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Statistical Analysis of the equilibrium configurations of W7-X stellarator using Function Parametrization

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Introduction

W7-X is a 5-period, fully optimised stellarator under construction at IPP-Greifswald, Germany. It has a standard magnetic configuration, with five islands at the boundary where iota=1, produced by a set of 2×5 modular field coils (MFC) in each period. The boundary iota value can be varied between 5/6 (low iota case) and 5/4 (high iota case) using 2×2 additional planar coils (PLC) per period. An important goal of W7-X is to investigate the steady state capabilities of fusion devices. For stellarators this essentially implies a real time monitoring of the discharges which have long pulse lengths, of the order of minutes. But for a real time study one must have means to generate a magnetic configuration in seconds, while 3-D computer codes, which simulate stellarator configurations, do so in hours with a strong demand on computational resources. This implies the use of methods which are fast and accurate.

For W7-X we have planned a sequence of in-depth analyses of the magnetic configurations which, ultimately, will lead to a proper understanding of plasma equilibrium, stability and transport. The first step in that sequence involved a study of the W7-X vacuum configurations with magnetic islands [1] where we had used the statistical, inverse mapping method of Function Parametrization (FP) [2] to recover the physical properties of the configurations. This paper reports the initial results on the next step of analysis, the scenario at finite beta where there is a plasma equilibrium. The study [1] was encouraging enough to use FP again.

Details of database generation and selection of the FP model

Simulated W7-X plasmas were produced by VMEC2000, a 3-D equilibrium code [3] that assumes only nested flux surfaces in a configuration, thereby neglecting magnetic islands. The geometry and the magnetic field on the flux surfaces are given as Fourier coefficients (FC's) with a modest number of harmonics. A database of about 8000 such configurations was calculated on the same parameter space for the coil current ratios as in [1]. The parameters which were varied randomly and independently consist of the external (six) coil current ratios $i_2, \ldots, i_5, i_A, i_B$, the parameters of the profiles (as functions of normalised toroidal flux s) of plasma pressure and toroidal current (4+4) and the plasma size (a_{eff}), which is required to vary the plasma volume, giving a total of N_m=15 measurements. The plasma parameters were varied to allow a good FP for their expected values in W7-X: volume-averaged $<\beta>$ of up to 5% and toroidal net-current of up to ± 50 kA for a mean field strength of about 2 T throughout the database. The profiles of pressure and plasma current were chosen as a sequence of polynomials in the following forms: $p(s) = \sum_{i=1}^{n} a_i b_i(s)$ (normalised so that $\int_0^1 p(s)ds = 0.5$) and $I(s) = \sum_{i=1}^{n} c_i d_i(s)$, respectively, where $b_i(s)$ and $d_i(s)$ are moment-oriented polynomials of degree i in s. We chose

n=4 in either case. $b_1(s)$=$1-s$ is the only moment contributing to $\int p(s)ds$, thus relating a_1 with the volume-averaged pressure; $b_2(s)$ allows for pressure peaking variation which is inferred from a_1 and a_2; with $d_1(s)$=s, c_1 defines the total plasma current. The higher moments were constructed so as not to alter the contribution of the lower moments. Fig 1 shows some of the normalised pressure profiles used in the analysis. The criteria for deciding upon the usable cases for analysis were: (a) convergence of the code; (b) 0.16$\leq \iota_0 \leq$ 1.62; (c) 0.62$\leq \iota_b \leq$ 1.32; (d) restricting $\beta_0 \leq 12\%$, as large β_0 results in low volume-averaged $<\beta>$; (e) $a_{eff} \leq 60$ cm, as it is unlikely to exceed 55 cm in experiments. Fig 2 shows the configuration space in the i_A-i_B plane. The void at the lower left corner is caused by restrictions (b), (c) and (e) corresponding to the high-ι region. The points in the rest of the space are more or less uniform.

