Monte Carlo δf Calculation of the Ion Polarization Current at a Rotating Island and of the Bootstrap Current at a Large Pressure Gradient

A. Bergmann, E. Poli, A. G. Peeters, S. D. Pinches, E. Strumberger
Max-Planck-Institut für Plasmaphysik, IPP–EURATOM Association
85748 Garching, Germany

The following neoclassical phenomena are studied with drift-kinetic simulations: (i) the ion polarization current at a rotating island of a neoclassical tearing mode and (ii) the bootstrap current at a large gradient, e.g. H mode pedestal.
(i) Neoclassical tearing modes (NTM) play an important role in long-pulse tokamak experiments, since they often set the dominant limitation to the maximum pressure achievable in a given magnetic field. The NTM occurs when a sufficiently strong magnetic perturbation creates a 'seed island' in the plasma. It is driven by the loss of the bootstrap current inside the island (due to the flattening of the pressure profile), but is possibly damped by the polarization current in the time dependent electric field of the rotating island (the ion current is dominant due to the larger gyro radius). The polarization current is strongly enhanced due to the finite banana orbit width. Recently we have shown that due to finite orbit size there is no complete loss of the bootstrap current in small islands[1]. Here, we report on studies of the effect of the finite orbit size on the polarization current. Also, the dependence of the polarization current on the collisionality was studied. From the analytic theory only the limits for very low and for high collisionality are known.
(ii) The neoclassical theory of the bootstrap current is based on the assumption that the babana orbit width is small compared to the gradient length. We calculated the ion contribution to the bootstrap current in the opposite case of steep density and temperature profiles.

The neoclassical contribution to the rate of change of the island half-width w is proportional to an integral over the helical part of the parallel current,

$$\left.\frac{dw}{dt}\right|_{\rm neo} \sim \frac{1}{w}\int_{-1}^{\infty} d\Omega \iint \frac{j_{\|\rm neo}\cos\xi}{\sqrt{\Omega+\cos\xi}}\, d\xi d\theta, \tag{1}$$

where $\Omega = (q_s'/2q_s\tilde{\psi})(\psi-\psi_s)^2 - \cos\xi$ is the normalized helical flux, $\Omega = 1$ defines the island separatrix, $\Omega = -1$ the O-point. ψ is the unperturbed poloidal flux, $\tilde{\psi}\cos\xi$ is the perturbation that causes the island to form, and the prime denotes the derivative with respect to ψ. The index s denotes the resonant surface, where $q = m/n$, and $\xi = m\theta - n\phi + \omega t$ is a helical angle, θ and ϕ are the poloidal and toroidal angles in Boozer coordinates, $\omega/2\pi$ is the rotation frequency of the island. One contribution to the current $j_{\|\rm neo}$ in Eq. (1) is the parallel electron current that closes the ion polarization current which is directed across the helical flux surfaces. For our study we take an analytically defined equilibrium magnetic field with flux surfaces of circular cross section and a safety factor varying as $q = 1 + 2(\psi/\psi_{\rm edge})$. Also, we assume a perturbation of time independent size with mode numbers $m = 3$ and $n = 2$ resulting in an island of half-width $w = w_\psi\, dr/d\psi$ with $w_\psi = (3\psi_{\rm edge}\tilde{\psi})^{1/2}$.

Simulations of rotating islands were performed with different sets of parameters $\nu, \omega, \varepsilon, T_i$ and with flat density and temperature profiles such that no bootstrap current is driven. The drift-kinetic equation is solved numerically with the guiding-centre δf code HAGIS[2]. The deviation δf of the distribution function from a prescribed Maxwellian is represented by marker particles whose equations of motion are solved in Boozer's magnetic coordinates. The pitch angle part of the Fokker-Planck collision operator is modeled by a Monte Carlo procedure[3,4]. However, here the conservation of momentum is maintained not on average over the flux surface, but locally with respect to the helical angle. In the following the banana orbit width is denoted by $w_b = q\rho_i/\varepsilon^{1/2}$, ρ_i is the ion gyro radius and $\varepsilon = r/R$. We use the following form of the time-dependent electric potential of the rotating island[5],

$$\Phi = -\frac{q\omega}{m}\left\{(\psi-\psi_s) - \mathrm{sign}(\psi-\psi_s)\,\frac{w_\psi}{\sqrt{2}}\,(\sqrt{\Omega}-1)\,\Theta(\Omega-1)\right\} \tag{2}$$

