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Development of Function Parametrization on W7-X stellarator

A.Sengupta[1], P.J.Mc Carthy[1], J.Geiger[2], A.Werner[2]
[1] *Dept. of Physics, University College Cork, Association EURATOM-DCU, Cork, Ireland*
[2] *Max-Planck-Institut f. Plasmaphysik, Euratom Association, Greifswald, Germany*

Introduction

W7-X, under construction at IPP-Greifswald, Germany, is a fully optimized stellarator (average $R_0 = 5.5$ m; average $a = 55$ cm). It will be constructed with superconducting coils to show the steady state capability of stellarators. Stellarators have an equilibrium magnetic configuration even without a net toroidal current and thus have the inherent capability for steady state operation which is crucial for the device to be used as a fusion reactor. There is, however, a price to pay. Flux surfaces in a stellarator have a full 3-dimensional (3-D) geometry. The computational effort for 3-D equilibrium calculations is orders of magnitude higher compared to similar calculations for tokamaks. Long pulses need real time monitoring and control as well as data evaluation. Therefore, a fast method for monitoring, controlling and on-line data analysis, which does not need to provide a full magnetohydrodynamic (MHD) equilibrium, is necessary as an initial step for study. This led us to attempting a fast recovery of equilibrium parameters of the vacuum magnetic configuration of W7-X. We used the method of Function Parametrization (FP) [1,2] of the equilibrium parameters expressed, as quadratic or cubic polynomials including interaction terms, in terms of a set of measurements. In our case, these measurements were the currents in the external coils producing the magnetic configuration.

Database and model selection

In the present study, the database contains vacuum magnetic field configurations for W7-X. The basic details of the database generation was described in [3]. The database was since expanded to include more configurations of $\iota_b \in [5/4, 5/6]$ (ι_b = boundary rotational transform). This was necessary for a good statistical analysis. Thus we now have a main database of 1210 vacuum configurations including 175, 316 and 343 cases, respectively, of $\iota_b \in [5/4, 5/5, 5/6]$.

To generate the detectable major islands in the configurations of the main database, an island database, in addition to the main database, was generated. This database contains the basic parameters describing a magnetic island, namely, its location and width, for each of the islands at the major (low order) rational surfaces of the same configurations included in the main database. The incidence of island chains in the island database were as follows: 442 cases of $\iota = 5/5$ and 506 cases of $\iota = 5/6$ island chains. The higher incidence of 5/5 and 5/6 cases in this database, as compared to those in the main database, was due to the frequent presence of one of the major resonances as an internal island chain, followed by the other at larger r_{eff}. The island location and width were calculated from the r_{eff} values of the inner and outer island separatrices, denoted by $r_{eff}^{(i)}$ and $r_{eff}^{(o)}$, respectively. The island location is given by $r_{eff}^{(is)} = \frac{1}{2}(r_{eff}^{(i)} + r_{eff}^{(o)})$. On the other hand, the difference in the radii of the two separatrices was assumed to give a measure of the island width: $w^{(is)} = r_{eff}^{(o)} - r_{eff}^{(i)}$.

The FP model used to represent the vacuum (scalar) parameters (y) was a "mixed" quadratic, i.e., including interaction terms $y = \sum_{i=0}^{N} \sum_{j=0}^{i} a_{ij} x_i x_j$ or a "mixed" cubic $y = \sum_{i=0}^{N} \sum_{j=0}^{i} \sum_{k=0}^{j} a_{ijk} x_i x_j x_k$. For modelling the profiles of iota and V' we chose a radial dependence on r_{eff}^2, rather than r_{eff}, because according to MHD theory, iota and V' are functions of the toroidal magnetic flux which varies as r_{eff}^2. The model, therefore, is a polynomial of degree P in r_{eff}^2, each of whose coefficients is either a mixed quadratic function of the coil current ratios, resulting in the model $y = \sum_{k=0}^{P} \sum_{i=0}^{N} \sum_{j=0}^{i} a_{ij} x_i x_j (r_{eff}^2)^k$, or a mixed cubic function, giving the model $y = \sum_{l=0}^{P} \sum_{i=0}^{N} \sum_{j=0}^{i} \sum_{k=0}^{j} a_{ijk} x_i x_j x_k (r_{eff}^2)^l$.

