Review of experiments on the diffusion of plasma across a magnetic field.

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0 Introduction.

Since the work on controlled thermonuclear fusion research started, a crucial question has arisen whether the diffusion of plasma across a magnetic field is determined only by the collisions which the plasma particles suffer, or other more complicated effects are operative. The "collisional diffusion" gives a lower limit ot the diffusion rate, and any other process which may cause a larger diffusion rate is considered to be a threat for the fusion. For that reason many experiments have been set up, during the last years, in order to decide whether the diffucion of plasma particles across a magnetic field is "collisional" (or normal) or "anomalous" and what are the factors which determine if the plasma behaves one way or the other. However up to now very little progress in made in answering this question.

The first experimental work on the diffusion of plasma across a magnetic field was known from the work of Bohm et al. He stated that in their experiments an anomalous high diffusion rate was present which he called "drain diffusion". Since 1955 many attempts have been made to measure the diffusion rate of a plasma across a magnetic field in a great variety of apparatusses with different methods. The results so far obtained seem to depend very much on the experimental set up and the way the diffusion coefficient is measured. Some ways of measurement do not seem to be very suitable at all. On top of the difficulty of making proper measurements comes the fact that the values of many plasma parameters which are necessary to compare the experimental data with theory are not known and may only be guessed roughly. This explains the fact that no conclusive experiments have been made in this field and that many contradictory results were obtained.

Ever since Bohm's first publications on the subject the word "anomalous diffusion" or "drain diffusion" has fallen many times at the wrong place and probably was at the wrong place already for Bohm's own experiments. Here like in some other cases the diffusion was most probably found to be anomalously high because of a wrong theoretical interpretation of the experimental data. The word "anomalous" or "abnormal diffusion" has been used also in cases where macroscopic plasma instabilities were responsible for a fast loss rate of plasma particles. The concept "diffusion", however, has only some meaning in a macroscopic stable plasma with a pressure gradient over a distance which is large compared to the dimensions of the instability. In order to prevent confusion it is desirable to be cautious with the use of the word "anomalous diffusion" in cases where the cause of a large charged particle loss is not known. Anomalous diffusion, if it exists, may be explained by microscopic plasma instabilities, plasma oscillations, enhanced interaction, etc. Up to now there is no case known where an experimentally found abnormal high plasma loss rate is theoretically explained quantitatively by one of these mechanisms. As Ecker has pointed out, possibly, "enhanced interactions" in the plasma may manifest themselves clearer in other plasma paramters than in the always very indirectly measured diffusion coefficient.

Collisional diffusion is detected in a rather convincing way only in few experiments. Many experiments where collisional diffusion was claimed to be detected, gave doubtfull results. Partly because the theory on which they were based was not unobjectionable, partly because the theoretical assumptions were not fulfilled. Very often the finiteness of the ion gyro radius was overlooked.

An attempt is made to bring together the data of all experiments which, up to now, have been made trying to measure the plasma particle diffusion across a magnetic field. In order to make evaluation and discussion of the results easier, the different parameters which exist in the various experiments are brought together in tables. It is clear that the tables contain only approximate numbers.

The experiments are devided into several groups. The classification is made according to the discharge conditions and the accompanying way the measurements were made. In low pressure discharges the radial plasma density distribution, $n(r)$, is measured in dependence on the magnetic field, B (chapter 3). In glow discharges the axial electric field strength, E_z, is measured as function of B (chapter 4). In decaying plasmas ($\frac{\delta n}{\delta t} \neq o$) the decay time, τ, is measured in the afterglow of a discharge (chapter 5). The theoretical foundations of the different methods of measurement is given in chapter 2. A summary of the various expressions for the perpendicular diffusion coefficient is given in chapter 1.

List of used symbols.

n_o	neutral particle density in cm^{-3},
n_e, n_i	electron, ion particle density in cm^{-3}, ($= n$ if $n_i = n_e$),
B	magnetic induction in gauss,
E_r	radial electrical field strength in V/cm,
E_z	axial electric field strength in V/cm,
ω_{ce}, ω_{ci}	electron, ion cyclotron frequency in sec^{-1},
$\nu_{e,n}$, $\nu_{i,n}$	collision frequency of an electron, ion with neutral particles in sec^{-1},
$\nu_{e,i}$, $\nu_{i,i}$	collision frequency of an electron, ion with ions in sec^{-1}
$v_{e,t}$, $v_{i,t}$	thermal velocity of the electrons, ions in cm/sec,
$v_{e\perp}$, $v_{i\perp}$	perpendicular transport velocity of the electrons, ions in cm/sec, ($= v_\perp$ if $v_{e\perp} = v_{i\perp}$),
$D_{e\perp}$, $D_{i\perp}$	electron, ion diffusion coefficient perpendicular to the magnetic field in cm^2/sec,
$\mu_{e\perp}$, $\mu_{i\perp}$	electron, ion mobility perpendicular to the magnetic field in cm^2/V.sec,
$j_{e\perp}/j_{i\perp}$	ratio of electron to ion current for a plane probe perpendicular to the magnetic field,
kT_e, kT_i	electron, ion temperature in eV,
L	length of the plasma in cm,
R	distance of center to the wall in cm,
r_{pl}	radius of the plasma beam in cm,
λ_e, λ_i	mean free path length of electrons, ions in cm,
r_{ce}, r_{ci}	electron, ion cyclotron radius in cm,
r_{ci}^{max}	ion-cyclotron radius at the smallest values of B used in the experiment,
q	e-folding length of the radial plasma density distribution $n(r)$ in cm,
a	probe radius in cm,
p, p_n	plasma, neutral gas pressure in ~~erg~~ eV.cm^{-3},
P_o	neutral gas pressure in Torr.

1 The perpendicular diffusion coefficient.

If an ionized gas is put in a magnetic field, the diffusion of the plasma particles becomes anisotropic. In the direction of the magnetic field the velocity of diffusion of the plasma particles is practically equal to its value if no magnetic field is present. Perpendicular to the magnetic field the velocity of diffusion is reduced. Plasma particles may diffuse through a magnetic field by collisions. The theoretical results which are obtained for collisional diffusion are mentioned below respectively for a weakly ionized gas (Coulomb collisions neglegible compared to collisions with neutral particles), a fully ionized gas (only Coulomb collisions) and a strongly ionized gas (both type of collisions present). Any case where the diffusion rate is larger than may be expected from collisions only, is called enhanced diffusion.

In this theoretical introduction mainly the collisional diffusion will be reviewed. As will be shown in the later sections, there are only very few measurements made which are theoretically unobjectionable and these experiments all indicate the presence of collisional diffusion. As no measurements are made which unambignously indicate the presence of enhanced diffusion, we will only mention this subject superficially (section 1.4). Also we will not review macroscopic instability theories which explain the onset of instabilities which make diffusionmeasurements impossible. Excellent reviews on the diffusion of plasma in a magnetic field where this subject is treated are given by Lehnert [1] and Hoh [2].

1.1 Weakly ionized gas (Coulomb collisions neglegible).

Chapman and Cowling [3] derived expressions for the diffusion of plasma particles in a single component ionized gas, where a pressure gradient $\nabla_\perp p$ and an electric field $E_\perp$ are present in a direction perpendicular to the magnetic field B. In this direction the velocity of diffusion of the ions is reduces in the ratio: 1: $\left[1 + (\omega_{ci}/\nu_{i,n})^2\right]$ and for the electrons in the ratio:1: $\left[1 + (\omega_{ce}/\nu_{e,n})^2\right]$

So the perpendicular diffusionscoefficient of the ions is equal to:

$$D_{i\perp} = \frac{D_{io}}{1 + (\omega_{ci}/\nu_{i,n})^2} \text{ where } D_{io} = \tfrac{1}{3} v_{it} \lambda_i = \tfrac{1}{3} \nu_{i,n} \lambda_i^2 \quad (1)$$

$$D_{i\perp} \simeq \tfrac{1}{3} \nu_{i,n} r_{ci}^2 \quad \text{if } \omega_{ci} >> \nu_{i,n}$$

and analogous for the electrons:

$$D_{e\perp} = \frac{D_{eo}}{1 + (\omega_{ce}/\nu_{e,n})^2} \quad \text{where } D_{eo} = \tfrac{1}{3} v_{et} \lambda_e = \tfrac{1}{3} \nu_{e,n} \lambda_e^2 \quad (1a)$$

$$D_{e\perp} \simeq \tfrac{1}{3} \nu_{en} r_{ce}^2 \quad \text{if } \omega_{ce} >> \nu_{e,n}$$

In add ition to this ordinary or direct diffusion there is a flow of particles perpendicular both to B and $\nabla_\perp p$, which they call transverse diffusion and which is $\omega_{ci}/\nu_{i,n}$ (respectively $\omega_{ce}/\nu_{e,n}$) times the direct. Similar results hold for the direct and transverse electric currents and for the direct and transverse conductivity of the plasma. Only the direct diffusion is considered in this review.

In a weakly ionized gas ions and electrons may diffuse with different speeds through the magnetic field, their diffusion does not need to be coupled. The diffusion mechanism depends on the discharge conditions and on the boundary conditions.

In the special case of ambipolar diffusion, the ions and the electrons diffuse through the magnetic field with the same speed, which is given by (see e.g. [23]):

$$n\, v_{a\perp} = - D_{a\perp} \nabla_\perp n = - \frac{\mu_{i\perp} D_{e\perp} - \mu_{e\perp} D_{i\perp}}{\mu_{i\perp} - \mu_{e\perp}} \nabla_\perp n \qquad (2)$$

Between the diffusion coefficient D and the mobility μ exists the Einstein relation $\mu = \frac{eD}{kT}$, so that:

$$Da_\perp = \frac{D_{i\perp} D_{e\perp} \left(\frac{1}{kTi} + \frac{1}{kTe}\right)}{\frac{D_{i\perp}}{kT_i} + \frac{D_{e\perp}}{kT_e}}$$

As in all cases of interest, B is so high that $D_{i\perp} \gg D_{e\perp}$, it ist practically the electrons which determine the diffusion rate,and eq (2) reduces to:

$$n\, v_{a\perp} \simeq - \left(1 + \frac{kTi}{kTe}\right) D_{e\perp} \nabla_\perp n \qquad (2a)$$

The electrons move with much more difficulty through the magnetic field than the ions, so that in a direction perpendicular to the magnetic field an ambipolar diffusion is expected with the ions pulling on the electrons. A small space charge separation may provide for the transverse ambipolar electric field:

$$E_{a\perp} = \frac{D_{i\perp} - D_{e\perp}}{\mu_{i\perp} - \mu_{e\perp}} \frac{\nabla_\perp n}{n} \qquad (3)$$

or in the approximation of eq (2a):

$$E_{a\perp} \simeq \frac{kTi}{e} \frac{\nabla_\perp n}{n} \qquad (3a)$$

An expression like eq (2) is only valid of no temperature gradients are present; but as Schlüter [4] has pointed out also gravity- and interaction forces must be neglegible, together with the Lorenz force of the current and its interaction with the neutral gas. He also derived the relation:

$$a\, v_{a\perp} = \nabla_\perp\, p_n \tag{4}$$

$p_n = n_o\, kT_n$ and $a = n\,(\nu_{i,n} m_i + \nu_{e,n} m_e)$.

If $T_n \simeq T_i$ and $\nabla_\perp T_n = 0$:

$$n\, v_{a\perp} \simeq D_{io} \nabla_\perp\, n_o \tag{4a}$$

This equation prescribes the gradient in the neutral gas pressure, which arises by the force which the plasma particle flux exerts on the neutral gas.

If the end plates are conducting the electrons may destroy the ambipolar space charge field by diffusing along the magnetic lines of force and moving through the conducting end plates. The possibility of this "short circuiting effect" was first pointed out by Simon [5]. If the "short circuiting effect" for the electrons is operative, the transverse ambipolar electric field is removed and the ions move through the magnetic field with their own intrinsic rate:

$$n\, v_{i\perp} = -\, D_{i\perp} \nabla_\perp\, n \qquad \text{see eq (1)}$$

Characteristic for the diffusion coefficents in a weakly ionized gas is the fact that they are independent of the plasma particle density and proportional to the neutral particle density.

1.2 In a fully ionized gas the velocity of diffusion of the ions and of the electrons perpendicular to B must be the same, because the equation of motion (see litt. [6] and [7]) requires that $j_\perp \cdot \nabla_\perp p = 0$. If like particle collions are neglegible the plasma diffusion through the magnetic field is caused by collisions between ions and electrons and it is easy to deduce from the plasma equations the well known formula (see e.g. [7]):

$$v_\perp = - \frac{\eta}{B^2} c^2 \nabla_\perp p \tag{5}$$

$\eta = \frac{m_e \nu_{e,i}}{n e^2}$ is the resistivity of the plasma.

In case $\nabla T = o$, the diffusioncoefficient $D_\perp$ can be expressed as:

$$D_\perp \simeq \frac{1}{3} \nu_{e,i} \, r_{ce}^2 \tag{5a}$$

If not too large gradients in the electron temperature are present, this result is approximately equal to the relation:

$$n v_\perp = - \frac{\nu_{e,i}}{m_e \omega_{ce}^2} \left[(kT_e + kT_i) \nabla_\perp n - \frac{1}{2} n \nabla_\perp (kT_e) + n \nabla_\perp (kT_i) \right] \tag{6}$$

which was derived from the Boltzmann equation by Braginskii [8].

Collisions between like particles produce no flux in first order (proportional to ∇n). Ion-ion collisions produce a flux proportional to third derivative of the particle density [9] and [10] :

$$v_\perp = \frac{3}{8} \frac{r_{ci}^4}{\tau_{i,i}} \nabla_\perp \left(\frac{1}{n} \nabla_\perp^2 n \right) \tag{7}$$

Note that there is no like particle flux if n is linear or exponential. Also that Ficks law is not obeyed and that $D_\perp$ has lost its meaning.

It's is easily found from eq (5) and eq (7) and assuming $\lambda_{i,i} \simeq \lambda_{e,i}$, that the ratio of the flux from ion-ion collisions to the flux from ion-electron collisions is of the order of $(m_i/m_e)^{1/2}(r_{ci}/q)^2$. This ratio may be > 1, so that ion-ion collisions cannot be neglected tacitly. Also in strongly ionized plasmas, ion-ion collisions may not be overlooked. The ratio of the flux due to the electron-electron collisions to the flux from unlike particle collisions is of the order of $(r_{ce}/q)^2$ and is generally quite small.