($\Theta(x) = 1$ for $x > 0$, $\Theta(x) = 0$ for $x < 0$) which consists of two contributions. The first part is constant on the unperturbed flux surfaces and causes the plasma in the island to co-rotate with the island. The second part is constant on helical flux surfaces, rotates with the island and vanishes inside. Far away from the island, where the plasma is at rest, the two contributions to the electric field cancel each other. The resulting electric field along a radius through the O-point of the island is shown by the green curves in Fig. 1. Outside the separatrix the electric field has a scale length proportional to the island width. The first consequence of the radial electric field E_r is a parallel flow of magnitude $u_\parallel = \langle E_r/B_p\rangle$ (the brackets denote the flux surface average, B_p is the poloidal magnetic field), because the trapped particles on average cannot follow the poloidal $E \times B$ rotation, and by collisions the rotation of the passing particles is damped, too. In Fig. 1 the parallel flow is shown for two cases with islands of different sizes. If the island width is not large compared to the orbit width, the parallel flow is reduced inside the island and spread out to the plasma outside the separatrix; hence, the relation $u_\parallel = \langle E_r/B_p\rangle$ is not valid then.

The second effect of the time-dependent electric field is a polarization current across flux surfaces which is strongly enhanced over the classical value $nm_i\dot{E}_r/B^2$ due to the finite banana orbit width. According to the analytic theory [5], in the collisional regime $\nu \gg \omega$ it is $j_p = (nm_i/B_p)\,du_\parallel/dt$, the enhancement factor is q^2/ε^2. In the low-collisionality regime, the parallel flow is to leading order constant on the flux surface, only the next order contribution leads to a polarization current which is smaller by a factor $\varepsilon^{3/2}$. The polarization current is carried by the ions since their orbits have a much larger radial extent than the electron orbits, and it is closed by electron currents parallel to the magnetic field which contribute to the current in the integral in Eq. (1).

From $\nabla_\parallel j_\parallel = -\nabla_\perp \cdot \vec{j}_\perp$ we obtain $j_\parallel \sim j_\perp/w^2$, since the parallel gradient $k_\parallel\,\partial/\partial\xi$ is proportional to the island width w, whereas the perpendicular gradient is inversely proportional

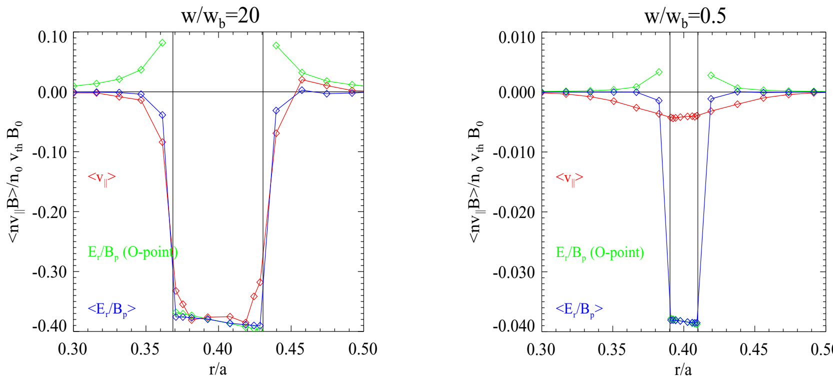
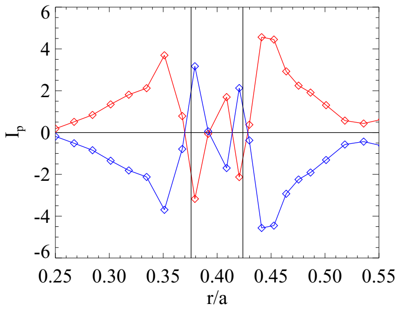


Fig. 1: Parallel flow in the island versus radius through the O-point (red) compared to E_r/B_p (green) and to the flux surface average $\langle E_r/B_p\rangle$ (blue) for two ratios of island size to orbit width.



