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Fast recovery of vacuum magnetic configuration of W7-X stellarator using Function Parametrization and Neural Nets

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Introduction

W7-X, a five-period, fully optimized stellarator with an average major radius $R_0 \sim 5.5$ m and a minor radius $a \leq 55$ cm, currently under construction at IPP-Greifswald, Germany, is built with super-conducting coils to show the steady state capability of stellarators. However, steady state needs continuous equilibrium information for monitoring and controlling the discharges. Although the time scales are long compared to tokamaks, the computational effort for calculating 3-dimensional MHD equilibria is also orders of magnitude higher. This has led us to start the development of a fast equilibrium recovery for W7-X. As a starting point and also to investigate the richness of magnetic configurations, of which only 9 physically interesting examples have been examined till now, a fast recovery of vacuum magnetic configurations, described by the flux surface geometry and profile parameters, is carried out using the method of Function Parametrization (FP). For some of the parameters, the quality of FP recovery is compared with the method of Artificial Neural Network (ANN), where the vacuum parameters are non-linearly regressed in terms of a linear combination of the coil currents. The results of these are shown in this paper.

FP, ANN and database generation

The FP method relies on a statistical analysis of a large database of simulated experiments. This aims at obtaining a simple functional representation for the intrinsic physical parameters of interest of a system in terms of accessible measurement values (for details see [1,2]). The parameters are expressed as polynomials, in terms of the most strongly varying, uncorrelated linear combinations (known as Principal Components PCs) of the measurements. The database is used as the basis of a statistical regression in order to determine the coefficients of the polynomials.

The ANN [2,4], on the other hand, is a non linear mathematical function that can generate an inverse mapping by means of non-linear regression. This mapping is effected by what is called the training (which is also central to FP), where a large database of equilibrium states (known as examples), comprising of both the measurement space and the parameter space, is applied to the network and certain adjustible parameters, called weights, are first randomly initiated and subsequently optimized.

In this study, the database contains the vacuum magnetic field configurations for W7-X. The database is created by using a field line tracing code delivering the relevant parameters such as profiles and geometry of flux surfaces and the magnetic field variation on the flux surfaces. The latter information is given as Fourier coefficients with respect to magnetic coordinates for flux surface geometry and the magnetic field strength B. Each flux surface has its own set of Fourier coefficients which are therefore profile parameters (i.e., those with a radial dependency on the effective radius r_{eff}). Other profile parameters such as the rotational transform $\iota\!\!\!-$ and magnetic well depth $V' = \frac{dV}{d\psi_T}$ (V= flux surface volume; ψ_T=toroidal flux), as well as the effective minor radius of the configuration a_{eff} are also calculated. Additionally, we extract information about the major natural islands, in terms of their location (r^{is}_{eff}) and width (w^{is}). The importance of this lies in the fact that for low shear stellarators, islands play a crucial role in the equilibrium, stability and transport, and the islands at the boundary have special relevance for the functionality of divertors. These vacuum configuration parameters are recovered as functions of the measurements that produce them.

In a stellarator, vacuum field configurations are generated entirely by external coil currents. For W7-X, these are generated by a system of seven independent coils, consisting of 5 modular field coils (3-D shaped) carrying currents $I_1, I_2, \ldots, I_5$ and 2 planar coils with currents I_A, I_B. The characteristics of the vacuum magnetic configuration do not depend on the absolute values of the coil currents but only on their (six) ratios $i_2, \ldots, i_5, i_A, i_B$ (normalized to I_1 held constant at 12 kA). The current ratios span the space of possible W7-X configurations defined by $i_{2,\ldots,5} \in [0.6, 1.2]$, $i_{A,B} \in [-1, 1]$. The ranges are derived from engineering limits for the coil currents. Summary statistics, comprising the population mean, standard deviation σ about the mean, and the extreme values, for the 6 current ratios are shown in Table 1. The point $i_{2,\ldots,5} = 1, i_{A,B} = 0$ represents the standard case for W7-X, with $\iota\!\!\!-$ at the configuration boundary $\iota\!\!\!-_b$=1. In this study the space is covered by a database of 715 configurations with coil current ratios randomly chosen within the above ranges. This ensures that the database covers all experimentally accessible configurations.

