

$$(14) \qquad \varkappa^2 = \frac{\omega^2}{c^2}\left(1 - \frac{4\pi i}{\omega Z(\omega)}\right)$$

of the complex dielectric constant $\varkappa$ in a wave equation (8). With the splitting in a real inimaginary part and little formal changes $(\omega << \omega_p)$

$$(15) \qquad Z(\omega) = R(\omega) + i X(\omega)$$

$$(16) \qquad \omega R(\omega) = 4\pi \left(\frac{\pi}{2}\right)^{-1/2} \frac{\omega Z e^2}{6 m_e u_0}\left[\ln\left(\frac{2k_{max}^2 u_0^2}{\omega^2}\right) - \gamma\right]$$

$$(17) \qquad \omega X(\omega) = \frac{4\pi\omega^2}{\omega_p^2}\left(1 - (2\pi)^{1/2}\frac{Z e^2 \omega_p^2}{6 m_e u_0^3 \omega}\right)$$

where $\gamma = 0.577...$ is the Euler constant, u_0 is the velocity maximum of the equilibrium distribution and k_{max} is the maximal value of the argument of a Fourier transform. Especially Dawson and Oberman put

$$(18) \qquad k_{max} = \sqrt{2}\, 10^7 \frac{\omega_p}{u_0}$$

The value of the absorption coefficient is then

$$(19) \qquad K_{DO} = 2\frac{\omega}{c}\frac{1}{\sqrt{\omega^2R^2 + \omega^2X^2}}\sqrt{\frac{1}{2}\left(\sqrt{\omega^2R^2 + (\omega X - 4\pi)^2} - \omega R\right)}$$

In Fig. 11 the values of K_{DO} with a temperature $T = 10^4$ eV and Z = 1 are calculated under the assumption (18) for electron densities n_e between 10^{21} cm^{-3} and 10^{24} cm^{-3} and are compared with the absorption coefficients K, calculated in the former section by the macroscopic theory. Taking into account the complexity of the propositions of Dawson's and Oberman's treatment with the microscopic theory, one may say that the agreement of K and K_{DO} is quite good.

In the case $n_e < n_{eo}$ the values (15) to (17) are not applicable. Instead of using the concerning values given by Dawson and Oberman[7] for this range of validity we take the following approximation formula, evaluated by Dawson[3]

$$(20) \qquad K_{DO} = \frac{32\pi^3}{3} \frac{Z^2 n_e n_i e^6 \ln\Lambda}{c\,\omega^2 (2\pi m_e kT)^{3/2} \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}}$$

This is identical with the here given approximation formula (13) of the same range of validity except the numerical factors of both formulas

$$(21) \qquad \frac{K}{K_{DO}} = \frac{1.18}{\gamma_E(Z)}.$$

and with the neglection of the impact frequency ν in the formula (13). Proposing $\nu \ll \omega$ this can be done, and in (20) the same proposition is made like in other case of microscopic theory to study impact processes on the basis of the Vlasov equation[11], where the duration of the studied processes ($\frac{1}{\omega}$) must be small compared with the collision time ($\frac{1}{\nu}$).

5. Comparison with the Theory of the Bremsstrahlung.

The quantum mechanical treatment of Bremsstrahlung and its application under the condition of Kirchhoff's law leads to an absorption coefficient for the case $\omega >> \omega_p$. Taking into account stimulated emission the absorption coefficient is with the Gaunt factor g [12])

(22) $$K_B = \frac{64\pi^4}{3\sqrt{3}} \frac{Z^2 n_e n_i e^6 g}{c\,\omega^2 (2\pi m_e kT)^{3/2}}$$

when $h\nu/kT$ is very much smaller than 1; this is always the case because the quantums $h\nu$ of the ruby lasers are of 1.77 eV energy and here $h\nu$ -values much higher than 10 eV are of interest. Comparing the result (13) of the macroscopic theory with $\omega_p << \omega$ and $\nu << \omega$ one finds an identical equality besides of a factor

(23) $$\frac{K}{K_B} = 0.324 \frac{\ln \Lambda}{\gamma_E(Z)\, g}$$

the dependence of which from n_e, T and Z is very slightly. It seems to be very remarkable that this correspondence is only fulfilled if the stimulated emission is considered. Otherwise an other power of the dependence of ω and T is given than in (22). Herein it is shown that the macroscopic theory bears implicitly the collective character of the plasma. In the microscopic theory the collective properties of the plasma must be introduced more or less explicitly. The absorption coefficient from Dawson's and Oberman's treatment for the case $\omega_p << \omega$ also gives a good parallelity comparable with (22) and leads to

(24) $$\frac{K_{DO}}{K_B} = 0.276 \frac{\ln \Lambda}{g}$$

Using a special representation of the Gaunt factor, whose validity is limited like that of the formula (22) [13] with $[T] = eV$ and $[n_e] = cm^{-3}$

$$(25) \qquad g = 1.2695\left(7.45 + \log T - \frac{1}{3}\log n_e\right)$$

and writing with the same dimensions after equ. (5)

$$(26) \qquad \ln\Lambda = 3.45\left(6.692 + \log T - \frac{1}{3}\log n_e - \frac{2}{3}\log Z\right)$$

we find

$$(27) \qquad \frac{K}{K_B} = \frac{0.883}{\gamma_E(Z)}\left(1 - \frac{0.76 + \frac{2}{3}\log Z}{7.45 + \log T - \frac{1}{3}\log n_e}\right)$$

and

$$(28) \qquad \frac{K_{DO}}{K_B} = 0.75\left(1 - \frac{0.76 + \frac{2}{3}\log Z}{7.45 + \log T - \frac{1}{3}\log n_e}\right)$$

With respect to $\gamma_E(Z)$ which goes monotoneously from 0.582 to 1 when Z changes from 1 to ∞ one can say that the agreement of the quantum mechanical formula (22) with the formula of the macroscopic theory is better than with K_{DO}. Furthermore, the fact that the macroscopical values K are heigher than K_{DO}, as shown in equ. (21), gives those a better physical reliability.

6. Calculation of Refractive Index.

In connection with the section 2) the refractive index n is given by

$$(29) \quad n = \sqrt{\frac{1}{2}\left(\sqrt{\left(1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right)^2 + \left(\frac{\nu}{\omega}\,\frac{\omega_p^2}{\omega^2+\nu^2}\right)^2} + 1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right)}$$

where the dependence of the plasma frequency ω_p and the impact frequency from the electron density n_e and the temperature T is given by (3) to (5). The validity of the formula (29) is again given by (10) and (12). In the Fig. 11 to 21 the refractive indices for fully ionized gases with Z = 1 to 10 are calculated in dependence of the temperature T with the atom density as parameter. The curves are fully drawn in the case of fulfilling both conditions (10) and (12) and are dashed continued when (10) still holds but the condition of Boltzmann statistics is not longer true.

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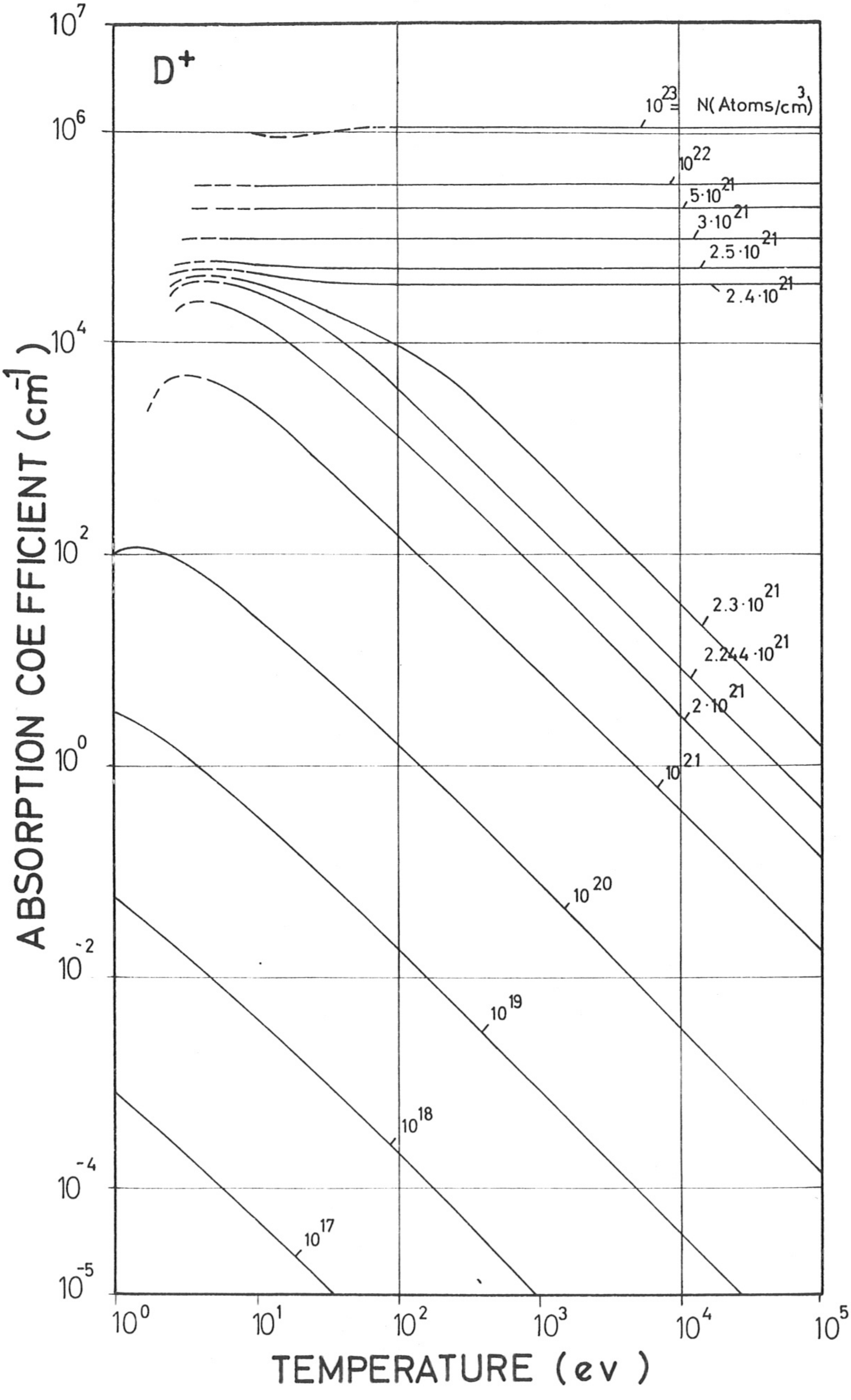
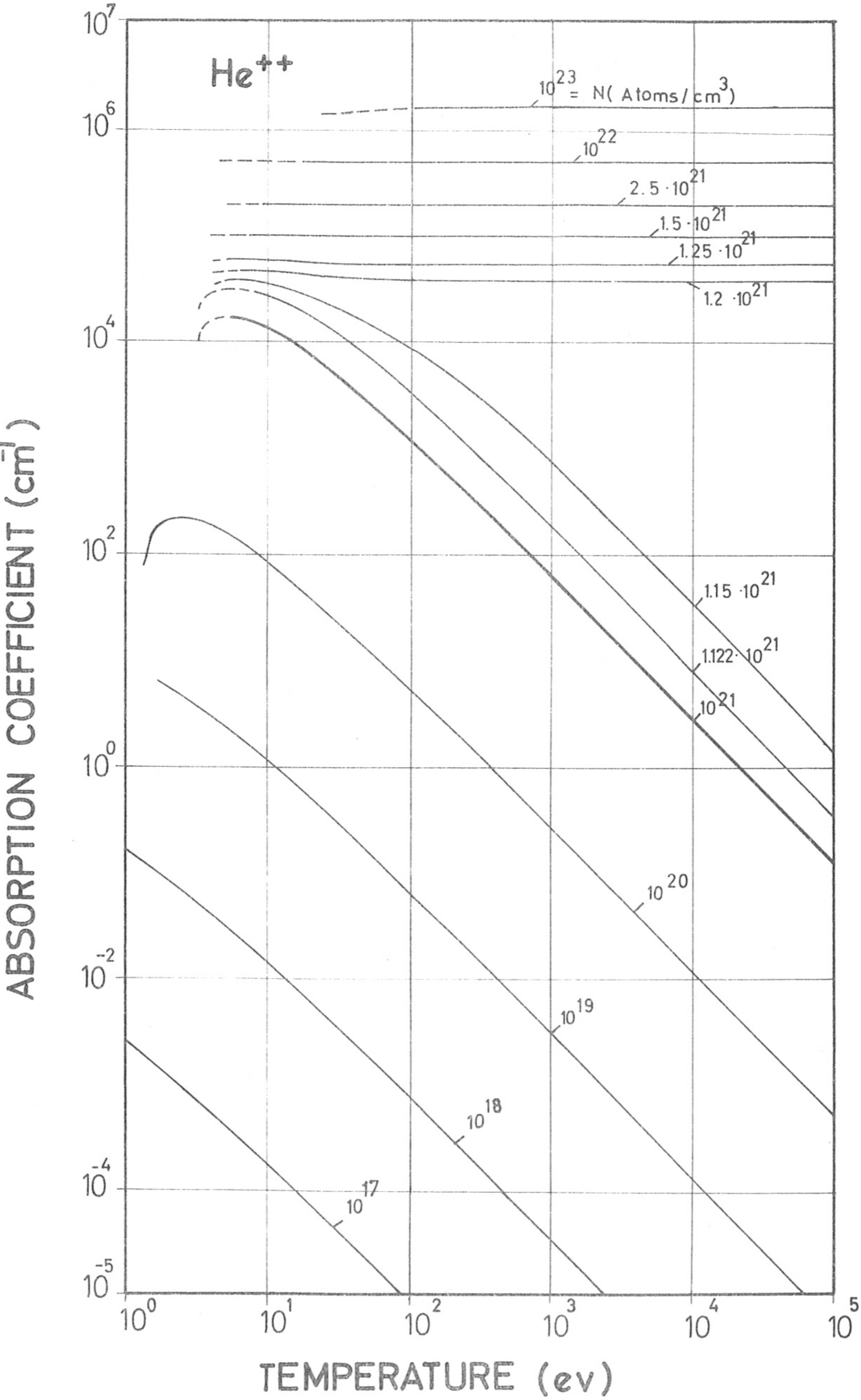


