

ELIMINATION OF THE BOOTSTRAP CURRENT FACTOR IN STELLARATORS WITH POLOIDALLY CLOSED CONTOURS OF THE MAGNETIC FIELD STRENGTH

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Abstract The value of bootstrap current in stellarator configurations depends strongly on the type of configuration. For stellarators with mod- B contours closed in the poloidal direction, one expects that bootstrap current value can be made nearly vanishing as, e.g. in W7-X. In the present paper, the effect of the behaviour of the contours of B on the magnetic surfaces on the value of the structural factor describing the bootstrap current in the long mean free path ($lmfp$) is analyzed numerically by using analytical expressions. The effect of such parameters as the number of periods and the value of $\langle \beta \rangle$ on the structural factor is studied. It is found that in the type of stellarators considered, it is possible to make the $lmfp$ bootstrap current negligible without violation of good neoclassical confinement properties.

Introduction

The value of bootstrap current in stellarator configurations depends strongly on the topography of the surfaces $B = \text{constant}$. For stellarators with axial quasisymmetry, $B = B(s, \theta_B)$, where s, θ_B, ζ_B are magnetic flux coordinates, this current causes the rotational transform to increase [1]. In quasihelically symmetric stellarators [2], when $B = B(s, \theta_B - N\zeta_B)$, the bootstrap current increases the rotational transform counted relative to the direction of quasisymmetry, $\theta_B = N\zeta_B$, thus, it diminishes the tokamak-like rotational transform and can impede plasma equilibrium. For the third type of stellarators, in which the contours of B on magnetic surfaces close poloidally [3], the direction of the bootstrap current cannot be predicted a priori (see Fig. 1).

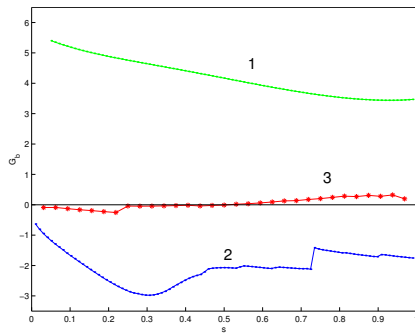


Fig. 1. Structural factor of bootstrap current for examples of tokamak (1), quasi-helically symmetric (2) and quasi-isodynamic (3) stellarator configurations.

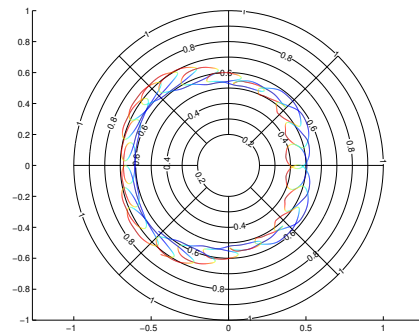


Fig. 2. Particle drift trajectory in near quasi-isodynamic configuration with $B=\text{constant}$ lines on the magnetic surfaces in the poloidal direction.

The characteristic feature of the particle drift trajectories in near quasi-isodynamic configurations with $B=\text{constant}$ lines in the poloidal direction on the magnetic surfaces [3] is seen from Fig. 2, where the projection of drift motion of one guiding centre on the plasma cross-section is shown. The red (blue) color corresponds to positive (negative) parallel velocity. One can conjecture that the same particle during its drift motion should contribute to the bootstrap current

with different signs on the inward and outward parts of the trajectory. Thus, by appropriate choice of the positions of points with zero banana size one can make the bootstrap current vanishing. This argument can be illustrated also by the example of a mirror type configuration with mirror symmetry, when all magnetic field lines are closed and the rotational transform is equal to zero [4]. In this case, it is forbidden to have non-zero current in cross-sections of mirror symmetry, so that the bootstrap current should be equal to zero exactly in order to conserve the mirror symmetry.