The basic plasma parameters chosen for analysis were the profiles (as functions of an effective flux surface radius r_{eff}) of ι and the Fourier coefficients of the magnetic field B_{mn}, the geometry (R_{mn},Z_{mn}) and the periodic stream function λ_{mn} which is included in the Clebsch representation of B [3]. m and n are, respectively, the poloidal and the toroidal Fourier mode number. Before setting up the FP model for statistical analysis, the measurements were tested for possible correlations. A Principal Component Analysis (PCA) showed that there were no insignificant variances in the data. However, strong correlations were observed in the coefficients a_2, a_3 and a_4 of the pressure profile. This was possibly due to the need to maintain monotonicity and the negative gradient of the pressure profile and to have $\beta_0 \leq 12\%$. Nevertheless, the PCA step was skipped.

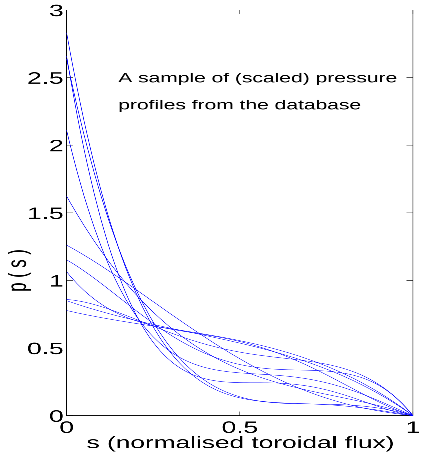

Figure 1: Scaled pressure profile forms with $\int_0^1 p(s)ds = 0.5$

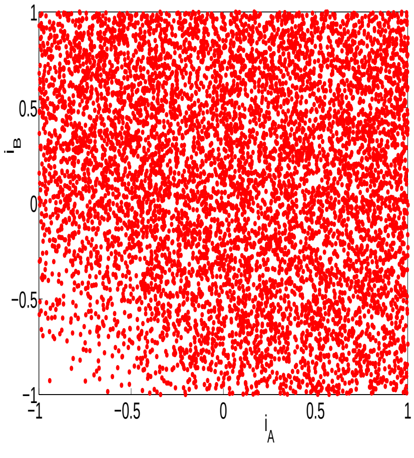

Figure 2: The i_A-i_B *space in external current configuration.*

As our output plasma parameters to be recovered are all profiles, the usual FP model would be a radial polynomial. However, a different approach was followed here because of the unrealistic size of the radial polynomial model. The profile variables were taken at 21 radial points and subjected to a PCA. The Principal Components (PC's) form a set of radial functions replacing the global radial polynomial model. The PC's with significant variance were then regressed (as scalar parameters) as "mixed" quadratic (q-FP) or "mixed" cubic (c-FP) polynomials in the measurements [1]. The number of model coefficients for scalar parameters were $\frac{(N_m+1)(N_m+2)}{2}$=136 (q-FP) and $\frac{(N_m+1)(N_m+2)(N_m+3)}{6}$=816 (c-FP). Thus, with about 5000 cases chosen for "training" there is a sufficient number of degrees of freedom in either model for a reliable fitting. Once the model was set up, it was tested for validation. The model coefficients, together with the measurements in a different set (the test dataset) of about 900 observations were used to recover the radial PC's of the plasma profile variables in this dataset by simple calculations from the model. The recovered PC's were then combined with the corresponding matrix elements of the eigenvectors (of the covariance matrix) to get the (recovered) plasma parameters at the required radial points. Finally the recovered plasma parameters were compared with the observed ones (from VMEC2000) for the recovery statistics. The test part was further carried out with different levels of random errors in the measurements. These errors were assumed to have a uniform distribution whose extrema were expressed as a percentage of the root-mean-square (rms) values of the measurements.

Results

As reported in the previous section, the recovered plasma parameters were obtained from the regression of the significant PC's of the profile variables. Table 1 shows the number of significant PC's required to account for the total variance of the profile variables. For the FC's there is a slight overestimate as the number varies with harmonics. The significance of this number in the FP model is that it estimates the order of the radial polynomial if that approach had been taken. Thus $B_{mn}(r_{eff})$ would have been modelled by at least a cubic polynomial in r_{eff}. The FC's for R, Z and λ clearly have a more complex behaviour and would need a polynomial of a very high order.

