27th EPS Conference on Contr. Fusion and Plasma Phys. Budapest, 12-16 June 2000 ECA Vol. **24B** (2000) 436 -439

Pressure profile recovery on the W7-AS stellarator with external magnetic information using function parameterization

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Introduction

In this paper we address the feasibility of pressure profile recovery on a stellarator using only external magnetic information, a topic which has been the subject of debate for a number of years [1–5]. Determining the pressure profile is central to identifying an MHD equilibrium and it would naturally be attractive if this were possible solely using external magnetic measurements, since these form a routinely available, robust diagnostic on fusion experiments. The difficulty lies in the fact that magnetic measurements react only to the aggregate field, which in mathematical terms is specified purely by the plasma boundary and the value of the field there, and is independent of the internal details. This is further complicated through the degeneracy introduced by a finite current profile, in that pressure- and current-induced external field effects cannot be distinguished *a priori.* However, physics considerations lead to constraints on the accessible profile range (e.g. a strongly hollow pressure is difficult to achieve experimentally), so it may still be possible to glean some profile-relevant information from external measurements. Whereas previous investigations dealt only with restricted profiles and configurations, our goal here is to establish the extent of this information for W7-AS through statistical regressions on a large equilibrium database containing diverse configurations, pressure and toroidal current profiles. This is done using function parameterization (FP) methodology with simulated diagnostic information from an extensive set of 3-D magnetic probes. The impact of measurement errors on the recovery is considered.

Database description

The study is carried out over a 700 case equilibrium database generated with NEMEC [6], spanning variable vacuum coil current configurations, limiter positions, a flexible parameterization for the pressure profile $p_{\rm eq}$ and, expanding on previous work [7], a variable toroidal current profile $I_{\rm tor}$. This represents a significant advance since $I_{\rm tor}$ has a non-negligible effect on the external magnetic field, particularly in the presence of a net toroidal current. The parameterization for $p_{\rm eq}$ is identical to that in [7], having 3 shape parameters and a variable value on-axis allowing from near-vacuum to high-β equilibria. For $I_{\rm tor}$ we use 8 free parameters c_1–c_8:

$$\frac{\pi}{2} I_{\rm tor}(s) = c_1 \tan^{-1}\left(\frac{c_2 s^{c_3}}{(1-s)^{c_4}}\right) - c_5 \tan^{-1}\left(\frac{c_6 s^{c_7}}{(1-s)^{c_8}}\right)$$

where s is the normalized toroidal flux. Database equilibria with a wide range of current profile shapes and a net toroidal current up to 30% of the main coilset currents are permitted. Note that this parameterization is sufficiently flexible for our initial purposes but may need refinement in order to better reflect calculated experimental profiles.

Simulated diagnostic measurements

The magnetic field due to the plasma is calculated with DIAGNO [1] for each database equilibrium at a large number of points near the vacuum vessel contour, here we have chosen an array of 24 probes evenly distributed in the poloidal angle, each yielding 3 components (radial, poloidal and toroidal) of the field at 10 values of the toroidal angle. This is a more extensive setup than is likely to be feasible in reality and is intended to be sufficiently dense to capture all important field variations. Statistical analysis is used to reduce this set of 720 measurements to its principal components (PCs) [8], orthogonal linear combinations of the original data ordered by decreasing variance. The eigenvalue (variance) profile is shown in Fig. 1. As the rapid fall-off of the first 6 eigenvalues from roughly 370 $[\mathrm{mT}]^2$ to 1 $[\mathrm{mT}]^2$ demonstrates, the vast majority of the variance ($> 99.5\%$) is contained in these leading order PCs, thus only a comparatively modest number of PCs need be retained in models. The maximum number of useful PCs for a given measurement accuracy can also be deduced from the graph. For example, an accuracy limit of 0.1 mT means that at most 10 PCs can be used. The PCs of the magnetic data contain the combined effects of both p_{eq} and I_{tor} profiles, thus their physical interpretation is not straightforward. However, in the simple case of just one field component in a single toroidal plane, their structure is analogous to Fourier-like moments of the field.

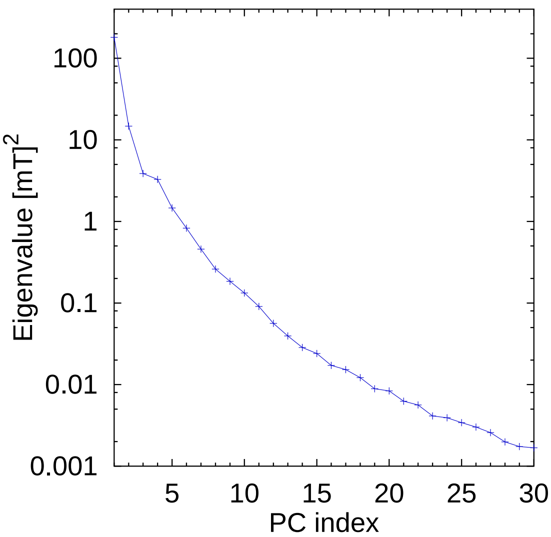

Figure 1: Eigenvalues (variances) of the leading PCs of the magnetic measurements