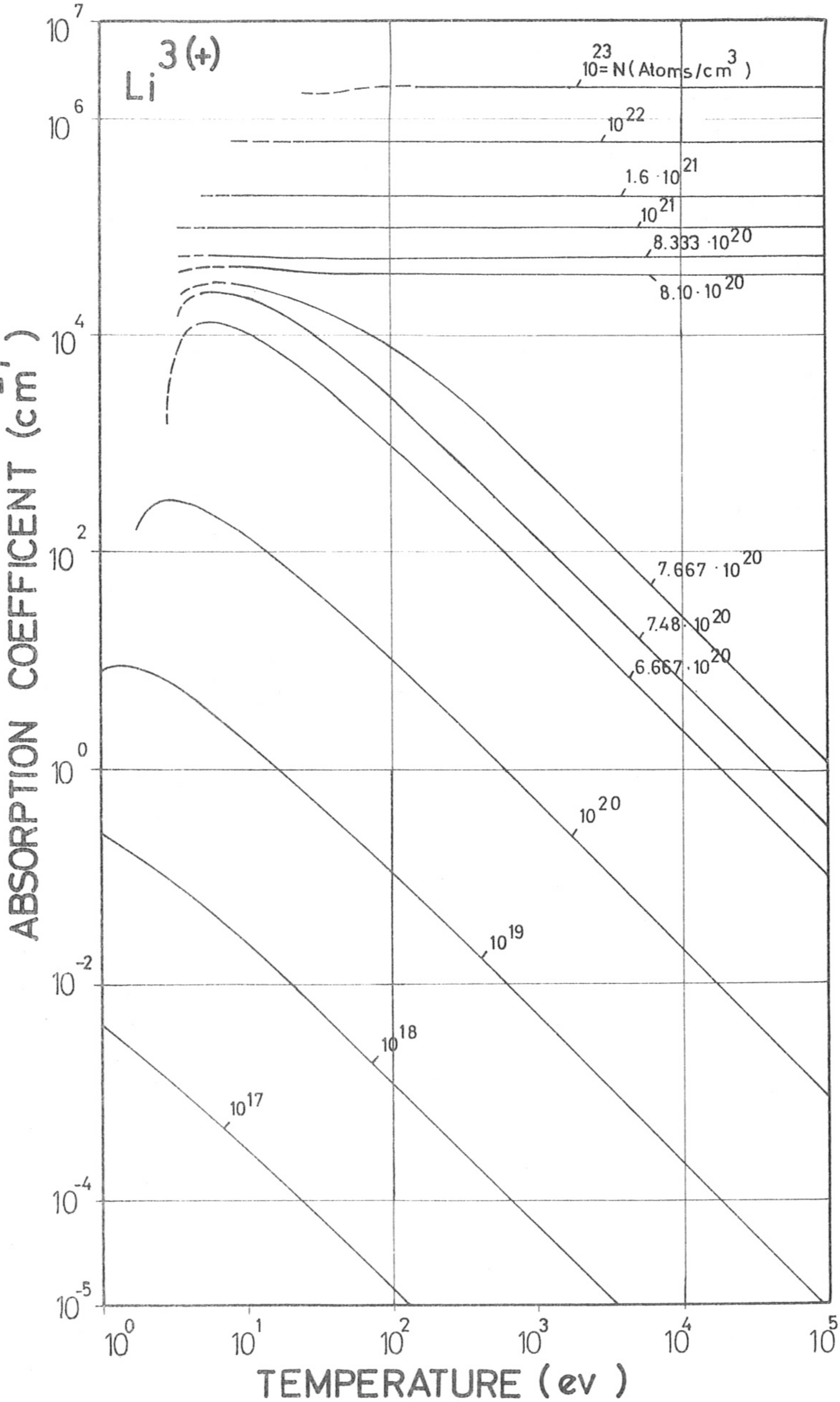
D+
ABSORPTION COEFFICIENT (cm-1)
TEMPERATURE (ev)
N(Atoms/cm3)
10^23
10^22
5·10^21
3·10^21
2.5·10^21
2.4·10^21
2.3·10^21
2.244·10^21
2·10^21
10^21
10^20
10^19
10^18
10^17
10^7
10^6
10^4
10^2
10^0
10^-2
10^-4
10^-5
10^0
10^1
10^2
10^3
10^4
10^5

Fig. 1



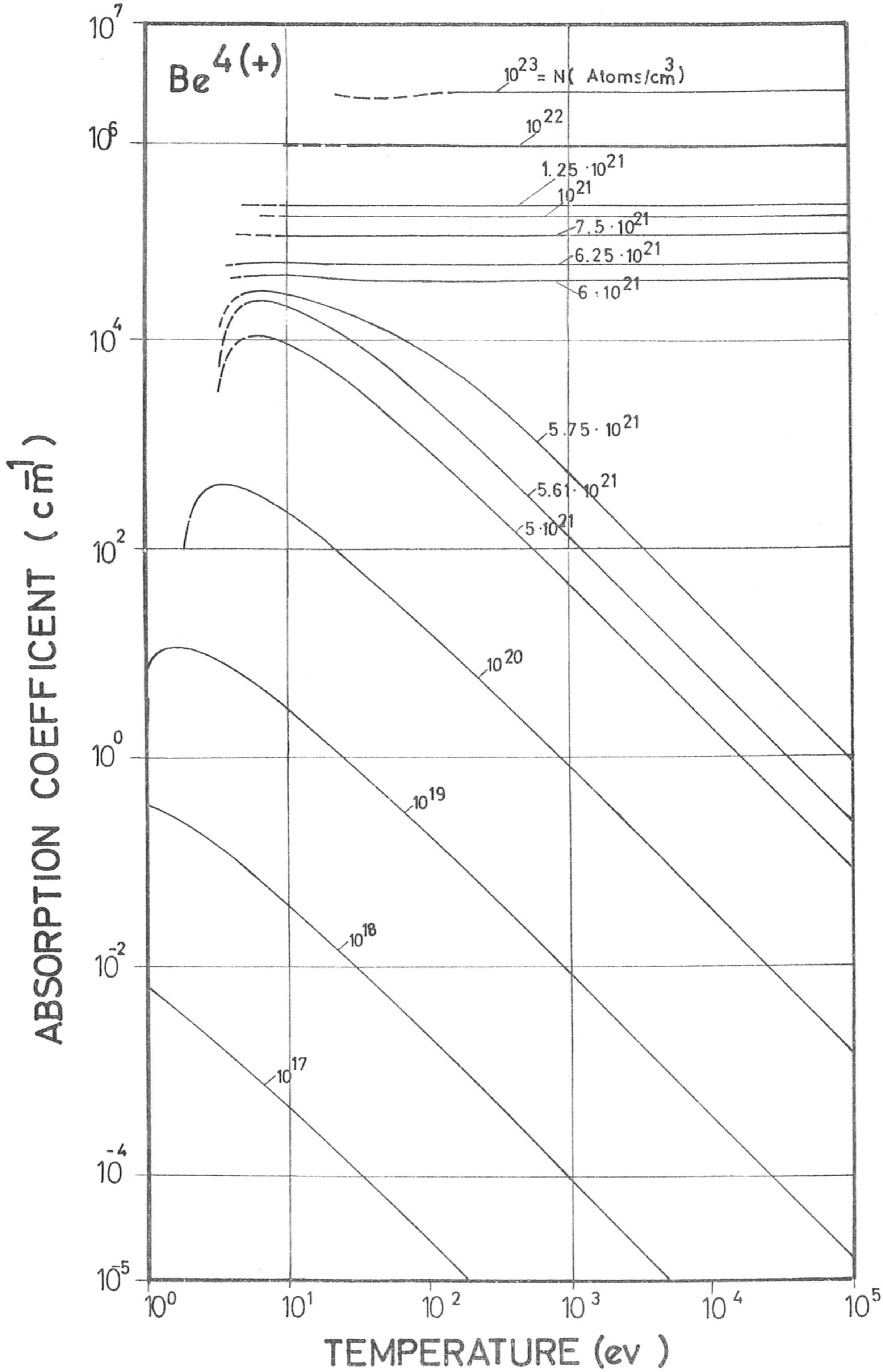
He++
10^23 = N(Atoms / cm^3)
10^22
2.5 · 10^21
1.5 · 10^21
1.25 · 10^21
1.2 · 10^21
1.15 · 10^21
1.122 · 10^21
10^21
10^20
10^19
10^18
10^17
ABSORPTION COEFFICIENT (cm^-1)
TEMPERATURE (ev)