Results

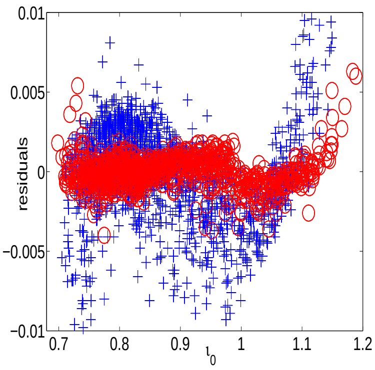

Figure 1: Comparison of the recovery quality of q-FP with that of c-FP for the central iota value ι_0. '+': q-FP. 'o': c-FP. Clearly much larger scatters of the q-FP residuals are seen. In addition, a structure rather than a random scatter, is observed in the q-FP residuals.

Table 1 displays the details of FP-recovery of parameters connected to the magnetic axis, where ι_{ax}: rotational transform; $V' = \frac{dV}{d\psi_T}$: specific volume (ψ_T=toroidal flux); $B_{ax}^{(av)}$: average magnetic field; $B_{ax}^{(m)}$: mirror field normalised to $B_{ax}^{(av)}$; $R_{ax}^{(av)}$: average axis position. The rms error and the percentage spread error, defined as 100(rms error)/(spread), form the statistics of the recovery. The spread of the observed values of a parameter about the mean is denoted by σ. $rmse_q$ is the rms error from the quadratic model (q-FP) and $rmse_c$ is the same for the cubic model (c-FP). $perr_{q,c}$ is the percentage spread error for the same models. From the table we observe that for these parameters $perr_q$ is in the low single figures. This suggests that q-FP, by itself, was a reasonable success. This is not surprising, because the axis is a well defined entity and the parameters related to it are also stable and do not suffer from the topological complexities of regions populated by magnetic islands. However, the residual plot of fig 1 indicates that even though q-FP performs very well for regressing the axis parameters, it is still outperformed by c-FP by a significant margin.

Table 1: FP recovery of axis parameters

parameter	Unit	Mean	σ	$rmse_q$	$rmse_c$	$perr_q$	$perr_c$	$rmse_q/rmse_c$
ι_{ax}	–	0.875	0.110	3.89e-03	1.04e-03	3.54 %	0.95 %	3.74
V'_{ax}	cm/Tesla	287.85	33.51	0.94	0.20	2.82 %	0.59 %	4.78
$B_{ax}^{(av)}$	Tesla	2.09	0.2138	2.42e-04	5.03e-05	0.11 %	0.02 %	4.97
$B_{ax}^{(m)}$	–	0.084	0.075	9.82e-04	1.63e-04	1.31 %	0.21 %	6.33
$R_{ax}^{(av)}$	cm	555.98	4.90	0.05	0.01	1.02 %	0.30 %	3.43

Table 2 a) displays the recovery details of the island parameters. Regressions of island location and width are reported. The q-FP model gave a very poor recovery of the island parameters. The c-FP model, in spite of improving upon the q-FP recovery quality, was also found to be inadequate. To clarify the reasons for this difficulty, we constructed a 1-D scan of the configuration space in order to observe the behaviour of $r_{eff}^{(is)}$ and $w^{(is)}$ on the coil current ratios.

We varied only the planar coil current I_A from 3 kA up to 24 kA. A set of 91 configurations consisting of 5/6 islands was generated.

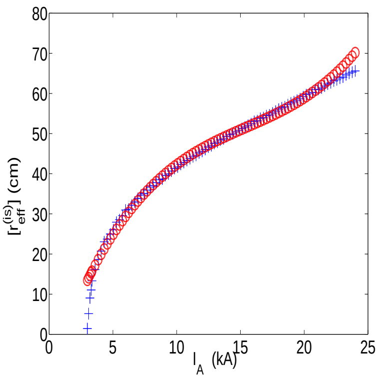

Figure 2: Variation of $r_{eff}^{(is)}$ with I_A. '+': observed data. 'o': c-FP fit. The square root nature of the variation (the '+' symbol) is clearly observed, showing that $r_{eff}^{(is)} \sim (I_A - I_{A0})^{1/2}$.