Profile recovery results

A predictor set of all possible quadratic combinations of the vacuum information (external coil currents and limiter position) and the PCs is constructed and used in regressions for p_{eq} at fixed values of the radial coordinate s. The same treatment also holds for I_{tor}, but for brevity we consider only p_{eq}. For convenience, we define $\langle\delta_{\mathrm{p}}\rangle$, a profile-averaged figure of merit to facilitate easy model comparisons. We denote the root-mean-square (RMS) recovery error of p_{eq} at a gridpoint s_i as δ_i. Then,

$$\langle\delta_{\mathrm{p}}\rangle = \sqrt{\frac{1}{N}\sum_{i=1}^{N} {\delta_i}^2}$$

measures the overall profile recovery error. We plot $\langle\delta_{\mathrm{p}}\rangle$ in Fig. 2 versus the number of PCs in the model for different levels of pseudo-random Gaussian noise added to the

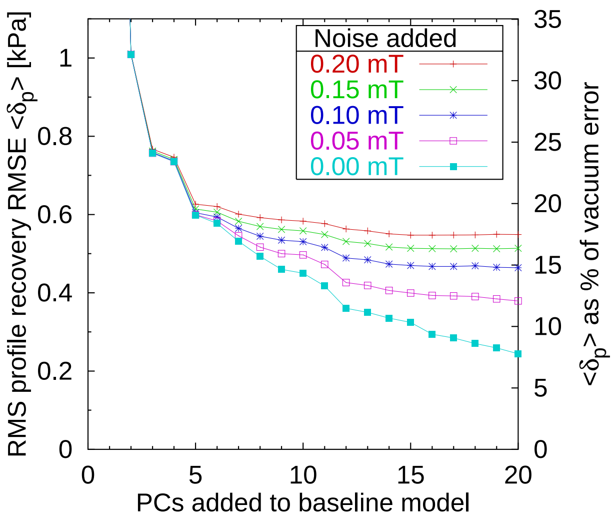

Figure 2: RMS profile recovery RMSE $\langle\delta_{\mathrm{p}}\rangle$ for models with varying numbers of included magnetic PCs and noise

measurements, with the error in kPa on the left-hand axis and on the right the error as a percentage of the vacuum error, i.e. that for a model with only the vacuum information and zero PCs of the magnetic measurements. Note that although it is more usual to perturb each measurement with noise proportional to its standard deviation, here we prefer to use an absolute error instead since all measurements are of identical type and their uncertainties (possibly arising due to stray fields etc.) should likewise be comparable. At least two PCs must be included for a significant improvement over the vacuum-only error of 3.14 kPa, which is in contrast with our experience from the previous I_{tor}-free database [7], where the leading PC was almost perfectly correlated with the plasma energy and was therefore sufficient to identify the average p_{eq} profile. Here, investigations find the first 2 PCs to contain mixed information regarding both the net current $I_{\mathrm{tor}}(s = 1)$ and the plasma energy. In the noise-free case, $\langle \delta_{\mathrm{p}} \rangle$ improves steadily, however there remains a significant residual error of almost 8% at the maximum number of retained PCs considered, revealing a fundamental limit of information in the PCs. The level at which $\langle \delta_{\mathrm{p}} \rangle$ saturates rises sharply for even low noise levels, indicating that much of the predictive information is contained in fine variations which are easily lost in the measurement errors. To put these in context, the vacuum field on W7-AS can be up to 2.5 T so a 0.05 mT error corresponds to a demanding field measurement accuracy of 2 parts in 10^5. In theory, magnetic diagnostics on a stellarator can be performed more accurately than on a tokamak, since they need only begin sampling when the main field has reached steady state. Nevertheless, it is questionable if such accuracy is attainable in practice, since even small variations in the nominally flat-top vacuum field would induce very large signals in the probes, requiring careful stray field compensation.

Pressure eigenvector recovery

Although $\langle \delta_{\mathrm{p}} \rangle$ is useful as an overall measure of the profile recovery, it is revealing to cast results in terms of a class of profile features which are identifiable using the magnetic data. We thus examine the individual recovery of radial eigenvectors of p_{eq} constructed by performing a PC analysis of the pressure on the same grid as used in the regressions above. The resulting polynomial-like eigenvectors (scaled by the square root of the corresponding eigenvalue to highlight their relative significance) are drawn in Fig. 3 along with their recovery error versus retained PCs for noise levels up to 0.2 mT. The first PC (representing an average profile) is robustly described with 2 or more PCs. The second PC (a profile peaking factor) has a best-case error of roughly 14% of the vacuum error and exhibits more noise sensitivity, but is sufficiently determined at least to distinguish between peaked and broad profiles. The third PC is very sensitive to the background noise and even with 20 noise-free PCs, retains 40% of the vacuum error.

