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Numerical Simulation of Particle Flux in a Poloidally Rotating Tokamak Plasma

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Abstract. Parallel viscosity $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ as a function of radial electric field E_r (or, equivalently, non equilibrium poloidal rotation) in a tokamak is studied using the 5D (3D in configuration space and 2D in velocity space) Monte Carlo code ASCOT. It is shown that $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ changes sign when the Mach number $M_p = |E_r/v_t B_\theta|$ is of the order of unity. Here, v_t is the thermal velocity and B_θ is the poloidal magnetic field.

Introduction. In the standard neoclassical theory of tokamaks, the neoclassical transport is automatically ambipolar and independent of the radial electric field E_r. As discussed in Ref.[1], this follows from momentum conservation and is valid only in the absence of momentum sources. In the presence of forces, such as an externally applied radial electric field [2] or torque by the orbit losses [3], neoclassical transport depends on E_r, and various expressions for the neoclassical ion flux and parallel viscosity have been derived [4, 5]. When studying the L–H transition theory in Ref. [3], it was important to expand the validity of the expression for the parallel viscosity to the region where $M_p \geq 1$. Here, $M_p = |E_r/v_t B_\theta|$ is the Mach number, B_θ is the poloidal magnetic field, and $v_t = (2k_B T/m)^{1/2}$ is the thermal velocity, where m is the ion mass and T the zeroth order temperature (no poloidal variation). An expression for $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ was derived assuming an incompressible plasma flow and poloidally constant density ($\mathbf{B}$ is the magnetic field, $\mathbf{\Pi}_i$ is the viscosity tensor, and $\langle \ \rangle$ indicates a flux surface average). It was found that the viscosity has a maximum at $M_p \approx 1$ and decays to zero without changing sign when M_p increases. Similar result was obtained for $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i/n \rangle$ in Ref. [6], where the effect of poloidal variation of density n and compressibility were included. Here, the behavior of $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ was not investigated. Because $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ is a frequently used quantity in the literature and appears in a majority of formulations of rotation dynamics and momentum balance in tokamak theory, the study of $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ including the poloidal density variation is of importance. We show numerically that the standard expression for the parallel viscosity changes sign when the Mach number increases, provided that the variation of density in poloidal angle is taken into account consistently.

Model. The standard expression for the parallel viscosity in terms of pressure anisotropy is

$$\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle = \left\langle (p_\perp - p_\parallel) \frac{\mathbf{B} \cdot \nabla B}{B} \right\rangle, \tag{1}$$

where $p_{||}$ and $p_\perp$ are the parallel and perpendicular components of the pressure, respectively. Various expressions for the pressure components and pressure anisotropy exist in the literature. The analytic theory presented in Refs. [4, 7] includes poloidal variation of the electrostatic field, density and temperature, and is based on full velocity integrals. To simplify the problem, we here neglect the radial density and temperature gradients as well as the poloidal electric field. Thus we can write the density as $n = n_0 + n_1(\theta)$, where θ is the poloidal angle.

In the numerical simulation, the 5D Monte Carlo code ASCOT [8] is used. The guiding-centre orbits of the ions are followed in a tokamak geometry, and a binary collision model is used to model ion-ion collisions. The collision operator conserves the number of particles, the total momentum, and the total energy, quasi-locally. The parallel viscosity from Eq.(1) is calculated directly from the code in terms of the statistically measured pressure components $p_{||} = \int m(v_{||} - u_{||})^2 f d^3v$, and $p_\perp = \int (m|\mathbf{v}_\perp - \mathbf{u}_\perp|^2/2) f d^3v$. Here, $v_{||}$ and $v_\perp$ are the parallel and perpendicular velocity components, respectively, and $u_{||}$ and $u_\perp$ are the corresponding flow velocities. All the flow velocity components, as well as $p_{||}$ and $p_\perp$, are calculated from the code on a (r,θ)-grid as time and ensemble averages of particle velocities. Using the momentum conserving collision operator with a fixed radial electric field and excluding other forces generates a test particle flow, $U_{||}$, parallel to the magnetic field to compensate the poloidal rotation. This mean flow velocity, $U_{||} = \int u_{||}(\theta)(R/R_0) f d\theta$, is driven by viscous processes and its build up occurs on a collisional timescale. Thus, in order to compare the results to analytic estimates obtained for $U_{||} \approx 0$, the measurement has to be done before a significant mean parallel velocity has developed.

Results. Parameters similar to those of ASDEX Upgrade, $a = 0.5$ m, $I_{pl} = 1$ MA and $B_\phi = -2.5$ T, are used for the minor radius, plasma current, and toroidal magnetic field, respectively. Since the analytic results were derived in the large aspect ratio limit of a quasitoroidal configuration, a larger value ($R_0 = 3$ m) for the major radius and co-centric circular magnetic surfaces on a poloidal cross-section are chosen.

As an example of the importance of convection and compressibility in the calculation of total parallel viscosity, in Fig. 1 we compare the numerically obtained total parallel viscosity, i.e., the rate of change of the flux surface averaged parallel momentum density dotted with $\mathbf{B}$, to the standard parallel viscosity $< \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i >$ calculated numerically from Eq.(1) and analytically from Ref.[3]. Here, the density is $n = 5 \times 10^{19}$ m^{-3}, and temperatures a) 100 eV and b) 200 eV correspond to the collisionalities $\nu_{*i} = 46$ and 12, respectively, the first one being in the Pfirsch-Schlüter regime, and the latter in the plateau regime. Both in the Pfirsch-Schlüter regime and in the plateau regime, for large poloidal Mach numbers, the standard parallel viscosity $< \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i >$ has a different sign than the total parallel viscosity, i.e., its effect is to resist the growth of the parallel rotation which is driven by the other terms. The total viscosity remains positive definite, which leads to the decay of poloidal rotation. Only when the poloidal density variation is neglected, one obtains a positive definite result for $< \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i >$ irrespective of the value of M_p. For a small poloidal rotation, the density perturbation is insignificant and the standard neoclassical result is found with all methods. The effect of a poloidal electric field has also been tested by running the ASCOT code by solving the poloidal electric field from the assumption of quasi-neutrality and Boltzmann distribution of the electrons on a magnetic surface. Again, the results were not changed qualitatively.