Fig 2 explains the poor regression results with our models. In particular, a simple polynomial, including the cubic, was inadequate for modelling the square-root-like dependence of $r_{eff}^{(is)}$ on the coil currents for low values of the currents. We then tried to regress $r_{eff}^{(is)2}$. This gave significantly better results as is seen from Table 2 b). The quantity $\sqrt{r_{eff}^{(is)2}}$ refers to the rms error in $r_{eff}^{(is)}$ with respect to a regression of $r_{eff}^{(is)2}$ and similarly for $w^{(is)}$. The models were tested again on the same 1-D data, but now for the $r_{eff}^{(is)2}$ fit. The difference in fitting accuracy is seen from Fig 3, where $\sqrt{r_{eff}^{(is)2}}$, instead of $r_{eff}^{(is)}$, is plotted against I_A. The entire variation is found to be fitted very well, something which is also described by rms errors and percentage errors in Table 2 b). The island width also showed a good improvement, though not as much as $r_{eff}^{(is)2}$, when $w^{(is)2}$ was regressed.

Table 2 a): FP recovery of island parameters

parameter	Unit	Mean	σ	$rmse_q$	$rmse_c$	$perr_q$	$perr_c$	$rmse_q/rmse_c$
$r_{eff}^{(is)}(5/5)$	cm	47.83	11.87	1.23	0.66	10.39 %	5.59 %	1.86
$r_{eff}^{(is)}(5/6)$	cm	42.88	12.39	1.61	0.91	13.02 %	7.31 %	1.78
$w^{(is)}(5/5)$	cm	6.69	1.90	0.67	0.44	35.03 %	23.00 %	1.52
$w^{(is)}(5/6)$	cm	3.74	1.91	0.44	0.35	23.24 %	18.15 %	1.28

Table 2 b): Improved FP recovery of island parameters

parameter	$rmse_q$	$rmse_c$	$perr_q$	$perr_c$	$\frac{rmse_q}{rmse_c}$
$\sqrt{r_{eff}^{(is)2}}(5/5)$	0.84	0.38	7.09 %	3.16 %	2.24
$\sqrt{r_{eff}^{(is)2}}(5/6)$	1.18	0.52	9.56 %	4.23 %	2.26
$\sqrt{w^{(is)2}}(5/5)$	0.67	0.35	35.06 %	18.55 %	1.89
$\sqrt{w^{(is)2}}(5/6)$	0.41	0.24	21.36 %	12.77 %	1.64

Table 3 : iota-profile recovery with

	coefficients quadratic in current ratios		**coefficients cubic in current ratios**	
P	$rmse_q$	$perr_q$	$rmse_c$	$perr_c$
1	0.0088	7.35%	0.0076	6.40%
2	0.0057	4.80%	0.0035	3.00%
3	0.0052	4.36%	0.0028	2.45%
4	0.0050	4.24%	0.0028	2.24%

Table 4 : V'-profile recovery with

	coefficients quadratic in current ratios		**coefficients cubic in current ratios**	
P	$rmse_q$ (cm/Tesla)	$perr_q$	$rmse_c$ (cm/Tesla)	$perr_c$
1	1.0268	3.16%	0.48	1.41%
2	0.9289	2.83%	0.23	0.68%
3	0.9255	2.82%	0.22	0.64%
4	0.9253	2.82%	0.21	0.63%

Remarks

A polynomial quadratic in r_{eff}^2 is sufficient to model the V'-profile. The modelling is crucial also from the viewpoint of the configuration having a magnetic well or hill. This can be shown by observing the variation of V' with r_{eff}^2 as calculated from the model. This is justified, as the toroidal flux $\psi_T \sim r_{eff}^2$.