It is to be noted that for some of the parameters which are likely to be influenced by the conditions near the configuration boundary, the database has been subdivided into separatrix-bound and limiter-bound configurations. The separatrix cases correspond to the major resonances with $\iota\!\!\!-_b$ ∈ [5/4, 5/5, 5/6]. These subsets of the original population include 33, 307 and 121 cases respectively. 249 cases are recorded for the limiter configurations (indicated as lim in Table 3 and 4 later). Since more than one internal major island chain has often been detected in a single configuration, the island database contains 40 cases of 5/4 chain, 455 cases of 5/5 and 161 5/6 island chains.

Predictor and model selection

From statistical terminology, 'predictor' refers to the arguments of the polynomial that models a dependent variable. In this work, the coil current ratios are generated independently, and hence are only weakly correlated ($\leq$ 0.09) due to the finite size of the (randomly generated) database. So the dimension reduction step in the PC analysis (PCA) is unnecessary here.

The FP model used to represent the vacuum (scalar) parameters (y) is either a mixed quadratic [3] or a mixed cubic given by $y = \sum_{i=0}^{N}\sum_{j=0}^{i}\sum_{k=0}^{j} a_{ijk}x_i x_j x_k$.

For the profile parameters, the quadratic model equation is modified as $y = \sum_{k=0}^{P}\sum_{i=0}^{N}\sum_{j=0}^{i} a_{ij}x_i x_j r^k_{eff}$. Here N = 6, the number of current ratios; P = order of polynomial in r_{eff}. The number of model coefficients are $\frac{(N+1)(N+2)}{2}$ =28 (quadratic) and $\frac{(N+1)(N+2)(N+3)}{6}$=84 (cubic), much less than the 715 cases in the database.

For the ANN model, let x_i and y_k be the ith input and kth output with N_i inputs and N_{out} outputs in the model. Let there be N_h neurons in a single hidden layer. Then, if w_{ji} be the input weight corresponding to the jth hidden neuron and ith input, and w_{kj} be the output weight corresponding to the jth hidden neuron and kth output, then $y_k = \sum_{j=1}^{N_h} w_{kj} F(\sum_{i=1}^{N_i} w_{ji}x_i)$, where F is the non-linear transformation function which, in our case, is a symmetric sigmoid:

$$F(x) = \frac{2}{1+exp(-x)} - 1.$$

Results

The results of FP-recovery of the vacuum configuration parameters are listed in Table 2 and 3. Table 4 shows that some of the parameters are much better recovered by an ANN model. In Table 2, 3 and 4, and also in the text below, 'q', 'c' and 'nn' refer to the quadratic, cubic and ANN models respectively; $E^{(mo)}_{rms}$ means the rms error of regression according to model $mo \in [q, c, nn]$; R^2_{mo} denotes the R^2-statistic for model mo, adjusted for the degrees of freedom in the model, given by 1-$\left(\frac{E^{mo}_{rms}}{\sigma}\right)^2$. The ratios of the values of $\sqrt{1 - R^2_{mo}} = \frac{E_{rms}}{\sigma}$, for two models numerically shows the degree of improvement in the model indicated in the denominator of the ratio.

Table 2 shows the recovery accuracy of the profiles of $\iota\!\!\!-$ and the leading fourier coefficients as functions of r_{eff}. We find that the quantities are well recovered by a q-FP model in the current ratios. A cubic polynomial in r_{eff} was generally found sufficient. The improvement observed with a c-FP model (in current ratios) is not worth the increase of model size.