It was assumed in these treatments however that space charge does not occur as a result of ion-ion collisions, because of the short circuiting effect. Kaufman showed [11] that in a system isolated, so as to allow charge separation the resultant electrical field $E_\perp$ grows to such a value as to effectively destroy the flux caused by ion-ion collisions. In this case the electrical field builds up to the value of an ambipolar electric field in a weakly ionised gas:

$$E_\perp = \frac{kT_i}{e} \frac{\nabla_\perp n}{n} \qquad \text{compare eq (3a)}$$

Thus also in the case of a fully ionized gas an important question is whether the short circuiting effect is operative or not.

Finally reference may be made to the work of Post [12] and Taylor [13] which shows the great influence that impurities may have on the diffusion of a fully ionized gas across a magnetic field.

In a fully ionized gas the diffusion coefficient is proportional to the plasma particle density. This makes the solution of the diffusion equation more difficult.

1.3 <u>In a strongly ionized gas</u> the velocity of diffusion of the plasma across a magnetic field may be described with an ambipolar diffusioncoefficient. Golant [14] derived for this case, assuming that $m_e \nu_{e,i} \ll m_i \nu_{i,n}$

$$n\nu_{a\perp} = - \frac{D_{ao}}{1 + \frac{\omega_{ce}\omega_{ci}}{\nu_{i,n}(\nu_{e,i}+\nu_{e,n})}} \nabla_\perp n \tag{8}$$

where D_{ao} is the coefficient of ambipolar diffusion if no magnetic field is present:

$$D_{ao} = \frac{D_{io}D_{eo}\left(\frac{1}{kTi} + \frac{1}{kTe}\right)}{\frac{D_{io}}{kT_i} + \frac{D_{eo}}{kT_e}} \simeq D_{io}\left(1 + \frac{kTi}{kTe}\right) \simeq \frac{kTi + kTe}{m_i \nu_{i,n}}$$

The equations for the limiting cases, ambipolar diffusion of a weakly ionized gas, $\nu_{e,i} = 0$ (section 1.1) and diffusion of a fully ionized gas, $\nu_{i,n} = \nu_{e,n} = 0$ (section 1.2) may easily be obtained from
Golant's formula taking into consideration the approximations which are made in order to derive the equations.

1.4 <u>Enhanced diffusion</u>, also called anomalous diffusion, is a collective noun for all cases where the diffusion rate is larger than may be expected from collisions. The causes of such an "enhanced diffusion" are often not exactly formulated. As the word "anomalous diffusion" ist not strictly defined, it has caused much confusion. It was used in cases where the reason of particle losses, higher than could be attributed to collisional diffusion, was not known. In some of these cases it was shown afterwards that the high loss rate could be explained by macroscopic instabilities with a dominant long wavelength. It is senseless however to speak about diffusion if instabilities are present in a region which is not small compared to the distance $n/\nabla n$ over which the particle density decreases. In this review we will not go into much detail about the possible causes of enhanced diffusion. Only two formulae are given which are used to describe enhanced diffusion.

Bohm [15] has given a formula for the perpendicular diffusioncoefficient of the electrons:

$$D_{e\perp} \text{ (drain)} = \frac{10^8}{16B} kT_e \tag{9}$$

or:

$$D_{e\perp} \text{ (drain)} \simeq \frac{1}{3} \nu_{ce} r_{ce}^2 \quad (\nu_{ce} = \omega_{ce}/2\pi) \tag{9a}$$

Comparing eq. (9a) with eq.(1), eq.(1a) and eq.(5a) shows that Bohm's formula for the drain diffusion coefficient may be found by taking as usual the product of a characteristic length squared (in this case r_{ce}^2) and a characteristic frequency. For this latter was taken the electron cyclotron frequency in stead of a collision frequency. For the cases of practical interest, $\nu_{ce} \gg \nu_{e,i}$ or $\nu_{e,n}$, this yields a diffusion coefficient which is much larger than the collisional one. Recently several efforts are made to give this formula a more solid theoretical foundation.

Taylor [16] found with stochastic methods an expression for the perpendicular diffusioncoefficient of the ions, which contains a factor $\beta/\beta^2+\omega_{ci}^2$, where β is the coefficient of dynamical friction of the ions. If $\beta = \nu_{i,n}$ the expression reduces to $D_{i\perp}$ (eq.1) . If $\beta = \omega_{ci}$ a formula like eq (9) ist found, though with a factor 1/4 instead of 1/16 and with $k(T_i+T_e)$ instead of kT_e. As $\beta/\beta^2+\omega_{ci}^2$ attains its maximum value for $\beta = \omega_{ci}$, this case corresponds to a maximum diffusion rate. The derivation of Taylor does not yield an equation of the form of eq. (9), without leaving still some questions unanswered.

Yoshikawa and Rose [17] derived an expression which describes qualitatively both the normal collisional diffusion and the enhanced diffusion of Bohm. The effect on which their calculations are based is a nonlinear interaction proceeding in the following physical mechanism: $\nabla_\perp n$ (or $E_\perp$) gives rise to a transverse current in the Hall direction. If there is any small density fluctuation, $\langle(n'-n)^2\rangle_{av}$, in the Hall direction, a space charge field in this direction must be set up. This additional field, crossed with B, gives motion in the direction of the original pressure gradient $\nabla_\perp n$ (or electric field $E_\perp$). If the diffusion is normal or anomalous depends on the ratio of two quantities: $S = \langle(n'-n)^2\rangle_{av}/n^2$ and $\alpha = \nu_{e,i}/\omega_{ce}$. If $S \gg \alpha$, the system obeys eq (9), on the other hand if $\alpha \gg S$, the diffusion is collisional. Therefore one expects that anomalous diffusion takes place if the pressure is low or the magnetic field is high.

A qualitative derivation of a formula of the form of eq (9) has been given by Spitzer [18] , assuming that a fluctuating $E_\perp$ field in the plasma may be preliminary responsible for the perpendicular diffusion. This theory has been critisized by Stix [80] .

Drummond and Rosenbluth [19] calculated the anomalous diffusion caused by electric field fluctuations, arising form a particulary strong micro instability - the two stream ion-cyclotron instability. In a plasma in which a Maxwellian distribution of electrons with thermal velocity v_{et} is drifting relative to a Maxwellian distribution of ions with thermal velocity v_{it} in such a way that the drift velocity v_d is given by $5\, v_{it} \lesssim v_d \lesssim v_{et}$, a diffusion coefficient of the electrons is found given by $D_{e\perp} = \alpha\, D_{e\perp}$ (drain) where α is given by a constant x $(v_d / v_{et})^5 (T_e/T_i)^2$ and is $\ll 1$. Thus the diffusion coefficient obtained is small compared to the Bohm diffusion coefficient, although it has the same dependence on B.

Ecker came to a geeneral formula [20] :

$$D_{\sigma\perp} = \frac{D_{ao}}{1 + \dfrac{\dfrac{\omega_{ci}\omega_{ce}}{\nu_{i,n}\nu_{e,n}}}{1+\sigma_{+}+\sigma_{-}} \cdot \dfrac{\mu_{+}+\varepsilon\mu_{-}}{\mu_{-}+\varepsilon\mu_{+}}} \qquad (10)$$

This formula covers both ambipolar diffusion ($\varepsilon = 1$) and Simon diffusion ($\varepsilon = 0$). The interaction parameter σ is not calculated, but introduced as a quantity which describes the enhanced interaction. Formula (10) reduces to (2) or (8) if $\sigma = 0$ and $\varepsilon = 1$, and to (1)*) if $\sigma = 0$ and $\varepsilon = 0$. It is clear that for the case $\sigma \neq 0$, the decrease of the diffusion coefficient caused by the magnetic field is reduced.

*) Apart form a factor 2, which is also the difference between Simon's and Tonk's results (see 2.2.1).

2 Methods of measuring the perpendicular diffusion coefficient.

In practically all experiments the plasma is cylindrical with B in the axial direction. And one has tried to measure the direct diffusion, i.e. the velocity of diffusion in radial direction. The transverse diffusion is in azimuthal direction and is of less interest. The transverse diffusion or Hall diffusion causes electrons and ions to move in opposite direction. The azimuthal electric current which is associated with Hall diffusion is connected to the radial pressure gradient by the relation $\bar{j} \times \bar{B} = \bar{\nabla} p$. As this current is not hindered to flow in the cylindrical geometries which were used in the experiments (except perhaps in the arc chamber used by Bohm et al.) it is not of interest in the scope of this review. For this reason we will use cylindrical coordinates (r, φ, z) with B in the z direction and limit our attention to particle currents in radial and axial direction. These directions are also denoted with ⊥ and ∥ signs respectively. The theoretical basis for the methods of measurement of the perpendicular diffusion coefficients will be only illustrated here for a weakly ionized gas.
The methods which are used for highly ionized gases are very similar and short reference to the used theories will be made at the place where experiments with highly ionized gases are discussed.

2.1 The diffusion equation is derived from the particle conservation equation, which reads for the ions:

$$\frac{\delta n_i}{\delta t} = - \operatorname{div}(n_i v_i) + \beta_i n_i - \alpha n_i^2 \qquad (11)$$

α is the recombination coefficient, and βn_i the production rate of charged particles.

The particle current equations may be derived from Schlüter's equations [4] for the case of a weakly ionized gas and by neglecting inertia and gravity forces. A very good treatment of these equations (12) may be found in the book of Rose and Clark [21]. The approximations are no longer valid if Coulomb collisions are non neglegible and care must be taken if there is no stationary state, so that $\frac{dv_i}{dt}$ may be comparable with $\frac{v_i}{V_{i,n}}$.

As mentioned before we disregard the transverse particle currents in azimuthal direction, so that we have:

$$n_i v_{i\perp} = - D_{i\perp} \frac{\delta n_i}{\delta r} + n_i \mu_{i\perp} E_r \qquad (12a)$$

$$n_i v_{i\parallel} = - D_{i\parallel} \frac{\delta n_i}{\delta z} + n_i \mu_{i\parallel} E_z \qquad (12b)$$

$$D_{i\perp} = \frac{D_{io}}{1+(\omega_{ci}/\nu_{i,n})^2}$$

$$\mu_{i\perp} = \frac{\mu_{io}}{1+(\omega_{ci}/\nu_{i,n})^2}$$

$$D_{i\parallel} = D_{io}; \quad \mu_{i\parallel} = \mu_{io}$$

Between the diffusion coefficient D and the mobility μ exists the Einstein relation $\mu = \frac{eD}{kT}$.

In a weakly ionized gas the diffusioncoefficient and the mobility may be considered as constants, independent on the plasma particle density. But these quantities depend still on the energy of the particles, and thus in principle on r and z. If no temperature gradients are present, substitution of equations (12a) and (12b) in equation (11) yields:

$$\frac{\delta n_i}{\delta t} = D_{io} \frac{\delta^2 n_i}{\delta z^2} + D_{i\perp} \frac{1}{r} \frac{\delta}{\delta r} \left(r \cdot \frac{\delta n_i}{\delta r}\right) - \mu_{io} \frac{\delta}{\delta z} (n_i E_z)$$
$$- \mu_{i\perp} \frac{1}{r} \frac{\delta}{\delta r} (r \cdot n_i E_r) + \beta_i n_i - \alpha n_i^2 \qquad (13a)$$

For the electrons one derives in the same way:

$$\frac{\delta n_e}{\delta t} = D_{eo}\frac{\delta^2 n_e}{\delta z^2} + D_{e\perp}\frac{1}{r}\frac{\delta}{\delta r}\left(r\cdot\frac{\delta n_e}{\delta r}\right) + |\mu_{eo}|\frac{\delta}{\delta z}(n_e E_z) + |\mu_{e\perp}|\frac{1}{r}\frac{\delta}{\delta r}(r\cdot n_e E_r) + \beta_e n_e - \alpha\, n_e^2 \tag{13b}$$

with $|\mu_{eo}| = -\mu_{eo}$ and $|\mu_{e\perp}| = -\mu_{e\perp}$.

These equations describe in every volume element of the plasma the particle balance due to particle currents parallel and perpendicular to B, and eventual production and loss of particles. Though electric fields may be present, caused by space charge separation, it is justified to put $n_i = n_e$ in these equations.

In order to integrate these equations the boundary conditions have to be known exactly. If by example the plasma is in contact with insulating end plates, one expects an ambipolar transverse diffusion of the plasma particles with important E_r fields. Whereas conducting end plates make the short circuiting effect possible with vanishing E_r fields and $\mu_\perp \nabla_\perp nE \to 0$.

A way to prevent short circuiting effects is by choosing the plasma long enough, so that moving to the end plates take the electrons more time $(t_\parallel \simeq L^2/D_{ao})$ than diffusing through the magnetic field $(t_\perp \sim R^2/D_{a\perp})$. $D_\parallel \nabla_\parallel^2 n$ and $\mu_\parallel \nabla_\parallel nE_\parallel$ are then neglegibly small compared with $D_\perp \nabla_\perp^2 n$ and $\mu_\perp \nabla_\perp nE_\perp$ and another term must remain, like $\beta\, n$ (see section 2.3) or $\frac{\delta n}{\delta t}$ (see section 2.4). What length is required is not exactly known. Simon [28] has given the condition:

$$L \gg \frac{\omega_{ce}}{\nu_{e,n}}(qR)^{1/2} \tag{14a}$$

requiring $D_{a\perp}/R^2 \gg D_{ao}/L^2$ and taking $\lambda_e \simeq \lambda_i$ yields:

$$L \gg \frac{\omega_{ce}}{\nu_{e,n}} R \left(\frac{v_i}{v_e}\right)^{1/2} \quad \text{or} \quad \frac{L}{\lambda_i} \gg \frac{R}{r_{ce}}\left(\frac{v_i}{v_e}\right)^{1/2} \tag{14b}$$

The other possibility of preventing the short circuiting effect by using insulating end plates does not seem to be very well investigated experimentally. The difference with the preceding case lies in the fact that $D_{\parallel} \nabla_{\parallel}{}^2 n$ and $\mu_{\parallel} \nabla_{\parallel} nE$ have not to be neglected.

Another aspect which is not investigated up to now is the role which wall sheaths may play. Potential drops which generally occur in the sheath hinder the electrons in reaching the end plates and allow only fast electrons to provide for the short circuiting effect.