Fig. 2



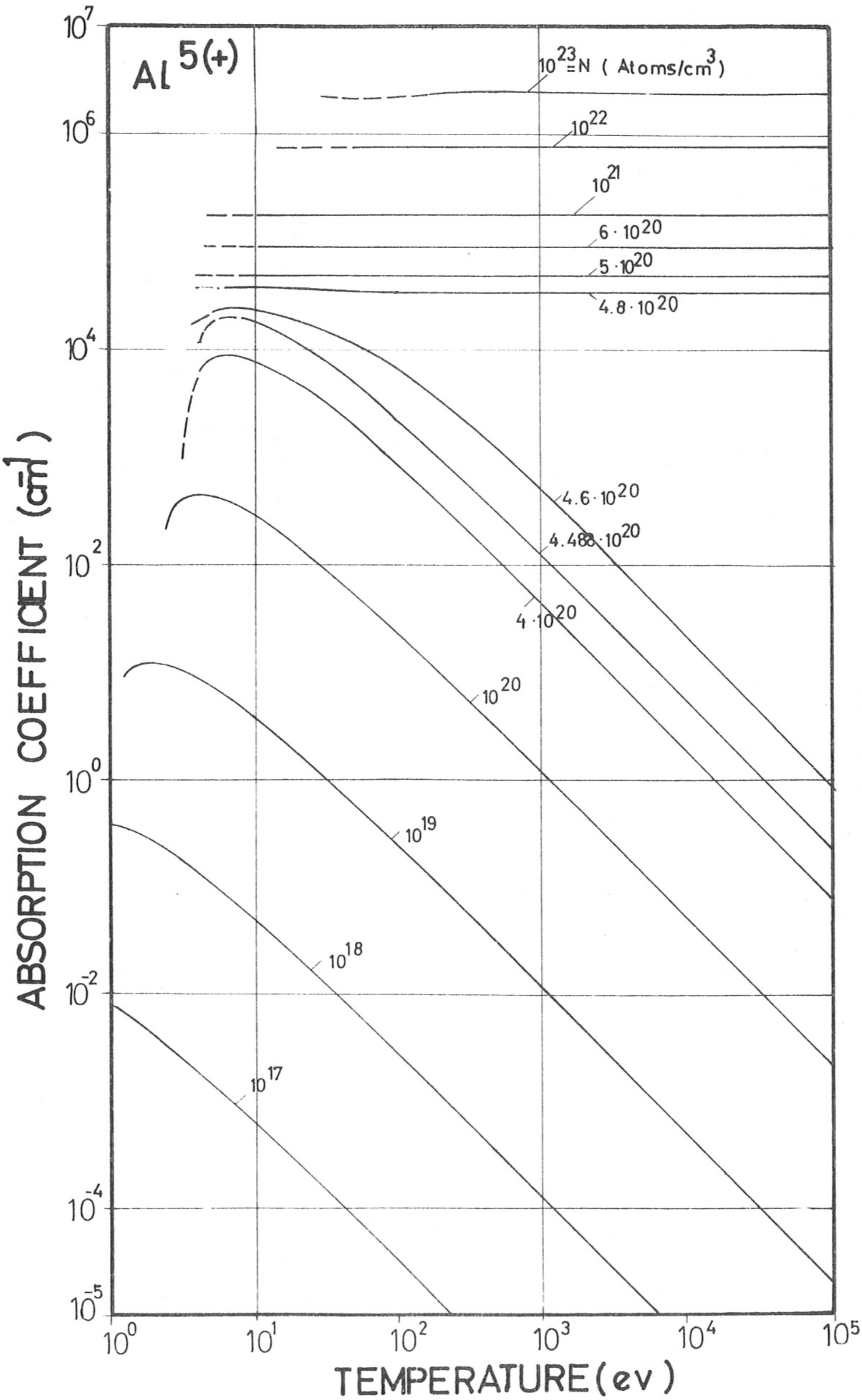
Li 3(+)
10 = N (Atoms/cm 3) 23
10 22
1.6 · 10 21
10 21
8.333 · 10 20
8.10 · 10 20
7.667 · 10 20
7.48 · 10 20
6.667 · 10 20
10 20
10 19
10 18
10 17
ABSORPTION COEFFICENT (cm -1)
TEMPERATURE (ev)

Fig. 3



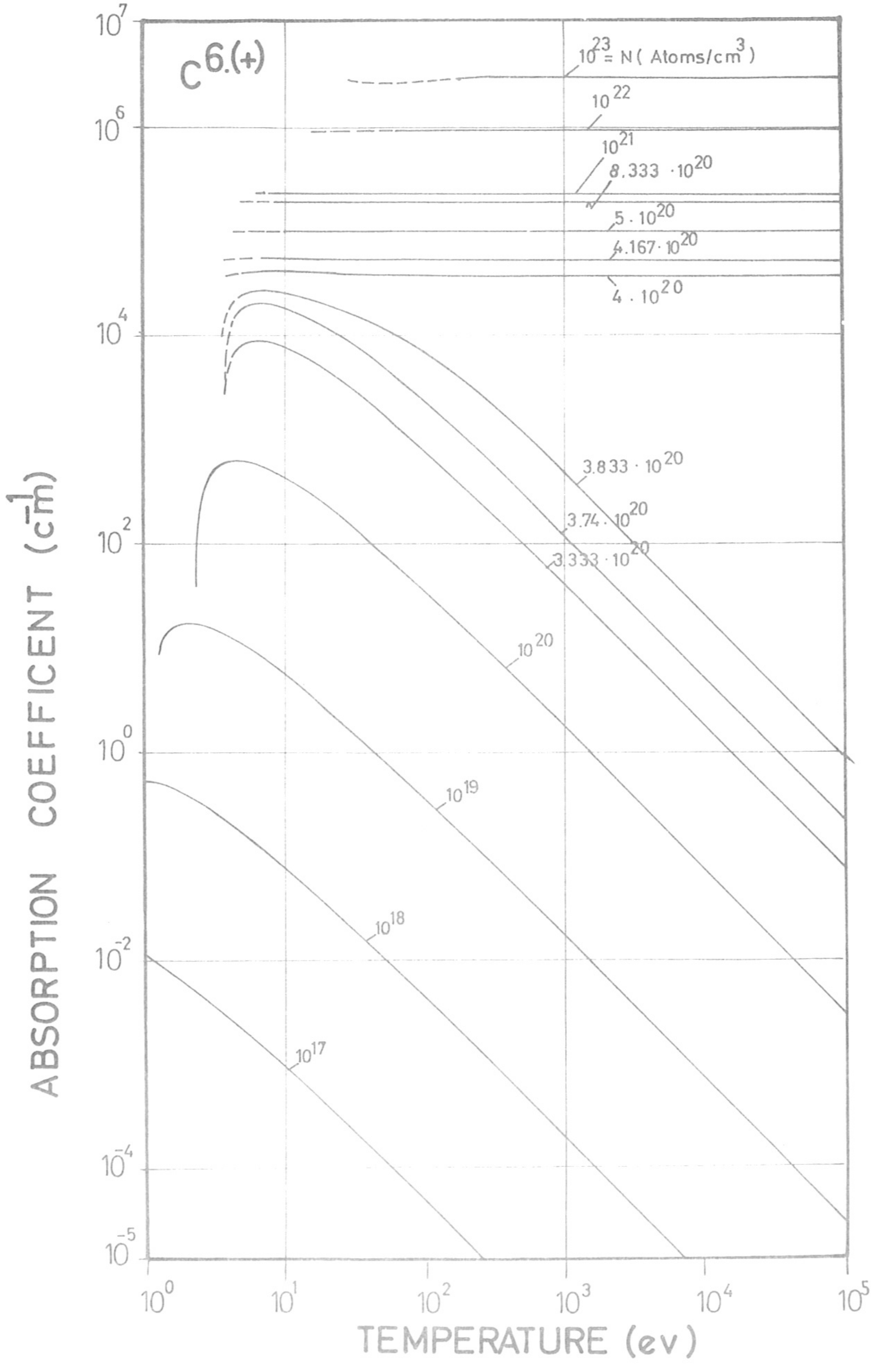
Be4(+)
10^23 = N(Atoms/cm^3)
10^22
1.25 · 10^21
10^21
7.5 · 10^21
6.25 · 10^21
6 · 10^21
5.75 · 10^21
5.61 · 10^21
5 · 10^21
10^20
10^19
10^18
10^17
10^7
10^6
10^4
10^2
10^0
10^-2
10^-4
10^-5
10^0
10^1
10^2
10^3
10^4
10^5
ABSORPTION COEFFICENT (cm^-1)
TEMPERATURE (ev)

Fig. 4



Al 5(+)
10^23 = N (Atoms/cm^3)
10^22
10^21
6 · 10^20
5 · 10^20
4.8 · 10^20
4.6 · 10^20
4.488 · 10^20
4 · 10^20
10^20
10^19
10^18
10^17
10^7
10^6
10^4
10^2
10^0
10^-2
10^-4
10^-5
ABSORPTION COEFFICENT (cm^-1)
10^0
10^1
10^2
10^3
10^4
10^5
TEMPERATURE (ev)

Fig. 5



C6.(+)
10^{23} = N (Atoms/cm^3)
10^{22}
10^{21}
$8.333 \cdot 10^{20}$
$5 \cdot 10^{20}$
$4.167 \cdot 10^{20}$
$4 \cdot 10^{20}$
$3.833 \cdot 10^{20}$
$3.74 \cdot 10^{20}$
$3.333 \cdot 10^{20}$
10^{20}
10^{19}
10^{18}
10^{17}
10^{7}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-5}
ABSORPTION COEFFICENT (cm^{-1})
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
TEMPERATURE (ev)

