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Navier-Stokes Neutral and Plasma Fluid Modelling in 3D

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1 Introduction

Coupled modelling of plasma and neutral fluids is a standard technique for the description of edge plasma phenomena in fusion devices. An exact treatment requires the solution of three Navier-Stokes equations for the neutral momentum. However, in most cases only a simplified treatment of the neutral particle momentum is used. Such a simplified ansatz considers a 1D Navier-Stokes equation for the neutral parallel momentum while the perpendicular momentum is assumed to be driven by the neutral particle pressure and the corresponding velocities are determined in the framework of a diffusive model.

In this work, the 3D scrape-off layer (SOL) transport code BoRiS is tested on three different neutral models (diffusive, parallel Navier-Stokes and full Navier-Stokes).

The tests were conducted with a standard SOL physics model with equations for the plasma (electrons - e, ions - i) and the neutrals (0) [1, 2]. Both ions and neutrals represent the heavy species ($a = i, 0$) having the same mass $m_0 = m_i$ (hydrogen mass) and the same temperature $T_0 = T_i$.

With parallel and perpendicular velocities according to $\vec{V} = \vec{V}_{\|} + \vec{V}_{\perp} = \vec{u} + \vec{v}$ we have the following set of equations:

$$\frac{\partial}{\partial t}(n_a) + \vec{\nabla} \cdot (n_a \vec{V}_a) = S_n^a \tag{1}$$

$$\frac{\partial}{\partial t}\Big(m_a n_a \vec{u}_a\Big) + \vec{\nabla} \cdot \Big(m_a n_a \vec{u}_a \vec{V}_a - \eta_a \vec{\nabla} \vec{u}_a\Big) = -\vec{\nabla}_{\|} p_a + \vec{S}^a_{m\|} \tag{2}$$

$$\frac{\partial}{\partial t}\Big(m_0 n_0 \vec{v}_0\Big) + \vec{\nabla} \cdot \Big(m_0 n_0 \vec{v}_0 \vec{V}_0 - \eta_0 \vec{\nabla} \vec{v}_0\Big) = -\vec{\nabla}_{\perp} p_0 + \vec{S}^0_{m\perp} \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p_e\right) + \vec{\nabla}\cdot\left(\frac{5}{2}T_e n_e \vec{V}_e - \kappa_e \vec{\nabla} T_e\right) = \vec{u}_e \vec{\nabla}_{\parallel} p_e + Q_{ei} + S_q^e \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}\sum_a p_a\right) + \vec{\nabla}\cdot\left(\frac{5}{2}T_i n_a \vec{V}_a - \sum_a \kappa_a \vec{\nabla} T_i\right) = \vec{u}_a \vec{\nabla}_{\parallel} p_a - Q_{ei} + S_q^i \quad . \tag{5}$$

The above equations are written with respect to the physically relevant directions defined by the left-handed curvilinear system $\vec{B}$, $\vec{\nabla}s$, $\vec{\nabla}s \times \vec{B}$ $(\parallel, \perp_1, \perp_2)$. The plasma properties $n_i = n_e$, u_i, T_e and T_i are described by equations (1) - plasma continuity, (2) - plasma parallel momentum, (4) - electron internal energy and (5) - ion-neutral internal energy respectively. The perpendicular plasma velocities follow from $\vec{V}_{\perp,i} = \vec{v}_i = -D_\perp \vec{\nabla}_\perp n_i$, where $D_\perp$ is the anomalous diffusivity. For full Navier-Stokes neutrals $(a = 0)$ in this slab setup we have equations (1) - neutral continuity, (2) and (3) - neutral parallel and perpendicular momentum respectively again together with (5) - ion-neutral internal energy. The equations contain viscosities η_a, thermal conductivities $\kappa_{e,i}$ and sources S_n^a, $\vec{S}^a_{m\parallel}$, $\vec{S}^0_{m\perp}$ and $S_q^{e,i}$ which account for ionization and charge exchange. Q_{ei} is the heat exchange between ions and electrons.
If we assume the propagation of neutrals only being driven by their thermal pressure, equations (2) and (3) are replaced by the neutral velocity in the diffusive model

$$\vec{V}_0 = \vec{u}_0 + \vec{v}_0 = -D_0(\vec{\nabla} n_0/n_0 + \vec{\nabla} T_i/T_i) \quad . \tag{6}$$

The more advanced parallel Navier-Stokes model solves equation (2) for u_0 and determines v_0 from the corresponding part of (6).