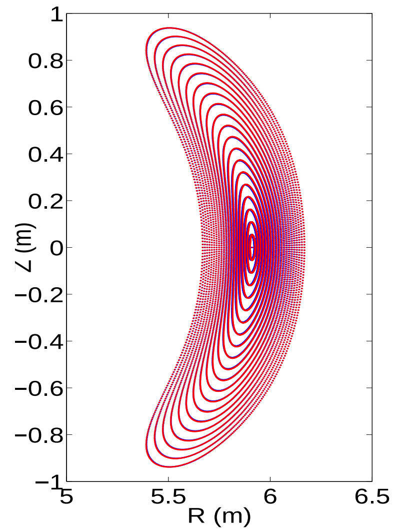

Figure 3: The bean-shaped flux surfaces. blue:VMEC surfaces; red:cubic-FP-recovered surfaces.

Table 2 lists the summary and the output statistics for the profiles of ι and the low order FC's B_{mn}, R_{mn}, Z_{mn} and λ_{mn}. The rms error and the R^2-measure of fit are compared for q-FP and c-FP models, where $R^2_{mo} = 1-(rmse_{mo}/\sigma)^2$, adjusted for the degrees of freedom. σ is the spread in the data about the mean, while $rmse_{mo}$ is the rms error for the model mo. We observe significantly smaller values of error with c-FP model for most of the FC's. Also shown here are the recovery qualities when measurement errors were introduced. These errors were on the database rms values of the measurements, which, for the coil currents, were 11 kA (MFC) and 6.9 kA (PLC), so 0.1% noise corresponds to extrema of ±11 amps for MFC and ±6.9 amps for PLC. The relative accuracy of the coil current measurement in W7-X is 2.0e-04 (0.02 %) at 20 kA, that corresponds to a measurement error of 4 amps, so the similarity of the errors up to 0.5% noise signifies a general stability of the models up to 10-15 times the estimated error. Next, the FC's were combined in a Fourier series of the forms

$X(r_{eff}, u, v) = \sum_{m=0}^{M} \sum_{n=-N}^{N} X_{mn}(r_{eff})\ cos(mu - nv)$

$Y(r_{eff}, u, v) = \sum_{m=0}^{M} \sum_{n=-N}^{N} Y_{mn}(r_{eff})\ sin(mu - nv)$

Here X stands for R and B, while Y represents Z and λ. u is a poloidal angular coordinate $(0 \le u \le 1)$ and v is a toroidal angular coordinate $(0 \le v \le N_p)$. For W7-X, N_p = 5, the number of toroidal periods of field and geometry. We also found $M = \mid N \mid =$ 6 sufficient to construct the flux surfaces from the FC's. Fig 3 shows the bean-shaped cross section of the W7-X flux surfaces at v=0 plane. The VMEC flux surfaces are shown in blue, while the FP-recovered (c-FP model) surfaces are in red. The latter compare well with the former, except for some deviation in the inner parts of the profile. The positive aspect of this recovery is the fitting of the indentation, while the uncertainty of the regions around the magnetic axis are not surprising in a global configuration recovery because the boundary is more integral and the axis is more stable if recovered locally. A statistic for this recovery is the (rms) deviation of r_{eff} of the recovered flux surfaces from the VMEC surfaces, defined as $(\Delta r_{eff})_{rms} = \sqrt{\frac{\sum_{j=1}^{N_{obs}} \sum_{i=1}^{N_{rad}} \left[r^{obs}_{eff_{i,j}} - r^{rec}_{eff_{i,j}}\right]^2}{N_{obs}.N_{rad}}}$ the mean taken over all N_{obs} observations and all N_{rad} (=21) radial points. Table 3 shows a comparison of this quantity for q-FP and c-FP models. For ideal measurements (noise = 0) the uncertainty in r_{eff} is about 0.9 mm for c-FP, while it is 1.3 mm when q-FP was used. The latter is worse by a factor of 1.42 — "statistically" significant! Also shown in Table 3 are the respective values in presence of noise. Table 4 shows the recovery of B and λ.