It has to be remembered that equations (13a) and (13b) have only a meaning if $\lambda_i, \lambda_e \ll L$ and $r_{ci}, r_{ce} \ll R$. The latter conditions may be easily fulfilled by experimenting with high enough magnetic fields. In many experiments with low pressure discharges, the condition $L \gg \lambda_i, \lambda_e$ is not fulfilled however. In that case the particle loss in the z direction cannot be described by the diffusion term $D_{\parallel}\frac{\delta^2 n}{\delta z^2}$ and the conduction term $\mu_{\parallel}\frac{\delta nE_z}{\delta z}$. The particles move freely to the end plates with thermal velocity which is more or less changed under the influence of E_z *). This causes an anisotropy in the velocity distribution of the particles, which makes the problem rather complex. As a first approximation the diffusion term $D_{\parallel}\frac{\delta^2 n}{\delta z^2}$ is simply replaced by $\frac{nv_t}{L}$ and the conduction term by $\frac{ne}{2mv_t}\frac{V}{L}$, where V is the potential drop in z direction.

A summary of the possible solutions of equation (13) which are investigated theoretically and experimentally up to now is given in table I. The terms which are neglected are indicated as zero.

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*) If the approximation $kT \gg e\int E_z dz$ is justified, has to be checked in every specific experiment.

Table I.

	$\frac{\delta n}{\delta t}$	$D_{\parallel}\nabla_{\parallel}^{2}n$	$D_{\perp}\nabla_{\perp}^{2}n$	$\mu_{\parallel}\nabla_{\parallel}nE_{\parallel}$	$\mu_{\perp}\nabla_{\perp}nE_{\perp}$	βn	αn^2
Low pressure g.d. with short circ.	0				0	0	0
Low pressure g.d. without short circ.	0					0	0
Bohm's calc.		0		0		0	0
Glow discharges	0	0		0			0
Decaying plasma		(0)*		(0)*		0	0
(Cs plasma)	0	0		0	0	0	

*) In most experiments the plasma is chosen so long that $D_{\parallel}\nabla_{\parallel}^{2}n$ and $\mu_{\parallel}\nabla_{\parallel}nE_{\parallel}$ may be neglected, but this is not necessary so.

The Cs plasmas are also mentioned in this table, though they represent the class of fully ionized gases, to which eq(13) is strictly not applicable.

2.2 Low pressure gas discharges ($\lambda_i \gg r_{ci}$).

2.2.1 Density profile determined by diffusion. The most proper solutions of the diffusion equations (13a) and (13b) for the case of a steady state ($\frac{\delta n}{\delta t} = 0$) low pressure discharge, are given by Tonks [22]. The calculations are made under the assumption that $L \gg \lambda_i, \lambda_e; \lambda_i \gg r_{ci}$ (or $\omega_{ci}/\nu_{i,n} \gg 1$) and $\lambda_e \gg r_{ce}$ (or $\omega_{ce}/\nu_{e,n} \gg 1$) whereas βn and αn^2 are neglegible. The boundary conditions are taken into account by assuming either conducting or insulating walls to be present. Complications which may arise from the presence of wall sheaths are assumed to be neglegible.

Like in earlier calculations of Simon [5] it is assumed that the density in the z direction varies as $n(0,z) = n_o \sin(\frac{\pi z}{L})$. This assumption makes the solution of the diffusion equation simple, but it in many experiments it is not fullfilled. Generally the density will decrease monotonicelly from the cathode, so that the term $D_{\parallel} \frac{\delta^2 n}{\delta z^2}$ is postive, instead of negative. For this reason in many discharges the results obtained with this theoretical assumption may not be valid. The validity of this assumption should at least be checked experimentally.

The theory predicts a dependence of the plasma density on the radius, which is in principle exponentially. The solution is $n_i(r) = A. K_o(r/q)$, with A constant and K_o a Hankel function. For Values of $r \gtrsim 1.5q$, this function is already very much like $e^{-r/q}$, so that it is experimentally impossible to distinguish if the density profile goes with $K_o(r/q)$ or $e^{-r/q}$.

For the case that the end plates are conducting, so that the short circuiting effect is possible, the transverse e-folding length of the plasma density is found to be:

$$q_c = \frac{1}{\pi} \frac{L}{\lambda_i} r_{ci} \tag{15}$$

As is mentioned before (see eq (3)), the diffusion is not ambipolar in this case, but the ions move with their own intrinsic rate across the magnetic field. The electrons move along the magnetic lines of force and through the conducting end plates. What influence potential drops in the wall sheaths near the end plates may have on this picture is not clear however. Formula (15) differs a factor $\sqrt{1+T_i/T_e}$ from the one originally derived by Simon [5, 23].

The validity of formula (15) is limited for small values of L/λ_i by the finiteness of the ion gyro radius, which requires $q_c \gtrsim r_{ci}$ (section 2.2.2). At the higher side L/λ_i is limited by the condition that the short circuiting effect is possible, eq(14).

In case that the end plates are <u>insulating,</u> Tonks predicts a much smaller value of the e-folding length:

$$q_i = \left(\frac{\mu_{io}}{\mu_{eo}}\right)^{1/2} \frac{1}{\pi} \frac{L}{\lambda_i} r_{ci} \tag{16}$$

It is clear that if no "short circuiting effect" is present, radial electric fields may build up and that the perpendicular conduction term $\mu_\perp \frac{1}{r} \frac{\delta}{\delta r} (r\, nE_r)$ may not be neglected. The radial diffusion of the ions is decelarated by the electrons, which leads to smaller e-folding lengths of the plasma density.

The case $\lambda_i, \lambda_e \gg L$ <u>(free streaming)</u> is not treated by Tonks. A crude analysis, assuming the presence of the "short circuiting effect", was given by Simon [23]. He arrived at an e-folding length:

$$q_L \simeq \sqrt{\frac{LD_i}{2v_{it}}} \quad \text{or} \quad q_L \simeq 0.4 \sqrt{\frac{L}{\lambda_i}}\, r_{ci} \tag{17}$$

This equation corresponds to conducting end plates.

An earlier analysis by Bohm [15] assuming that the plasma column is infinite in the z-direction - so that the z derivaties are zero and the perpendicular diffusion is ambipolar - yielded:

$$q_L = \sqrt{(1+\frac{T_i}{T_e})\quad\frac{LD_{e\perp}}{\gamma}} \tag{18}$$

where $\gamma \simeq 2\, v_{it}$ x) and where according to Bohm for $D_{e\perp}$ not its collisional value (eq.1) must be taken, but $D_{e\perp}$ (drain) as given by eq (9). This equation is however derived in a way which strictly speaking violates the particle conservation equation (11) as in the plasmas under consideration $\frac{\delta n}{\delta t} = 0$ and $\beta n = 0$. His derivation may be considered as for a case where $\frac{\delta n}{\delta t} \neq 0$.

It has to be stressed that equations (15) to (18) are derived under very simplifying assumptions and for certain conditions, which are not necessarely present in the experiments. But there ist still another circumstance which makes the usefullness of the determination of perpendicular diffusion coefficients, from radial density profiles only, very questionable. This is the fact that the finite gyroradii of the ions may not be neglected.

2.2.2 Density profile determined by ion gyroradii. It is clear that, if $\lambda_i \gg r_{ci}$, an ion may emerge from the central plasma column up to a distance equal to its gyro radius. The radial distribution of the emerging ions is smeared out because of the temperature distribution and the spatial distribution of the guiding centers in the plasma column. Depending on the latter one finds somewhat different density profiles, but in all cases an exponential decrease with an e-folding length q of some ion gyroradii. For a Gauss distribution of the guiding centers and a Maxwellian velocity distribution, Pfirsch and Biermann found [24] :

$$n_i(r) = \text{Const.}\; e^{-\frac{\alpha}{1+\alpha}\left(\frac{r}{r_{ci}}\right)^2} \quad \text{where } \alpha > 0 \tag{19}$$

x)" γ is the mean velocity, parallel to the magnetic field, of those electrons which have sufficient velocity, in the same sense to penetrate through the anode sheath to the anode ([15] page 200)."

If the guiding centers are distributed as a step function in the central plasma column, the radial density distribution outside the beam is given by errorfunctions. An exponential distribution in the plasma beam gives also an exponential distribution in the density of the surrounding plasma.

The easiest way for the electrons to compensate the space charge which is build up by the emerging ions is provided by the "short circuiting effect" ^x).

For the case of free streaming (λ_i, $\lambda_e \gg L$) the diffusion theory predicts e-folding lengths, which are always smaller than r_{ci} (17). Thus in this case Simon's theory can never by checked on its validity within a distance of some ion gyroradii from the central plasma column- and Bohm's formula (18) no more..... At larger distances the plasma density should fall much steeper than near the plasma column. However anticipating on the discussion of the experimental results it may be remarked that such a density profile was never found.

If $L \gg \lambda_i$, λ_e, the possibility to check Simon's theory is somewhat larger. The e-folding length q is given in this case by equation (15) or (16), depending on the end plates. Eq(15) predicts a linear relationship between q/r_{ci} and L/λ_i as depicted in Fig.1. The ion gyroradius effect gives a lower limit to q/r_{ci} of about .4. Thus one must expect a deviation from eq(15) in the region where $\frac{L}{\lambda_i} \lesssim 20$. Another condition for the validity of eq(15) is $\lambda_i \gg r_{ci}$.

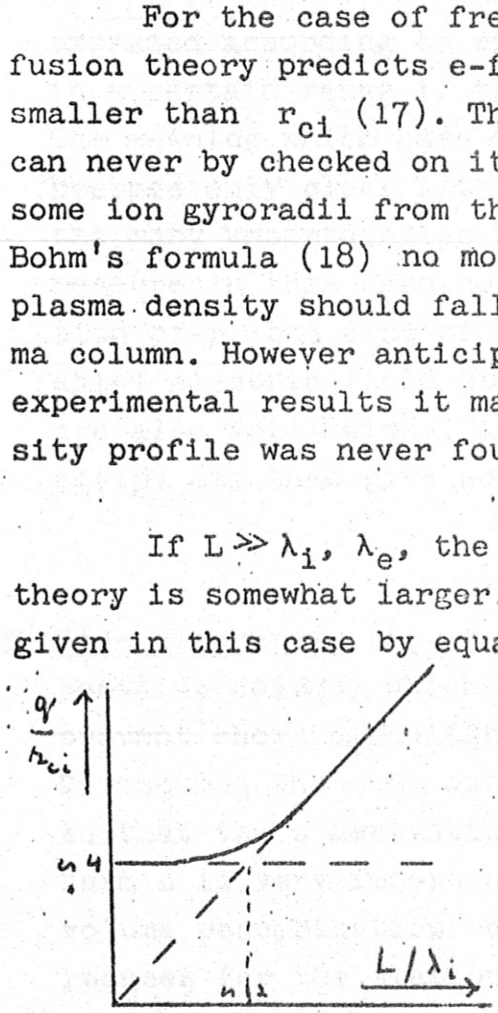


Fig. 1

^x) As mentioned before, this mechnism is not investigated very well. But what is good for the goose is good for the gander.

Concluding it seems questionable if it is possible to determine in low pressure, weakly ionized gases, the perpendicular diffusion rate of plasma particles across a magnetic field from measurements of transverse plasma density profiles only. The fact that one finds $q \sim \frac{1}{B}$ is by no means a guarantee that collisonal diffusion is operative. Firstly because from the ion gyro-radius effect the same dependence is expected, and secondly because the first part of a square root function (which would be expected according to eq(18)) may also appear to be linear in a certain range if the constants are chosen properly. The meaning which have to be given to such a relationship becomes only clear after quantitive consideration. With the many uncertainties which exist, it seems necessary to measure in this case not only the plasma density n as function of r, but also of z and additionally the radial and axial electric field strengths. If the boundary condtions are also well defined, and the plasma column is long enough, eq(13) may then give some information about $D_{\perp}$.

2.3 <u>Glow discharges</u> ($\lambda_i \ll r_{ci}$). The experiments are made in the positive column, which must be made very long in order to prevent short circuiting of the electrons (eq 14). Then it may be assumed that the axial flux of particles is neglegible, so that the z derivaties in eq(13) are zero. The production term β is very important in glow discharges, whereas the volume recombination αn^2 may be neglected. Eq(13) then reduces for the stationary state simply to [25]:

$$\beta\, n = - D_{a\perp} \nabla^2 n \tag{20}$$

with $D_{a\perp}$ as defined in eq(1). The transverse diffusion is ambipolar, and a radial electric field should be present. The solution of (20) which is regular at the axis (r = 0) is:

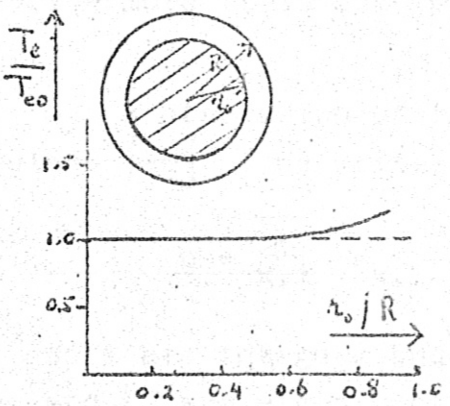
$$n(r) = n(o) \quad J_o \left[(\beta/D_a)^{1/2} r \right] \tag{21}$$

The boundary condition $n(R) = 0$ gives:

$$D_{a\perp} = \left(\frac{R}{\alpha_0}\right)^2 \beta, \quad \alpha = 2{,}405 \text{ --} \tag{22}$$

β is a function of the electron temperature T_e and may be determined from measurements of the axial electric field E_z, because E_z is also connected with T_e. So it is possible, in principle, to determine $D_{a\perp}$ from E_z - measurements, however in a rather indirect and insensitive way.

Furthermore Ecker [20] has pointed out that it is not possible to draw reliable conclusions about the perpendicular diffusion from E_z measurements. As an extreme case he considered the situation that $D_\perp \to \infty$ up to a distance r_o from the axis, but $D_\perp$ attaining its normal value in the outer shell between r_o and R. T_e was calculated as function of r_o/R; the result is shown in Fig.2.



r_o limits the region where $D \to \infty$; R is the radius of the discharge tube. T_e/T_{eo} is the relative electron temperature, T_{eo} being its value for $r = 0$.