2 Setup and boundary conditions

The setup is motivated by an experiment/simulation described in [3]. We consider a slab with $L_x = 0.1$m, $L_y = 0.1$m and $L_z = 4.0$m. In BoRiS, this slab is internally represented by a corresponding slab in generalized (magnetic) coordinates (s, θ, ϕ) which are normalized to unity. For simplicity (x, y, z) are perfectly aligned with s, θ, ϕ respectively and the magnetic field is along the z axis. Boundary conditions are as follows: The $\phi = 0$ ($z = 0$m) boundary represents a wall with zero-flux conditions and a slit along θ (y) around the center at $s = 0.5$ ($x = 0.05$m). For the slit region both electron and ion temperatures are fixed to $T_e = T_i = 15$eV. The plasma parallel momentum flux is set to zero and the plasma density varies between $3\cdot 10^{18}\mathrm{m}^{-3}$ $(\theta = 0)$ and $2.7\cdot 10^{18}\mathrm{m}^{-3}$ $(\theta = 1)$. At $\phi = 0$ the neutral parallel velocity is set to zero while both the neutral density and the neutral perpendicular velocities face zero-gradient conditions.
The $\phi = 1$ ($z = 4$m) boundary represents a target plate with appropriate sheath conditions for all quantities. Accordingly, T_e and T_i are determined by their corresponding heat fluxes $\Gamma_q^{e,i} = \delta_{e,i} T_{e,i} \Gamma_n$ with the sheath coefficients $\delta_e = 5$ and $\delta_i = 3.5$. For the ion heat flux an additional contribution $\Gamma_q^i = -1.5(T_e + T_i)\Gamma_n R_E$ accounts for energy recycled by the neutrals with a recycling coefficient $R_E = 0.3$. The plasma parallel velocity is set to $u_i = c_s$ ($c_s = ((T_e + T_i)/m_i)^{1/2}$ being the local ion accoustic speed).

The neutral particle flux at the plate is $\Gamma_o = -R_0 \Gamma_n$ and the neutral parallel velocity is set to $u_0 = -R_M u_i$ with particle and momentum recycling coefficients $R_0 = 1.0$ and $R_M = 0.3$ respectively. Again, the neutral perpendicular velocities have zero-gradient conditions.
The side walls at $s = 0$ and $s = 1$ have zero-flux conditions for the plasma and zero-gradient conditions for the neutrals except for the neutral perpendicular velocity $v_{0,1}$ which is fixed to $v_{0,1} = 0$. Likewise, $\theta = 0$ and $\theta = 1$ have zero-flux and zero-gradient conditions for the plasma and the neutrals respectively except for $v_{0,2} = 0$.

