Theta-Pinch

+

ON STELLARATOR "PUMP-OUT"

N. D'Angelo +)

IPP 2/31 May 1963

INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

"Theta-Pinch"

INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

ON STELLARATOR "PUMP-OUT"

N. D'Angelo +)

IPP 2/31 May 1963

+) On leave from Plasma Physics Laboratory, Princeton, N.J., USA

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Institut für Plasmaphysik GmbH und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

ON STELLARATOR "PUMP-OUT"

N. D'Angelo

In stellarator experiments plasma loss rates have been measured largely in excess of those predicted on the basis of the so-called "classical" diffusion theory (1). This phenomenon of enhanced plasma loss is usually referred to as "pump-out". Loss rates in excess by several orders of magnitude over the classical rates should arise, according to Bohm (2), if fluctuating electric fields of large enough amplitude are present.

Stellarator experiments by Stodiek, Ellis and Gorman (3) have shown plasma confinement times τ proportional to B/T_e (B = magnetic field intensity, T_e = electron temperature) as predicted for Bohm's diffusion. Furthermore, the absolute values of τ were in good agreement with Bohm's predictions. No statement was made, however, by Stodiek and co-workers as to the mechanism of excitation of fluctuating electric fields which might account for the observed loss rates.

Spitzer has advanced the suggestion (4) that fluctuating electric fields, with components in the azimuthal direction, might be present in stellarator plasmas, as a result of ion wave propagation along the magnetic lines and in phase over areas of the size, perhaps, of an ion Larmor radius, but otherwise uncorrelated from one "tube of force" to the next.

Experiments by Motley (5) and some additional preliminary evidence from cesium thermal plasmas in a stellarator geometry seem, however, to point against the picture of ref. (4).

"In retrospect .... one must admit the failure .... of the straight-forward theory to account for the wide spectrum 'hash' which is observed whenever current is passed through a plasma and which evidences the presence of instability. While the low current $T_i \ll T_e$ hash may find a satisfactory fit in the fast cold-ion acoustic overstability, the $T_i \sim T_e$ case is badly wanting for a theory. For this case, not only are the theoretical minimum unstable currents much too high, but the real part of the frequency also comes out much too high (electron plasma frequency)." [6]

In this note we wish to point out that for a fully ionized plasma in a stellarator geometry an instability might, perhaps, occur which gives rise to quasi-electrostatic ion waves propagating across the magnetic lines. The excitation does not require a relative drift between ions and electrons along the magnetic lines. The growth rate of the oscillations appears to be fast enough to account for the observed "pump-out" phenomena.

This instability might be present not only in stellarators, but, in general, in other devices which make use of curved magnetic lines for confinement of the plasma. A more complete report will be given at a later date.

Consider a fully-ionized plasma in a stellarator geometry, with very small value of ß (ratio of material to magnetic pressure), and a cross section of the plasma, say in the middle of a U-bend, with a plane perpendicular to the magnetic axis. Assume that, in the absence of any perturbation and with no zero-order electric fields, the plasma is confined and has a certain radial density distribution, cylindrically symmetric. Because $\underline{B} \cdot \nabla B \neq 0$ and the magnetic lines are curved, the centers of gyration of the ions have a drift velocity in the vertical direction. In the absence of any density gradient a drift of the centers of gyration will not result in any macroscopic velocity of the ions. However, if density gradients are present, macroscopic velocities will also arise in connection with the drifts of the centers of gyration.[7]

We shall assume, for simplicity, that the magnetic field is uniform throughout the entire plasma cross section, but the magnetic lines are curved with a radius of curvature R. The effect of $\underline{B} \cdot \nabla B \neq 0$ should be treated in a similar way.

In the case of straight magnetic lines and a $\underline{B}$ field of uniform intensity, if the density distribution of the plasma is cylindrically symmetric, the lines of force of the macroscopic velocity field are circles and coincide with the lines of constant density.

When the magnetic lines are bent, this is no longer true. For a density distribution which is still cylindrically symmetric, there are regions along a line of constant density where the macroscopic velocity vector is no longer tangent to the line. The fact that a non-vanishing component of the macroscopic velocity exists along the direction of the density gradient might then give rise to an instability.

We shall only consider the plasma motion in two dimensions (plasma cross section in the middle of a U-bend) and account for the curvature of the magnetic lines and the presence of a rotational transform by means of two "source" terms in the equation of continuity and momentum for the ions, which we shall write as

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\,\underline{v}) = \varepsilon\, n\,, \qquad (1)$$

$$nm\,\frac{\partial \underline{v}}{\partial t} + nm\underline{v}\cdot\nabla\underline{v} + \varkappa T\,\nabla n + qn\,\nabla\varphi - qn\underline{v}\times\underline{B} = \alpha mn\underline{v}\,. \qquad (2)$$

The terms εn and $\alpha mn\underline{v}$ represent rates of change of density and momentum and the coefficients α and ε could, in general, depend on coordinates, particle velocities, curvature of the magnetic lines, etc. They will, of course, vanish, when the magnetic lines have no curvature. However, α and ε must not vanish if a macroscopic drift is present from the curvature of the magnetic lines. It is at least difficult to specify ε and α from what actually happens in a stellarator. We can leave them entirely undetermined and let the zero-order equations specify them.