Fig. 2

In case the normal relation $E_z(T_e)$ is used, as calculated by Lehnert, $E_z/E_{zo} \approx T_e/T_{eo}$ and it would be impossible to obtain information about the diffusion coefficient over the whole cross section from E_z measurements. $E_z(T_e)$ is however strongly effected by the enhanced interaction which is responsible for the enhanced diffusion, so that the situation may be somewhat more favourable than would be concluded from Fig.2. The best way of determining the interaction parameter σ (see eq.(10)) is by measuring the radial potential distribution.

It may be concluded that measurement ot the longitudinal electric field only in glow discharges cannot give unambignous information on the perpendicular diffusion rate. Moreover the discharge becomes unstable at higher B values, so that the plasma particle loss is no longer diffusion determined.

2.4 <u>Decaying plasmas.</u> $\frac{\delta n}{\delta t} \neq 0$ and the production term ß n may be put zero. If the plasma density is low enough $\alpha\, n^2$ may be neglected. If there is no short circuiting and assuming ambipolar diffusion in both directions the diffusion equation becomes:

$$\frac{\delta n}{\delta t} = D_{ao} \frac{\delta^2 n}{\delta z^2} + D_{a\perp} \frac{1}{r} \frac{\delta}{\delta r} (r \frac{\delta n}{\delta r}) \qquad (23)$$

with D_{ao} and $D_{a\perp}$ given by eq (8) and eq (2) respectively.

It has to be remembered that this equation is only valid for a weakly ionized gas, with $L \gg \lambda_i$, λ_e and $R \gg r_{ci}, r_{ce}$. Furthermore it is assumed that $\frac{dvi}{dt} \ll \frac{vi}{\nu i,n}$ and $\frac{dve}{dt} \ll \frac{ve}{\nu e,n}$.

A discussion of the solution of eq (23) is given by Golant [26] . He writes:

$$n = \sum_{k,l} A_{k,l}\, e^{-\left(\frac{D_{a\perp}}{\Lambda^2_k} + \frac{D_{ao}\,\pi^2 l}{L^2}\right)t} n_k(r) \sin \frac{\pi k}{L} z \qquad (24)$$

k and l are integers and A_{kl} constant coefficients. n_k and Λ_k are eigenfunctions and eigenvalues of the equation:

$$\frac{1}{r} \frac{\delta}{\delta r} r \frac{\delta nk}{\delta r} + \frac{nk}{\Lambda^2_k} = 0 \qquad (25)$$

If at t = 0 the plasma density is diffusiondetermined, i.e. n(r) is given by (25) and $n(z) = n(o) \sin \frac{\pi z}{L}$, only the first term of eq (24) remains. The spatial distribution of the plasma is not time dependent in that case, and the plasma decays exponentially with a decay time given by:

$$\frac{1}{\tau} = \frac{D_{a\perp}}{\Lambda^2_o} + \frac{D_{ao}\pi^2}{L^2} \qquad (26)$$

If at t = 0 the plasma is distributed arbitrarily, the fundamental mode appears in a later stage of the plasma decay, as the higher modes of eq(24) are reduced faster with time than the fundamental mode.

The diffusion determined plasma density distribution is given by the solution of eq(25) for k = 0:

$$n(r)_{k=o} = n(o)_{k=o} \; J_o\left(\frac{r}{\Lambda_o}\right) \; ; \; \Lambda_o = \frac{R}{2{,}405} \qquad (27)$$

A decaying plasma is expected to be more quiescent than a stationary plasma, because the forces by which new particles are generated may introduce instabilities and oscillations. Also a sinusoidal axial density distribution, which is expected some time after the onset of the plasma decay, is a much more realistic assumption in this case than for a stationary low pressure gas discharge. It seems therefore that decay time measurements on extinguishing plasmas provide a good way to obtain information on the perpendicular diffusion coefficient.

Finally it may be mentioned that as a variant on making measurements with a decaying plasma (with exponential time dependence of the density) one may also consider to make measurements with a modulated plasma source.ᴷ⁾

ᴷ⁾ Such measurements are made in our institute by Mr. Geißler.

3. Low pressure, low temperature discharges.

To this group belongs a great diversity of discharges which have in common a low neutral gas pressure ($P \lesssim 10^{-3}$Torr), so that λ_i is comparable with L and/or R or even larger; the B range is such that $\lambda_i/r_{ci} \gg 1$. The thermal Cs plasmas are also taken in this group, though the high degree of ionization of these plasmas causes very low values of λ_i, and λ_i/r_{ci} is in the range 0.1 - 50.

The axial electrical field E_z is generally very small and reliable E_z measurements are very difficult. For this reason E_z is not measured in this class of experiments, though the knowledge of E_z fields present in a plasma may be of importance for the correct interpretation of the measurements.

The transverse diffusion coefficient is estimated from radial density distribution measurements (section 2.2.1) and from the ratio $(j_{e\perp}/j_{i\perp})$ of electron to ion currents to a plane probe pointing perpendicular to B; both methods were first applied by Bohm et al. (section 3.1). Rough estimates may be obtained also from the ratio $(j_e/j_i)A$ of the electron to ion currents which flow to the anode, (3.3), from the plasma density as function of the discharge current n(I), (3.6), and from the radial current $I_\perp$ to a ring electrode around the plasma as function of $E_\perp$(3.8). A review of the diffusion measurements on low pressure, low temperature devices is given in table II.

It has to be remembered that in many experiments the plasma density n_e is measured with Langmuir probes and that the applicability of these in magnetic field is a problem which is not solved up to now. Generally however, reasonable results are obtained, so that they are used for the better. The same is true for T_e, whereas T_i is not measured at all, but may only be estimated. For that reason the listed values of T_i should not be taken too serious, with exception of the values for the thermal Cs plasma. Characteristic lengths are L, R, λ_i, r_{ci} and r_{pl} from which only the first two are known very well. For r_{ci} is given its maximum value corresponding to the lowest values of B used in the experiment. As r_{ci} varies with $T_i^{1/2}$, the lack of knowledge of T_i is not too disturbing.

If the radial plasma density distribution function is diffusion determined, the perpendicular diffusioncoefficient, $D_\perp$, is connected to the e - folding length q by the relations (see section 2.2.1):

$$D_\perp = D_{i\perp} = D_{io} \frac{\pi^2}{L^2} q^2 \quad \text{if} \quad L \gg \lambda_i, \text{ with short circ.} \tag{15a}$$

$$D_\perp = D_{i\perp} = \frac{2 v_{it}}{L} q^2 \quad \text{if} \quad L \ll \lambda_i, \quad \text{"} \quad \text{"} \quad \text{"} \tag{17a}$$

$$D_\perp = D_{io} \frac{\mu_{eo}}{\mu_{io}} \frac{\pi^2}{L^2} q^2 \quad \text{if} \quad L \gg \lambda_i, \text{ without short circ.} \tag{16a}$$

In all cases where $n_e(r)$ is measured, in a certain range of r near the central plasma beam, an e-folding length q is found which is inversely proportional to B.*) This is what one expects if collisional diffusion is operating, but also for the ion gyroradius effect. For this reason the experimentally determined factor $(q/r_{ci})_{exp.}$ is listed together with the ratio $(q/r_{ci})_{Simon}$ which one would expect from eq(15) or eq(17). In the greater part of the experiments the ratio $(q/r_{ci})_{exp.}$ is found to be between 1 and 4, as is expected if the density distribution is determined by ions emerging from the central beam. The ratio (q/r_{ci}) Simon is generally much smaller, so that a density distribution determined by

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*) This presumes that the logarithmicly drawn n(r) curve shows a straight part near the central plasma column. Generally such an evaluation may be made without too much fantasy is required.

collisional diffusion cannot be observed (see section 2.2.2). Only in the experiments with Cabinet (3.4) in the 10^{-3}Torr pressure range both $(q/r_{ci})_{exp.}$ and $(q/r_{ci})_{Simon}$ are larger because of a relatively long plasma column, so that in this case perhaps a collisional diffusion determined density distribution is measured. In the thermal Cs plasmas collisional diffusion density profiles are measured with a high probability in the range $\lambda_i/r_{ci} \ll 1$, where the ion gyro radius effect cannot occur.

Finally is also listed the ratio of the experimental q value to the q value which may be expected from Bohm's drain diffusion theory as given by eq(18). This ratio changes with B, because $q_{exp.} \sim \frac{1}{B}$ and $q_{drain} \sim \frac{1}{\sqrt{B}}$. For that reason it is given for the maximum and minimum B value used in the experiment. It is clear that $q_{exp.}/q_{drain}$ should be $\ll 1$ in order to be sure that the gyroradius effect does not obscure a distribution which may be expected from drain diffusion. In many experiments this is not the case, so that it is even questionable if drain diffusion is operative or not.

3.1 <u>Bohm et al. [15] .</u> An arc was run inside a graphite box, which was placed between the poles of a strong magnet (figure 3). The arc chamber was rectangular, not cylindrical like in the other experiments. Electrons emitted from a filement entered the box through a small collimating slot. The filement was usually run about 150 V negative relative to the box and currents of a few amps were flowing. Most experiments were made with Ar, at a pressure of about 10^{-3} Torr. The plasma density profiles were measured with negatively biased Langmuir probes.

The theoretical formula derived for q is:

$q \simeq \sqrt{\frac{LD_{e\perp}}{\gamma}}$, where γ is a quantity which is about equal to v_{it} (see eq(18)). According to this formula $D_{e\perp}$ was found to be two orders of magnitude larger than the theoretical value $D_{e\perp}(\nu)$, calculated with the formula $D_{e\perp}(\nu) = \frac{D_{eo}}{\omega_{ce}^2/\nu_{e,n}^2}$. From these measurements, and from $j_{e\perp}/j_{i\perp}$ measurements to be mentioned later, it was concluded that the diffusion was anomalous, and for this so-called "drain diffusion" an alternative formula for $D_{e\perp}$ was given which bothers the minds up to now: $D_{e\perp}(\text{drain}) = \frac{10^8}{16B} kT_e$ (see eq(9)).

But no measurements were made to check the B^{-1} dependence of the diffusion coefficient $D_{e\perp}$(drain); all measurements were made at B = 3700 gauss, and so there was not given a convincing proof of the correctness of this formula. Moreover as mentioned in section 2.2.2 Simon has pointed out that Bohm's formula corresponds to ambipolar diffusion which cannot occur because of the "short circuiting" effect. Instead of the electrons, the ions determine the diffusion rate, and Simon arrived at the formula: $q \simeq \sqrt{\frac{LD_{i\perp}}{2\ v_{it}}}$ (see eq(17)). Thus Bohm derived $D_{i\perp}$ from q and took this value abusively for $D_{e\perp}$ which he thought to be unexpectedly high. Unfortunately, the question has not come to an end with this remark, because the gyroradius effect overshadowsboth explanations, as may be seen in table I.

In other experiments the ratio of electron to ion current to a plane probe, oriented perpendicular to B, $j_{e\perp}/j_{i\perp}$, was measured and found to be 6 times larger than expected from theory:

$$j_{e\perp}/j_{i\perp} \simeq \frac{v_e}{v_i} \frac{r_{ci}}{a} \tag{28}$$

This formula is based on the idea that the ions reach practically unhindered the probe (with radius a), whereas the electron motion is essentially determined by diffusion (with steps r_{ce}) which would give a factor $(\frac{r_{ce}}{a})^2$. However, because of the ease with which the electrons move along the magnetic field, they are collected from a long distance in this direction, whereas they are hindered greatly in their transverse motion. The net result of this combination is to cause current collection to be a sort of geometric mean between its normal value and the value it would have, if it would depend on transverse diffusion alone; and so the factor would become $\frac{r_{ce}}{a}$ [15]. The theoretical derivation is not very accurate and the experiments are not so either (because of the non-saturation of the electron current). Thus the factor 6 should not be taken too serious.

3.2 Simon and Neidigh. [27], [28], [23] . The experiments were set up in order to reconsider the results obtained by Bohm et al. in the light of Simon's short circuiting theory. A plasma was formed in an experimental arrangement which was made similar to the one described above (fig 3). The arc chamber was however cylindrical and made out of copper. Most experiments were made with nitrogen at pressures of about 10^{-3} Torr.

Using Simon's short circuiting theory, one expects an e-folding length $q_c \simeq \frac{1}{\pi}\frac{L}{\lambda_i} r_{ci}$ if $L \gg \lambda_i$ (eq.15) and $q_L \simeq 0.4\sqrt{\frac{L}{\lambda_i}}\, r_{ci}$ if $L \ll \lambda_i$ (eq 17). In the experiments $L \simeq 5\,\lambda_i$, so that the expected e-folding length $q \simeq 1.5\, r_{ci}$. It is found to be about $4\, r_{ci}$, however, over a very wide range of B.*) This seems to indicate that the ion gyroradius effect overshadows possible diffusion effects. Also the fact that q is proportional to r_{ci} over the whole B range does not exclude Bohm's drain diffusion, because the ratio $\frac{q_{exp.}}{q_{drain}}$ lies between 2.3 and 1.

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*) The curves shown by Simon and Neidigh give the impression that an exponential decrease of the plasma density is not found over the whole radius of the discharge chamber.

It seems contradictionary with the statements made in this work of Simon and Neidigh, but in agreement with the comments made above, if Simon states at another place [9]: "It is extremely unlikely that the ion density n, in any practical problem, can fall off faster than with an e-folding length equal to the Larmor radius r_{ci}. In fact the actual e-folding length should usually be considerably larger than this."

3.3. Zharinov. Fig 4 shows a diagram of the discharge apparatus [29]. The arc chamber is a thick-walled copper cylinder, whose ends are covered by a molybdenum anode and a diaphragm. The cathode is a tungsten cylinder opposite an aperture of diameter 3 mm in the diaphragm. In the anode six apertures are made at 1 cm from its center. Six collectors are placed, one opposite each aperture, at about o,5 mm from the rear face of the anode; these are used to make probe characteristics. This apparatus was placed inside a metal vacuum-chamber which was situated in the gap of an electromagnet, so the experimental method was similar to those described above. The gas used was nitrogen at pressures of about 10^{-3} Torr.