Fig. 6

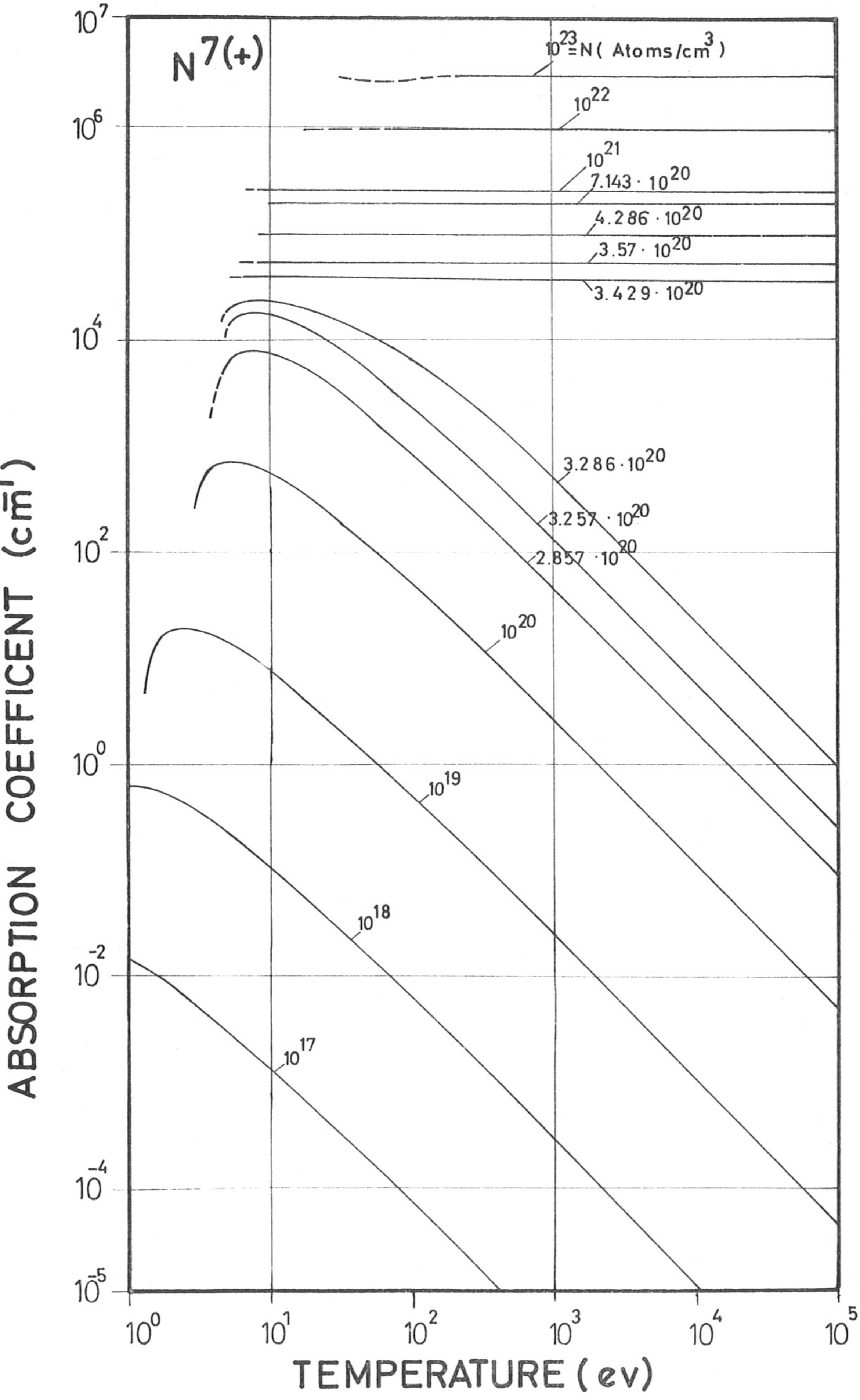
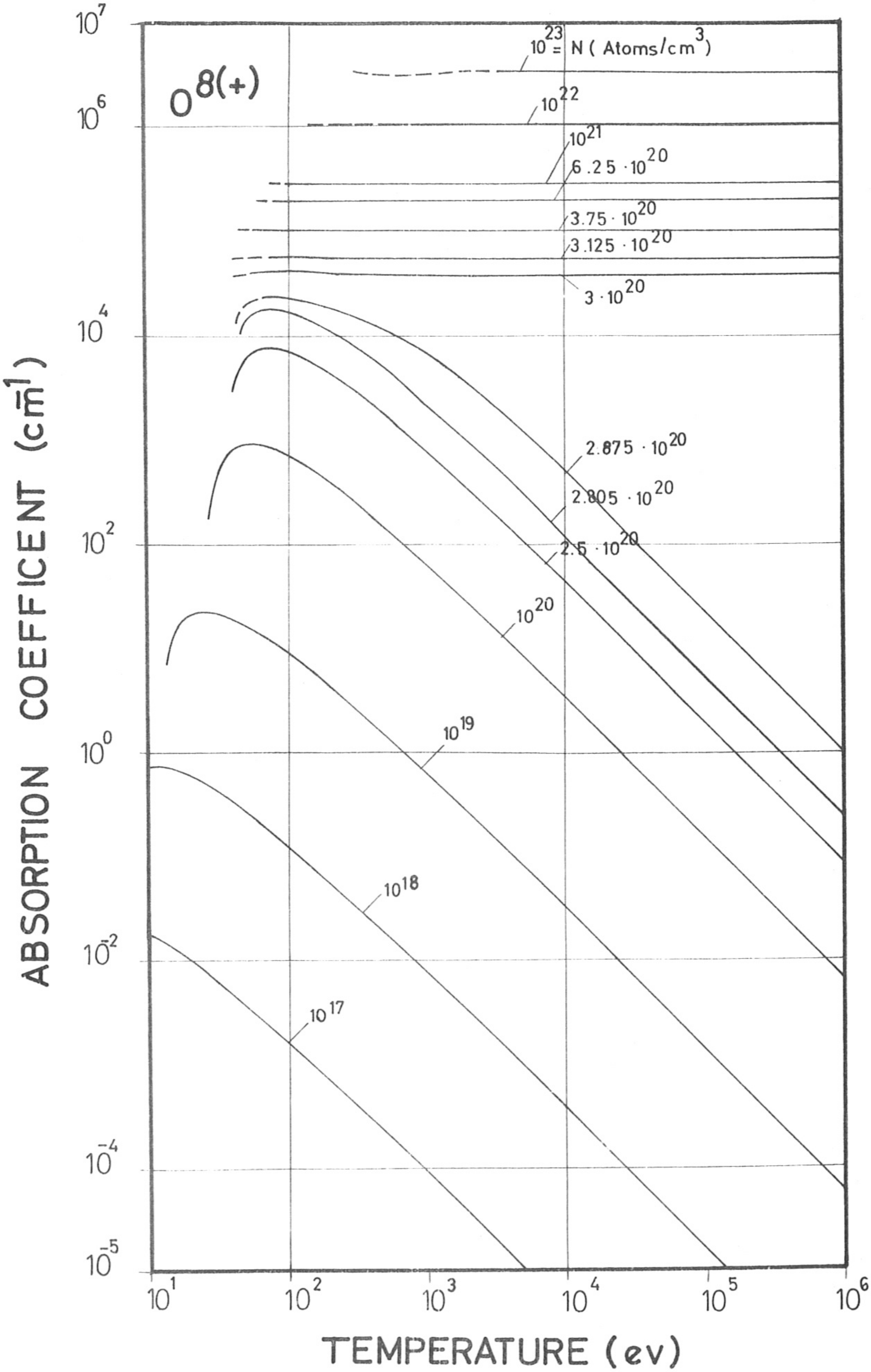
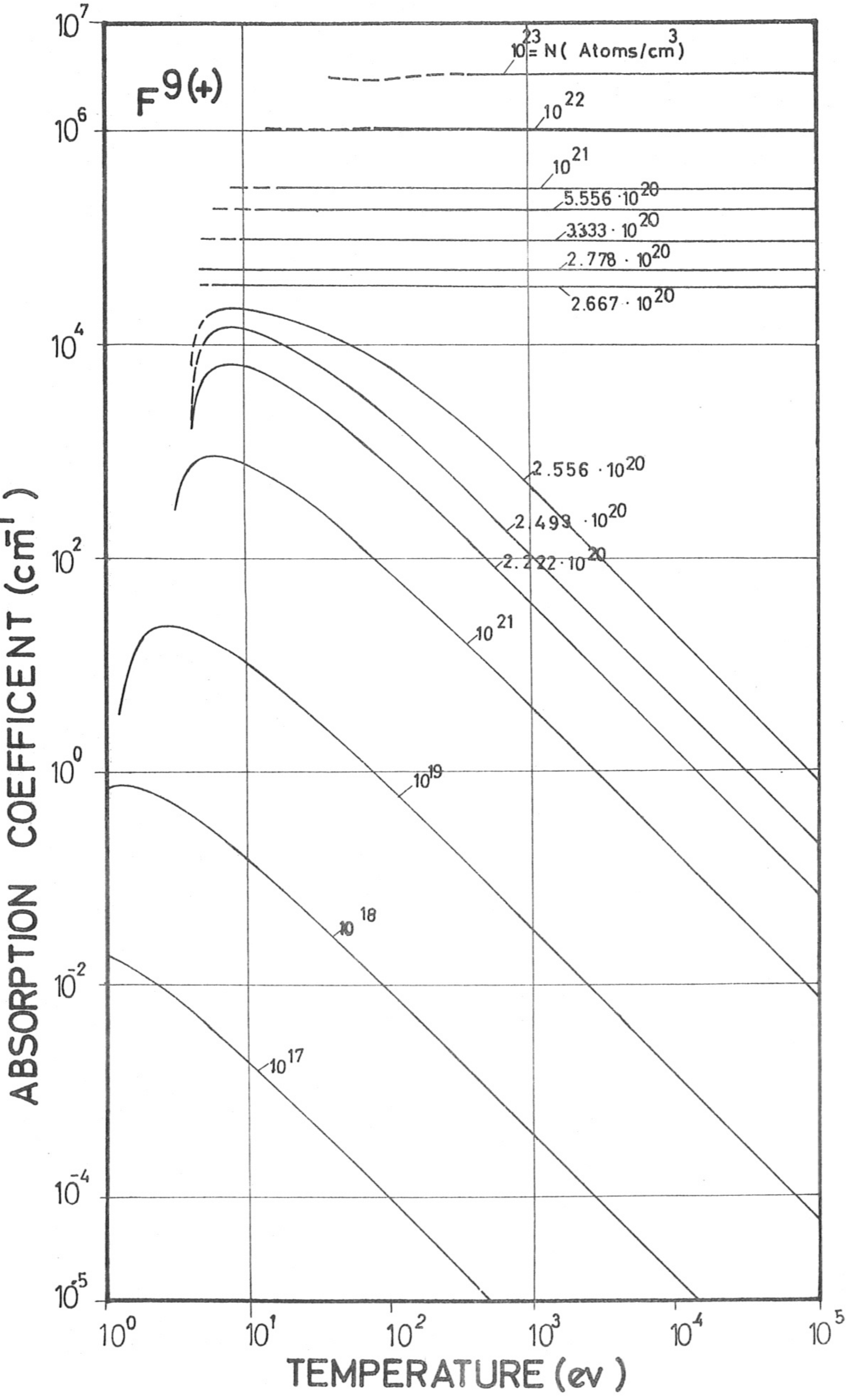

N 7(+)
10^23 = N (Atoms/cm^3)
10^22
10^21
7.143 · 10^20
4.286 · 10^20
3.57 · 10^20
3.429 · 10^20
3.286 · 10^20
3.257 · 10^20
2.857 · 10^20
10^20
10^19
10^18
10^17
ABSORPTION COEFFICENT (cm^-1)
10^7
10^6
10^4
10^2
10^0
10^-2
10^-4
10^-5
10^0
10^1
10^2
10^3
10^4
10^5
TEMPERATURE (ev)

Fig. 7



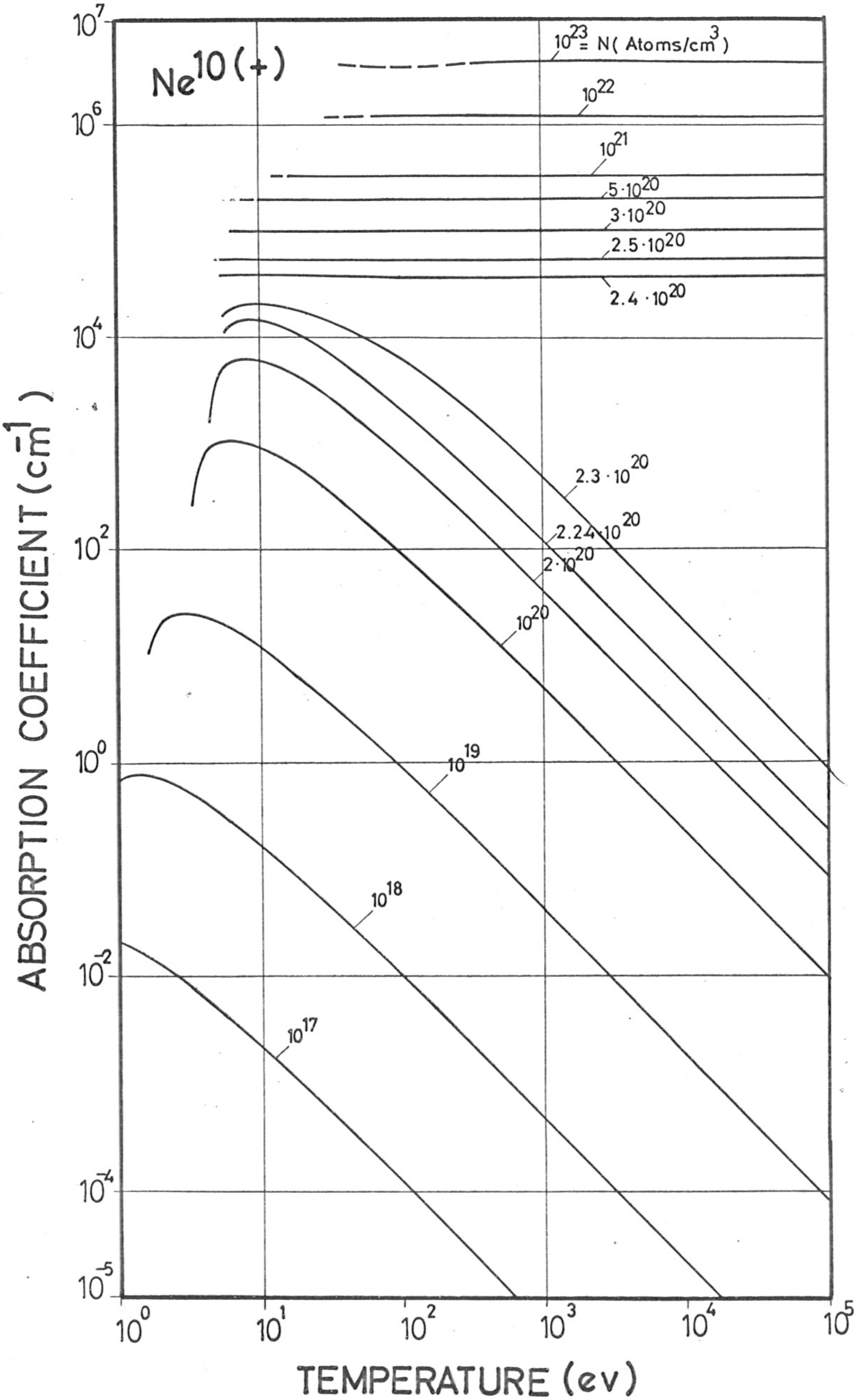
$O^{8(+)}$
10^{23} = N (Atoms/cm^3)
10^{22}
10^{21}
$6.25 \cdot 10^{20}$
$3.75 \cdot 10^{20}$
$3.125 \cdot 10^{20}$
$3 \cdot 10^{20}$
$2.875 \cdot 10^{20}$
$2.805 \cdot 10^{20}$
$2.5 \cdot 10^{20}$
10^{20}
10^{19}
10^{18}
10^{17}
ABSORPTION COEFFICENT (cm^{-1})
TEMPERATURE (ev)
10^7
10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-5}
10^1
10^2
10^3
10^4
10^5
10^6