In addition, it is known that the value of bootstrap current is nearly vanishing in the optimized stellarator W7-X which is close to being quasi-isodynamic. It was shown also [5] that for a six-period stellarator optimized with respect to quasi-isodynamicity [6], the bootstrap current changes the rotational transform only slightly. In the present paper, the possibility to nullify the structural (geometric) factor G_B of the bootstrap current is analyzed numerically by using the optimization procedure based on codes for calculation of equilibrium (VMEC code [7]), transition to magnetic (Boozer) coordinates (JMC code [8]) and on calculations of the G_B -factor from analytical expressions. Some $N=6$ and $N=9$ configurations optimized with respect to quasi-isodynamicity are studied for different values of $\langle \beta \rangle$. The calculations were performed on the supercomputers Himiko (Germany) and Prometeo (Switzerland).

Results of the optimization

The bootstrap current model in $1/\nu$ regime that has been used in this paper has been very compactly described in [9]. The formulas presented there have evolved from previous research on the subject [10]. The specific equations have the form:

$$\langle \mathbf{jB} \rangle = G_b(L_1 \frac{dp}{d\Phi} + L_2 \frac{dT}{d\Phi}),$$

where the structural factor G_B can be expressed as:

$$G_b(s) = \frac{1}{f_t} [\langle g_2 \rangle - \frac{3 \langle B^2 \rangle}{4B_{max}^2} \int_0^1 d\lambda \lambda \frac{\langle g_4 \rangle}{\langle g_1 \rangle}],$$

$$g_1 = \sqrt{(1 - \lambda B/B_{max})}$$

$$\mathbf{B} \cdot \nabla(g_2/B^2) = \mathbf{B} \times \nabla\Phi \cdot \nabla B^{-2},$$

$$\mathbf{B} \cdot \nabla(g_4/g_1) = \mathbf{B} \times \nabla\Phi \cdot \nabla g_1^{-1},$$

$$g_2(B_{max}) = g_4(B_{max}) = 0.$$

To calculate G_B in magnetic coordinates (which are used in the optimization), one needs to know only the flux functions and the magnetic field strength B . In the present paper, the structural factor was calculated for net current free configurations, so that the effect of bootstrap current itself on the geometric factor was not taken into account. Configurations with numbers of periods $N = 6$ and $N = 9$ and $\langle \beta \rangle = 10\%$ are optimized with respect to vanishing bootstrap current structural factor. The configurations obtained were investigated then for different values of $\langle \beta \rangle$. It is worth to note that during the optimization only the requirement of elimination of the bootstrap current was imposed. In spite of this, the main favourable properties (collisionless particle confinement and local interchange-mode stability) were conserved.

N=6 configuration. The 3D view of the $N = 6$ configuration [6] is shown in Fig. 3. Fig. 4 shows the cross-sections of the optimized configuration and some flux functions. The history of the optimization procedure is shown in Fig. 5 (compare the scales of G_B in Fig. 1 and Fig. 5). Fig. 6 shows that the quality of closure of contours of the second adiabatic invariant $\mathcal{J} = \int V_{||} dl$ is not violated seriously during the optimization.