Zharinov has compared the measured ratio of electron to ion current which reaches the anode at about r = 1 cm from the axis with a roughly estimated theoretical value [30]. In this investigation the ions could reach the wall practically unhindered by the magnetic field, so that $D_{e\perp}$ is measured. Zharinov gives for the ratio the formula:

$$\left(\frac{j_{e\perp}}{j_{i\perp}}\right)_A \simeq \frac{L D_{e\perp}}{r^2 v_{it}} \qquad (29)$$

which amounts to:

$$\left(\frac{j_{e\perp}}{j_{i\perp}}\right)_A \simeq \frac{1}{3}\frac{v_{et}}{v_{it}}\frac{L}{\lambda_e}\left(\frac{r_{ce}}{r}\right)^2 \qquad (29a)$$

From this formula $(\frac{j_{e\perp}}{j_{i\perp}})_A$ is expected to be about $5 \cdot 10^{-4}$, whereas $6 \cdot 10^{-2}$ is found experimentally. The discrepancy would indicate that $D_{e\perp}$ is greater than may be accounted for by collisions.

In the formula is not taken into account, however, the geometrical averaging as used by Bohm to derive $j_{e\perp}/j_{i\perp}$ for probe measurements (see eq(28)). Doing so yields:

$$(\frac{j_{e\perp}}{j_{i\perp}})_A \simeq \frac{v_{et}}{v_{it}} \frac{L}{\lambda_e} \frac{r_{ce}}{r} \tag{30}$$

and a theoretical value which is in good agreement with the measured one.

Zharinov also measured the ratio of electron to ion current to a plane probe oriented perpendicular to B [30]. Up to a value of B of 3000 gauss the ratio $j_{e\perp}/j_{i\perp}$ showed a B^{-1} dependence which may be expected according to eq(28) if collisional diffusion occurs; even quantitative agreement is found. The reason for the steep increase (about a factor 6) at B = 3000 gauss is not understood. In later experiments [31] Zharinov investigated the occurence of the critical magnetic field B_c as a function of gas pressure for different gases (H_2, N_2 and Ar). B_c was found to depend on the type of gas and to increase almost linearly with pressure in the interval of 10^{-3} - 10^{-2}Torr*). It was observed that near B_c a transverse plasma jet was formed in the discharge, rotating on the ion side around the primary electron beam. The rotational velocity of the jet is $1\text{-}3 \cdot 10^5$ cm $\sec^{-1}$ and decreases with increasing B. Low frequency oscillations in the same frequency range were also reported by Bohm et al. [15], so that perhaps the same effects are operative as in Bohm's experiments. The rotating plasma torch was also studied with a "plasmascope", a device which was developed for graphic study of the motion of plasma across

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*) This is in agreement with the theory of Yoshikawa and Rose, where density fluctuations are assumed to be responsible for anomalous diffusion (see section 1.4).

a magnetic field [32].

An explanation for these observations may perhaps be given in terms of instabilities as described by Kadomtsev and Nedospasov (see section 4.1). If an instability occurs, like in the positive column, it is not quite clear how this would effect the ratio $j_{e\perp}/j_{i\perp}$.

3.4. Boeschoten and Schwirzke [33]. The experiments were made with Cabinet, an arrangement in which is continuously drifting plasma from a duoplasmatron into a vacuumchamber which is placed inside a solenoid, which produces a magnetic field up to 4ooo gauss parallel to the cylinderaxis (fig 5). The central plasma column is thus not a current carrying arc, like was the case in the experiments mentioned before. Radial plasma density measurements were made in two pressure ranges:

10^{-5}Torr ($L \ll \lambda_i$). The plasmaparticles drift unhindered from the duoplasmatron into the vacuumchamber ($\lambda_e, \lambda_i \simeq$ 1o m). The plasma density in the center is about 10^{10}cm-3. kT_e is found from Langmuir probe measurements to be about 12 eV and rather constant over the radius; kT_i is estimated to be about 10 eV. An exponential decrease of the plasma density with an e-folding length $q \simeq 4\ r_{ci}$ was found between 1 and 3 - 4 cm from theaxis. It has to be kept in mind, of course, that r_{ci} depends on $\sqrt{T_i}$ and that T_i may only be estimated very roughly. A factor 4 is however much more than may be expected from eq(17) and one may assume with a high degree of probability that the radial density distribution is determined by the ion gyroradius effect (section 2.2.2). The plasma is on a positive potential relative to the end plates, so that only the fast electrons which are present can provide for the short circuiting effect.

At 3 - 4 cm from the axis (depending on B) a "kink" occurs in the curves and from there on the density decreases much slower with radius. It is not clear what mechanism determines the density profile in this part of the recipient.

10^{-3} Torr $(L \gg \lambda_i)$. The particles which emerge from the duoplasmatron collide with the neutral gas particles $(\lambda_e, \lambda_i \simeq 5$ cm) and form a secondary plasma. The plasma density in the center increases to about 10^{13} cm^{-3}. Langmuir probe measurements indicate $kT_e \simeq 1$ eV and spectroscopic measurements indicate $kT_i \lesssim$ o.4 eV. Because of the many collisions with neutral particles which the plasma particles suffer in order to move from the axis, such a low temperature seems to be quite reasonable. Probably one has to calculate in this case principally with molecular ions of temperature $kT_i \simeq$ o.1 eV. The same kind of density distribution curves were obtained in this pressure range as in the 10^{-5} Torr range. For hydrogen both $(\frac{q}{r_{ci}})_{exp.}$ and $(\frac{q}{r_{ci}})_{Simon}$ are clearly larger than 4, so that perhaps a collisional diffusion determined density distribution is measured in this case. It has to be noted, however, that $(\frac{q}{r_{ci}})_{exp.}$ may be 2 times smaller (if kT_i = o.4 eV). $(\frac{q}{r_{ci}})_{Simon}$, however, is much larger than in other experiments, because L is relatively large (eq(15)). As $L/\lambda_i \simeq 20$ the measurements are made under rather favourable conditions in this respect (fig 1).

3.5 D'Angelo and Rynn. The measurements were made on thermal Cs and K plasmas [34], [35]. A schematic is shown in figure 6. Two hot tungsten plates are placed at both ends of a cylindrical vacuum-chamber and the output of a cesium or potassium atomic beam oven impinges on the center of one of them. The metallic vapor is singly ionized by the hot tungsten plate, which is hot enough to emit an electron flux of about the same magnitude as the incoming metal flux. The entire device is placed inside a solenoid which produces an uniform magnetic field, parallel to the axis. The walls of the vacuum-chamber are cooled to approximately -10°C to condense neutral cesium (or potassium). Since the neutral background can be made quite small, the apparatus is capable of producing a plasma with a percentage of ionization ranging from about 40 % at a density of 10^{10} cm^{-3} to better than 99 % at a density of 10^{12} cm^{-3}.

In contrast to the other experiments, in this case Coulomb collisions dominate. λ_i is very small and λ_i/r_{ci} is in the range between 0.1 and 50. We have here the case of a fully ionized gas (section 1.2), which is expected to diffuse through the magnetic field with a velocity given by eq(5). The diffusion coefficient is in this case proportional to the plasma density. The plasma particles are assumed to be lost by volume recombination. For $T_i = T_e$ and constant over the radius, the particle conservation equation yields:

$$\frac{1}{r}\frac{\delta}{\delta r}\left(r\, D_\perp \frac{\delta n}{\delta r}\right) = \alpha\, n^2 \qquad (31)$$

$$\text{with} \quad D_\perp = 2\,\frac{\gamma\, c^2 kT}{B^2}\, n \quad \text{(see eq(5))}.$$

In this case not n, but n^2 is expected to be proportional to $K_o(r/q')$, or $e^{-r/q'}$ for large enough values of r/q'. The e-folding length of the radial plasma density, $q = 2\,q'$, is given by:

$$q = \sqrt{\frac{2D_\perp}{n\alpha}} \simeq \sqrt{\frac{\nu_{e,i}}{n\alpha}}\; r_{ce} \qquad (32)$$

The experiments show a B^{-1} dependence of q, as expected for the collisional diffusion theory. A B^{-1} dependence of q was found both in the case $\lambda_i/r_{ci} < 1$ as in the case $\lambda_i/r_{ci} > 1$. In the first case the ion gyro radius effect cannot occur, so that it seems that here is clearly shown a case of a collisional determined density profile. In case $\lambda_i/r_{ci} > 1$, the fact that the ratio q/r_{ci} is found to be 2 - 4 , makes it impossible to exclude the ion gyro radius effect.

In order to be sure that no Bohm's drain diffusion is present, it is very important to obtain also quantitative agreement. Therefore the recombination coefficient α must be known. A good estimate was made as follows: density measurements were taken with both plates hot (double ended) and with the reflector plate cool (single ended). α may than be estimated within a factor 2 or 3 from the ratio of the measured plasma densities [35]. Within the limits of the uncertainties, the experimentally determined value of the e-folding length q was in agreement with the value calculated from eq. (32)[xx)].

Though it seems possible to obtain from radial plasma density measurements in Cs plasmas more reliable information about the perpendicular diffusion coefficient than from similar measurements in the other experiments described in this chapter, there is still one uncertainty.This is caused by the possible role which ion-ion collisions may play. As mentioned in section 1.2 the ratio of the flux from ion-ion collisions to the flux from ion-electron collisions is of the order of $(\frac{m_i}{m_e})^{1/2}$ $(\frac{r_{ci}}{q})^2$. This ratio is clearly larger than one in this case, because of the high atomic mass of Cs and K. If there is no short circuiting effect, however, a radial electric field E builds up to such a value as to effectively destroy the flux caused by ion-ion collisions.

In this case we must expect according to eq(7) an electric field $E_{\perp}$ of about o.2 - o.6 V/cm. The measurements indicate however $E_{\perp} \lesssim$ o.1 V/cm [36]. An argument against the presence of the short circuiting effect is on the other hand the fact that no difference in behaviour between Cs and K is found.

Thus also in these experiments the crucial question is whether short circuiting of the electrons is possible or not. But like in the other experiments described in this chapter, no effort is made to obtain direct evidence if the short circuiting effect is present or not. A strange circumstance is that in the low pressure discharge experiments with weakly ionized gases the presence of the short circuiting effect is welcome in order to obtain agreement between experiments and theory, but rather large $E_{\perp}$ fields are measured which should not occur in this case. In the thermal Cs and K plasmas on the other hand the short circuiting effect is not welcome, but the $E_{\perp}$ fields which should be present are not found experimentally.

The theory which Ecker [37], [38] has given for the Cs discharge in the plasma volume between the core and the wall is not checked on its validity.

3.6. Geller and Pigache [39]. A PIG discharge was operated at very low values of B (1oo gauss), so that the ions could reach the wall practically unhindered (like in the experiments of Zharinov). The discharge was pulsed, with a pulse duration of about 1oo μ sec. During this time the current reaches an equilibrium value in the range of 4o - 200 Amps. In these experiments the plasma density n_e was derived from microwave measurements and not from Langmuir probe data, as was the case in the other experiments described in this chapter.

The perpendicular diffusion coefficient of the electrons $D_{e\perp}$ may be derived from the curve which gives the plasma density as function of the discharge current I_{eq}. Analysis of the experimentally found $n(I_{eq})$ curves yields values of $D_{e\perp}$ which are in agreement with values expected from collisional diffusion of the electrons. The degree of ionization seems to be so high that Coulomb collisions dominate and $D_{e\perp} \sim n$.

There are indications that the diffusion coefficient shows another behaviour, if B is raised to values higher than some hundreds of gauss, like was found in the following experiments of Bonnal et al, described in section 3.7.

3.7 Bonnal, Brifford, Grégoire, Manus, [40] , [41] . The experiments were made with a small PIG discharge of 11 cm length; later on a discharge of about 1 m length was used (fig 7). The pressures are generally one order of magnitude higher than those in the other experiments mentioned in table II; the degree of ionization being very low. In that respect the experiments belong in between the experiments with glow discharges and the experiments with low pressure devices.

The ratio of the plasma density outside and inside the plasma beam n_{ext}/n_o was taken as a measure of the diffusion as in the experiments of Bickerton and von Engel (section 4.1). Like in the experiments with glow discharges a critical B field was found, from which point on the ratio n_{ext}/n_o increases instead of decreases with B.

Measurement of n_{ext}/n_o is however a very rough procedure, it does not give as much information as continuous radial density distribution measurements which are already insufficient to make reliable statements on the perpendicular diffusion coefficient. In the small tube r_{ci} is not small compared with R.

All of this makes it difficult to compare these experiments with the other of table II, where no critical B_c field is found. The authors have noticed that the product of critical magnetic flux density and plasma radius, $B_c \cdot r_{pl}$, is about a constant, so that the discrepancies with the other experiments may be also connected with r_{pl}.

Unlike as is the case in the experiments with glow discharges, E_z is expected to be very small in the region where the measurements are made, so that not the same instability mechanism could be operative. For this reason one tries to find an explanation of the occurence of B_c with micro instabilities [42]. It must be remarked, however, that - though the E_z field in the central part of the discharge is very small- this is certainly not true for the two regions between anode and cathode, and it may be possible that the measured anomalies are excited in these regions.

3.8 Yoshikawa and Rose. Experiments were carried out in a hollow cathode discharge [17] and [40]. Fig 8 shows the apparatus. Argon gas was admitted through the hollow cathode. The initial pressure in the vacuum chamber was several 10^{-3} Torr. The main discharge was run between the cathode C and the anode A. The plasma thus generated is highly ionized ($\gtrsim$ 30 %) so that Coulomb collisions are dominating. A third ring electrode T was biased positive with respect to the anode to draw electrons from the main discharge. The Langmuir probe L, is used to detect the noise and the floating potential of the plasma, and L_z is used to determine n_e and T_e. The system is enclosed in a Pyrex tube of 10 cm diameter.

The magnetic field lines are divergent outside of two coils that maintain a slightly mirror type magnetic field in the discharge section. Field lines are terminated at glass walls where no plasma is visible. The authors hope that the short circuiting effect is avoided in this way.