Fig. 8



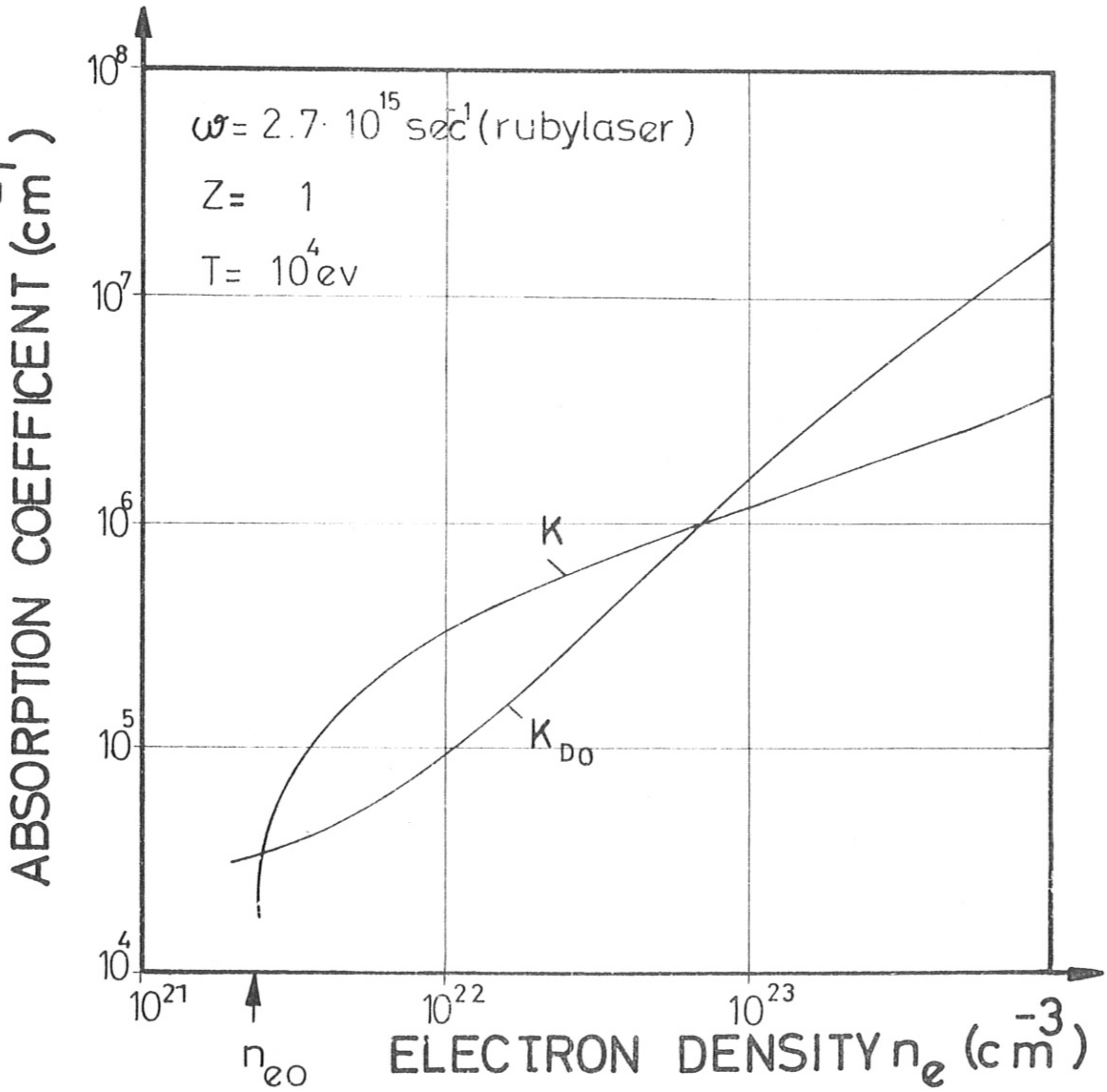
F 9(+)
10 23 = N (Atoms/cm 3)
10 22
10 21
5.556 · 10 20
3333 · 10 20
2.778 · 10 20
2.667 · 10 20
2.556 · 10 20
2.493 · 10 20
2.222 · 10 20
10 21
10 19
10 18
10 17
ABSORPTION COEFFICENT (cm-1)
TEMPERATURE (ev)

Fig. 9



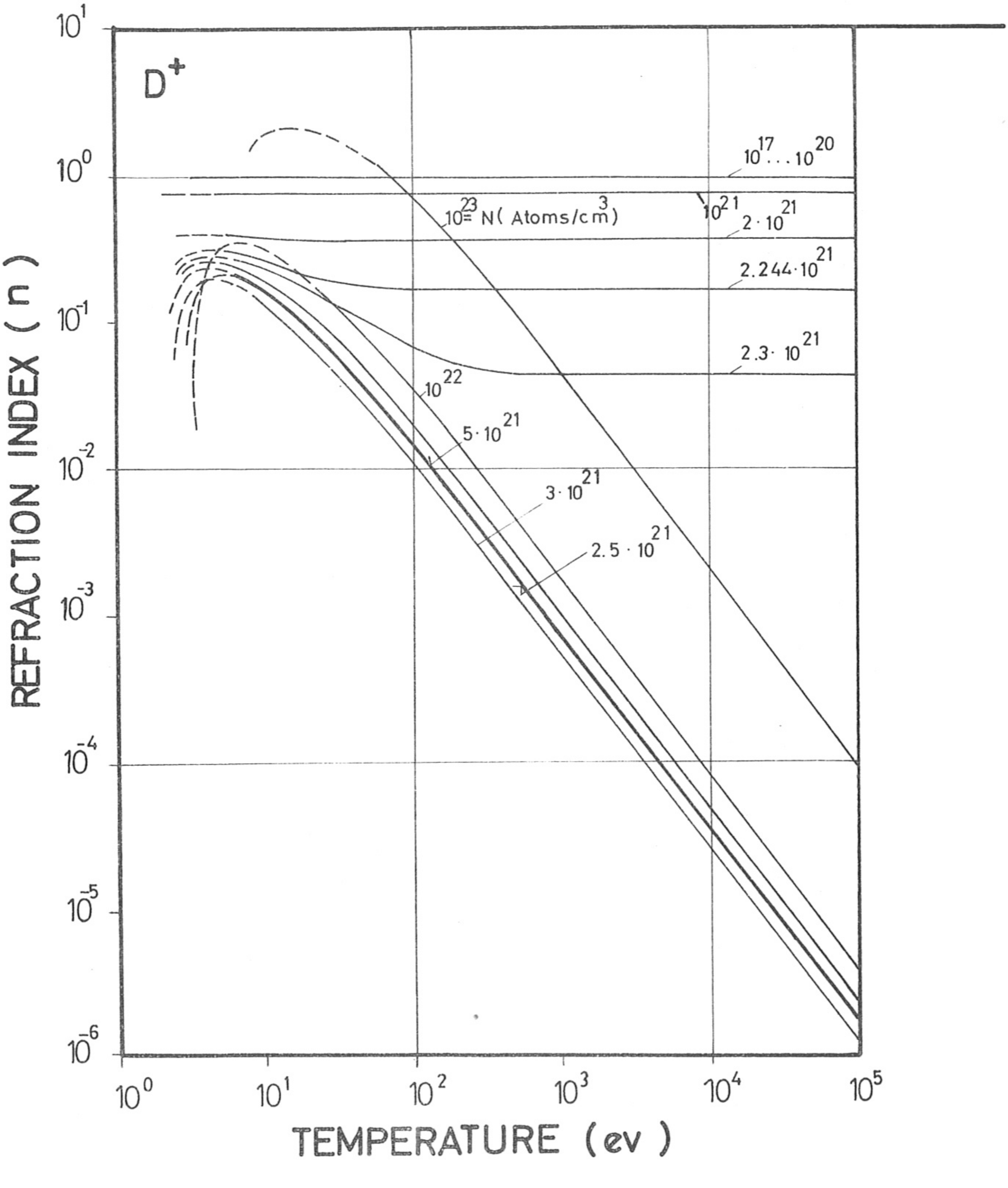
Ne10 (+)
10^{23} = N(Atoms/cm^3)
10^{22}
10^{21}
$5 \cdot 10^{20}$
$3 \cdot 10^{20}$
$2.5 \cdot 10^{20}$
$2.4 \cdot 10^{20}$
$2.3 \cdot 10^{20}$
$2.24 \cdot 10^{20}$
$2 \cdot 10^{20}$
10^{20}
10^{19}
10^{18}
10^{17}
ABSORPTION COEFFICIENT (cm^{-1})
10^7
10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-5}
10^0
10^1
10^2
10^3
10^4
10^5
TEMPERATURE (ev)