This analysis indicates that only profile features which are described by the first 2 eigenvectors can be deduced from the external data. Distinguishing shoulder-like features and finer profile structure is not possible since these require knowledge of the third and higher-order PCs.

Discussion

We conclude that, although pressure and current effects on the external magnetic field cannot be distinguished *a priori*, the FP approach provides a means of utilizing the mixed information to partially identify the pressure profile. We have shown that even from an extensive set of virtually error-free external measurements, only the average pressure profile and a peaking factor are readily identifiable. This ill-posed inverse problem is also

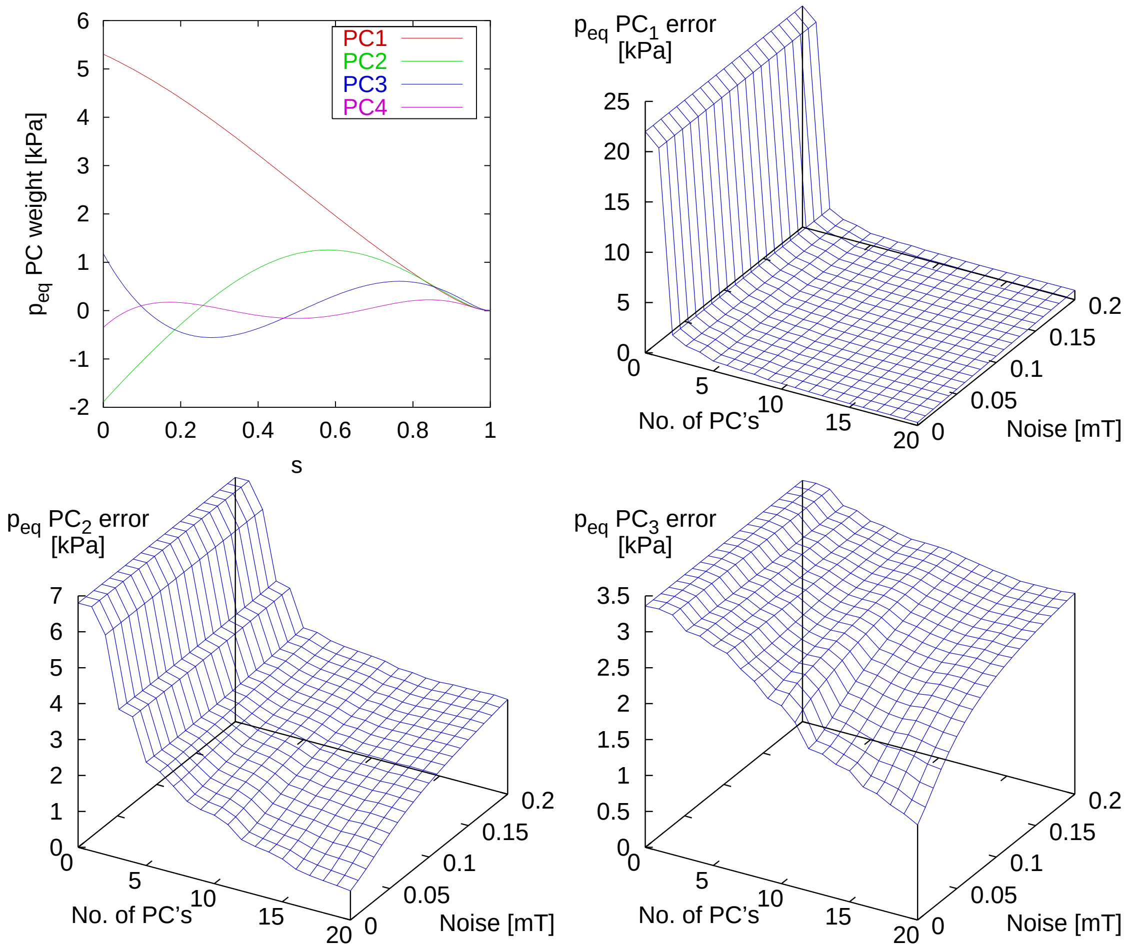

Figure 3: Leading pressure profile radial eigenvectors and their recovery errors for models with varying numbers of included magnetic PCs and noise

sensitive to measurement errors. However, it is significant that the p_{eq} recovery does not deteriorate dramatically compared to the I_{tor}-free case [9]. To our knowledge, these results have not been demonstrated previously in the context of a large equilibrium database. Ultimately, we must turn to internal profile measurements such as Thomson scattering data for more exact pressure profile identification and complete equilibrium recovery.

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