The magnetic field is relatively small (250 - 900 gauss), so that the ion cyclotron radius r_{ci} is comparable with the radius of the discharge chamber R. Thus the diffusion of electrons across the magnetic field is measured. The radial current $I_\perp$ to the third electrode T was measured as a function of the voltage $V_\perp$ on this electrode, with B as parameter. Theoretically one expects $I_\perp$ to be proportional to $E_\perp$; the ratio $dI_\perp/dE_\perp$ (which is $\backsim \mu_{e\perp}$) should be proportional to n/B if the diffusion is anomalous, as given by Bohm, and proportional to n^2/B^2 for collisional diffusion (see eq(4) and eq (9)). For small values of $V_\perp$, the experimental $I_\perp$ - $V_\perp$ curve is moderately straight. Evaluation of the curves found for different values of B, yields the points shown in figure 9. The experiments seem to favour anomalous diffusion. But one has to remember that the electrical field $E_\perp$ in the plasma cannot be simply derived from potential drops between electrodes, because of potential drops which occur in sheaths near the electrodes. The proportionality between $E_\perp$ and $I_\perp$ is not clearly shown in the experiments. Moreover the curve for anomalous diffusion is drawn to give the best fit with the experimental points, whereas the curve for collisional diffusion is not so exactly known, that a better fit with the experimental points can be excluded. Also more careful experiments are described, but these experiments again do not seem to support the presence of anomalous diffusion in a convincing way.

It may be remarked that in the experiments of Lidsky, Rothleder, Rose, Yoshikawa, Michelson and Mackin, with the same apparatus at B = 900 gauss, a density profile was found, which decreases exponentially with radius with an e-folding length of about one ion gyro radius [40].

3.9. Gardner, Barr, Kelly and Oleson. Finally the probe measurements made on P 4 remain to be mentioned [43]. No claim is made that the perpendicular diffusion was measured, but a radial density profile was found, which seems to decrease exponentially with radius with an e-folding length of about one ion gyroradius.

Table II demonstrates that up to now no conclusive measurements have been made on the diffusion rate of plasma particles across a magnetic field in low density gas discharges. In some experiments the limit set by the gyroradius effect is, however, low enough to make the presence of Bohm's drain diffusion rather improbable. The measurements on the thermal Cs and K plasmas show that collisional diffusion in these plasmas is very probable.

4. Glow discharges.

In the positive column of glow discharges λ_i is small compared to L and R, and $\omega_{ci}/\nu_{i,n}$ or $\lambda_i/r_{ci} \ll 1$, so that the magnetic field does not inhibit noticeable the motions of the ions, but acts only on the electrons.

4.1. Measurements of the longitudinal electrical field E_z may yield information about the ambipolar diffusion coefficient perpendicular to a longitudinal magnetic field, $D_{a\perp}$. The field is smaller if the diffusion rate of charged particles to the wall is less; thus, E_z is expected to decrease with increasing B (see section 2.3). In order to have for these kind of measurements clear conditions with a simple diffusion equation, it is necessary to prevent the short circuiting effect, thus to use a very long positive column (of some meters length). Furthermore the gas pressure must not be too high ($\omega_{ce}/\nu_{e,n}$ should be $\gg 1$) or too low (λ_e should be $\ll R$).

Experiments with glow discharges taking into consideration these requirements were started by Bickerton and von Engel [44]. No anomalies were found up to a (maximum) magnetic induction of 450 gauss. It does not seem to be certain that the tube was long enough to prevent short-circuiting effects.

Lehnert [25] started experiments from the same idea, but the magnetic induction B could be adjusted to much higher values, and the tube length was such that short circuiting effects were prevented *). He found a decrease of E_z with B (in agreement with "collisional diffusion") up to a critical value B_c. From this point on, E_z increased with increasing B, indicating that the plasma reached the wall much faster than may be expected from "collisional diffusion" (fig 10). The increased particle loss must be compensated by an enhanced production rate which requires an enhanced electron temperature. This reflects itself not only in an increase of E_z, but also in the power input and in the light intensity.

Numerous experiments were done by Lehnert and collaborators [45], [46], [47], and others, with different gases, gas pressures, tube radii and tube lengths (see table III). The experiments have a good reproduceability and in all

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*) Which condition must be fulfilled in order to prevent short circuiting effects does not seem to be quite clear (see section 2.1). It was observed experimentally by Hoh and Lehnert [46] that the critical point in Ar and Kr disappeared or at least shifted to much higher B values when L was made smaller than 50 to 100 tube radii.

cases a critical B field was found as long as the gas pressure was in the proper range.

An interesting feature of the experiments is that the product RB_c turns out to be roughly constant when the pressure, gas composition, and discharge current are fixed, which seems to indicate some analogy with the onset of turbulent flow in tubes [45]. And indeed, the onset of higher particle losses to the wall at a certain B_c is excellently explained by the turbulence theory of Kadomtsev and Nedospasov [48]. They showed - theoretically - that the plasma in the positive column becomes instable, if the magnetic field exceeds a certain critical value B_c. At higher values of B oscillations build up in the plasma which will eventually become irregular and chaotic, giving rise to a real turbulent state. In figure 11 the experimental results of Hoh and Lehnert are compared with this instability theory. It can be seen that there is not only qualitative agreement with the experimental points but, furthermore, good quantitative agreement up to a field B = 3 Kgauss.

The turbulence theory not only explains the earlier data obtained from glow discharges, but is also confirmed by additional experiments made by <u>Allen, Paulikas</u> and <u>Pyle</u> [49], [50], [51] and <u>von Gierke and Wöhler</u> [52]. The former obtained direct experimental evidence for the expected screw-like instability which could be photographed. The latter showed that a decrease of the E_z field (obtained by additional ionization from a H.F.-field) causes the critical B field to be shiftet to higher values as is expected from theory. This is also confirmed by measurements with the afterglow of positive columns, where E_z fields are certainly very small (see 5.1). The instability of Kadomtsev and Nedospasov was not found in this case.

A physical interpretation of the instability mechanism of Kadomstev and Nedospasov was given by Hoh and Lehnert [53] ; the theory was further extended by Hoh [54] and Johnson and Jerde [55] Kuckes found that the instability may also be present in a highly ionized gas [56] , and tried to explain the enhanced diffusion in stellerator experiments in this way (see section 5.3). Guest and Simon pointed out that the mechanism of Kadomstev and Nedospasov could also be present in low pressure gas discharges [57] . No experimental data are present, however, which make a careful checking of their theoretical results possible.

*)

At the time that the origin of the enhanced particle loss in the positive column was not known, the word "anomalous" or "abnormal diffusion" was used to indicate the observed effects. But this term was not used properly, because it seems to be sure, now, that the plasma becomes unstable at B_c. Speaking about diffusion one supposes, however, to deal with a macroscopic stable plasma. And for that reason it seems to be better to reserve the term "anomalous diffusion" for plasmas where enhanced diffusion is caused by microinstabilities in which case the plasma as a whole may be stable. More precisely: the plasma density gradient should be present over a distance $n/\nabla n$ which is large compared to the dimensions of the possible (micro)instabilities. Speaking of anomalous diffusion is the more undesirable, because in many cases it is connected with Bohm's "drain diffusion", and as was pointed out above, such a diffusion mechanism is not verified in the experiments of Bohm et al., and - as a matter of fact - is not verified in a convincing way in any experiment up to now.

The experiments with the positive column have also given some evidence for "collisional diffusion" below the critical value of the magnetic induction B_c. But the presence of collisional diffusion in the stable B region is not proven unequivocally either. As Ecker has pointed out [20], the longitudinal electrical field E_z is rather insensitive for the dimensions of the region in which enhanced diffusion eventually occurs (see fig 2). For excluding the presence of enhanced diffusion in a large part of the column the measurements are not exact enough (see fig 10). For this reason Ecker advises to make radial potential distribution measurements in order to find enhanced interaction effects.

In table III are summarized the several experiments of the described type with glow discharges together with the most important parameters used. Contrary to the experiments with low pressure gas discharges the findings are very much the same and satisfactory explained theoretically. Unfortunately they give us only little information on the diffusion rate of plasma across a magnetic field if one uses the word diffusion in the narrow sense.

4.2. Finally the measurements of Nedospasov remain to be mentioned [59]. These were started from the idea that the basic characteristics of the glow discharge are determined by ambipolar diffusion both in radial and axial direction. Of course the plasma must be short enough that $D_\parallel$ is of the same order of magnitude as $D_\perp$ (see section 2.1). For the ion current to the wall of the discharge tube was given the formula:

$$j_{iw} = C_1 \; e^{-C_2 \; z \sqrt{\frac{D_\perp}{D_\parallel}}} \qquad (33)$$

Where C_1 and C_2 are constants.

j_{iw} was measured as function of z in a glow discharge in dependence on B (up to about 1000 gauss). Argon gas was used at pressures of 0.25, 0.7 and 1.0 Torr. From these measurements the ratio $D_{\parallel}/D_{\perp}$ could be determined and was found to be in agreement with the expected value for collisional diffusion: $1 + \frac{\omega_{ce}\,\omega_{ci}}{\nu_{e,n}\nu_{i,n}}$ (see eq.8 and 10).

5. Decaying plasmas.

Diffusionmeasurements on decaying plasmas are made with such a different apparatusses as pulsed glow discharges, mirror machines and stellerators. The decay time τ of the plasma is measured as function of B and P_0. For weakly ionized gases the results may be compared with eq(26). Strongly ionized gases have a more complicated diffusion equation, which is not solved till now. In case that the diffusion in axial direction is neglegible (long tubes, torus geometries) the perpendicular diffusioncoefficient may be found from the decay time with the simple expression $D_{\perp} \approx R^2/6\tau$ under the condition λ_i or $r_{ci} \ll R$ and $r_{ce} \ll R$. The difficulty with these measurements is however, that one has to be sure that the diffusion time is smaller than other characteristic times connected with plasma loss processes.

5.1. Pulsed glow discharges and H.F.discharges. In table IV are brought together experiments with decaying plasmas, which were excited in rather long tubes by the passage of a pulsed current or with a H.F.puls. A division is made between the experiments with weakly ionized gases (sections 5.1.1 - 5.1.3) and with strongly ionized gases (section 5.1.4 and 5.1.5). In the experiments made with weakly ionized gases, the plasma density is measured by exciting the plasma in a microwave-cavity or a wave guide. For strongly ionized gases this is not possible and n_e must be measured from the phase shift of a microwave signal which is passed through the plasma.

In all experiments the condition $\lambda_i \ll R$ is fulfilled; λ_i/r_{ci} may be smaller or larger than one. Though the experiments were made with He, the influence of volume (reaction) processes on the decay time cannot be avoided at higher values of n_e. As may be seen from table IV, the experiments very strongly indicate the presence of collisional diffusion if care is taken that non-diffusional loss processes are avoided.

5.1.1 Bostick and Levine [60] measured decay times of a plasma in a microwave cavity with an annular magnetic field. This cavity had the shape of a torus (outer diameter 15 cm) with a rectangular cross section (5 x 3 cm). The electron density was measured by means of the frequency shift method in the afterglow following the breakdown of the gas in the cavity. The pressure of the He gas was in the range of 5.10^{-2} Torr to 1 Torr; B could be varied up to 1370 gauss.

The diffusion coefficient $D_\perp$ as obtained from the decay time, first seems to decrease with B, but at about 6 oo gauss $1/\tau$ increases rapidly. The dependence $1/\tau$ on B in the low field portion of the curves before the minimum was not evaluated exactly enough to decide whether $1/\tau$ varies proportional to 1/B or to $1/B^2$.

The existence of a ∇B force in the toroidal geometry -from which one expects $\tau \backsim 1/B$- may be responsible for the high field portion of the experimental curves. The minimum in the value found for $D_\perp$ would then come where the $\sim 1/B$ (or $\sim 1/B^2$) diffusion process and the formally derived $\backsim B$ process are equal. x

5.1.2 Golant and Shilinsky made measurements in a straight tube of 110 cm length and 0.8 cm radius which was placed in a circular wave guide, surrounded by solenoids [26]. Microwaves

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were propagated in axial direction; the wave guide with plasma was excited by microwaves with circular or linear polarisation. In both cases the electron-concentration may be determined from phase shifts of the waves propagated through the plasma. The measurements were made with He in the pressure range of 0.1 - 2 Torr, whereas B could be varied up to 1100 gauss.

The decay time τ was measured as function of B for several values of the gas pressure. The diffusion in axial direction may be neglected in these experiments (λ_i/L is very small), so that $D_\perp$ is simply given by $R^2/6\tau$. For B values above 200 gauss (where $r_{ci} \approx R$) the values found for $D_\perp$ in this way are in rather good agreement with the theoretical values as given by eq(2) or eq(8).*) For plasma densities above $5 \cdot 10^9$ cm^{-3}, $1/\tau$ was found to increase with increasing plasma density. This would indicate that the electron-ion collisions are not neglegible compared to the electron-neutral collisions in eq(8). Theoretically one would expect however this being the case for plasma densities which are one order of magnitude higher.

In later experiments Golant and Shilinsky [61] made measurements with the same apparatus in a larger range of gas pressure (2.10^{-2} - 0.8 Torr) and magnetic induction (up to 2400 gauss). In the low pressure range the decay time was found to be much smaller than expected if the plasma decay is determined by collisional diffusion, whereas in the high pressure range as before a rather good agreement was found. This result may perhaps be explained by the fact that in the low pressure range λ_i is comparable with R (which is rather small in these experiments), whereas the condition $R \gg r_{ci}$ is only fulfilled for higher B values.

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*) The value which is mentioned in this paper for the collision frequency: $\nu_{e,n} = 2{,}8 \cdot 10^8 . P_o$ [Torr], seems to be on the low side. Using the cross section which is given by Drawin [62] for thermal electrons with He atoms (10^{-15}cm^2), one finds six times higher collision frequencies for electrons of 1 eV energy. So the discrepancy of a factor 2,5 with the product $\nu_{en} \nu_{in}$ appearing in eq(8) seems to ly completely within the uncertainties about the collisionfrequencies.

Furthermore a very interesting experiment was made in order to measure the diffusion in axial direction. By making L small enough the diffusion in axial direction becomes more important (see eq(26)). L was roughly varied by changing the length of the magnetic field. The change in the measured decay time was found to be in agreement with eq(26). So here is given a proof that the plasma particle loss in axial direction is determined by ambipolar diffusion.

5.1.3 <u>Demirchanov, Alichanov, Komina, Podlesny and Chorassanov</u> [63]. made experiments similar to those of Bostick and Levine, but in a straight microwave cavity (L = 70 cm, R = 3.5 cm) instead of a toroidal one. Very pure He gas was used in the pressure range of 2.10^{-2}-2.10^{-1} Torr. Because of the relatively large tube radius, the measurements are made under favourable conditions in respect to the requirement λ_i or $r_{ci} \ll R$. It is possible to verify the measurements up to much smaller values of B than in the other experiments described in this section.