We shall treat the problem, for simplicity, in a Cartesian geometry. The uniform $\underline{B}_o$ field is in the direction of the positive z axis and a density gradient exists in the x direction. We take for the unperturbed density distribution $n_o(x) = \bar{n}_o\, e^{-\lambda x}$, assume that charge quasineutrality is preserved and neglect all variations of the $\underline{B}_o$ field arising from plasma motion (low-ß approximation).

For the zero-order macroscopic velocity we write

$$\underline{v}_o = \mu\, \hat{x} + \eta\, \hat{y} \quad , \qquad (3)$$

where $\hat{x}$ and $\hat{y}$ are the unit vectors in the x and y direction, and μ and η are constant. That μ must be a constant can be seen from the fact that $\mu = \mu_d f(\lambda \varrho)$, where μ_d is the drift velocity of the gyrocenters and ϱ is the ion Larmor radius. In our case $\lambda\varrho$ = const.

The zero-order equations are:

$$\lambda c^2 + \omega_c\, \eta + \alpha\mu = 0 \qquad \omega_c \mu - \alpha\, \eta = 0 \qquad (4)$$
$$\lambda\mu + \varepsilon = 0 \quad ,$$

with $c = (\varkappa T/m)^{1/2}$ and $\omega_c = qB/m$.

For the first order quantities (perturbation) we assume:

$$n_1 = \bar{n}_1\, e^{-\lambda x}\, e^{i(ky-\omega t)} \qquad \underline{v}_1 = \bar{v}_1\, \hat{x}\, e^{i(ky-\omega t)}$$
$$\varphi_1 = \bar{\varphi}_1\, e^{i(ky-\omega t)} \quad . \qquad (5)$$

A root of the dispersion relation obtained from eqs. 1, 2, 3, 4, and 5 is:

$$\omega_r = k\eta \qquad \omega_i = \alpha \quad . \tag{6}$$

This is a wave travelling in the y direction with a phase velocity equal to the ion drift velocity η of the zero-order motion and $\bar{v}_1 \neq 0$. For $\mu^2 \ll \eta^2$, one obtains from eq. 4:

$$\alpha \approx -\mu \frac{\lambda c^2}{\eta^2} \approx -\frac{\mu}{\lambda} \frac{1}{\varrho^2} \quad . \tag{7}$$

We have now to evaluate μ. In the stellarator geometry the x direction of our problem corresponds to the radial direction and we shall consider a radius making a certain angle with the vertical, say about 20^o or 30^o. Then:

$$\mu = \mu_d \, f(\varrho\lambda) \approx \mu_d \, f'(0) \, \varrho \, \lambda \approx \gamma \mu_d \, \varrho \, \lambda \, , \tag{8}$$

where γ will be, perhaps, of the order of unity (or larger). Then:

$$\tau_{growth} \approx \frac{1}{\gamma} \frac{\varrho}{\mu_d} \approx \frac{1}{\gamma} \frac{R}{c} \quad . \tag{9}$$

The last relation is obtained by keeping in mind that (1)

$$\mu_d \approx \frac{1}{\omega_c R} \; c^2 \quad . \tag{10}$$

In order to have growth it is necessary that $\alpha > 0$ or $\mu/\lambda < 0$. This means that the ions have to move, in the zero-order motion, along the x axis, <u>against</u> the density gradient. This condition will always be satisfied in some region of the plasma cross section. From eq. 9 one obtains for typical stellarator H plasmas $\tau_{growth} \approx 10\,\mu s$, whereas for Cs thermal plasmas in a stellarator geometry $\tau_{growth} \approx 1$ ms, taking for γ a value of about 1.

It should finally be added that waves similar to those described in this note have been actually observed in cesium and potassium thermal plasmas, with straight magnetic lines [8]. In that case, however, an "external" excitation mechanism was required to observe the waves and no appreciable "enhanced" diffusion seemed to arise from their presence. For straight magnetic lines one would actually obtain from the present calculation $\omega_i = 0$ and $\bar{v}_1 = 0$.

I wish to thank Drs. E. Canobbio, R. Croci, and D. Dimock for many helpful discussions.

Bibliography

1) L. Spitzer, jr.: Physics of Fully Ionized Gases, Interscience Publishers, Inc., New York (1956)

2) D. Bohm: The Characteristics of Electrical Discharges in Magnetic Fields, McGraw Hill, Inc., New York (1949)

3) W. Stodiek, R.A. Ellis, jr., J.G. Gorman: Nuclear Fusion, Supplement I, 193 (1962)

4) L. Spitzer, jr.: Phys. Fluids 3, 659 (1960)

5) R.W. Motley: Nuclear Fusion, Supplement I, 199 (1962)

6) T.H. Stix: The Theory of Plasma Waves, McGraw Hill, Inc., New York (1963)

7) L. Spitzer, jr.: Astr. Jour. 116, 299 (1952)

8) N. D'Angelo, R.W. Motley: Phys. Fluids 6, 422 (1963)