# 2D model for runaway electron energy amplification in a vertical disruption

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**Abstract**

During the current quench of a tokamak disruption, a substantial fraction of the initial plasma current can be converted into runaway electrons (REs). The ease at which this happens grows with the plasma current, making REs an issue of concern in ITER, where they may damage the first wall upon impact. Although they are highly relativistic, the total energy of the REs is initially much smaller than that of the pre-disruption plasma or the poloidal magnetic field. However, the latter provides a reservoir of free energy that can be converted to the RE population as the plasma drifts toward the wall. Following a suggestion by Putvinski [1], a computational 2D model for the energy conversion in a vertical tokamak disruption has been developed, assuming a runaway plasma with circular cross section drifting toward the surrounding conducting structures which include the vessel wall, poloidal field coils and a central solenoid. The motion of the plasma is calculated self-consistently with the evolution of the runaway current and the resistive diffusion in conductors. The energy transfer to the runaways is computed, and for an ITER-like tokamak the amount of energy deposited on the first wall by runaways is found to be of the order of 100 MJ. It is found that most of this energy conversion happens when the RE plasma gets in contact with the wall and is being scraped off.

## Introduction

The toroidal electric fields induced in a tokamak disruption can accelerate electrons to relativistic speeds and thereby form a current carried by runaway electrons (REs). In particular large tokamaks can have a major fraction of their current that gets converted into REs in this way, and ITER is expected to reach a current-conversion factor of about 2/3 [2, 3, 4, 5]. The total kinetic energy $W_\mathrm{k}$ carried by the REs is much smaller than the pre-disruption thermal energy of the plasma and also much smaller than the energy $W_\mathrm{m}$ of the corresponding poloidal magnetic field $B_\mathrm{pol}$. For a current $I$ the poloidal magnetic field energy per unit length of a cylindrical plasma column is

$$W_\mathrm{m} \sim \int \frac{B_\mathrm{pol}^2}{2\mu_0} 2\pi r\, dr \sim \frac{\mu_0 I^2}{4\pi} \quad ,$$

and if $I$ is carried by REs of energy $\gamma m_e c^2$, their total kinetic energy is $W_k \sim I(\gamma-1)m_e c/e$ leading to

$$\frac{W_k}{W_m} \sim \frac{(\gamma-1)I_A}{I} \ll 1$$

with the Alfvén current $I_A = 4\pi m_e c/\mu_0 e \simeq 17$ kA. Most runaways are generated by the avalanche mechanism, in which relativistic electrons create more REs in collisions with thermal electrons leading to an exponential growth according to

$$\frac{\partial J_r}{\partial t} \simeq \frac{J_r}{\tau_a}\left(\frac{\langle E_\varphi \rangle}{E_c} - 1\right) \quad , \tag{1}$$

where $\tau_a = \tau_a(\tau, \ln\Lambda, ...)$ is the avalanche growth time that in general depends on the electric field $E_\varphi$, $\tau = 4\pi\varepsilon_0^2 m_e c^3/ne^4 \ln\Lambda$ the collision time for relativistic electrons and $E_c = m_e c/e\tau$ the critical electric field below which no RE generation occurs [1, 6, 7]. Putvinski et al [1] have described and modelled a possible mechanism of energy transfer from the magnetic field into the runaways that are amplified in an avalanche during the vertical drift of the plasma in a disruption. Experimental evidence was recently found at JET [8]. Our model is a step toward a self-consistent simulation of the vertical plasma motion in a two-dimensional surrounding of external conductors (vessel wall, PF coils and central solenoid) that resemble the typical elements of a tokamak.

## Model description

We consider an axisymmetric geometry with cylindrical coordinates $(R, \varphi, z)$ and the magnetic field written as

$$\mathbf{B} = I(\psi, t)\nabla\varphi + \nabla\varphi \times \nabla\psi \quad , \tag{2}$$

where $\psi(R, z, t)$ is proportional to the poloidal magnetic flux. With the toroidal current density as the sum of the runaway current and the Ohmic current

$$J_\varphi = J_r + \sigma E_\varphi = \frac{\Delta^*\psi}{\mu_0 R} \qquad \left(\Delta^* = R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R}\right) \tag{3}$$

the Grad-Shafranov equation for the time evolution of the poloidal magnetic field in the plasma and the external conductors becomes

$$\sigma\mu_0\frac{\partial\psi}{\partial t} = \Delta^*\psi - \mu_0 R J_r - \sigma\mu_0\,\mathbf{v}\cdot\nabla\psi \quad . \tag{4}$$