A very good agreement with eq(26) was found over a very wide B range (20-4000 gauss) at different gas pressures. Only at the highest pressure used (2.10^{-1} Torr) a deviation was found, which could be explained satisfactory by the ion-molecular reaction $He^+ + 2\,He \rightarrow He_2^+ + He$. Control measurements were made in order to make sure that the plasma particle loss was not due to the attachment of electrons to electro-negative gas contaminations; the purity of the gas turned out to be a very important factor.

In this experiment the presence of collisional diffusion is demonstrated in a convincing way. The authors suggest that the increased escape rate of plasma particles as reported in other experiments may be related to the firing pulse which excites plasma oscillations. Whenever the initial perturbation is rapidly damped, the decay obeys the diffusion law.

5.1.4 Anisimov, Vinogradov, Golant and Konstantinov. [64].
The purpose of this investigation was to study the diffusion of plasma particles across a magnetic field if Coulomb collisions are important. Therefore the decay of a plasma of density $5.10^{11}-10^{13}$ cm^{-3} was investigated at a neutral gas pressure of 2.10^{-1}- 2.10^{-2} Torr. B could be varied up to 3 ooo gauss. The spatial distribution and the time variation of the electron density was measured with microwaves [65].

The decay rate of the plasma is not only determined by the diffusion of the plasma particles, but also by volume recombination. The latter process may be distinguished from the diffusion by the way that τ is found to depend on the gas pressure P_o. It was found to be practically independent on P_o and B. Thus the speed of decay, connected with diffusion may be found seperatly. Above 1ooo gauss this is difficult however, as the diffusion is reduced so much that the volume loss process predominate.

A difficulty is in this case that the diffusion equation is no longer linear as the diffusion coefficient depends on n (section 1.3). Therefore the theory is not worked out as easy as in the case of a weakly ionized gas and interpretation of the experiments is more difficult. Approximate integration of the nonlinear diffusionequation under conditions when the diffusioncoefficient is proportional to the plasmadensity, leads to concentration distribution curves close to those obtained in the experiment. (Fig). Thus up to 1ooo gauss the experimentally determined diffusionrate was found to be in agreement with collisional diffusion theory (eq.8). If B is larger than 1ooo gauss the diffusion is masked by the volume recombination, but the diffusioncoefficient does not exceed its value at 1ooo gauss, so that no Lehnert effect is present.

5.1.5. Ichimaru, Iida, Sekiguchi and Yamada [66] started the same kind of experiments as described above in section 5.1.4. In order to be sure to eliminate the short circuiting effect (see eq. 14), the tube radius was chosen extraordinary small (0.25 cm). The question arises of one did not losse on one side what was gained at the other side, becaus λ_i is not small compared with R over the whole range of plasma density and the condition $R \gg r_{ci}$ is not fulfilled either for B values smaller than 2 ooo gauss.

A preliminary analysis of the experimental results which was based upon a somewhat crude theoretical background, yielded experimental values of $D_\perp$ which are inversely proportional to B^2 and which are of the order of magnitude which is expected for a fully ionized gas. This agreement is only obtained in the region where B is high (above a few thousand gauss) as may be expected if the remarks made above are applicable.

5.2. Mirror machines.

The reason of the plasma loss in mirror machines is not well understood up to now. The containment time is expected to be of the order of magnitude of the collision time of the ions $\tau_{i,i}$. This may be considered as a "diffusiontime" in axial direction. The diffusiontime for collisional diffusion perpendicular to the magnetic field is given by: (see eq. 4a)

$$\tau_{D.coll} \simeq r_{pl}^2/6\, D_\perp \simeq r_{pl}^2/2\, \nu_{e,i}\, r_{ce}^2 \qquad (34)$$

If drain diffusion were operative, the perpendicular diffusion time is expected to be $\tau_{Ddrain} \simeq r_{pl}^2/6D_\perp(\text{drain})$

$$\simeq r_{pl}^2/2\, \nu_{ce}\, r_{ce}^2 \quad \text{(see eq 9).}$$

In table V are listed some data which are obtained from mirror machines which are in operation in the U.S.A. and the U.S.S.R.[x)] The containment times τ_{exp} are compared with the collision times of the ions $\tau_{i,i}$, the diffusion times calculated assuming "collisional diffusion" $\tau_{D\ coll}$, and calculated assuming "drain diffusion" $\tau_{D\ drain}$. The collision time of the ions $\tau_{i,i}$ is calculated with the iontemperatur T_i listed in the table. Its value will be smaller for the cases where $T_i \gg T_e$, because of cooling by the electrons ($\tau_{i,e} < \tau_{i,i}$) so that the actual values for τ_i will be lower and more realistic than those listed. It is evident that in all cases $\tau_{D\ coll} \gg \tau_{i,i}$, so that it is principally impossible to decide from these experiments whether the perpendicular diffusion $D_\perp$ is collisional or not.

On the other hand $\tau_{D\ drain} \ll \tau_{i,i}$, and drain diffusion can be excluded in cases where the theoretical containement time is obtained.

Formula (34) states nothing else as a well known prediction of the probability theory, that the mean square distance which a particle moves form the origin (r_{pl}^2) is equal to the length of one step squared (r_{ce}^2) times the number of steps ($\tau \nu_{e,i}$). If a short circuiting effect were possible one would expect the step length to be r_{ci} and:

$$\tau'_{D\ coll} \simeq \frac{r_{pl}^2}{r_{ci}^2 2\nu_{i,i}} \simeq \left(\frac{m_e T_i}{m_i T_e}\right)^{1/2} \tau_{D\ coll} \qquad (34a)$$

In case that eq.(34a) is valid the decay time expected from collisional diffusion theory would be reduced considerably and be closer to $\tau_{i,i}$.

Only in table top $\tau_{exp} \simeq \tau_{i,i}$, so that it can be concluded that drain diffusion is not present in this machine [67] . A similar conlusion cannot be drawn for the other experiments with mirror machines. It seems that

-55-

x) Most of these data were given by the authors themselves at the Salzburg Conference 1961 at invitation of Artsimovich.

instabilities which are not completely understood cause much lower containment times than is hoped for. In the mirror machines of the U.S.S.R. [68], [70], [71], [72] the low plasma densities make the collision time of the ions, τ_i, very high and the collisions with neutral particles of the rest gas will be relatively more important. Also "the collisional diffusion" time $\tau_{D\ coll}$ yields unrealistic numbers (which tendency is still accentuated by the large plasma radii r_{pl}). Because of the low electron temperature T_e, even the "drain diffusion" time $\tau_{D\ drain}$ is larger than the observed containment times τ_{exp}, to that in these machines drain diffusion cannot be excluded.

In all experiments but the one of Joffe and Yushmanov, the ion gyroradius r_{ci} is not very small compared to the plasma radius r_{pl}, a circumstance which will certainly have influence on "collision diffusion" determined phenomena.

5.3. Stellerator .

The plasma in the stellerators is lost much faster than may be expected from "collisional diffusion" theory or from a theory which takes into account the influence of the electrical conductivity on the equilibrium of the plasma [73]*). At this moment it is not quite clear what mechanism is responsible for these losses. During the ohmic heating of the plasma, the plasma density reaches a maximum and then decreases rapidly during a time when the ohmic heating field is still applied (B range 5-40 Kgauss, 10^{-3} to 10^{-4} Torr pressure range).

In the experiments with hydrogen discharges in the B-3 Stellerator, Stodiek, Ellis and Gorman showed

*) This effect (which is a non diffusional one) gives rise to a loss rate which is $8\,\pi^2/\iota^2$ (ι = rotational transform of B) times larger than what is expected from collisional diffusion. By choosing ι high enough its influence may be reduced.

that the enhanced loss rate prevails during the initial increase of the electron density, independent of the degree of ionization, and that the loss rate during the rise varies roughly as B^{-1} [74] . The loss rate during the decay of the electron density was also found to be more nearly proportional to B^{-1} rather than to $B^{-1/2}$, as stated in earlier experiments [76] .

Assuming a diffusion coefficient constant over the radius of the plasma column, the observed loss rate leads to a value of $D_{e\perp} \simeq 2.10^7 \frac{kTe}{B}$ in the range of $kTe = 1$ eV - 20 eV and $B = 5$ - 30 Kgauss. The data also indicate that the loss rate is inversely proportional to r_{pl}^2. This empirical formula found for $D_\perp$ resembles Bohm's formula $D_{\perp e} \simeq 6.10^6 \frac{kTe}{B}$ cm^2/sec (see eq. 9).

<u>Gorman</u> and <u>Rietjens</u> [77] measured the density profile with Langmuir probes and showed that the loss rate in the inner region ($r < R$), where no radial electric field exists, is very high and probably not caused by diffusion. In the outer region a diffusion coefficient was found which is about two times smaller than the one found by Stodiek et al.

In other Stellarator experiments one has tried to verify Spitzer's theory (see section 1.4) by identifying the fluctuation electric field with ion waves. It is then expected that a current which passes through the plasma may excite ion wave instabilities [78] , [79] . From the experiments of <u>Motley</u> with a recombining He plasma in B-1 Stellarator [75] it was found that the plasma loss rate increases sharply if the electron drift reaches a critical level. The critical current density was plotted as a function of the electron density for different gases at different pressures. In one case (He at 4.10^{-3}Torr) there is found agreement with theory. In the other cases there is a discrepancy of one order of magnitude (He at 6.10^{-4}Torr; Ar at 5.10^{-4}Torr) or even two orders of magitude (H_2 at 10^{-3}Torr). The fact that all the experimental curves lie close together for different cases, makes one thinking that another effect may be responsible for the observed losses. Thus no definite test of the ion wave instability theory is given.

Another possible explanation of the large plasma loss rates observed in stellerators may be given in terms of the electro-static plasma instability theory of Kadomtsev and Nedospasov [56] .

Acknowledgement.

I want to thank Dr.G.von Gierke for his stimulating interest in this work and the many clarifying discussions I had with him. Discussions with Dr. N. D'Angelo, Dr.D.Pfirsch, Mr.W.Stodiek and others have made special points clear to me in this still scarcely explored and known field.

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Several other explanations are given in order to explain the occurence of a critical B field in the diffusion experiments with the positive column. But none has been so successfull as the instability theory of Kadomstev and Nedospasov. An excellent review on this subject was given by Wöhler [58] .

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This elegant way of eliminating the unknown quantity α in eq.(32) makes the result independent on the exact value of α in a Cs plasma. Thus the conclusions about the diffusion are not influenced by the fact that a rather high value for α is found in this experiment.

Table II

Authors	Device	Gas	$n_o x 10^{12}$ cm^{-3}	$n_e x 10^{12}$ cm^{-3} in beam	B-range kgauss	Currents in z dir.	kT_e eV	kT_i eV
Bohm et al.	Arc in graphite box	Ar	30	3	3.7	yes	3	1
	"	"	"	"	"	"	"	"
Simon and Neidigh	Arc in copper cylinder	N_2	45	-	2.5-14	"	2	2
	and magn. bottle	"	"	-	3.5	-	"	"
Zharinov	Arc in copper cylinder	N_2	70	0.1	0.4-4.5	yes	1	1
	"	Ar	"	"	2.3	"	"	"
Boeschoten and Schwirzke	Plasma drifting out of duoplasmatron	H_2	0.3	0.01	0.7-3.5	very small $E_z < 0.1$ V/cm	12	10
	"	"	"	"	0.25-4	"	"	"
	"	He	"	"	0.7-3.5	"	"	"
	"	H_2	30	5	"	$E_z \lesssim 0.2$ V/cm	1	0.1
	"	N_2	"	"	"	"		
D'Angelo and Rynn	Thermal plasma	Cs	0.15-0.5	0.05-1.5	3-9	none	0.2	0.2
	"	K	"	"	"	"	"	"
Yoshikawa, Rose et al.	Hollow cathode discharge	Ar	30	1-30	0.9	yes	5	1.5
		"	"	"	"	"	"	"
Geller and Pigache	PIG discharge (pulsed)	H_2	30	10	0.1	?	7.5	1.5
Bonnal, Brifford, Grégoire, Manus	PIG discharge	H_2	600	0.05	0.15-4	?	3	0.1
	"	"	100	0.1	0.4-1.6	?	"	"
	"	N_2	"	"	"	?	"	"

T a b l e I I (continued)

L cm	R cm	λ_i cm	max. r_{ci} cm	r_{pl} cm	Method	$(\frac{q}{r_{ci}})_{exp.}$	$(\frac{q}{r_{ci}})_{gi}$	$\frac{q_{exp.}}{q_{drain}}$ B_{min}–B_{max}	Remarks
12	2.5	10	0.25	0.3	n(r)	1	0.4	1	B dependence not checked
"	"	"	0.3	"	$j_e^\perp/j_i^\perp$	-	-	-	$j_e^\perp/j_i^\perp$ 6x larger than expected
17	5	3	0.3	0.3	n(r)	4	1.5	2.3-1	-
"	"	"	0.5	"	n(r)	"	0.9	"	-
10	1.5	2	1.35	0.2	$j_e^\perp/j_i^\perp$	-	-	-	$j_e^\perp/j_i^\perp$ as expected up to B_c; then a steep increase
"	"	"	0.4	"	$(j_e^\perp/j_i^\perp)_A$	-	-	-	-
120	10	10^3	0.65	0.4?	n(r)	4	0.2	1.6-0.7	For r>3-4 cm: behaviour not clear
"	"	"	1.8	"	$j_e^\perp/j_i^\perp$	-	-	-	as expected
"	"	"	1.3	"	n(r)	3	0.2	1.6-0.7	-
"	"	5	0.09	0.4	n(r)	12	7.5	0.7-0.3	-
"	"	4	0.32	"	n(r)	3.5	9.5	0.35-0.15	-
70	1.5	0.1-2	0.24	0.2	n(r)	2	-	0.7-0.4	"Fully" ionized; in case $\lambda_i \ll r_{ci}$, strong proof for coll. diff.
"	"	"	0.13	"	n(r)	4	-	1.2-0.7	"
25 (var.)	5	0.2-6	1.2	0.2	n(r)	0.8	1.4	0.8	Strongly ionized
"	"	"	"	"	$I_\perp(V_\perp)$	-	-	-	$D_{e\perp}$ anomalous
100	1.5	0.6	1.7	1.5	n(I)	-	-	-	Strongly ionized; $D_{e\perp}$ normal up to 200 gauss
11	1.5	0.2	0.43	0.5-1.5	$\frac{n_{ext.}}{n_o}$	-	-	-	$\frac{n_{ext.}}{n_o}$ decreases with B up to B_c, then increases
100	2.5	1.2	0.16	2.5	"	-	-	-	"
"	"	1	0.6	"	"	-	-	-	"