Fig. 10



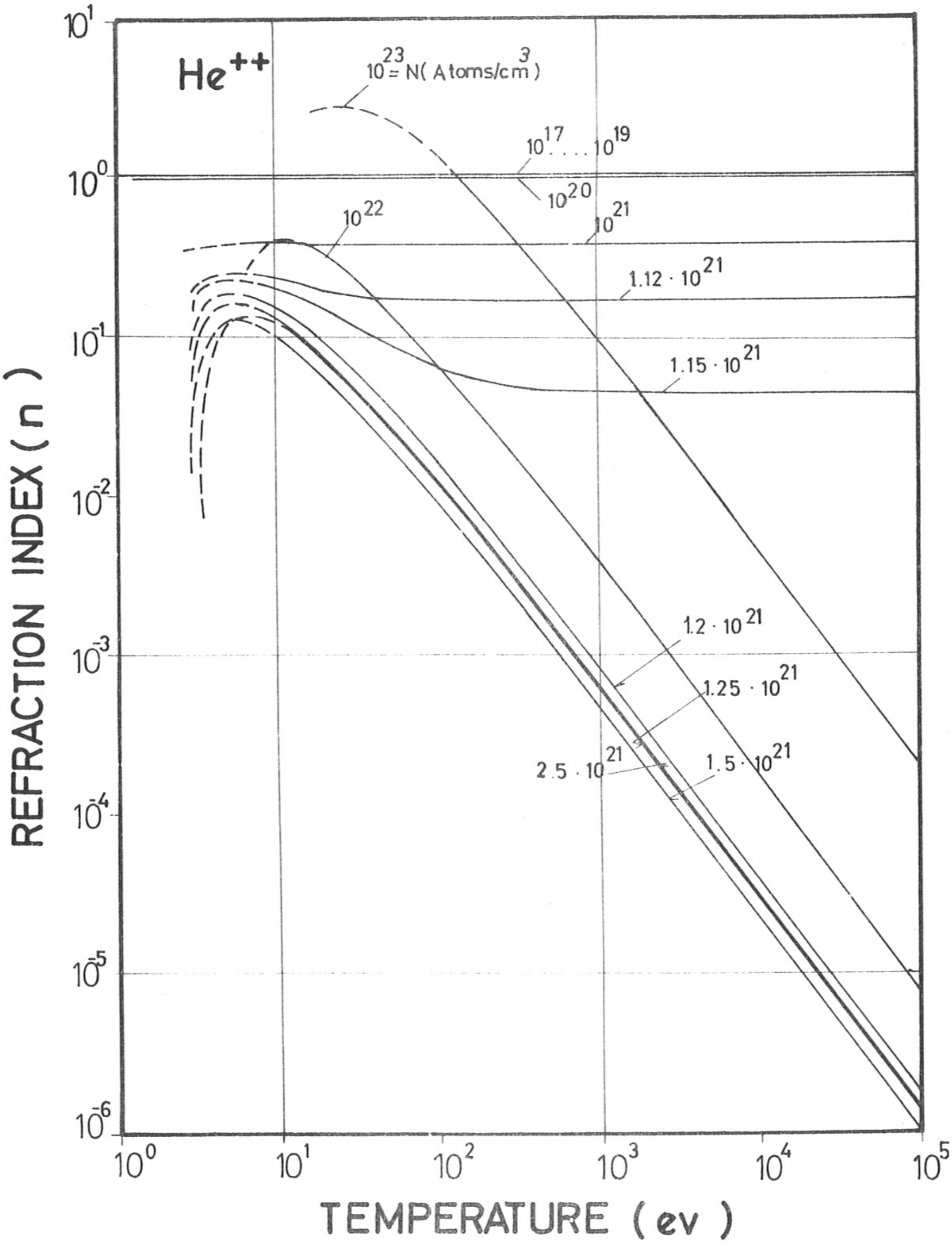
$\omega = 2.7 \cdot 10^{15}\ sec^{-1}$ (rubylaser)
Z = 1
$T = 10^4\ ev$
K
K_{Do}
10^8
10^7
10^6
10^5
10^4
ABSORPTION COEFFICENT (cm^{-1})
10^{21}
10^{22}
10^{23}
n_{eo}
ELECTRON DENSITY n_e (cm^{-3})

Fig. 11



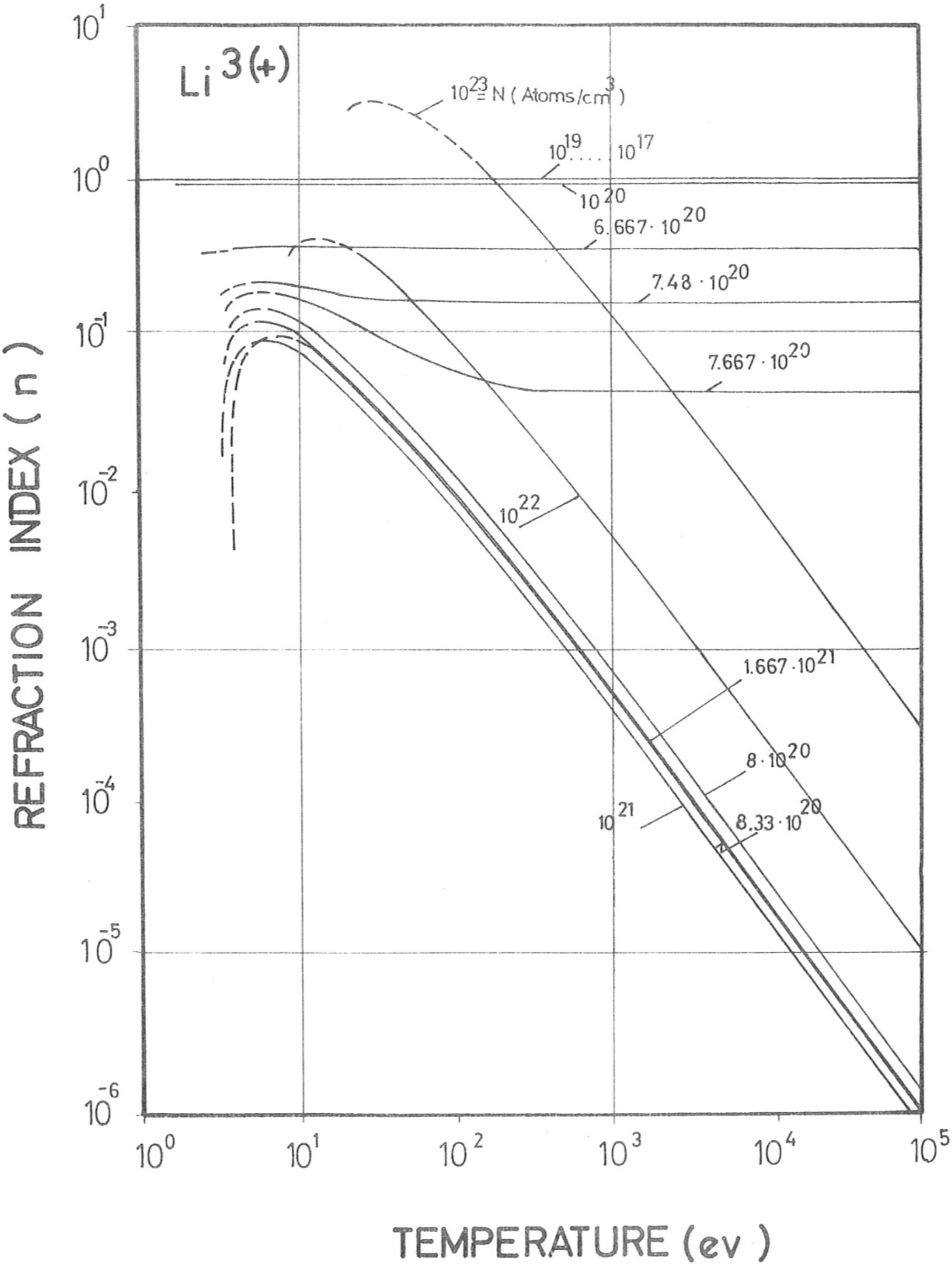
D+
10^17 ... 10^20
10^23 = N(Atoms/cm^3)
10^21
2·10^21
2.244·10^21
2.3· 10^21
10^22
5·10^21
3·10^21
2.5 · 10^21
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig. 12



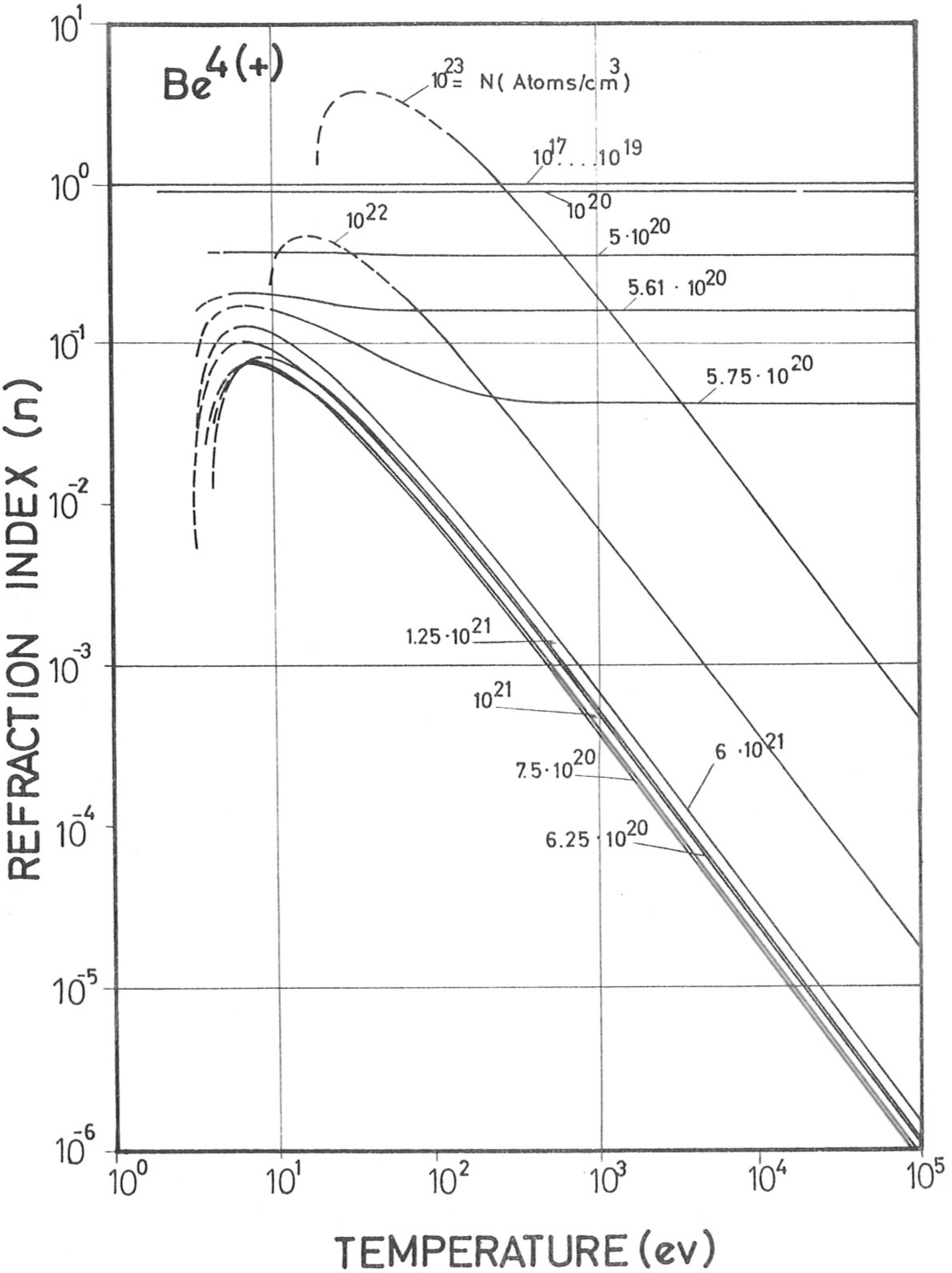
He++
10^23 = N(Atoms/cm^3)
10^17 . . . 10^19
10^20
10^22
10^21
1.12 · 10^21
1.15 · 10^21
1.2 · 10^21
1.25 · 10^21
2.5 · 10^21
1.5 · 10^21
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig. 13