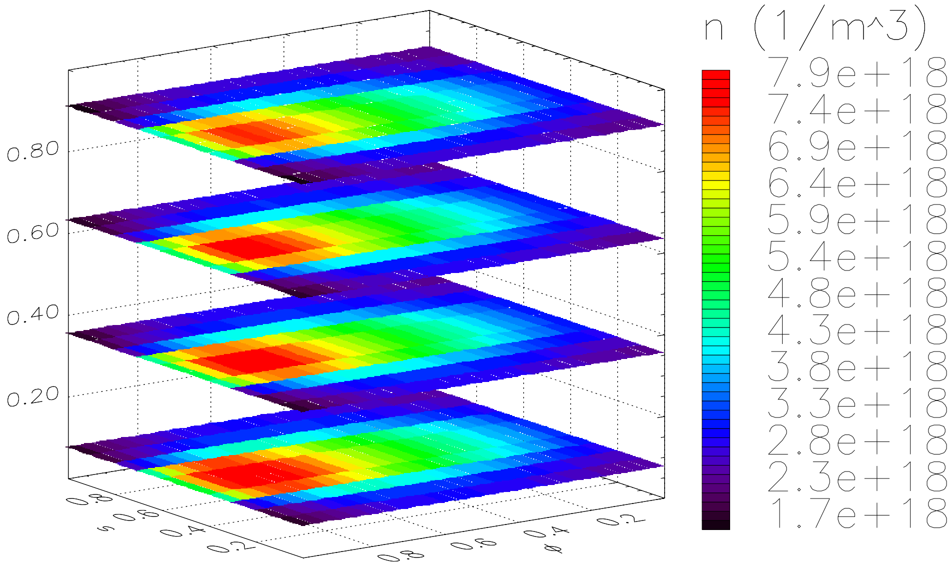

Figure 1: Plasma density $[m^{-3}]$ in a 3D slab in magnetic coordinates.

3 Results and discussion

The three different neutral models were tested and the results are compared to each other. The main characteristics for the plasma properties are found to be very similar with very little differences between the parallel and full Navier-Stokes model. Fig.1 illustrates the formation of the ionization zone with enhanced density in front of the target plate. For both the parallel and full Navier-Stokes model the momentum recycling coefficient of $R_M = 0.3$ limits the propagation of neutrals into the plasma and subsequently shifts the ionization zone towards the target plate. This effect is more pronounced than in the diffusive model where there is no such limitation of the neutral parallel velocity. The high density in front of the plate results leads to high viscosities for the neutrals. In the diffusive model the neutrals feel no friction and the velocity pattern reflects the presence of parallel and perpendicular pressure gradients (left part of fig.3). For the parallel Navier-Stokes model, friction leads to an equilibration of the parallel velocities but still allows for variations of the perpendicular components (middle of fig.3). Subsequently the full Navier-Stokes model (right part of fig.3) shows equilibration of both parallel and perpendicular velocities due to the high viscosities in front of the target plate, which results in a moderate variation of neutral velocities in parallel direction on the scale of the neutral mean free path (≈ 0.1m).

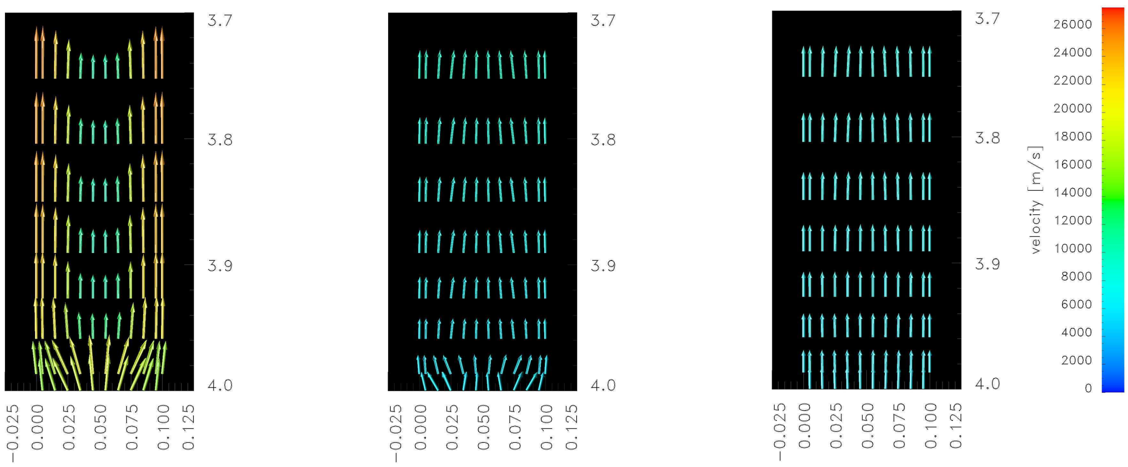

Figure 2: Neutral velocity $[m/s]$ in a 3D slab at the target plate ($z = 4$m).
(left - diffusive, middle - parallel Navier-Stokes, right - Navier-Stokes)

In BoRiS the coupled equations are solved simultanously with the Newton method utilizing a variety of sophisticated solvers [4]. The above results were obtained with a parallel version of the code using an iterative sparse solver *GMRES* in combination with ILU(0) preconditioning. The runs were performed with a false time-stepping procedure and convergence was found within ≈ 50 iterations. A stable convergence behaviour was found with the preconditioner being updated every five steps.

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