Table III

Authors	Gas	P_0 Torr	I Amp/cm^2	L cm	R cm	B_{Max} Kgauss
Bickerton and v. Engel	He	$2\cdot10^{-2}-1$	10^{-2}	70	2	0.6
Lehnert and Hoh	H_2, He, Ar, Kr, N_2	$10^{-1}-4$	$10^{-2}-10^{-1}$	350	0.5 - 1	10
Allen, Paulikas and Pyle	H_2, D_2, He, Ne, Ar	$10^{-2}-4$	$10^{-2}-1$	200	1 - 3	7
v. Gierke and Wöhler	He	$3\cdot10^{-2}-1$	$3\cdot10^{-3}$	170	3	1.2

Table IV

Authors	Gas	Puls	P_0 Torr	n_e cm^{-3}	B Kgauss	L cm	R cm	Remarks
Bostick and Levine	He	H.F.	$5\cdot10^{-2}$ – 1	10^{7} – 10^{10}	0.25 – 1.37	Torus, outer diam. 15cm; cross section 5 x 3 cm		τ increases with B up to 0,6 Kgauss, then decreases.
Golant a. Shilinsky I	He	1-2 μs	0.1 – 2	10^{8} – 10^{11}	0.2 – 1.1	110	0.8	Agreement with eq. (26); if $n_e > 5\cdot10^{9}$ cm^{-3}, τ dependend on n_e
Golant a. Shilinsky II	He	1-2 μs	$2\cdot10^{-2}$ – 0.8	10^{8} – 10^{11}	0.2 – 2.4	110	0.8	No agreement, in low pressure range decay too fast.
Demirchanov et al.	He	H.F. at ν_{ce}	$2.5\cdot10^{-2}$ – 0.2	10^{8} – 10^{10}	0.02 – 4	70	3.5	Agreement with eq. (26) in the whole B range if gas was very pure.
Anisimov et al.	He	20 μs	10^{-2} – 0.2	$5\cdot10^{11}$ – 10^{13}	– 3	60	4	Agreement with coll. diff. up to 1 K gauss, probably up to 3 K gauss.
Ichimaru et al.	He, Ne	20 μs	0.1 – 2	10^{11} – 10^{12}	– 3	120	0.25	Agreement with coll. diff.

Table V

Authors	Apparatus	B Kgauss	n_i cm^{-3}	kT_i keV	kT_e eV	L cm	R cm	r_{ci} cm	r_{pl} cm	τ_{exp} sec	τ_{ii} sec	$\tau_{D_{coll}}$ sec	$\tau_{D_{drain}}$ sec
Post, Ellis, Ford, Rosenbluth	Table Top	> 10	$10^{13}-10^{14}$	1	$2\cdot10^4$	45	7.5	< 0.65	2	10^{-3}	$3\cdot10^{-3}-3\cdot10^{-4}$	1.5	$5\cdot10^{-8}$
Coensgen, Cummins, Nexsen, Sherman	Toy Top (2 stages)	18	$10^{13}-10^{14}$	3.6	100	30	11	0.7	5	10^{-4}	$2\cdot10^{-2}-10^{-3}$	3	10^{-4}
Joffe, Yushmanov		5	$10^{9}-10^{10}$	2	10	80	25	1.3	20	10^{-4} x)	60-6	$3\cdot10^4$	$5\cdot10^{-3}$
Bogdanov, Golovin Kucheryaev, Panov	Ogra	3	$\lesssim 10^9$	80	100	1000	70	13.5	60	$> 10^{-3}$	$> 10^4$	10^6	$3\cdot10^{-3}$
Brevnov, Romanovski Tomashchuk	Ogrenok	3	$3\cdot10^7$	10	≤ 30	100	25	4.8	20	$5\cdot10^{-5}$	$2\cdot10^4$	$3\cdot10^7$	10^{-3}

+) With magnetic field trapping not only in longitudinal direction, but also in transversal direction ("Hybrid field"), substantially longer containment times are found.

Fig. 3: Schematic of the apparatus used by Simon and Neidigh (section 3.2). The magnet-pole pieces are 230 cm^2 and B variable from 0 to 14000 gauss. A = anode; F = filament heated to emission; C = copper cylinder; W = water cooling lines, soldered to the outside of the cylinder wall; S = hole in the cylinder end wall; P = probe.

Fig. 4: Schematic of the apparatus used by Zharinov (section 3.3). The discharge chamber was placed inside a metal vacuum chamber which was situated in the gap of an electro-magnet.

Fig. 5: Schematic of Cabinet (section 3.4). The plasma is drifting from a duoplasmatron into a vacuum chamber, surrounded by solenoids which produce a magnetic induction up to 4000 gauss. If $L \gg \lambda_i, \lambda_e$, a secondary plasma is formed.

Fig. 6: Schematic of the Q machine (highly ionized Cs and K plasma, see section 3.5).

Fig. 7: Schematic of the large PIG discharge used by Bonnal et al. (section 3.7). The distance between the cathodes is 1 m, the diameter of the discharge tube is 5 cm.

Fig. 8: Schematic diagram of the hollow-cathode discharge modified by Yoshikawa and Rose to measure the diffusion of electrons across the magnetic field (section 3.8). A = anode; C = cathode; T = third electrode; L_1, L_2 = Langmuir probes.

Fig. 9: Comparision of the measured $dV_\perp/dI_\perp$ values with both classical and drain diffusion theories as a function of the magnetic induction B (section 3.8).

Fig.10: Voltage drop measured across two probes 0.345 m apart as function of B. The full curves are calculated from the theory for molecular ions and the dashed curves for atomic ions (from Hoh and Lehnert, see section 4.1).

Fig.11: Comparision of the experimental results of Hoh and Lehnert at p = 0.89 Torr with the theory of Kadomtsev and Nedospasov (section 4.1).

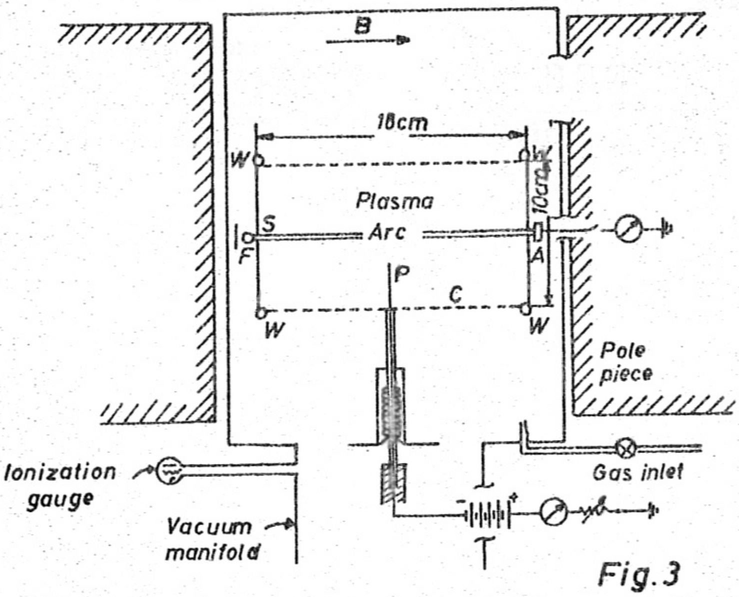

Fig. 3

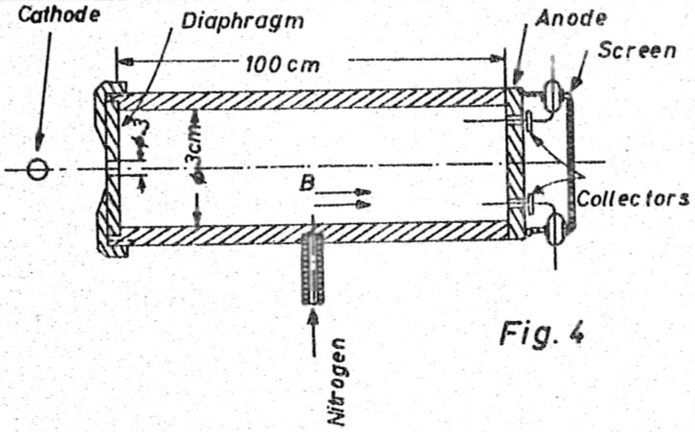

Fig. 4

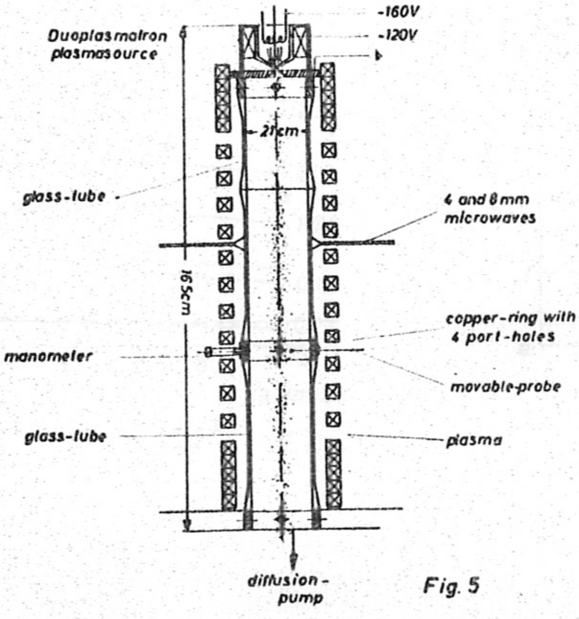

Fig. 5

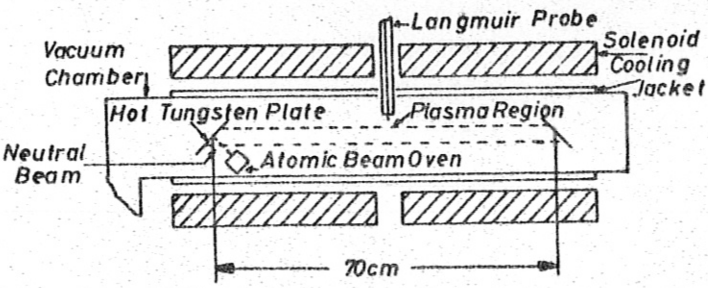

Fig. 6

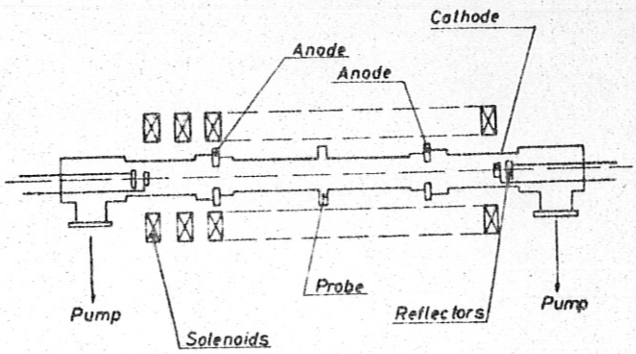

Fig. 7

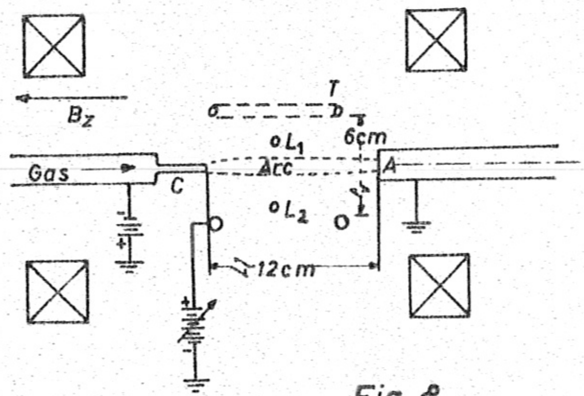

Fig. 8

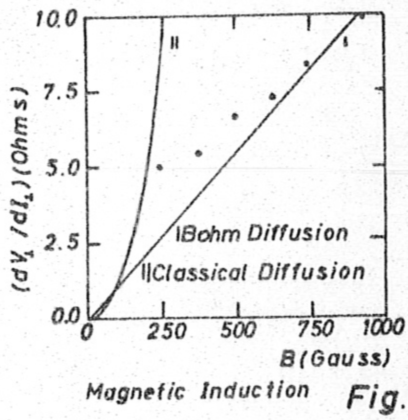

Fig. 9

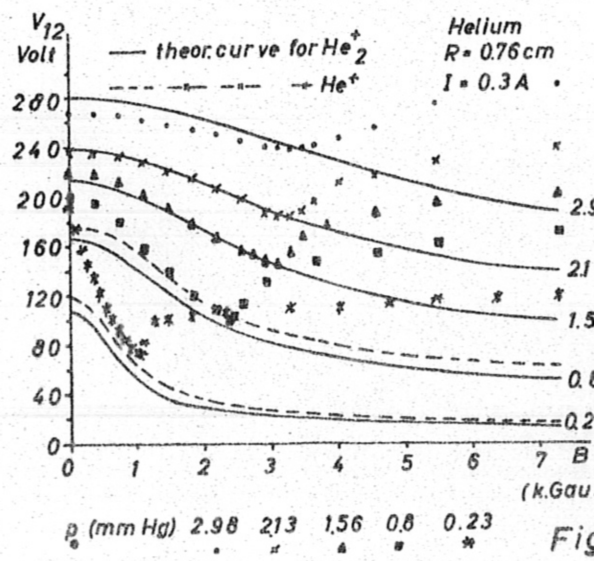

Fig.10

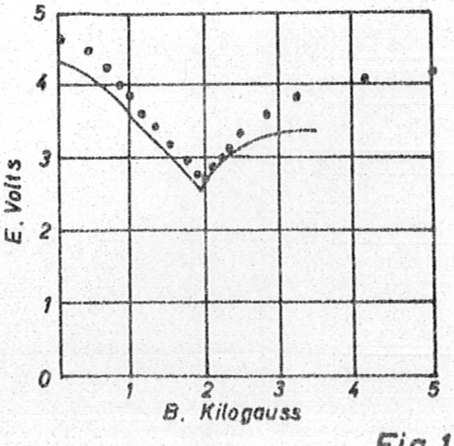

Fig.11