Table 3 displays the scalar parameter recovery details. Here $R_{0,36}$ refer to the axis positions at ϕ=0,36 degrees, and the difference is a measure of the radius of curvature in the bean-shaped plane (ϕ=0 degrees). The average axis position is $R_{av} = \frac{R_0+R_{36}}{2}$. $B_{0,36}$ gives the magnitude of the field strength on the magnetic axis at ϕ=0,36. Their sum is the average field B_{av} on the axis, while their difference is a measure of the mirror-field B_m on axis. Z_{18}, the vertical position of axis at ϕ=18, describes the helicity of the axis shape. The axis positions are in cm, while the magnetic fields are in Tesla. $\iota\!\!\!-_{0,b}$ and $V'_{0,b}$ are the $\{$axis,boundary$\}$ values of $\iota\!\!\!-$ and V' respectively. The scalar parameters are generally recovered significantly better with a c-FP than a q-FP model, as shown by the ratio of $\frac{E_{rms}}{\sigma}$ for the two models (last column).

Table 4 proves the dramatic improvement in the recovery of some of the parameters brought about by the non-linear regression with a sigmoidal function for the nn, as compared even with a c-FP model. Interestingly, these parameters are connected to the configuration boundary, which in our case is dominated by natural islands (or the separatrix). One exception has been the magnetic well depth at the boundary, $\frac{\delta V'}{V'(0)}$ for which a c-FP model is found to give the best results.

In an inverse transformation, we also try to recover the coil current ratios as functions of the configuration. This is important because it is often necessary to choose the coil currents corresponding to certain magnetic configuration properties. Here we choose the first 8 parameters listed in Table 3 as those describing physical properties of the configuration. A PCA shows 6 significant PCs to be used as independent variables in a q-FP model. Recovery is good, with R^2_q for the 6 current ratios found to be:

Current ratio:	i_2	i_3	i_4	i_5	i_A	i_B
R^2_q:	0.9999	0.9989	0.9967	0.9994	0.9999	0.9995

Attempting to find the most important field and geometric quantity for the current recovery, we proceed by removing one quantity in turn from the input set and regress all over again. We observe that except for V'_0, the chosen parameters were very important for the extraction of the current ratios, with their removal resulting in $\frac{E_{rms}}{\sigma}$ worsening (compared to the case with all 8 inputs) by a factor in the range of 1.35 to 3.87. Individually, it is seen that i_2 and i_B recovery is most influenced by the axis mirror field while the i_A-recovery is the worst when R_{av} is removed from the input set. On the other hand, removal of V'_0 influences only i_2 to any significant extent, with $\frac{E_{rms}}{\sigma}$ worsening by a factor of 1.41.

Table 1: Summary statistics for 715 vac. configs

current ratio	Mean	σ	Minimum	Maximum
i_2	0.9113	0.1744	0.6005	1.1984
i_3	0.9103	0.1702	0.6009	1.1992
i_4	0.8923	0.1756	0.6011	1.1992
i_5	0.8866	0.1727	0.6020	1.1981
i_A	0.0186	0.5629	-0.9973	0.9971
i_B	-0.0306	0.5529	-0.9815	0.9959

Table 2: Profile parameter recovery

parameter	$E_{rms}^{(q)}$	R_q^2	parameter	$E_{rms}^{(q)}$	R_q^2
$\iota(5/5)$	0.0012	0.9992	$R_{1,0}$	0.2545	0.9997
$\iota(5/6)$	0.0007	0.9991	$R_{1,1}$	0.0479	0.9985
$R_{0,1}$	0.0660	0.9989	$Z_{1,-1}$	0.1005	0.9997
$R_{0,0}$	0.0414	0.9995	$Z_{1,0}$	0.2943	0.9998
$R_{1,-1}$	0.1487	0.9996	$Z_{1,1}$	0.0370	0.9990

