# Interpretation of Plasma Dynamic Response to Additional Heating Power in ASDEX Upgrade and TCV 

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## Introduction

The study of the plasma dynamic response to additional heating power is very important for power deposition localisation and for perturbative transport investigations. Most of the time, data analysis based on Fourier transformation of the space-temporal signals is used to derive amplitude and phase profiles. This technique, though, is not capable of fully separating the contribution to the signals coming from the sawteeth and from the applied perturbation due to the strong phase locking between the two signals, leading to potentially misleading results. Hence, in order to treat the problem correctly, signal processing techniques that respect the spatially distributed character of the measurements are needed.

## Signal processing technique

A method based on a system identification using the singular value decomposition (SVD [1]) has been developed and is presented in this paper. In order to illustrate the method in a clear way, a simple example is used. To simplify, let us consider a purely diffusive particle transport model, cylindrical geometry and separable source terms. With these assumptions, the continuity equation becomes

$$
\begin{equation*}
\frac{\partial}{\partial t} y_{r}(t)=\sum_{r} L_{r r^{\prime}} y_{r^{\prime}}(t)+\sum_{m} S_{r m} u_{m}(t), \tag{1}
\end{equation*}
$$

where $L$ is the linearised transport operator, $S_{r m}$ the separable source term, $u_{m}$ the temporal inputs related to each spatial source term and $y_{r}$ are the observables. It is now convenient to perform a base transformation in which $L$ is diagonal, projecting equation (1) on the eigenvectors of $L$. In this base, the source terms can be expressed as linear combinations of the eigenvectors and the observables $y$ can be expressed as

$$
\begin{equation*}
y_{r k}=\sum_{l m} T_{r l m} h_{m l k}, \tag{2}
\end{equation*}
$$

where $T_{r l m}$ contains the amplitude associated to each eigenvector and $h_{m l k}$ describes the time evolution of each source term component induced by each temporal input. Figure 1


Figure 1. Spatial source terms $T$ and temporal inputs $h$ for the simple model $\left(v_{S T} \sim 1.8 v_{M E C H}\right)$.
shows the example treated here in which modulated ECH (MECH) and sawteeth excite two common eigenvectors with different amplitudes. Each excitation signal is characterised by a specific time constant related to the rise/decay time of the temporal input signal. Using (2), a simulated spatio-temporal set of signals is built. The goal of the signal processing method is, starting from the observables $y$, to be able to determine the source terms $T$. The first step consists in performing a SVD decomposition of $y$. The SVD decomposes the spatio-temporal signals into a unique set of orthonormal spatial and temporal eigenvectors, i.e.

$$
\begin{equation*}
y_{s k}=\sum_{q} V_{s q} D_{q} U_{q k} . \tag{3}
\end{equation*}
$$

$V$ is the orthonormal matrix containing the spatial eigenvectors, called topos, $U$ the temporal eigenvectors, called chronos, and $D$ is a diagonal matrix containing the singular values. Figure 2 shows the result of the application of the SVD to the model. Two conclusions can be drawn: the SVD is not capable of fully separating MECH and sawteeth components, but it is capable of isolating the subspace in which the two dynamics evolve,


Figure 2. First two topos and chronos resulting from SVD of the model signals, together with their singular values. i.e. the subspace spanned by the two excited eigenvectors, as seen by the two dominant singular values. The next step consists in decomposing the chronos $U$ so that

$$
\begin{equation*}
y_{s k}=\sum_{q} V_{s q} D_{q} U_{q k}=\sum_{l m q} V_{s q} D_{q} P_{q l m} h_{m l k} . \tag{4}
\end{equation*}
$$

$h$ is determined from a nonlinear minimisation problem, in which the minimisation variables are the above defined time constants, and the SVD chronos are the fitted signals. On the other hand, determining the elements of $P_{q l m}$ is a linear regression problem. Finally, the reconstructed spatial source terms can be calculated from


Figure 3. Top: fit (red) of SVD chronos (blue). Bottom: MECH and sawteeth reconstructed source terms.

$$
\begin{equation*}
T_{s m l}=\sum_{q} V_{s q} D_{q} P_{q l m} \tag{5}
\end{equation*}
$$

Figure 3 shows the results for the used model: the procedure allows to simultaneously reconstruct each source term.

## Experimental results

The above described procedure, called SI-SVD, has been applied to several MECH experiments. Let us first consider ECE measurements of an ASDEX Upgrade discharge in which the MECH was aiming on-axis. Figure 4 shows the calculated power density deposition profile calculated using (5), as well as amplitude and phase profiles after Fourier analysis at the first MECH harmonic of the raw and the treated signals, together with the profiles at the main sawtooth frequency. The power deposition location well corresponds to the expectations, while the treated amplitude and phase profiles clearly show the improvement in the quality of the results. A further advantage of using this method is related to the possibility of analysing the treated signals also at higher MECH harmonics, leading to narrower amplitude and phase profiles, hence to a better determination of the deposition. Another interesting, and for TCV important and necessary, result is related to the analysis of MECH discharges using soft X-ray (SXR) diagnostics. Figure 5 shows the results of a offaxis ASDEX Upgrade experiment. It is clearly visible that the power deposition location is well determined, hence it is possible to use SXR diagnostics for this purpose. To obtain useful



Figure 4. Top: power density profile (in red a fit using a Gaussian curve). Bottom: amplitude and phase profiles of a on-axis ASDEX Upgrade MECH discharge (black:: untreated; red: SI-SVD treated signals).


Figure 5. Amplitude and phase profiles of SXR treated signals at first three odd harmonics (blue: first; red: third; magenta: fifth).


Figure 6. Frequency scan comparison for the treated signals using different diagnostics. Modulation frequencies: 75 (black circles), 162 (blue triangles), 240 (magenta squares), 335 (red stars) and 718 Hz (green hexagons).
information on the phase, though, a tomographic inversion of the signals is needed. Taking in consideration these remarks, we can now focus on the analysis and interpretation of TCV MECH experiments using different diagnostics, in particular the ECE [2], soft X-ray tomography (XTOMO) and multiwire proportional X-ray detector (MPX [3]). Figure 6 shows the results of a discharge in which a MECH frequency scan was performed on-axis. It is clearly visible that the profiles are fully coherent between different diagnostics, hence the power deposition location can be determined using also SXR diagnostics. If studies such as transport investigations must be performed, dedicated experiments and deeper signal processing must be accomplished (tomographic inversion, density variations).
In conclusion, the SI-SVD procedure allows to simultaneously reconstruct MECH and sawteeth spatial source terms. Its application to ECH experiments in ASDEX Upgrade and TCV has pointed out in particular, with the parallel analysis and interpretation of different diagnostics data, the possibility of determining the power deposition location using also SXR diagnostics.

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## Bibliography

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