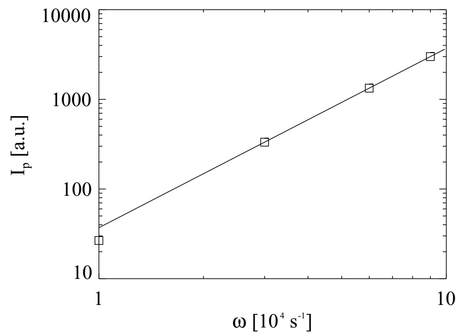


Fig. 2 (left): Flux surface averages $I_{p\pm}$ along the radius through the O-point.
Fig. 3 (right): $I_p = \langle(\nabla_\perp \cdot \vec{j}_\perp) \sin\xi\rangle$ varies quadratically with the island rotation frequency ω.

to w (the scale length of the electric field). The ion current across the flux surface is

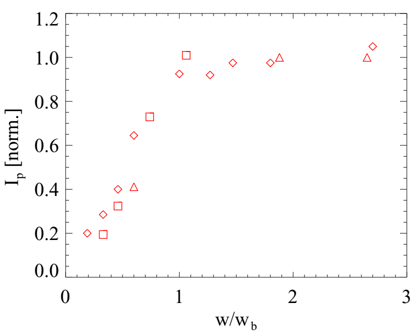
$$\vec{j} \cdot \nabla\Omega/|\nabla\Omega| = \int \vec{v}_d \cdot \nabla\Omega/|\nabla\Omega|\ \delta f d^3\vec{v} = \int \dot{\Omega}/|\nabla\Omega|\ \delta f d^3\vec{v}. \tag{3}$$

Here, $\dot{\Omega}$ is the rate of change of the helical flux Ω along the ion trajectory. We calculate integrals of $j_\perp$ over part of the helical flux surface

$$I_{p+} = \iint_{\sin\xi>0} \frac{\dot{\Omega}}{|\nabla\Omega|} \frac{d\xi d\theta}{\sqrt{\Omega + \cos\xi}} \qquad I_{p-} = \iint_{\sin\xi<0} \frac{\dot{\Omega}}{|\nabla\Omega|} \frac{d\xi d\theta}{\sqrt{\Omega + \cos\xi}} \tag{4}$$

which are of opposite sign and combine them as follows, $I_p = I_{p+} - I_{p-}$ (s. Fig. 2). We observe the quadratic dependence of I_p on ω (Fig. 3) derived in Ref. [5] for the regime $\omega \gg k_\| v_{\rm th}$ (note that $\omega_* = 0$ owing to the flat density and temperature profiles). Also, for big islands with $w \gtrsim w_b$ the scaling $I_p \sim w^0$ (i.e. $j_\| \sim 1/w^2$) from the thin-orbit theory is obtained (Fig. 4). At small islands, however,the polarization current is strongly reduced by a factor proportional to (w/w_b) (Fig. 4). This implies that in small islands the drive for the NTM [Eq. (1)] increases only like $1/w^2$, not like $1/w^3$. Hence, for $w < w_b$ not only is the drive by the loss of bootstrap current reduced[1], but a possible stabilizing contribution of the polarization current is reduced, too.

Next, the transition from the low-collisionality limit valid for $\nu/\varepsilon\omega \ll 1$ to the collisional limit for $\nu/\varepsilon\omega \gg 1$ is examined. We find that the transition occurs at about $\nu/\omega = 1$, hence



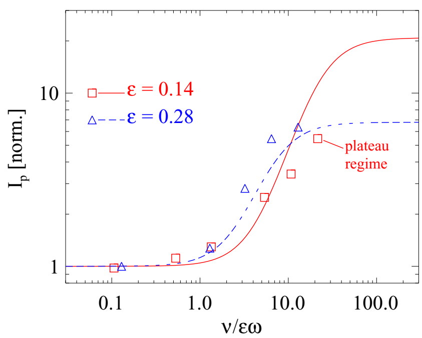
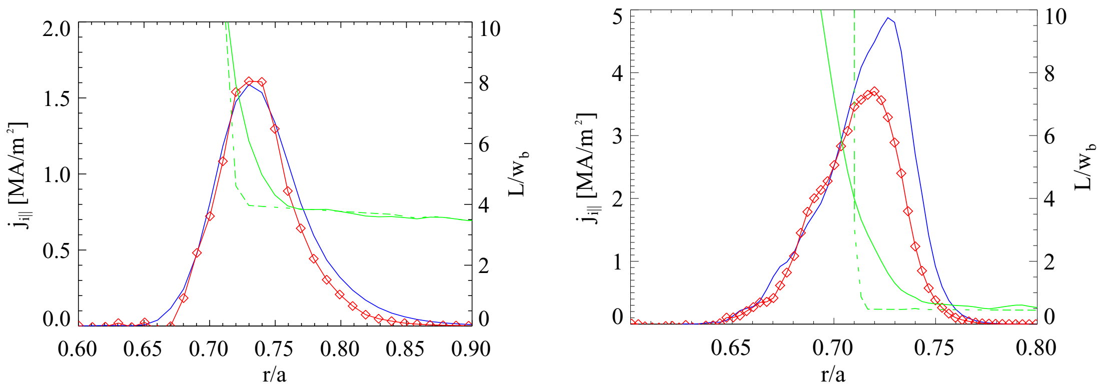