Table 3: Recovery of scalar parameters

parameter	$E_{rms}^{(q)}$	$E_{rms}^{(c)}$	R_q^2	R_c^2	$\sqrt{\frac{(1-R_q^2)}{(1-R_c^2)}}$
R_{av}	0.0541	0.0102	0.9999	1.0000	–
$\frac{R_0-R_{36}}{R_0+R_{36}}$	0.0001	0.00002	0.9997	1.0000	–
$\frac{Z_{18}}{R_0+R_{36}}$	0.00006	0.00001	0.9997	1.0000	–
ι_0	0.0028	0.0008	0.9989	0.9999	3.32
$\iota_b(5/5)$	0.003	0.003	0.8229	0.8503	1.09
V_0'	0.6113	0.1202	0.9995	1.0000	–
$V_b'(lim)$	1.7745	1.3609	0.9963	0.9978	1.30
B_{av}	0.0003	0.00005	1.0000	1.0000	–
B_m	0.0008	0.0001	0.9999	1.0000	–
$\frac{B_{18}}{B_0+B_{36}}$	0.0003	0.00003	0.9999	1.0000	–
$a_{eff}(5/5)$	0.9333	0.5611	0.9890	0.9960	1.66
$a_{eff}(5/6)$	0.9491	0.4975	0.9860	0.9961	1.90
$a_{eff}(lim)$	1.3234	0.9622	0.8833	0.9383	1.37
$\frac{\delta V'}{V'(0)}(5/5)$	0.0005	0.0002	0.9695	0.9951	2.80
$\frac{\delta V'}{V'(0)}(5/6)$	0.0003	0.0001	0.9801	0.9971	2.62
$\frac{\delta V'}{V'(0)}(lim)$	0.0056	0.0047	0.5637	0.7018	1.21
$r_{eff}^{is}(5/5)$	1.3624	1.1124	0.9862	0.9908	1.22
$r_{eff}^{is}(5/6)$	1.1619	0.6649	0.9873	0.9959	1.76
$w^{is}(5/5)$	1.2301	1.1011	0.6442	0.7150	1.12
$w^{is}(5/6)$	0.5951	0.5287	0.8598	0.8894	1.13
R_{00}^{axis}	0.0854	0.0434	0.9998	0.9999	1.41
R_{01}^{axis}	0.0581	0.0347	0.9941	0.9979	1.68
Z_{01}^{axis}	0.0434	0.0282	0.9962	0.9984	1.54

Table 4: Improvement with nn

parameter	$E_{rms}^{(c)}$	$E_{rms}^{(nn)}$	R_c^2	R_{nn}^2	$\sqrt{\frac{(1-R_c^2)}{(1-R_{nn}^2)}}$
$\iota_b(5/5)$	0.003	0.0007	0.8503	0.9597	1.92
$V_b'(lim)$	1.3609	0.1416	0.9978	0.9999	4.69
$a_{eff}(5/5)$	0.5611	0.0601	0.9960	0.9999	6.32
$a_{eff}(lim)$	0.9622	0.1830	0.9383	0.9977	5.19
$r_{eff}^{is}(5/5)$	1.1124	0.0628	0.9908	0.9999	9.59
$r_{eff}^{is}(5/6)$	0.6649	0.1715	0.9959	0.9996	3.20
$w^{is}(5/5)$	1.1011	0.2510	0.7150	0.9855	4.43
$w^{is}(5/6)$	0.5287	0.3105	0.8894	0.9515	1.51

Conclusions:

We have studied the recovery of the important magnetic and geometric parameters that describe the vacuum configuration of W7-X. FP with a full mixed quadratic model failed in several cases and a much larger model size involving a full mixed cubic model had to be used. The cubic model was found successful for most of the parameters. However, for a_{eff}, ι_b, V_b' and the island location and width, even a cubic model was found insufficient. Incidentally, these are the quantities strongly influenced by the boundary islands. We observe a dramatic improvement in the recovery of these parameters using the non-linear regression of the nn. An exception to this has been the magnetic well depth at the boundary. The accuracy of its recovery is found the best with a cubic model. On the other hand, the current ratios are well recovered by FP in terms of physical properties of the configuration. All quantities except V_0' prove important for this recovery. As a part of our future plans, we would like to include in our analysis other configuration properties, such as the effective helical ripple and trapped particle fraction which are related to neoclassical transport.

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