Moving the plasma self-consistently requires the velocity $\mathbf{v}$ to be determined from the condition that the force on the plasma

$$\mathbf{F} = \int_{V_{plas}} (\mathbf{J}\times\mathbf{B})\, dV \tag{5}$$

should vanish. The total energy transferred to the plasma over time $t$ is obtained as

$$W_{\mathrm{plas}} = \int_0^t dt' \int_{V_{\mathrm{plas}}} J_\varphi E_\varphi \, dV = \int_0^t dt' \int_{V_{\mathrm{plas}}} (\sigma E_\varphi + J_{\mathrm{r}}) E_\varphi \, dV = W_\Omega + W_{\mathrm{r}} \tag{6}$$

where the first contribution goes into Ohmic heating of the thermal background plasma and the second is the total energy transferred to the runaways. Runaways also experience collisional slowing down and would thermalise in a time $\tau_{\mathrm{d}}$ if they were not accelerated by fields exceeding the critical field strength $E_{\mathrm{c}}$, i.e. the energy

$$W_{E_{\mathrm{c}}} \approx \int_0^t dt' \int_{V_{\mathrm{plas}}} J_{\mathrm{r}} E_{\mathrm{c}} \, dV \tag{7}$$

is required to overcome losses du to the friction against cold bulk electrons. The final runaway energy that will hit the wall as kinetic energy of relativistic electrons is thus

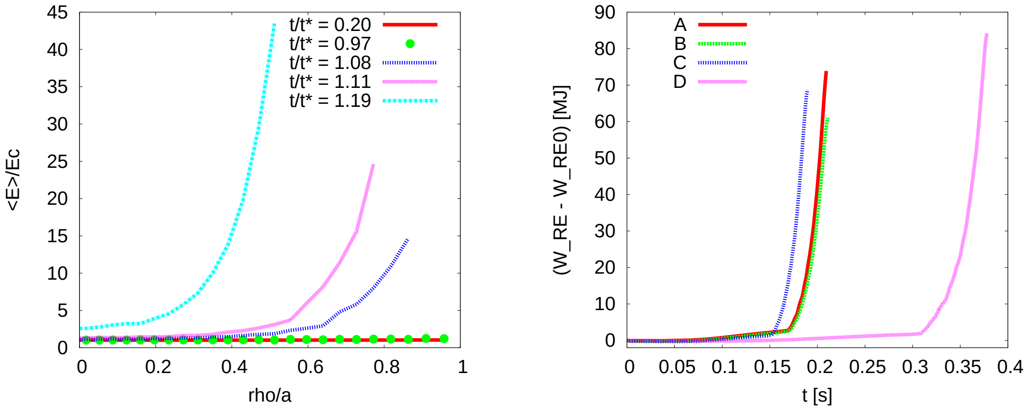
$$W_{\mathrm{RE}} = \int_0^t dt' \int_{V_{\mathrm{plas}}} J_{\mathrm{r}} (E_\varphi - E_{\mathrm{c}}) \, dV = W_{\mathrm{RE}}^0 + (W_{\mathrm{r}} - W_{E_{\mathrm{c}}}) \quad . \tag{8}$$

In our model we make a number of simplifying assumptions. Besides assuming a plasma with circular cross section, we only consider its vertical motion and thus avoid having to solve for two-dimensional trajectories. A large aspect ratio helps to simplify the expression for the vertical force on the plasma. All external conductors are assumed to be up-down symmetric. In addition we also apply an up-down symmetric co-oriented current in one pair of PF coils that provides an unstable equilibrium for a plasma positioned half-way between the upper and lower coil. We thereby neglect the stabilizing effects from counter-oriented currents in the remaining coils and thus consider our estimates as a worst-case model. None of these approximations should affect the mechanism of energy conversion qualitatively.

Initiated by a small vertical displacement of the plasma, a vertical disruption is simulated via a time-dependent solution of (4) on a rectangular subdomain of the $R-z$ plane. The vertical velocity of the plasma is determined in an iterative procedure.

## Results

Simulations were done for an ITER-like tokamak with expected post-disruption parameters ($T \sim 5$ eV, $n \sim 10^{21}$ m$^{-3}$) and a plasma current $I_{\mathrm{P}} \sim 2/3 I_{\mathrm{P}}^0 \sim 10$ MA, exclusively carried by runaways. The movement of the plasma from its initial position toward the vessel wall is separated into two phases by the time $t^*$ when the plasma first hits the wall. In the first phase $t < t^*$ the plasma moves at almost constant speed determined by the fact that the electric field in the plasma stays around the value of the critical field $E_\varphi \approx E_{\mathrm{c}}$. In the second phase $t > t^*$ the plasma is being scraped-off and gets accelerated which causes strong electric fields that penetrate the plasma on the scale of the skin depth, thereby amplifying the current density as to compensate for the current losses from the edge. Most of the energy conversion occurs in this phase. The final amount of runaway energy is mainly determined by the total initial current and is almost independent of the prior history of the discharge. Changing the plasma parameters or the conductivity of the first wall is found to have some effect on the process but does not change the runaway energy dramatically. The left figure shows the flux-surface averaged electric field over the plasma radius. For $t < t^*$, the field stays constant around $E_c$. For $t > t^*$, high field strengths occur at the edge and drive strong currents with a hollow profile over shrinking plasma radius.




The right figure compares the results for the reference case (A) – parameters as given above – with those for doubled temperature (B), doubled density (C) and doubled conductivity of the first wall (D).

Since the initial kinetic energy of the runaways is $W_{\mathrm{RE}}^0 \sim 20$ MJ, the runaway energy striking the first wall is of the order 100 MJ which would lead to substantial wall damage. Therefore apropriate measures should be taken in order to avoid or mitigate the formation of strong runaway currents.

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