Fig. 4 (left): $I_p/I_p(\nu \ll \varepsilon\omega)$ versus the ratio of island half-width to the banana orbit width. Different parameters ε, w, T.
Fig. 5 (right): Collisionality dependence of the polarization current. The curves show the function $g(\nu/\omega, \varepsilon) = (\nu^2/\omega^2 + \varepsilon^{1/2})/(\nu^2/\omega^2 + \varepsilon^{-1})$ [6].



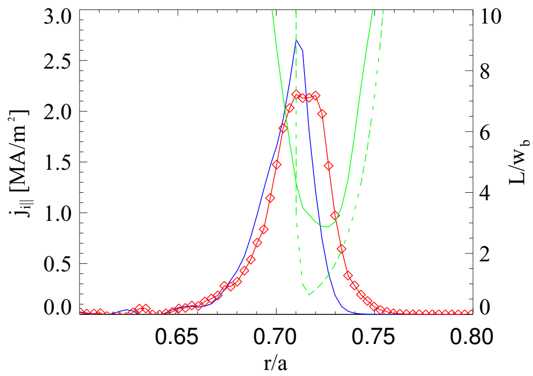


Fig. 6 (top): The bootstrap current at a steep ensity profile. Fig. 7 (left): The bootstrap current at a steep density and temperature profile (see text for colors).

at rather large values of $\nu/\varepsilon\omega$. The results are close to the relation derived in Ref.[6]. (Results reported in Ref.[7] were different, because the momentum was conserved during collisions only on average over the flux surface.) Within the regime $\omega > k_\parallel v_{\rm th}$ high values of ν/ω can be in the plateau regime $\nu_* = \nu q R/\varepsilon^{3/2} v_{\rm th} > 1$, then the high collisionality limit is not attained.

The same code, but without the helical field perturbation, is used to calculate the bootstrap current in the presence of a large density or temperature gradient. Large gradients exist, e.g., at the position of a transport barrier. A density profile of the form $n = n_c exp(-(\psi - 0.5\psi_{\rm edge})/L_\psi)$ for $\psi > 0.5\psi_{\rm edge}$ is assumed, and either a similar temperature profile or a flat temperature profile. Results for a steep density profile and a flat temperature profile are given in Fig. 6. Shown are the numerical results (red) and the results of the neoclassical theory (blue); also shown is the ratio of gradient length and orbit width (green) for both the prescribed profile of the distribution function f_0 (dashed) and the profile of the resulting distribution function $f = f_0 + \delta f$(solid). As compared to the neoclassical theory the current obtained from the simulations is reduced, if the gradient length is smaller than a few orbit widths. When also the temperature profile is steep (Fig. 7), the current can depend non-locally on the gradient; then it can be locally enhanced.

[1] E. Poli *et al.*, Plasma Physics Contr. Fusion **45** (2003) 71.
[2] S.D. Pinches *et al.*, Comp. Phys. Comm. 111, 133 (1998).
[3] Z. Lin, W.M. Tang, W.W. Lee, Phys. of Plasmas 2, 2975 (1995).
[4] A. Bergmann *et al.*, Phys. Plasmas **8** (2001) 5192.
[5] H.R. Wilson *et al.*, Phys. Plasmas **3** (1996) 248.
[6] A.B. Mikhailovskii *et al.*, Plasma Physics Contr. Fusion **42** (2000) 309.
[7] A. Bergmann *et al.*, Proc. Fus. Energy Conf. IAEA, Lyon 2002, TH-P1-01