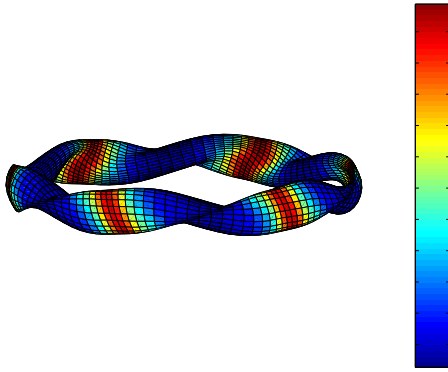
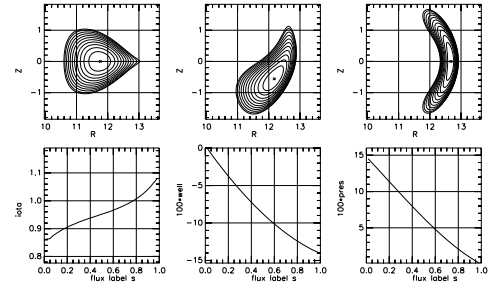
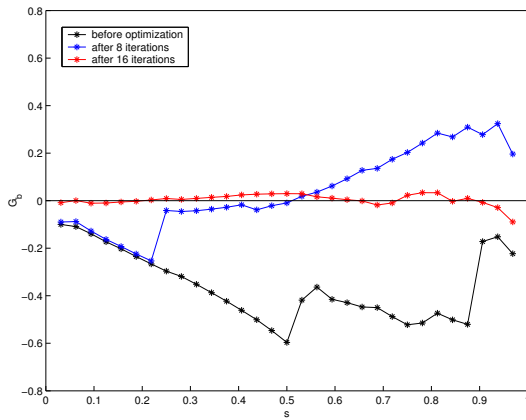
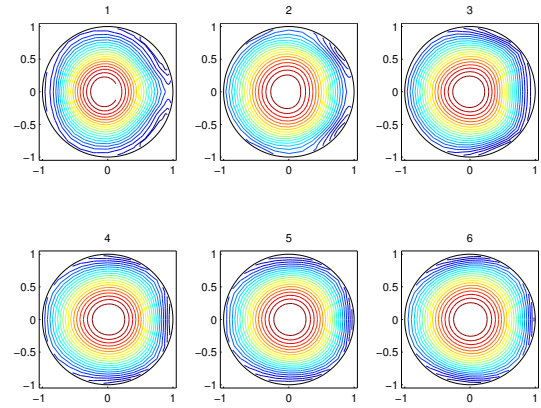
Fig. 3. 3D view of the $N = 6$ configuration.Fig. 4. Cross-sections of the $N = 6$ configuration.Fig. 5. Minimization of the geometric factor of the bootstrap current for the $N = 6$ configuration.

Fig. 6. Contours of the second adiabatic invariant.

$N=9$ configuration [11]. Fig. 7 shows the cross-sections and flux functions for the $N = 9$ configuration. The characteristic differences in comparison with the $N = 6$ system are the increased rotational transform and the negligibly small Shafranov shift for the large value of plasma pressure considered, $\langle \beta \rangle = 10\%$. The value of the structural factor here is larger than that for the $N = 6$ configuration (see Fig. 8).

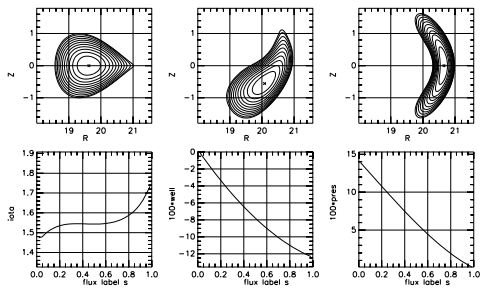
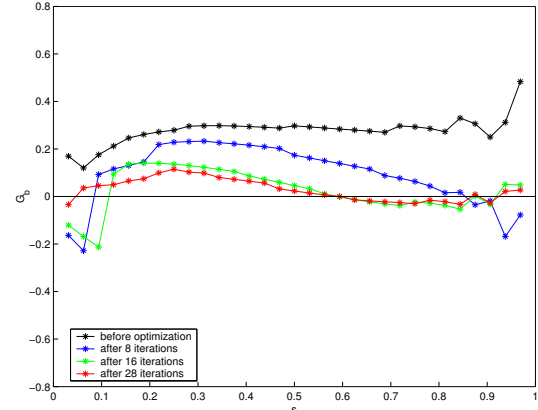
Fig. 7. Cross-sections of the optimized $N = 9$ configuration.Fig. 8. Minimization of the geometric factor of the bootstrap current for the $N = 9$ configuration.

Fig. 9 shows the Mercier and resistive mode stability properties of the configurations found. Finally, in Fig. 10, the dependencies of maximum $|G_B|$ on $\langle \beta \rangle$ for boundaries optimized at $\langle \beta \rangle = 10\%$ are shown.

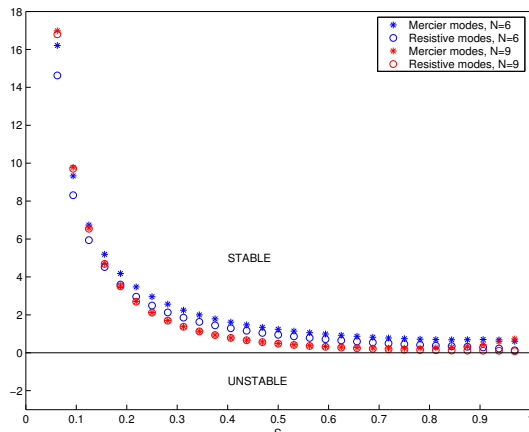


Fig. 9. Mercier and resistive mode stability properties for the configurations considered.