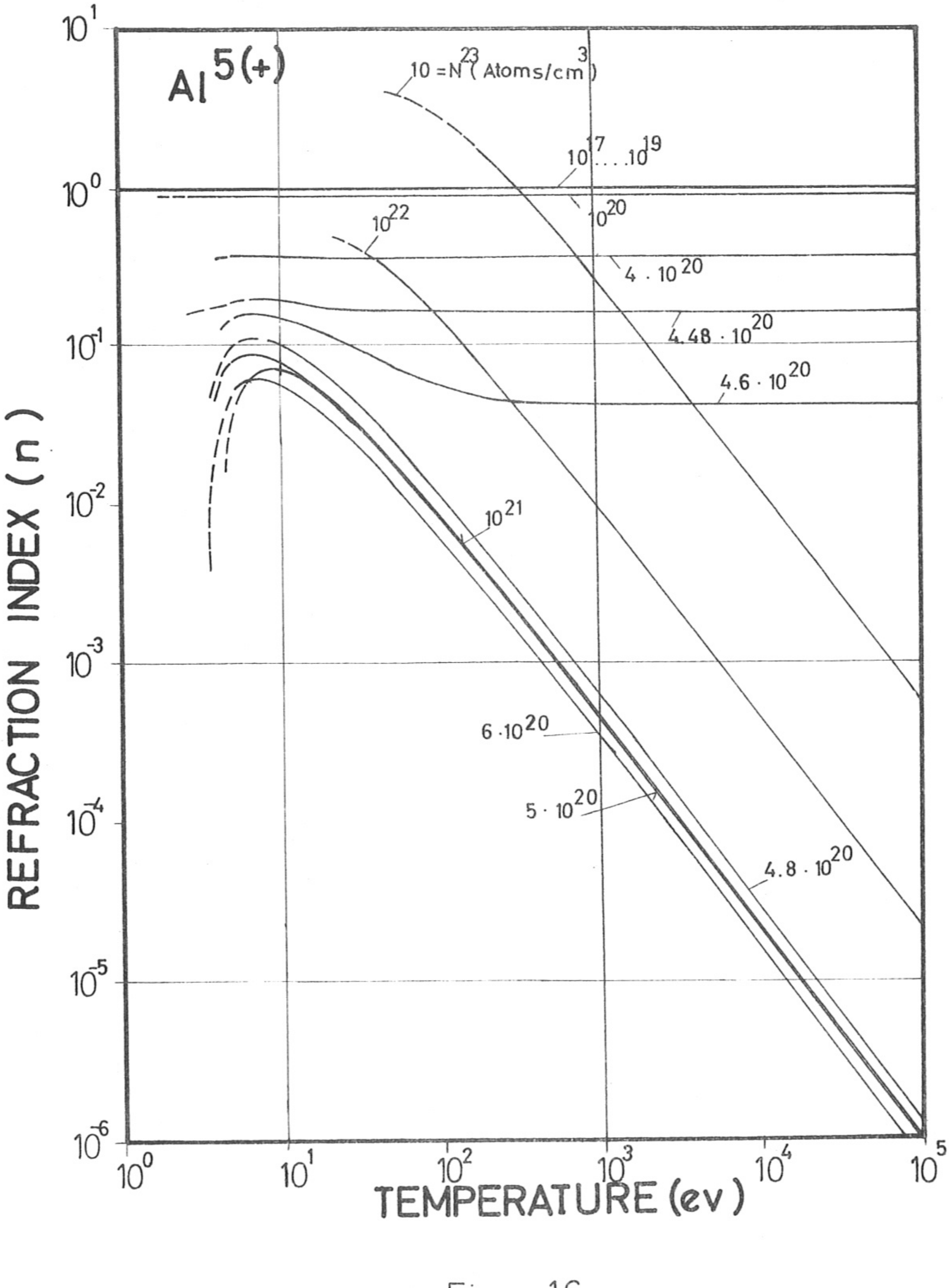
Li 3(+)
10^{23} = N (Atoms/cm^3)
10^{19} ..... 10^{17}
10^{20}
$6.667 \cdot 10^{20}$
$7.48 \cdot 10^{20}$
$7.667 \cdot 10^{20}$
10^{22}
$1.667 \cdot 10^{21}$
$8 \cdot 10^{20}$
10^{21}
$8.33 \cdot 10^{20}$
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig. 14



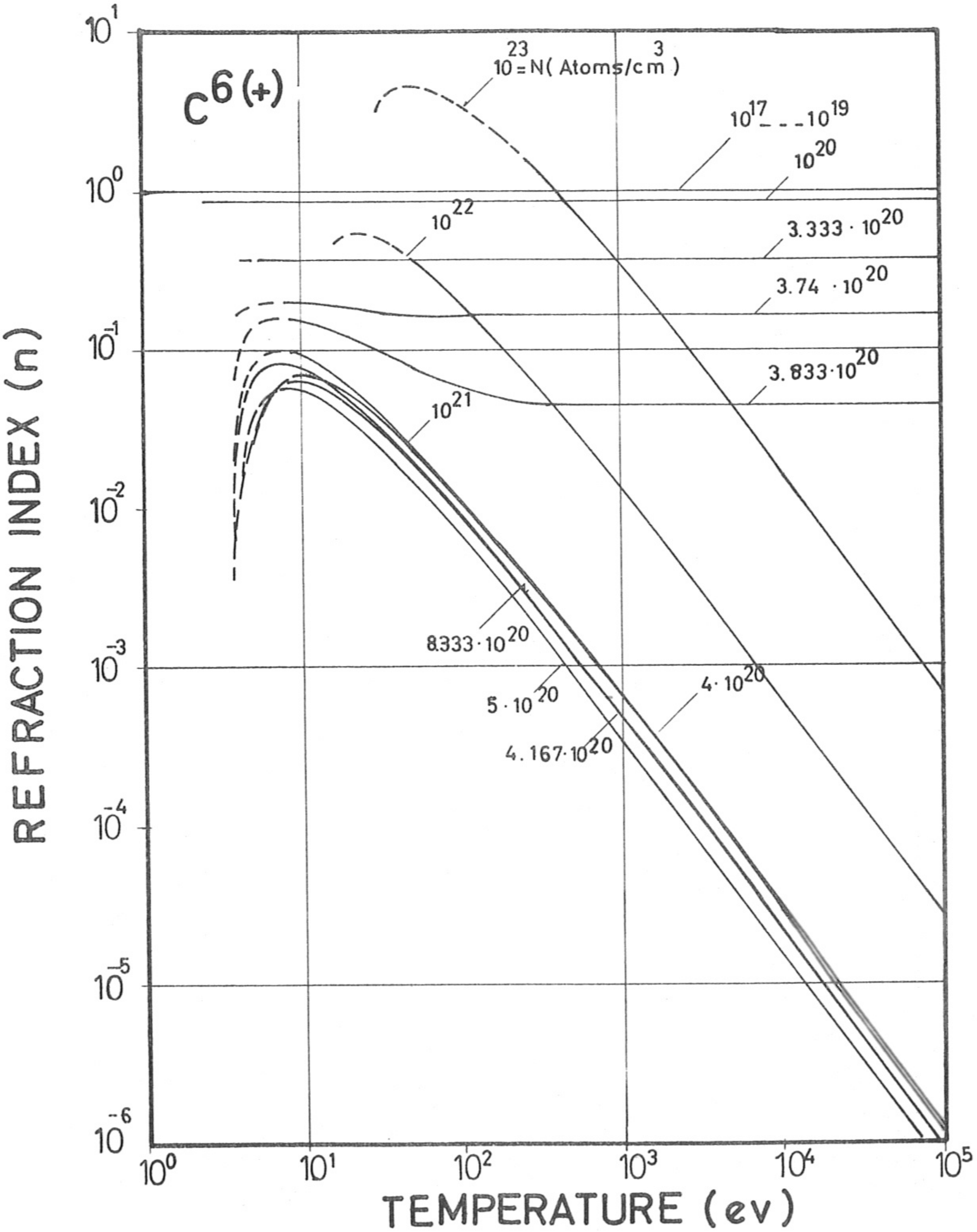
Be4(+)
10^23 = N(Atoms/cm^3)
10^17....10^19
10^20
10^22
5·10^20
5.61 · 10^20
5.75·10^20
1.25·10^21
10^21
7.5·10^20
6 ·10^21
6.25 · 10^20
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig. 15



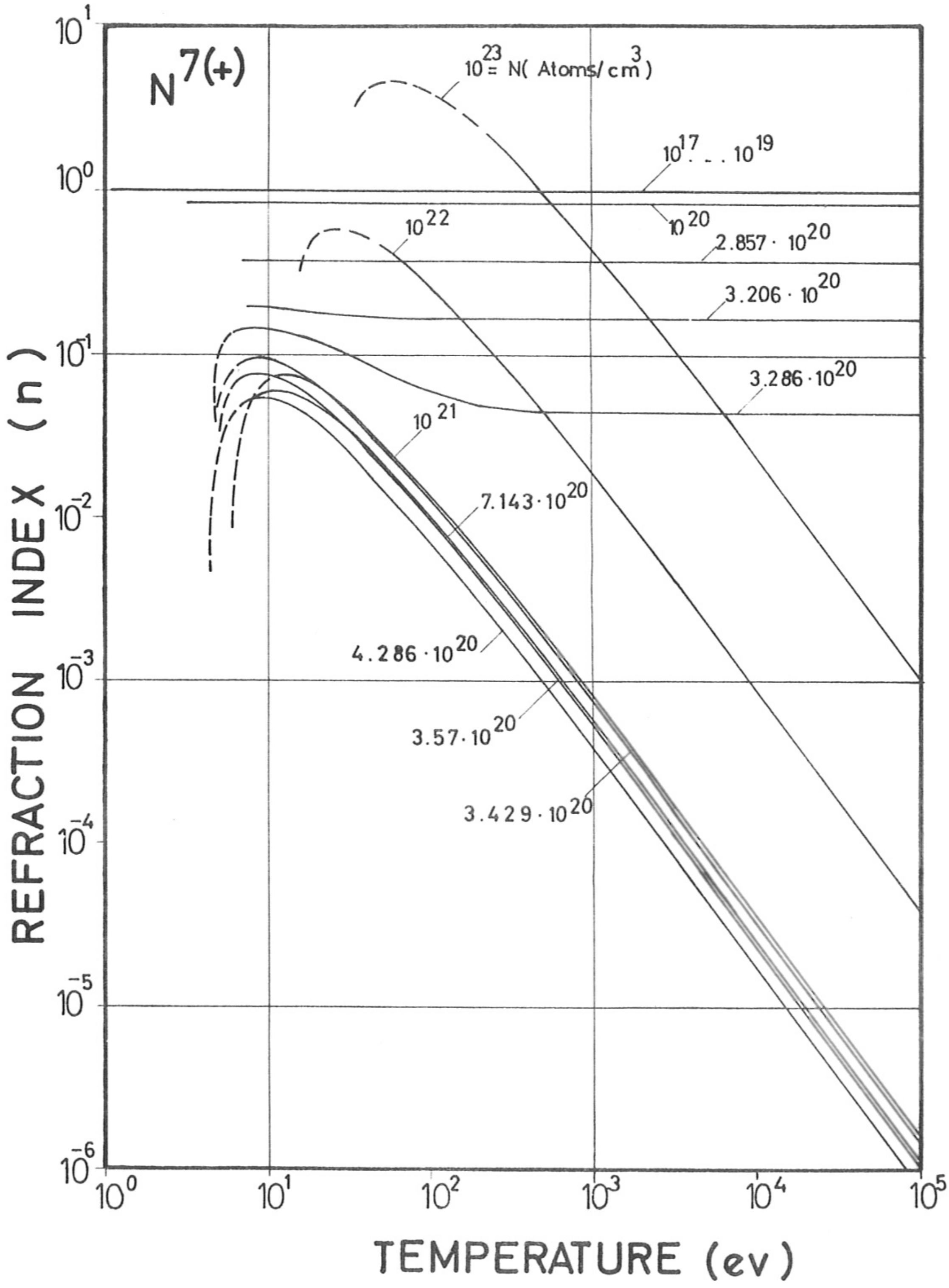
Al 5(+)
10 = N 23 (Atoms/cm 3)
10 17 .... 10 19
10 20
10 22
4 · 10 20
4.48 · 10 20
4.6 · 10 20
10 21
6 · 10 20
5 · 10 20
4.8 · 10 20
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig. 16