Although, according to the simulations, both convection and compression strongly affect the parallel momentum balance for large values of M_p, the effect of convection, in the case of zero radial current, can be shown to be weak in the poloidal part of the momentum equation. In Fig. 1(c), the same comparison is done for the low collisionality case, i.e., for a temperature $T = 300$ eV also with a density $n = 2 \times 10^{19}$ m^{-3}, corresponding to a collisionality $\nu_{*i} = 2.5$. Again, we find that the parallel viscosity from the code changes sign for large poloidal Mach numbers but, somewhat surprisingly, a fairly good agreement can be found between the Shaing's expression for the standard parallel viscosity and the ASCOT result for the total parallel viscosity.

Conclusions. In the present numerical study of parallel viscosity, it was found that the standard parallel viscosity changes sign when the Mach number is of the order of unity. However, the present ASCOT simulations show that the total viscosity, including the convection effect, remains positive definite for arbitrary M_p, and thus leads to the decay of poloidal rotation according to the known neoclassical expectation. Furthermore, one should note that the qualitative behavior of the effective viscosity $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i / n \rangle$ used in Ref. [6] is different from the behavior of the standard parallel viscosity $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ for large Mach numbers. The latter expression is more common in the theoretical analysis, although the effective viscosity is the quantity what is observed in experiments, when rotation velocities are measured.

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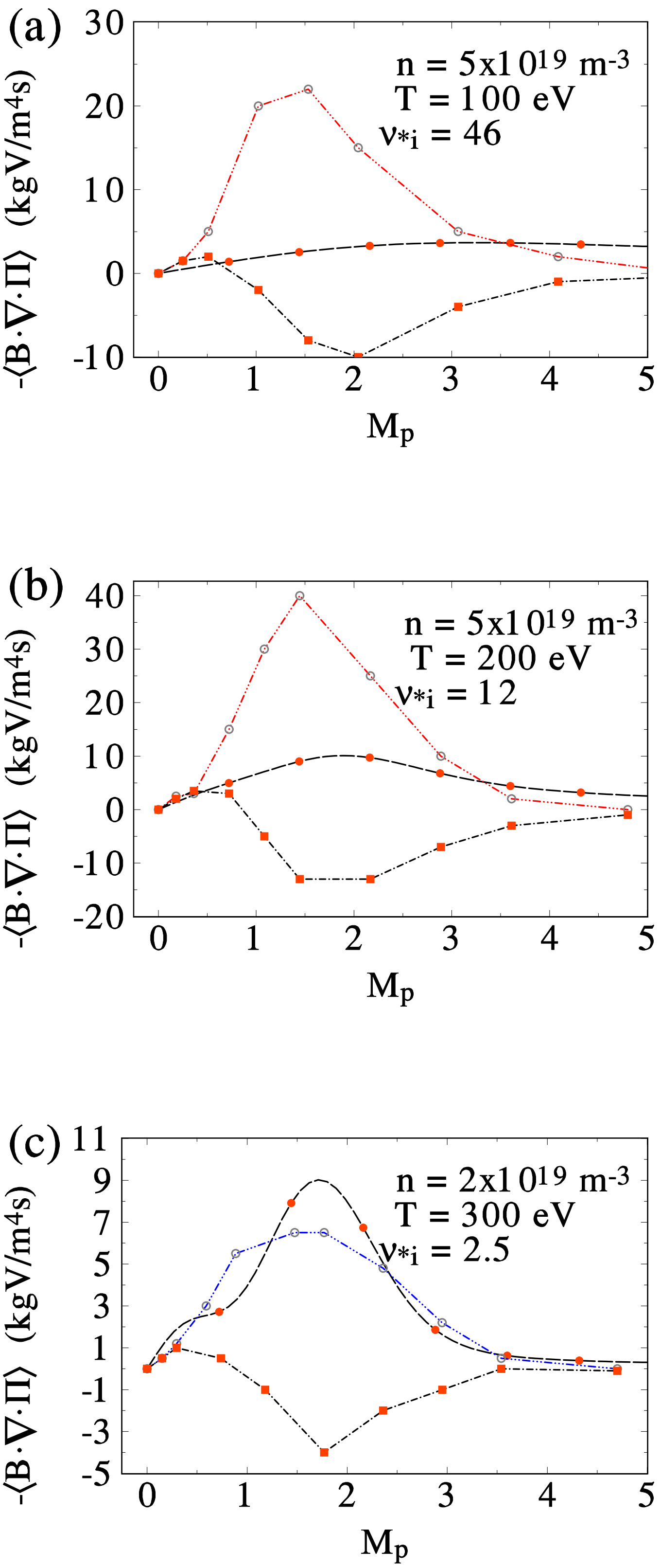

Figure 1: $<\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i>$ *as a function of the radial electric field for temperatures a)* 100 *eV and b)* 200 *eV, with the density* $n = 5\times10^{19}$ m^{-3} *and c) for low collisionality case with* $T = 300$ *eV and* $n = 2\times10^{19}$ m^{-3} *calculated with ASCOT (filled squares) and from the expression of Shaing (filled circles). The total parallel viscosity including convection is also shown (empty circles).*