The left half of Table 3 shows the recovery accuracy of the profile of ι modelled as a polynomial in r_{eff}^2 whose coefficients were quadratic functions of the coil current ratios. It is clear, however, that there is enough scope for improvement, so the model size was increased by including additional coefficients, which were cubic functions of the current ratios, in the r_{eff}^2 polynomial. These are reported in the right half of Table 3. In comparison with the model using quadratic functions of the current ratios, this shows a significant improvement in accuracy for the same P. Increasing the model size further, with a set of coefficients fourth order in the current ratios, we observed that, for $P = 3$, the rms error was 0.0028 and percentage error 2.24 which is an improvement by a factor of 1.09 over its value of 2.45 obtained with cubic current ratios. It can, therefore, be concluded that the model with cubic current ratios, with $P = 3$, is an adequate one for recovering the ι- profile. V'-profile recovery is described in Table 4.

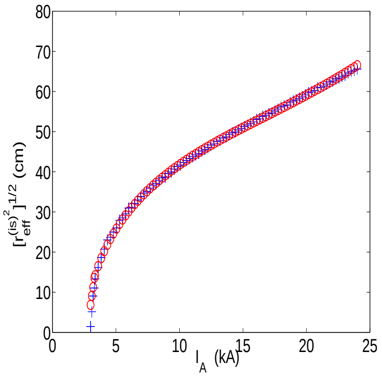

Figure 3: Variation of $\sqrt{r_{eff}^{(is)2}}$ with I_A. '+': observed data. 'o': c-FP fit.

The above results were obtained with ideal currents in the coils, with no assumption of measurement errors. However, in a realistic scenario these errors are inevitable and should be considered, so the regressions were tested with noisy inputs. The noise levels chosen were 0.1%, 0.5% and 1% of the nominal value of the coil currents (12 kA) that correspond to perturbations of 12 amps, 60 amps and 120 amps, respectively, in the coil currents. The relative accuracy of the coil current measurement aimed at in W7-X is 2.0e-04 (0.02 %) at 20 kA, that corresponds to a measurement error of 4 amps. Therefore, the minimum noise level of 12 amps considered here is already factor of 3 higher than that desired. At this level, the tests showed good stability and the noiseless results were found to be negligibly perturbed. Test results for a few scalar parameters are shown in Table 5.

Table 5: FP recovery of some of the scalar parameters with noisy inputs

parameter	Unit	σ	$rmse_c$				$perr_c$			
			0%	0.1%	0.5%	1.0%	0%	0.1%	0.5%	1.0%
ι_{ax}	–	0.110	1.04e-03	1.08e-03	2.20e-03	4.11e-03	0.95	0.99	2.01	3.75
V'_{ax}	(cm/Tesla)	33.51	0.20	0.42	1111.92	3.85	0.59	1.25	5.73	11.50
$B_{ax}^{(av)}$	(Tesla)	0.2138	4.91e-05	2.26e-03	0.0112	0.0224	0.02	0.92	4.56	9.13
$B_{ax}^{(m)}$	–	0.075	1.55e-04	4.99e-04	2.40e-03	4.82e-03	0.21	0.68	3.25	6.50
$R_{ax}^{(av)}$	(cm)	4.90	0.015	0.022	0.088	0.175	0.30	0.44	1.81	3.62
$\sqrt{r_{eff}^{(is)2}}(5/6)$	(cm)	12.39	0.52	0.53	0.77	1.36	4.23	4.26	6.20	10.95

Conclusions

We studied the recovery of the important magnetic and geometric parameters that describe the vacuum configuration of W7-X. The standard quadratic FP model, though sufficient to accurately recover most of the axis parameters, was inadequate for the parameters connected with the boundary (separatrix). The cubic FP model, involving a trebling of the model size, improved the quality of recovery of the scalar parameters significantly. The initial analysis with ideal currents was compared with different levels of measurement errors corresponding to different percentages of the nominal current in the coils. Up to an error of 0.1% at 12 kA the cubic FP model was able to keep the recovery errors within acceptable limits.

References

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