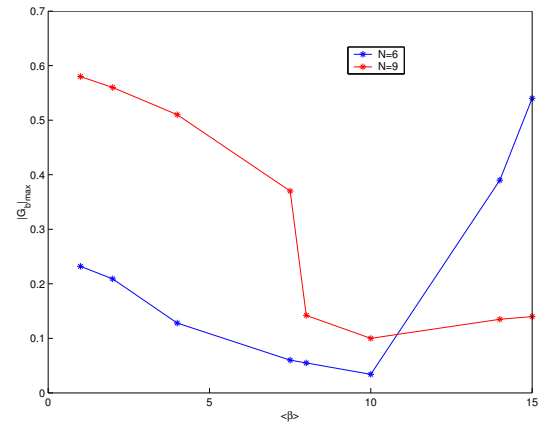


Fig. 10. Dependencies of maximum $|G_B|$ on $\langle \beta \rangle$ for $\langle \beta \rangle \geq 10\%$ optimized configurations.

Conclusions

It is shown by numerical optimization that in near quasi-isodynamic configurations with $B=\text{constant}$ lines on the magnetic surfaces in the poloidal direction, the bootstrap current can be eliminated with high accuracy.

The requirement of vanishing of the bootstrap current is not in contradiction with the conditions of improved collisionless particle confinement and local mode stability.

Acknowledgments This work was supported by INTAS Grant No 99-00592; by the Russian-Germany agreement WTZ-RUS-01/581; by Russian Federal program on support of leading scientific school researches, Grant No 2024.2003.2; by the Russian Fund for Basic Research, Grant No 03-02-16768; by the Fonds National Suisse de la Recherche Scientifique, Euratom; by Department of atomic science and technology, Minatom RF. One of the authors (M.I.M) would like to thank Prof. K. Yamazaki for helpful discussions during his stay in NIFS, Japan.

References

- [1] M.C. Zarnstorff et. al. Plasma Phys. and Controlled Fusion **43** A237 (2001).
- [2] J. Nührenberg, R. Zille, Phys. Lett. A **129** 113 (1988).
- [3] S. Gori, W. Lotz, J. Nührenberg, Theory of Fusion Plasmas (International School of Plasma Physics), Bologna: SIF 335 (1996).
- [4] W.A. Cooper, M.Yu. Isaev, M.F. Heyn et al., "New schemes for confinement of fusion products in stellarators" IAEA conference, Lyon, France, 2002.
- [5] W.A. Cooper, S. Ferrando i Margalet, S.J. Allfrey et.al. Plasma Phys. and Controlled Fusion **44** B357 (2002).
- [6] M.I. Mikhailov, V.D. Shafranov, A.A. Subbotin et.al., Nuclear Fusion **42** L23 (2002).
- [7] S.P. Hirshman and O. Betancourt, J. of Comput. Physics **96** 99 (1991).
- [8] J. Nührenberg, R. Zille, Theory of Fusion Plasmas (Varenna 1987), Editrice Compositori, Bologna 3 (1988).
- [9] J.L. Johnson, K. Ichiguchi, Y. Nakamura, M. Okamoto, M. Wakatami, N. Nakajima, Phys. Plasmas **6**, 2513 (1999).
- [10] K.C. Shaing and J.D. Callen, Phys. Fluids, **26** 3315 (1983); N. Nakajima and M. Okamoto, J. Phys. Soc. Jpn. **61** 833 (1992); K.Y. Watanabe et al., Nuclear Fusion **35** 335 (1995).
- [11] V.D. Shafranov, W.A. Cooper, M.Yu. Isaev et al., 30th EPS Conf. on Controlled Fusion and Plasma Physics, St. Petersburg, (2003).