Table 1

parameter	# PC's
ι	4
B_{mn}	4
R_{mn}	7
Z_{mn}	7
λ_{mn}	8

Table 3: Values of $(\Delta r_{eff})_{rms}$ (in metres) for q-FP and c-FP

noise (%)	q-FP	c-FP	q-FP/c-FP	Remarks
0.0	0.0013	9.2e-04	1.42	With increase in noise level the degradation
0.1	0.0013	9.4e-04	1.39	of q-FP with respect to c-FP decreases,
0.5	0.0016	0.0013	1.23	but in the most probable ranges of noise
1.0	0.0022	0.0020	1.11	the latter is significantly better.

Table 2: FP recovery of profile variables. Mean, σ on axis (a), mid (m) and last (l) flux surface.

parameter	Unit	Summary statistics		Global recovery statistics (all surfaces)					
		Mean	σ	noise (%)	$rmse_q$	$rmse_c$	R_q^2	R_c^2	$rmse_q/rmse_c$
R_{00}	m	5.63 (a)	0.09 (a)	0.0	0.0054	0.0025	0.9954	0.9990	2.16
		5.60 (m)	0.08 (m)	0.10	0.0054	0.0025	0.9954	0.9990	2.16
		5.54 (l)	0.07 (l)	0.50	0.0055	0.0027	0.9952	0.9990	2.08
Z_{10}	m	0.00 (a)	0.00 (a)	0.0	0.0022	0.0013	0.9961	0.9987	1.69
		0.31 (m)	0.03 (m)	0.1	0.0022	0.0013	0.9961	0.9986	1.69
		0.62 (l)	0.06 (l)	0.50	0.0025	0.0017	0.9950	0.9977	1.47
B_{00}	Tesla	2.01 (a)	0.18 (a)	0.0	0.01	0.01	0.9963	0.9964	1.00
		2.05 (m)	0.18 (a)	0.1	0.01	0.01	0.9963	0.9964	1.00
		2.08 (l)	0.18 (l)	0.5	0.01	0.01	0.9962	0.9962	1.00
λ_{10}	radian	0.00 (a)	0.00 (a)	0.0	0.023	0.011	0.9639	0.9914	2.09
		-0.25 (m)	0.15 (m)	0.1	0.023	0.011	0.9639	0.9914	2.09
		-0.23 (l)	0.12 (l)	0.5	0.023	0.011	0.9640	0.9913	2.09
ι	–	0.84 (a)	0.28 (a)	0.0	0.016	0.013	0.9928	0.9954	1.23
	–	0.85 (m)	0.15 (m)	0.1	0.016	0.013	0.9928	0.9954	1.23
	–	0.94 (l)	0.12 (l)	0.5	0.016	0.013	0.9927	0.9954	1.23

Table 4:Recovery of B and λ (as functions of magnetic coordinates) obtained from B_{mn} and λ_{mn}.

B (Tesla)		λ (radian)		Remarks
noise (%)	rms residual	noise (%)	rms residual	The rms residuals were calculated over all radial, poloidal
0.0	0.011	0.0	0.010	and toroidal points for about 800 observations with
0.1	0.011	0.1	0.010	cubic-FP model. B is determined with an uncertainty
0.5	0.011	0.5	0.010	of 0.01 Tesla. Up to 1% noise the results are very stable.
1.0	0.011	1.0	0.010	The results for λ do leave scope for improvement.

Conclusions

Earlier results on vacuum analysis were further corroborated by the initial studies at finite-beta regarding the importance of a cubic FP model to reconstruct the W7-X configurations, even without magnetic islands. The FP approach of PC regression as scalar variables, rather than the use of the radial polynomial, made the analysis faster by orders of magnitude and the model size was only slightly increased. The accuracy of the recovery of the flux surfaces and the magnetic field structure as well as the iota-profile were within acceptable limits. However, λ-recovery is clearly more challenging.

References

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