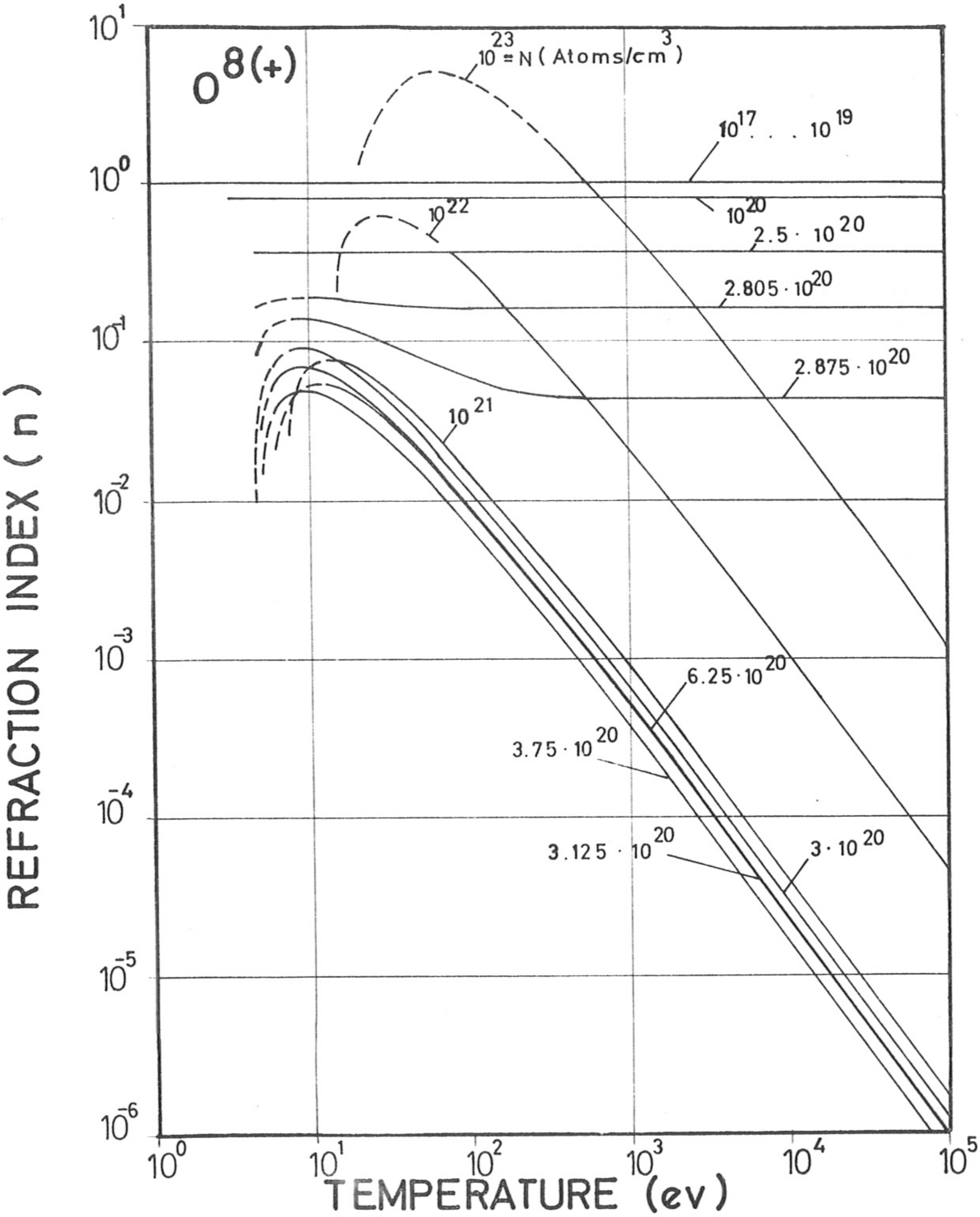
C6 (+)
10^23 = N(Atoms/cm^3)
10^17 - - - 10^19
10^20
10^22
3.333 · 10^20
3.74 · 10^20
3.833·10^20
10^21
8.333·10^20
5 · 10^20
4 · 10^20
4.167·10^20
REFRACTION INDEX (n)
TEMPERATURE (ev)
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^0
10^1
10^2
10^3
10^4
10^5

Fig . 17



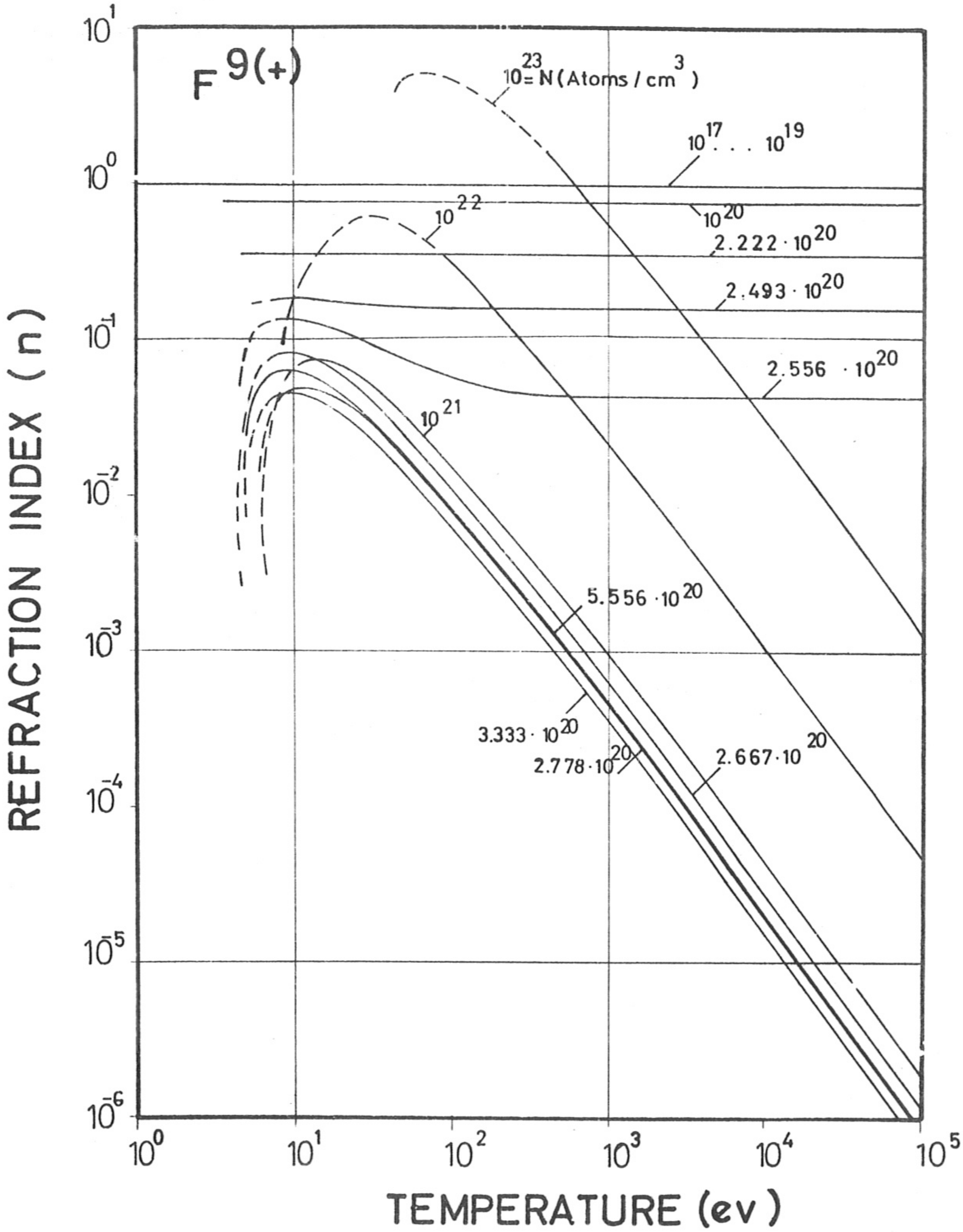
N 7(+)
10^23 = N(Atoms/cm^3)
10^17 . . . 10^19
10^20
2.857 · 10^20
10^22
3.206 · 10^20
3.286 · 10^20
10^21
7.143 · 10^20
4.286 · 10^20
3.57 · 10^20
3.429 · 10^20
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig. 18



O8(+)
10 23 = N (Atoms/cm 3)
10 17 . . . 10 19
10 20
10 22
2.5 · 10 20
2.805 · 10 20
2.875 · 10 20
10 21
6.25 · 10 20
3.75 · 10 20
3.125 · 10 20
3 · 10 20
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig. 19



F 9(+)
10 23 = N (Atoms / cm 3)
10 17 . . . 10 19
10 20
10 22
2.222 · 10 20
2.493 · 10 20
2.556 · 10 20
10 21
5.556 · 10 20
3.333 · 10 20
2.778 · 10 20
2.667 · 10 20
REFRACTION INDEX (n)
TEMPERATURE (ev)

Fig . 20

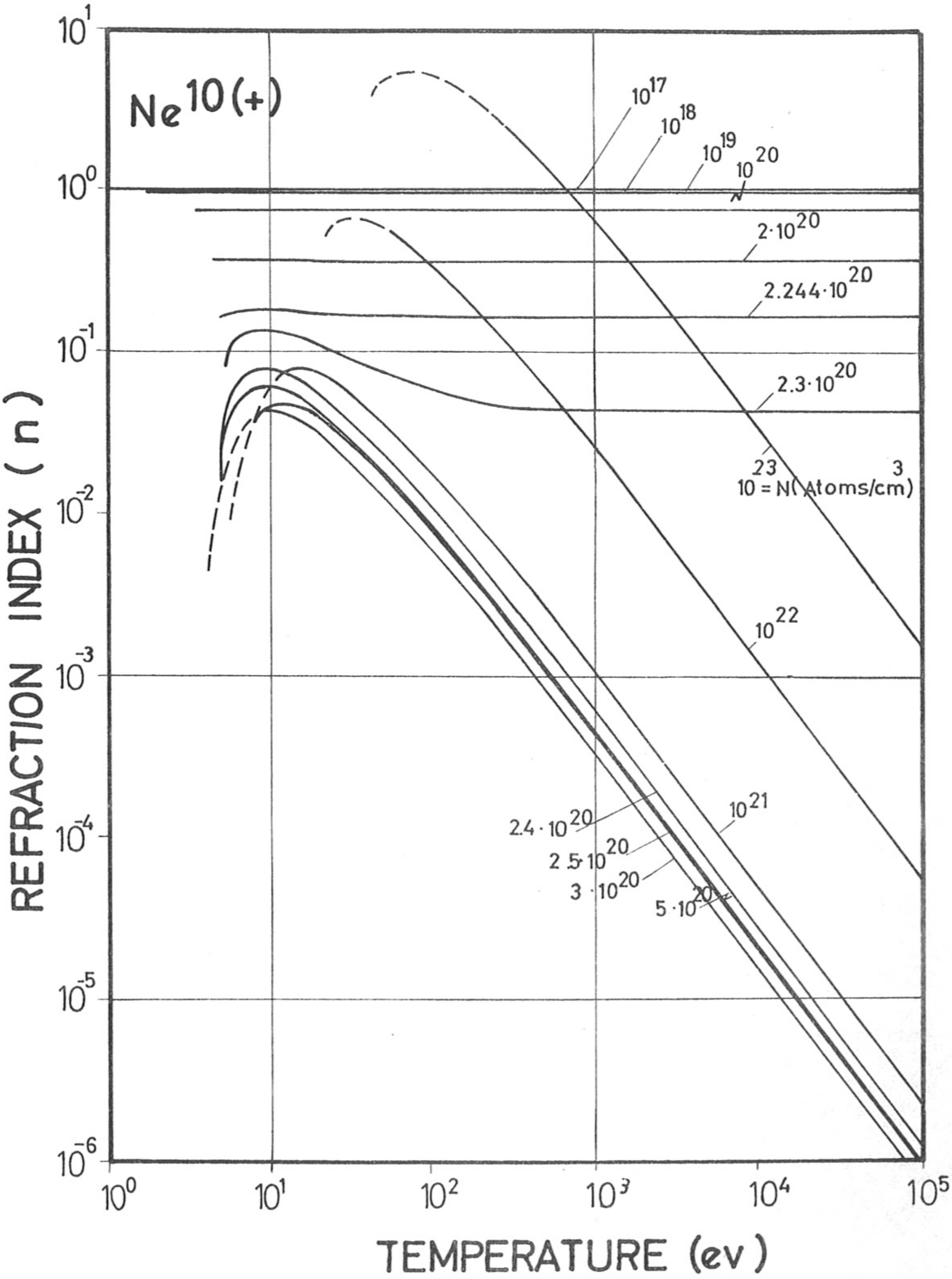


Fig 21